Puzzle: The AI Circus

(Puzzle in honor of Tuomas Sandholm's 50th birthday)

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Please send solutions to the author by e-mail, with the title of this puzzle in the subject header. By agreement with the editors, the best solution will be published in the next issue of SIGecom Exchanges, provided that that solution is of sufficiently high quality. Quality is judged by the author, taking into account at least soundness, completeness, and clarity of exposition. A solution may be published even if it does not solve all cases.

The AI circus is in town. Just that all the performers are robots—no animal abuse in this circus—does not mean that there is no mischief!

The monkey has stolen the clown's favorite red nose, and placed it on the top of the pentagonal tent (view from above: Figure 1). The clown would like to get her nose back, and can climb to the top of the tent along one of the corner sides (edges in the graph). Unfortunately for her, the monkey is still on the edge of the top of the tent; i.e., he is moving along the inner pentagon. If the monkey is at the same place (vertex) as the clown when she reaches the inner pentagon, he will throw her off. She will be fine, thanks to her inflatable clown suit, but the suit will be damaged, so she only has one attempt to get past the monkey and get her nose back. If she gets past the inner pentagon, the monkey will not follow her up and she will get her nose back. Besides climbing up, the clown can also ride her unicycle along the edge of the tent (outer pentagon). However, the decision to climb is irreversible; from that point, the only way down is to fall.

All moves are discretized: moving along an edge of either pentagon takes 1 unit of time. Note that the clown, on her unicycle, is much faster than the monkey, getting from one corner to the next in one time unit, whereas it takes the monkey two time units (in spite of being on the inner pentagon). The climb up from the outer to the inner pentagon takes $x \in \mathbb{R}$ units of time. Both the clown and the monkey are running the latest game-solving AI to achieve their objective in this zero-sum game. There is no deadline and the clown and monkey are in no rush; the audience will just have to wait until the game plays out. Assume they start at the northernmost vertices of their respective pentagons and alternate moves, with the clown moving first. I.e., first the clown moves for one time unit while the monkey stands still; then the monkey moves for one time unit while the clown stands still; etc. One is allowed to pass, i.e., stand still for a move. Once someone starts down an edge, she has to keep going (including the clown climbing up the side). The monkey will throw the clown off if they are ever in the same place, regardless of who got there first.

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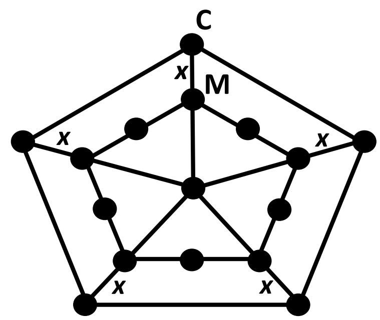

Fig. 1. View of the circus tent, from the top.

(1) Suppose this is a perfect-information game: i.e., both players know where the other player is at each point in time. For what values of x can each player win?

(2) Now suppose that the monkey cannot see across the tent (but the clown has perfect information). I.e., if the monkey is at a corner vertex of the inner pentagon, there are three outer pentagon vertices (and three edges along which the clown can climb) he can see, and if he is at a middle vertex, there are only two outer pentagon vertices he can see. I.e., he can see across a face of the graph but no further. If the clown is not at one of the visible vertices, the monkey does not know where she is (other than what he can infer from what he knew about where she was before).[2] For what values of x can each player win with certainty? Win with probability arbitrarily close to 1? Win with some other probability?

(3) Now suppose the monkey has perfect information, but the clown does not. (The clown can see the monkey in exactly the situations where the monkey could see the clown in the previous case.) Answer the same questions as before.

(4) Finally, suppose that neither player has perfect information. Answer the same questions as before.

[2]Note that to play this variant as a board game, one would need a referee or computer (as in the game of Kriegspiel).