One Equilibrium Is Not Enough: Computing Game-Theoretic Solutions to Act Strategically

0, 0	-1, 2
-1, 1	0, 0

0, 0	-1, 1
1, -1	-5, -5

2, 2	-1, 0
-7, -8	0, 0

1, 1	3, 0
0, 0	2, 1

Vincent Conitzer
Duke University

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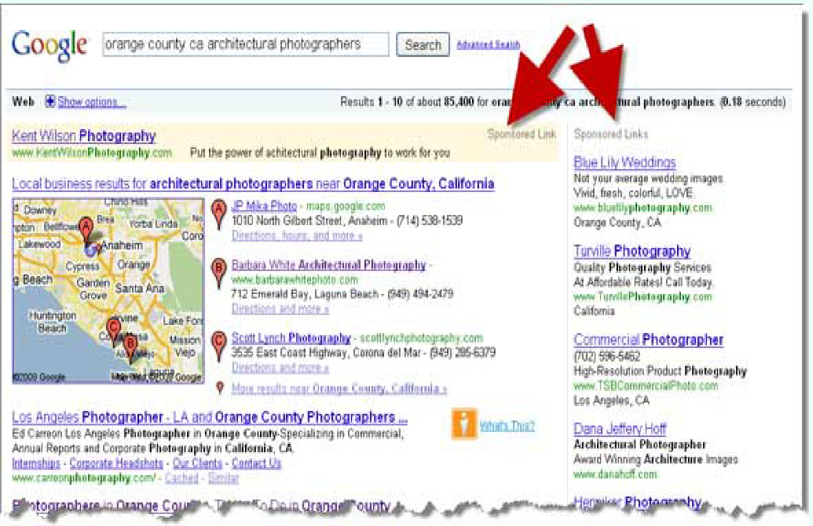
Multiple entities with different interests

How can AI help?

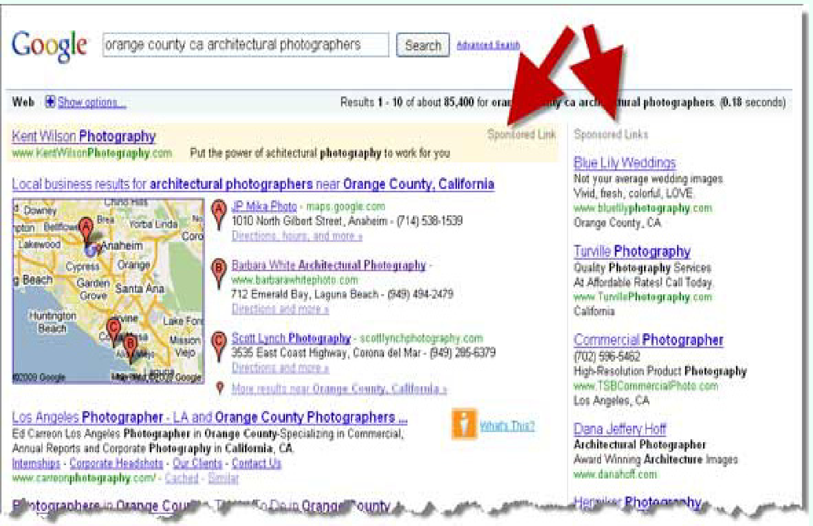
Multiple entities with different interests



Auctions

How can AI help?

Multiple entities with different interests

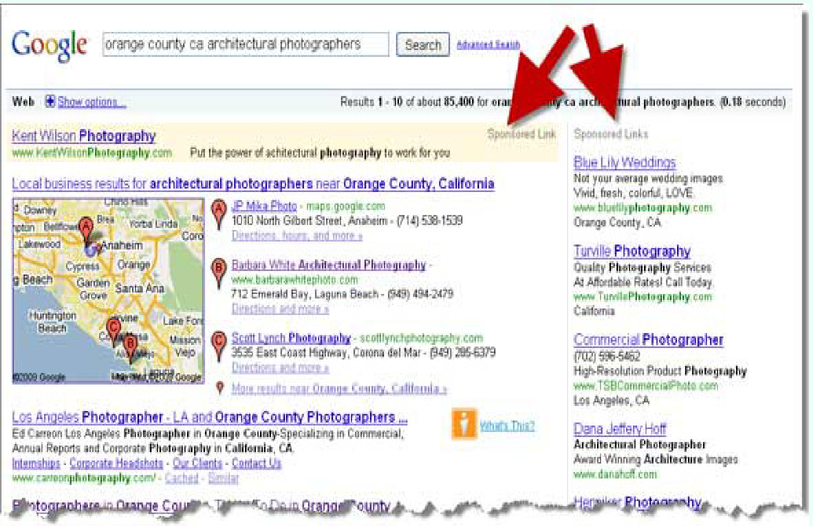


Auctions

Rating/voting systems

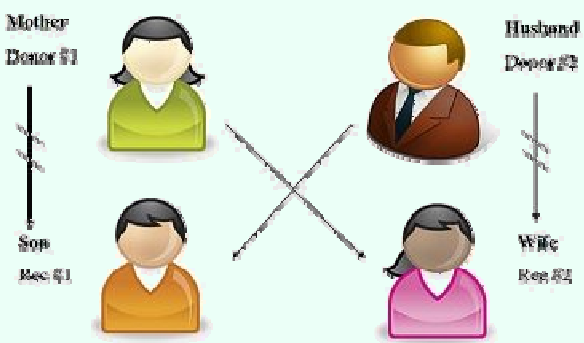
How can AI help?

Multiple entities with different interests



Auctions

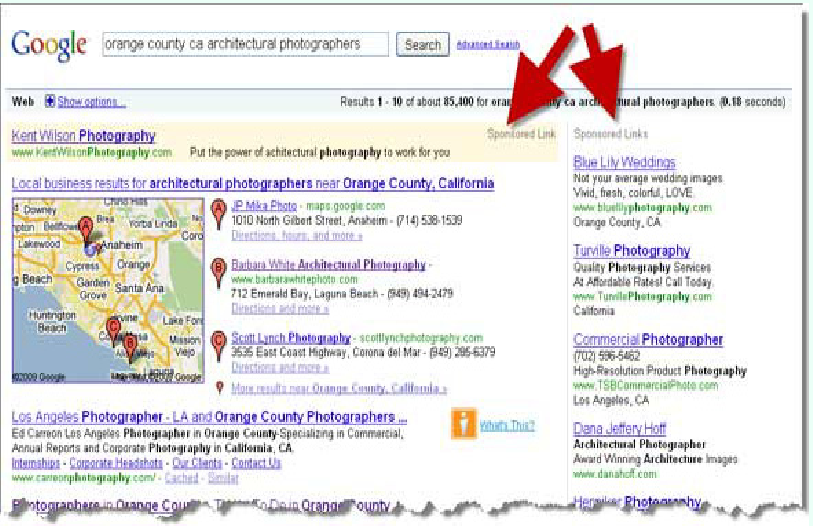
Rating/voting systems



Kidney exchanges

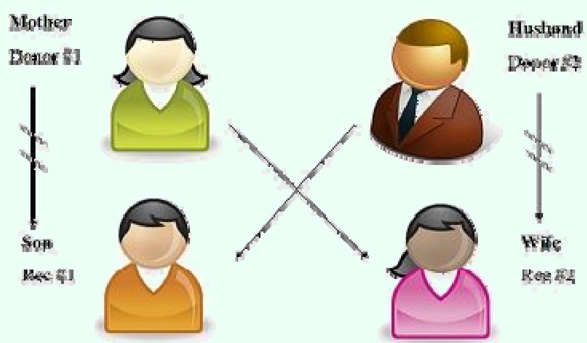
How can AI help?

Multiple entities with different interests



Auctions

Rating/voting systems

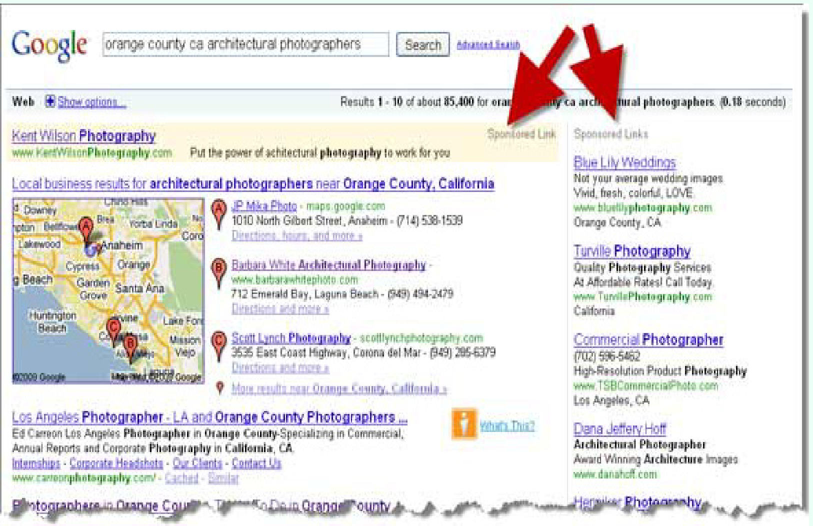


Kidney exchanges

How can AI help?

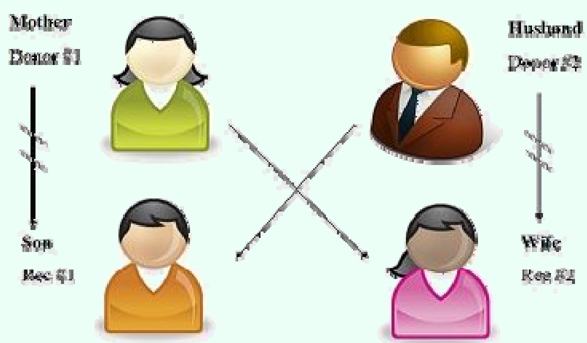
Prediction markets

Multiple entities with different interests



Auctions

Rating/voting systems



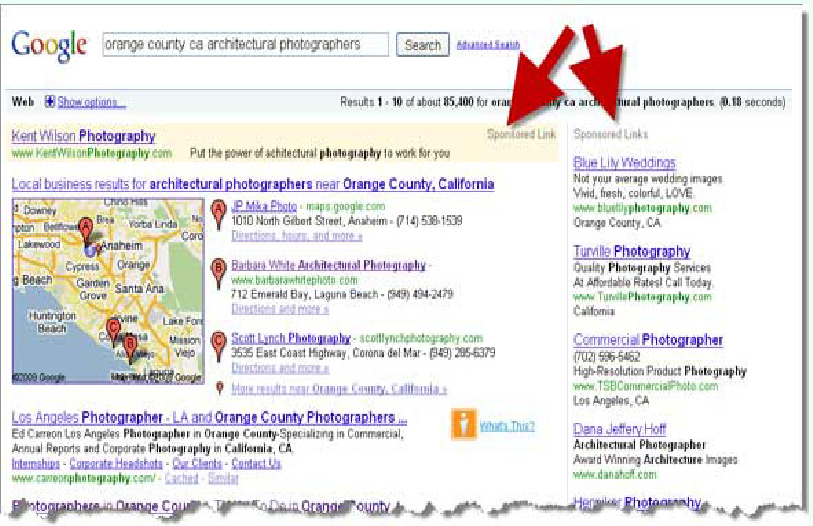
Kidney exchanges

Prediction markets

How can AI help?

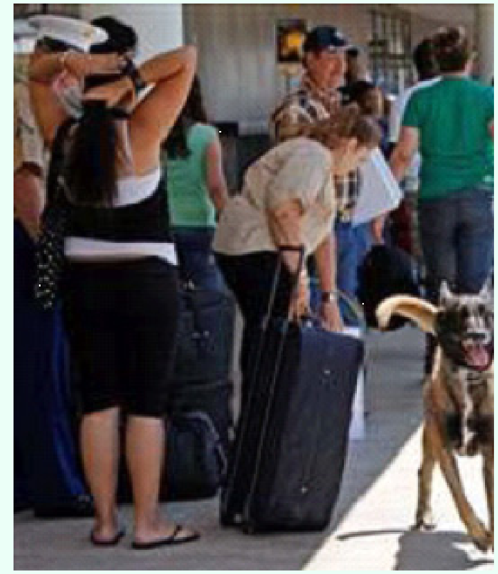
Donation matching

Multiple entities with different interests

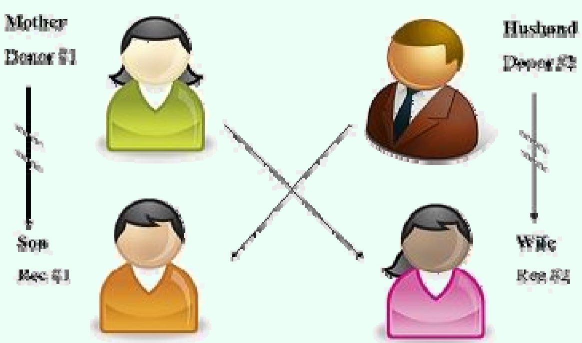


Auctions

Rating/voting systems



Security



Kidney exchanges

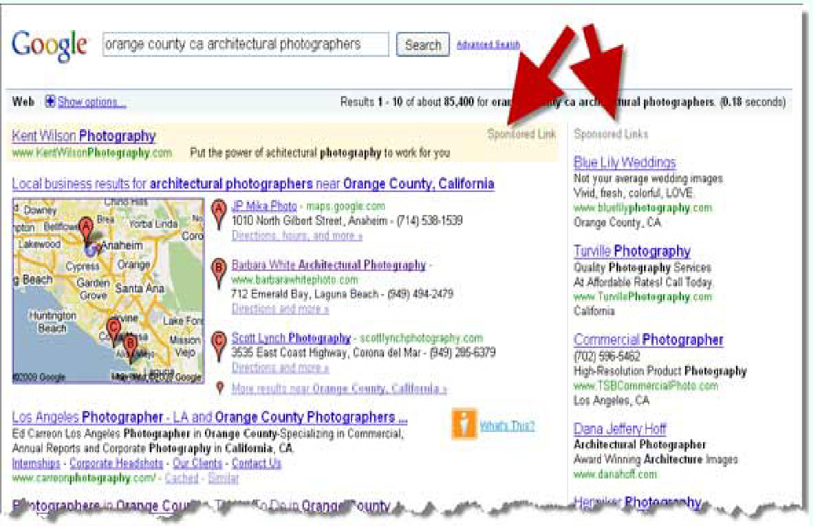
Prediction markets

Double Your Donation
Matching Gifts

Donation matching

How can AI help?

Multiple entities with different interests

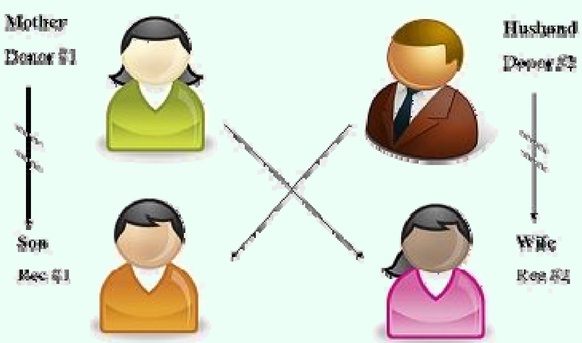


Auctions

Security

THIS TALK

Rating/voting systems



Kidney exchanges

Prediction markets

Donation matching

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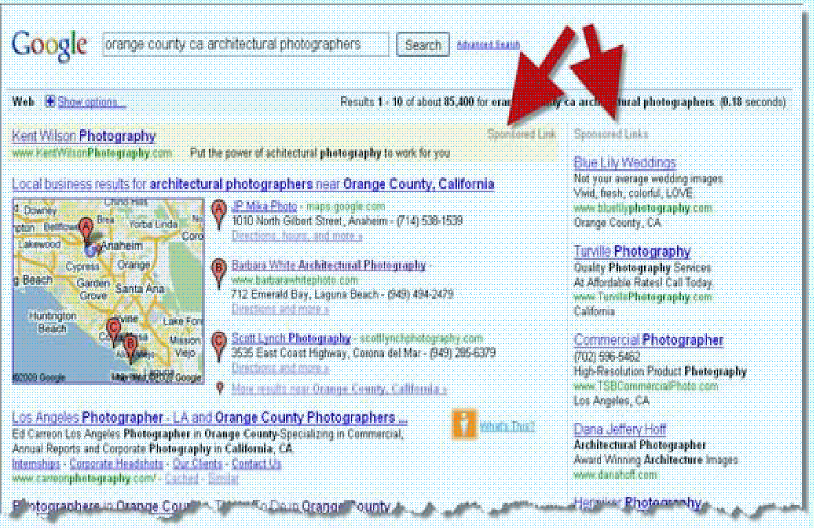
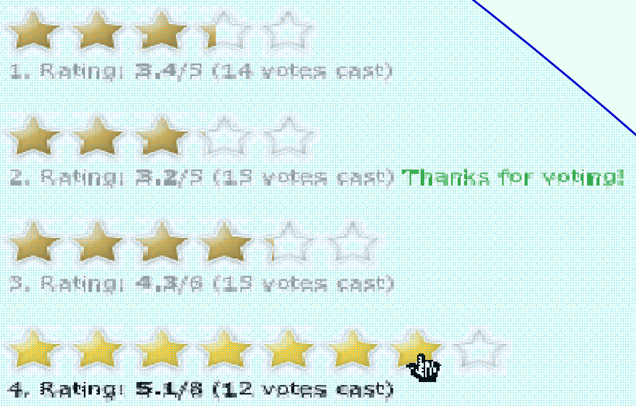


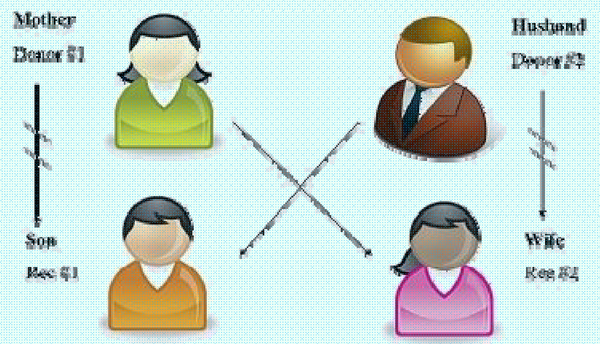
Auctions



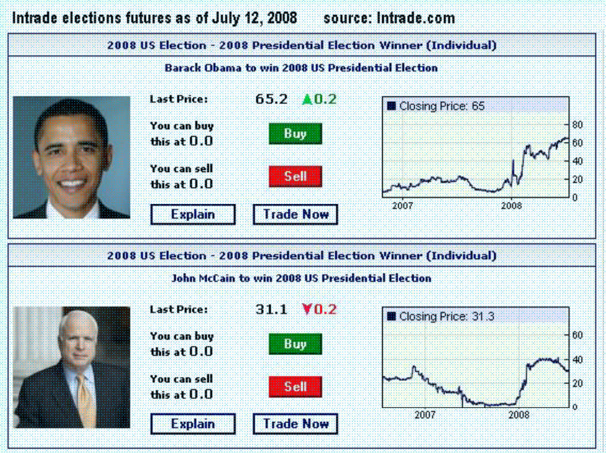
Rating/voting systems

Security

THIS TALK



Kidney exchanges



Prediction markets

overview: C., CACM March 2010

Double Your Donation
Matching Gifts

Donation matching

How can AI help?

Closer to home…

Closer to home…

Closer to home…

Game playing

Closer to home…

Game playing

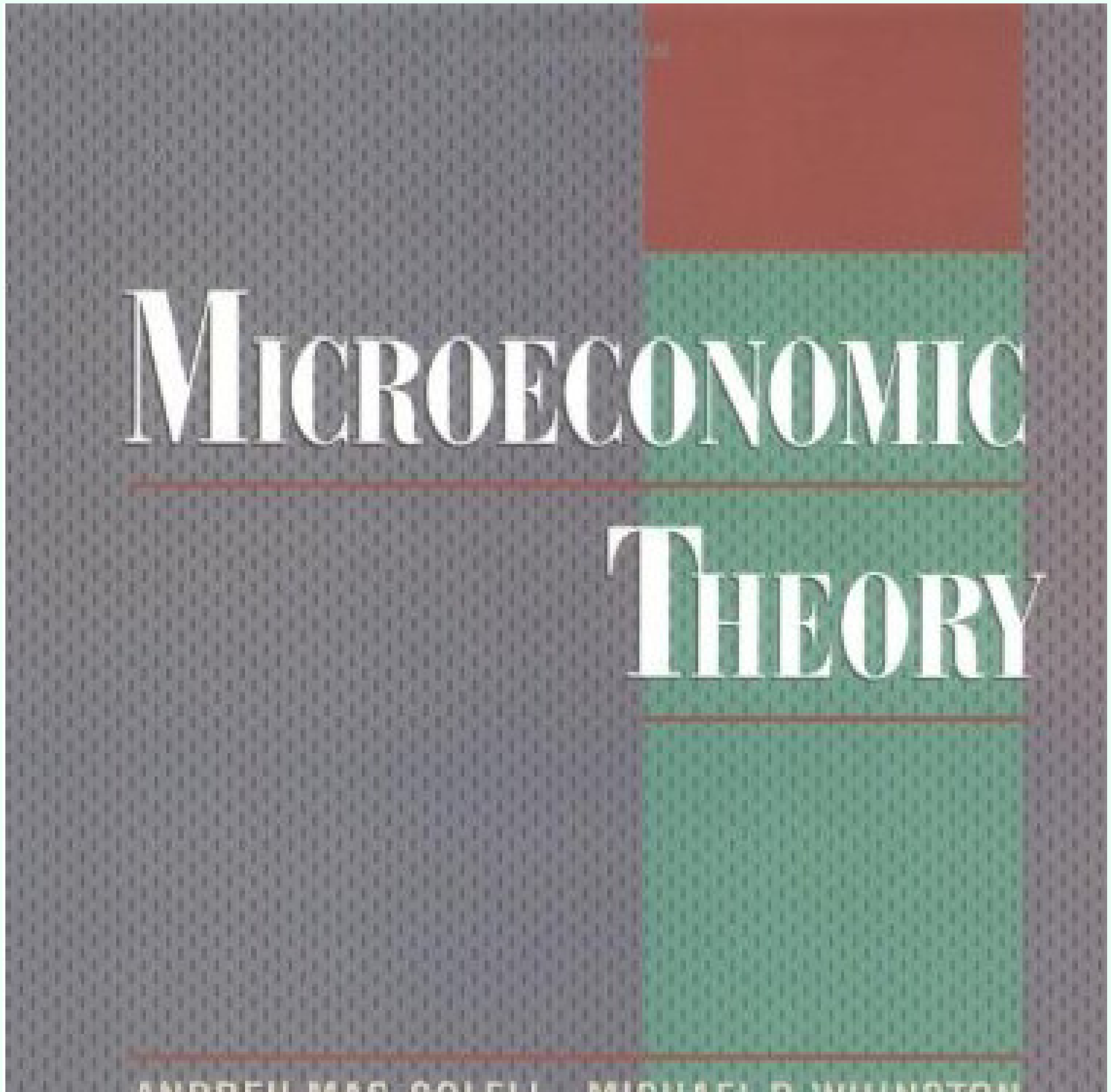
Closer to home…

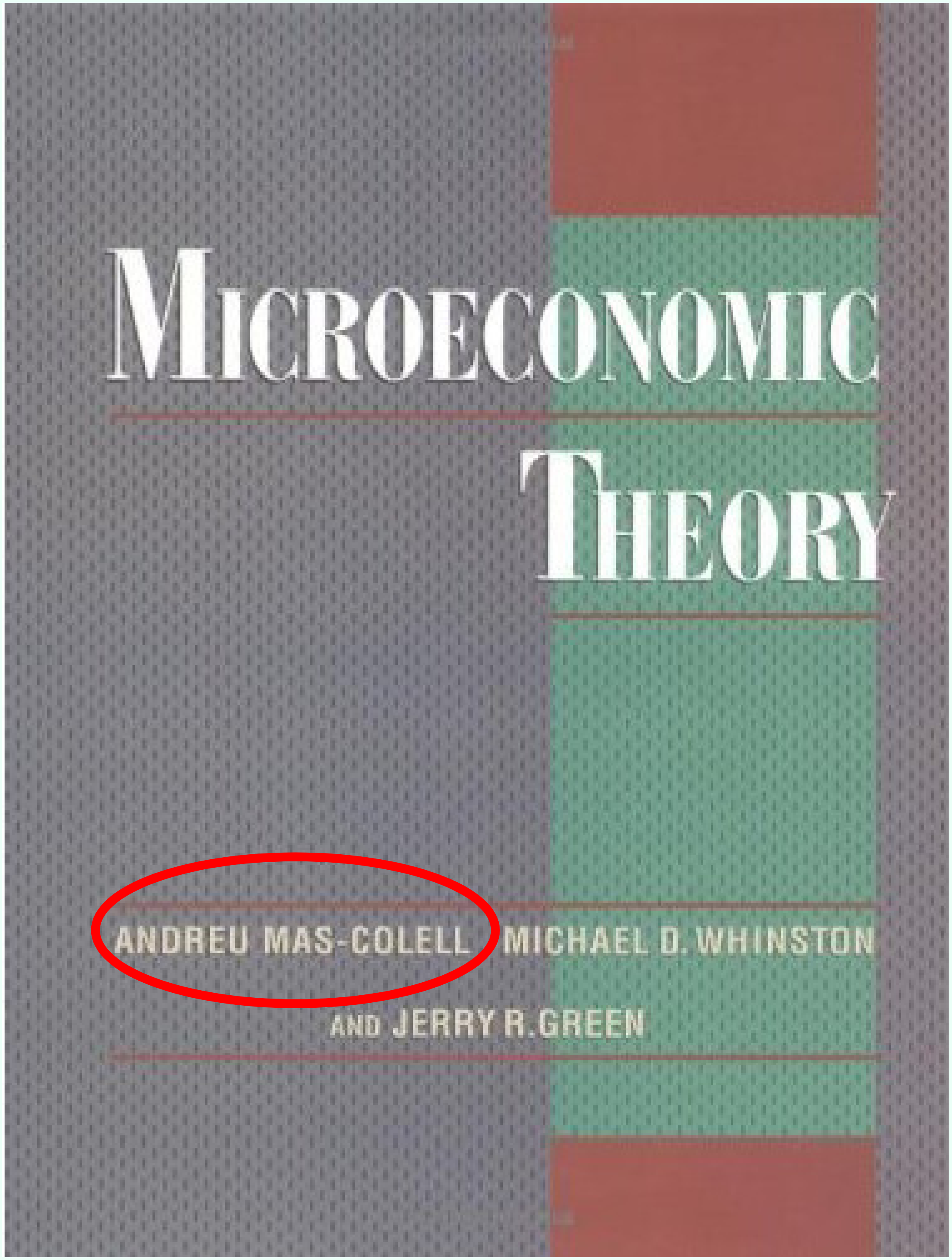
Game playing

Closer to home…

Game playing

Multiagent systems


MICROECONOMIC
THEORY
ANDREU MAS-COLELL MICHAEL D. WHINSTON
AND JERRY R. GREEN


MICROECONOMIC
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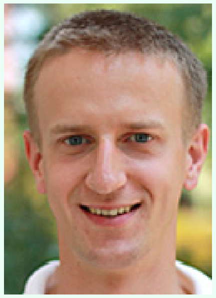
Some microeconomic theory tools for AI

GAME THEORY



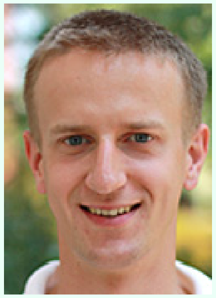
Dima Korzhyk

2, 2	-1, 0
-7, -8	0, 0

Josh Letchford

Some microeconomic theory tools for AI

GAME THEORY



Dima Korzhyk

2, 2	-1, 0
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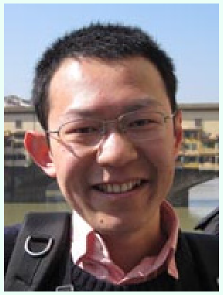
Josh Letchford

SOCIAL CHOICE

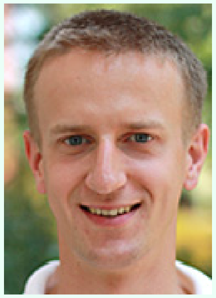
A>B>C
B>A>C
C>B>A
↓
B wins



Lirong Xia

Some microeconomic theory tools for AI

GAME THEORY



Dima Korzhyk

Josh Letchford

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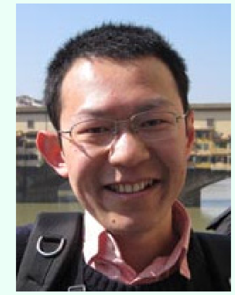
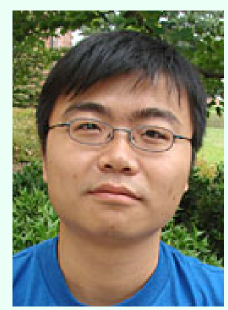


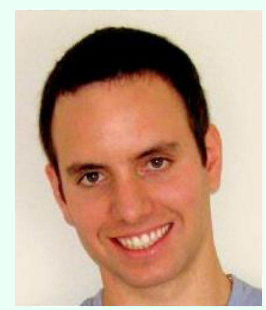
Lirong Xia

MECHANISM DESIGN

v_1=42
v_2=30
v_3=20 → 1 wins, pays 30



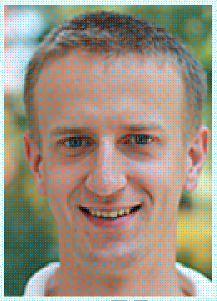
Mingyu Guo



Liad Wagman

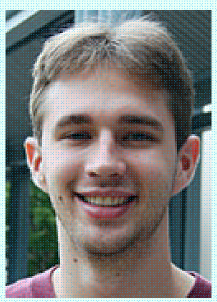
Some microeconomic theory tools for AI

GAME THEORY



Dima Korzhyk

2, 2	-1, 0
-7, -8	0, 0

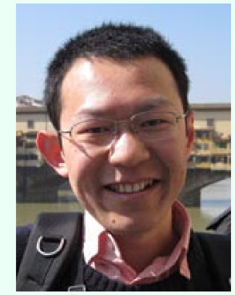


Josh Letchford

THIS TALK

SOCIAL CHOICE

A>B>C
B>A>C
C>B>A
↓
B wins



Lirong Xia

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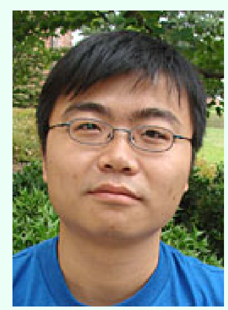
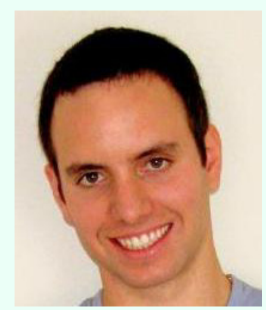
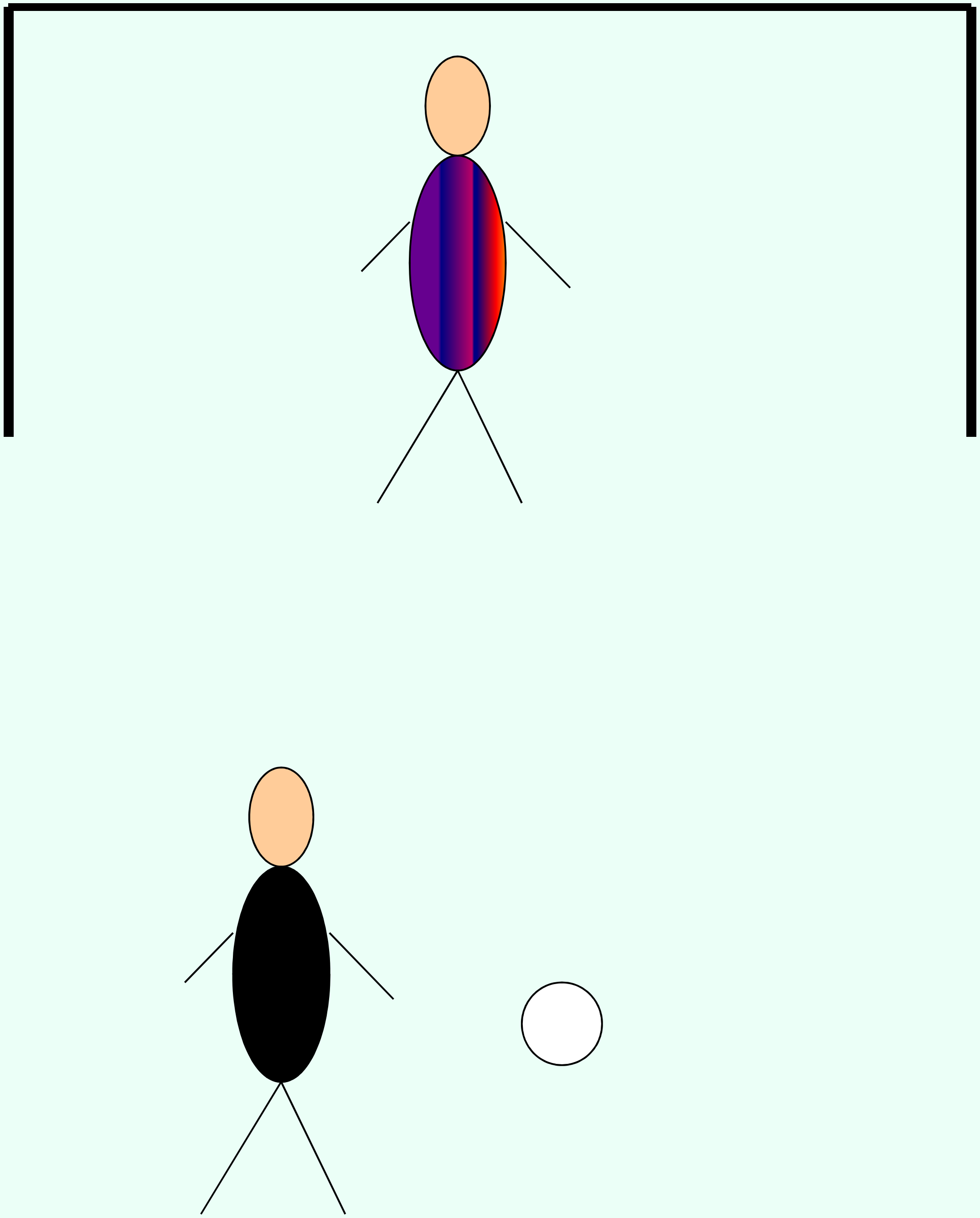


Mingyu Guo

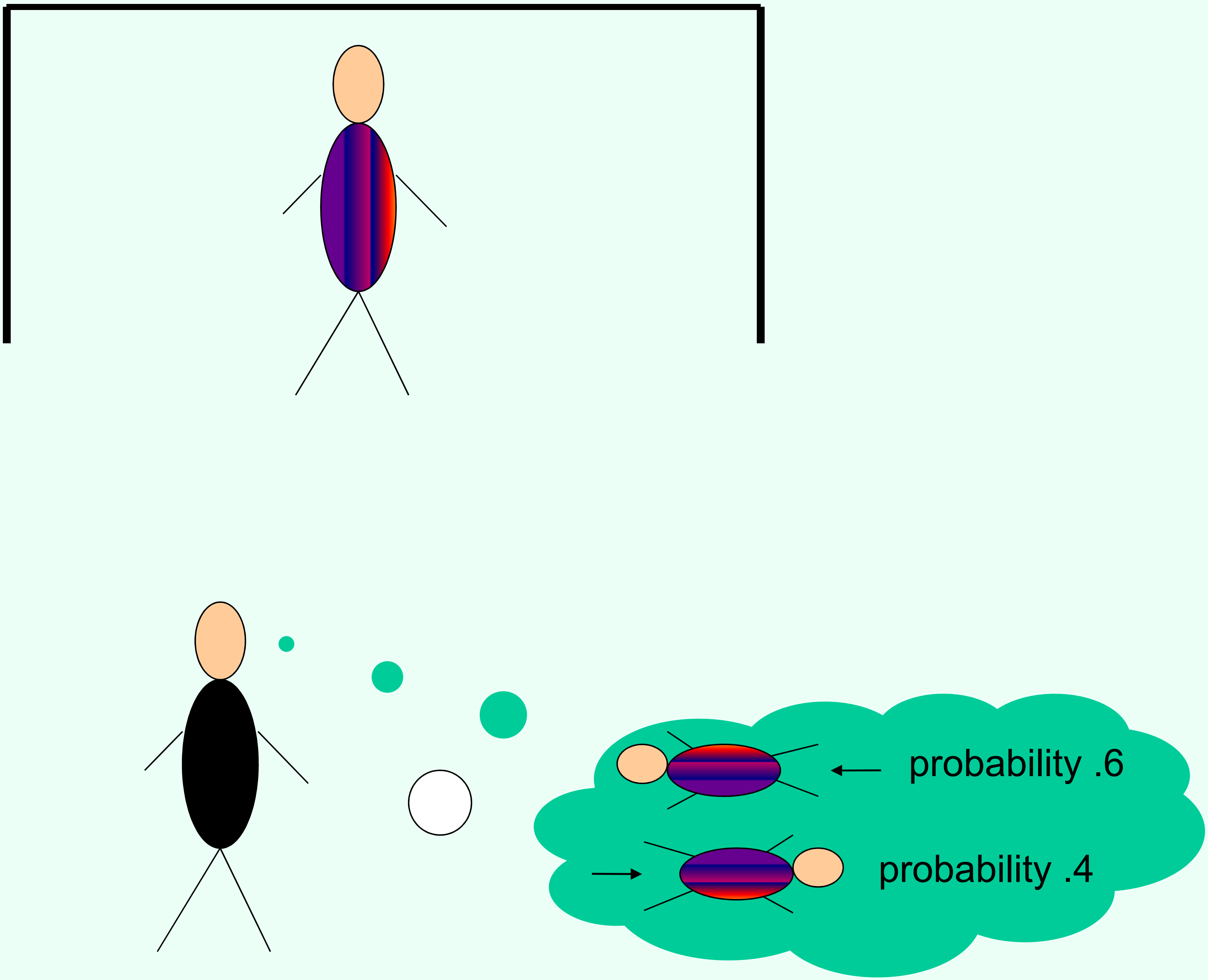


Liad Wagman

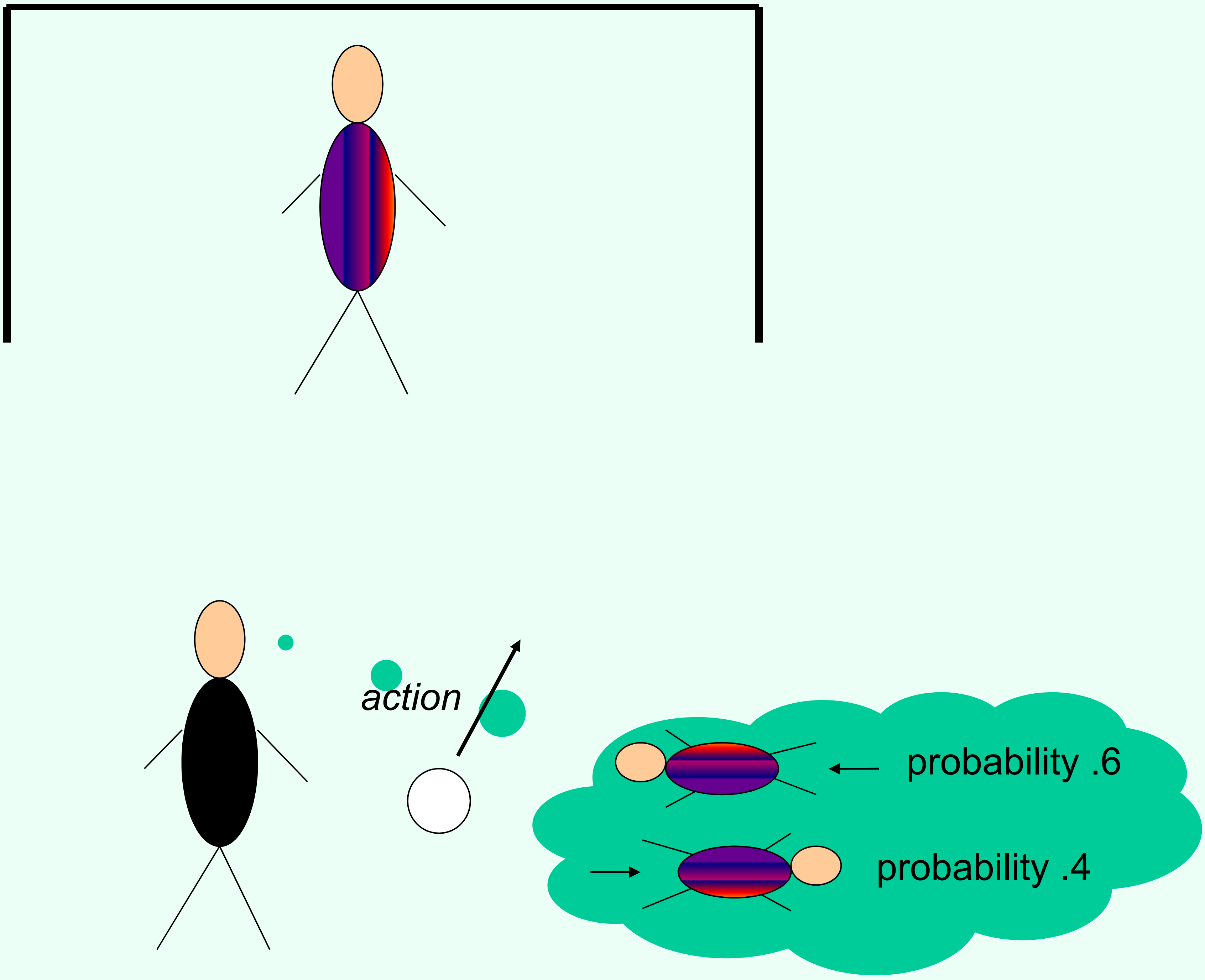
Penalty kick example



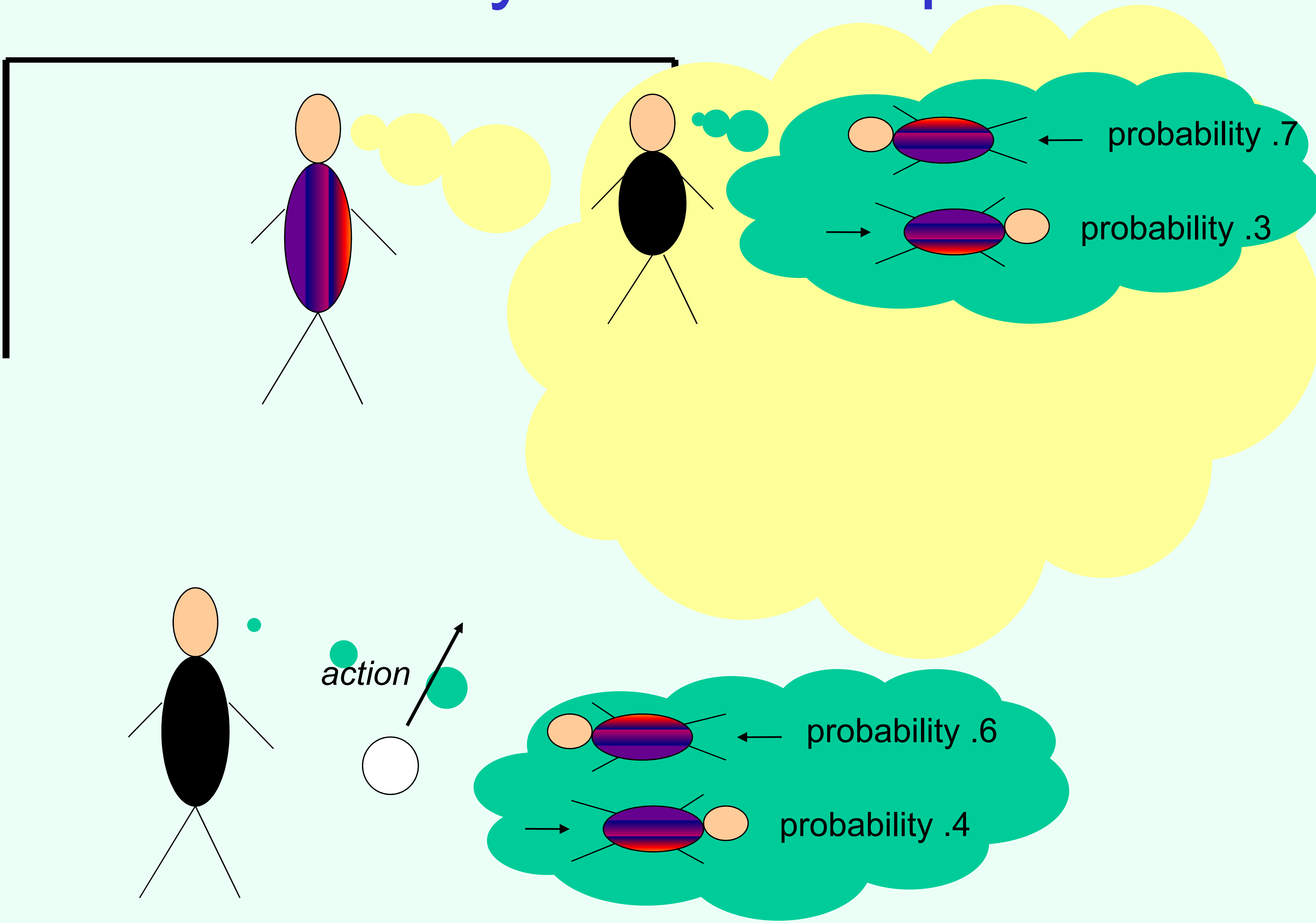
Penalty kick example



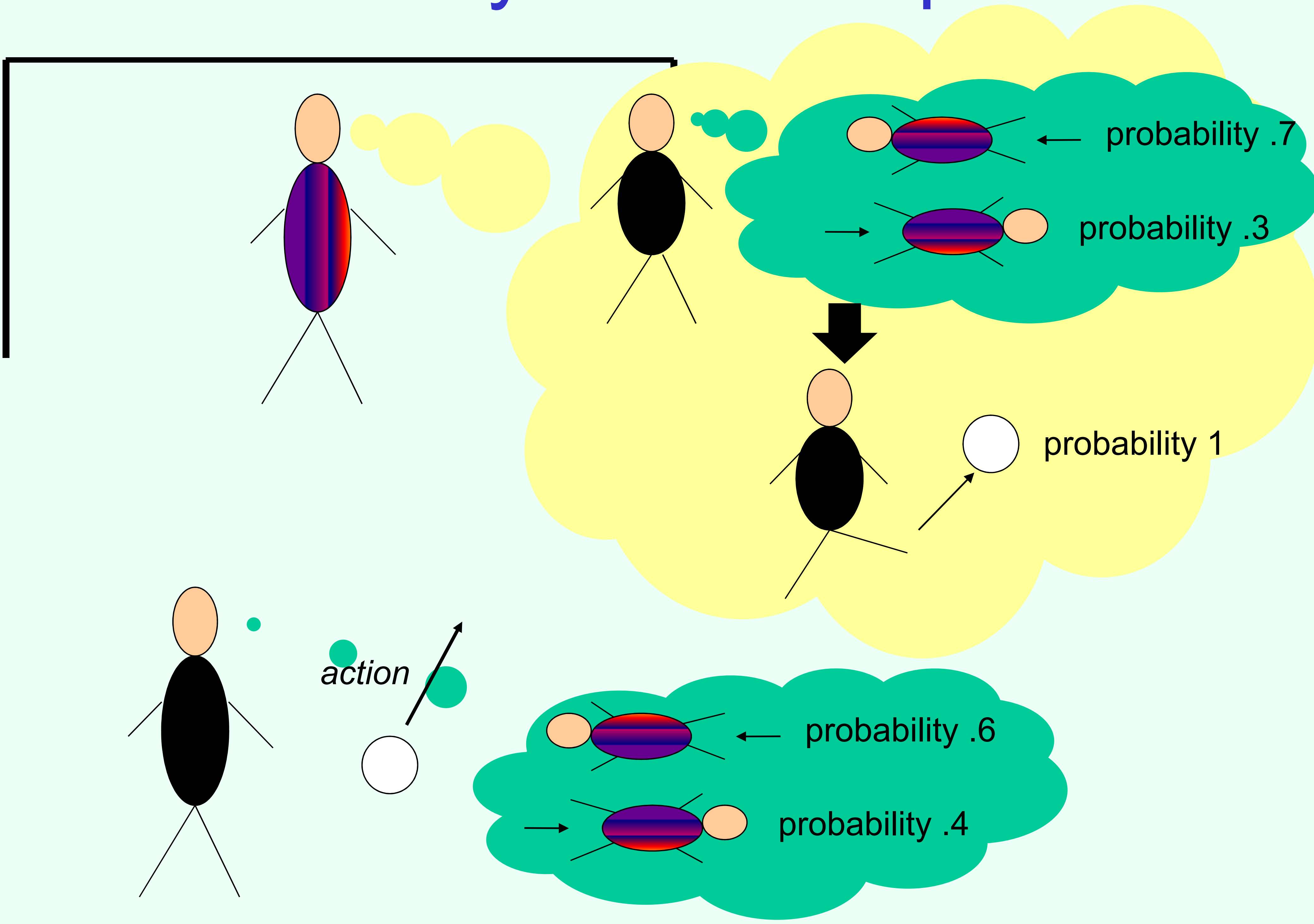
Penalty kick example



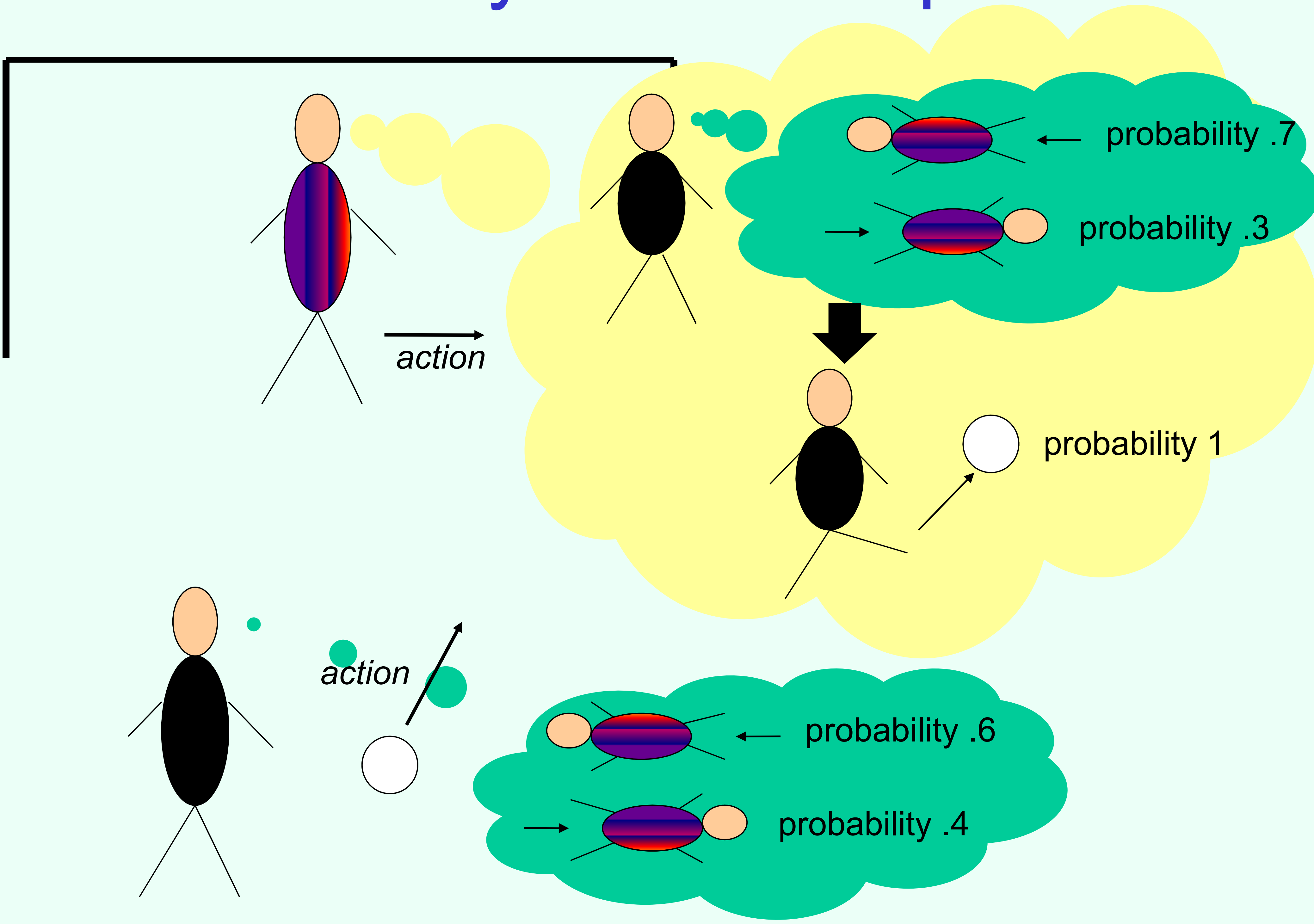
Penalty kick example



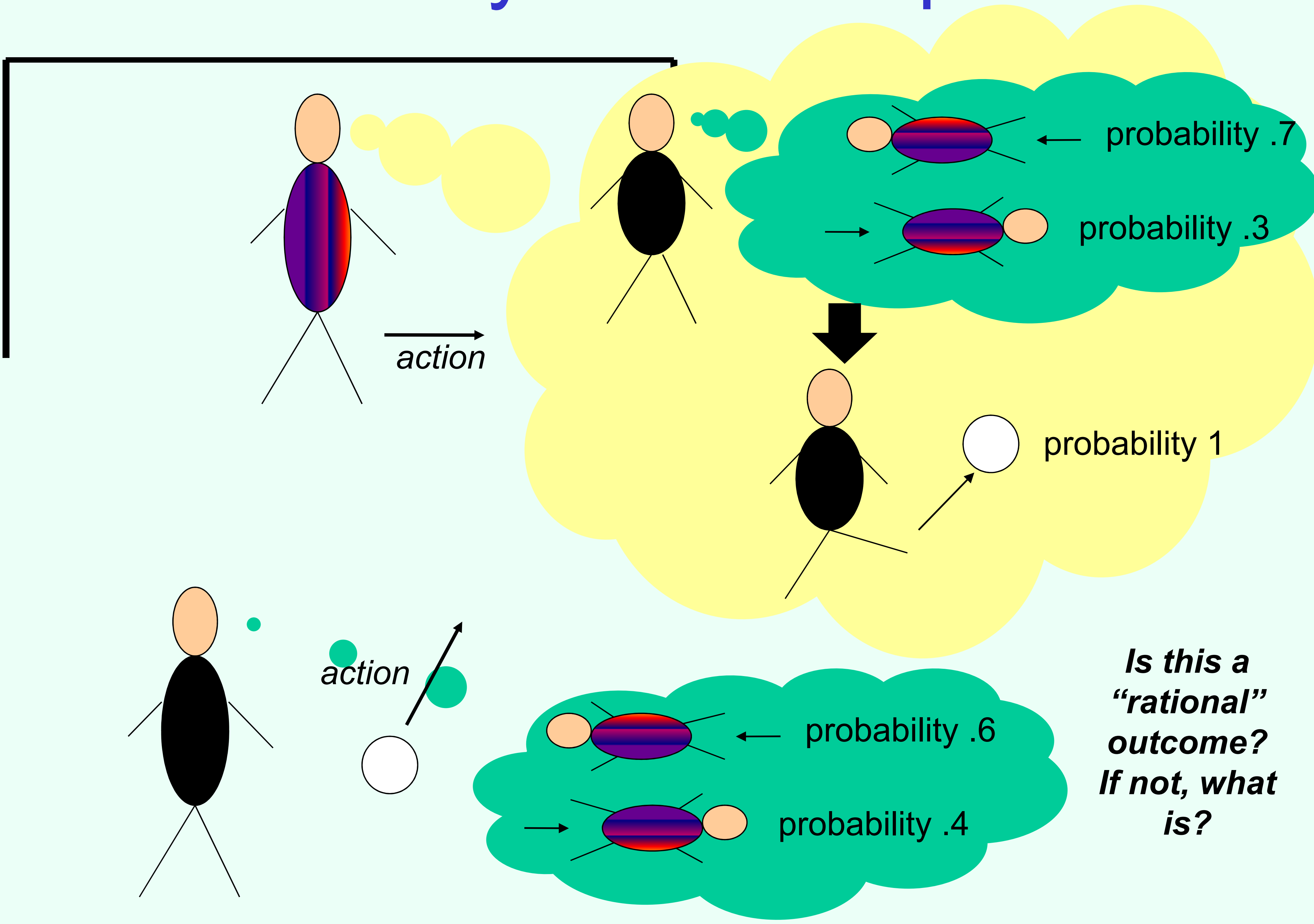
Penalty kick example



Penalty kick example



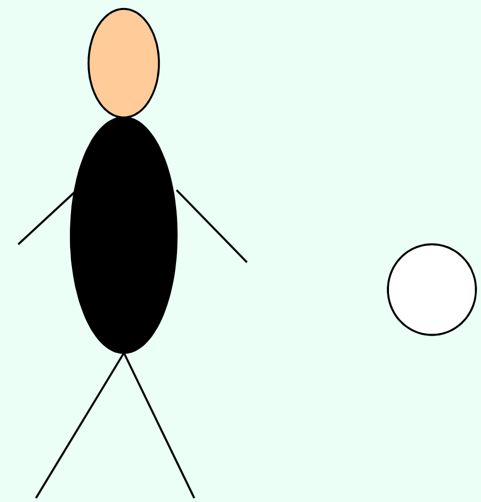
Penalty kick example

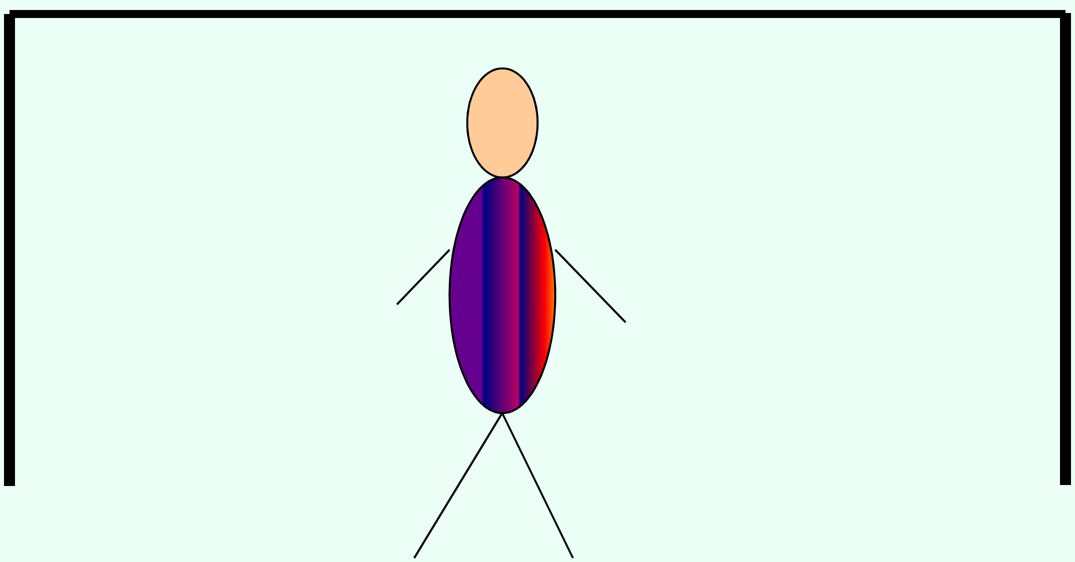


Is this a "rational" outcome? If not, what is?

Penalty kick

(also known as: matching pennies)

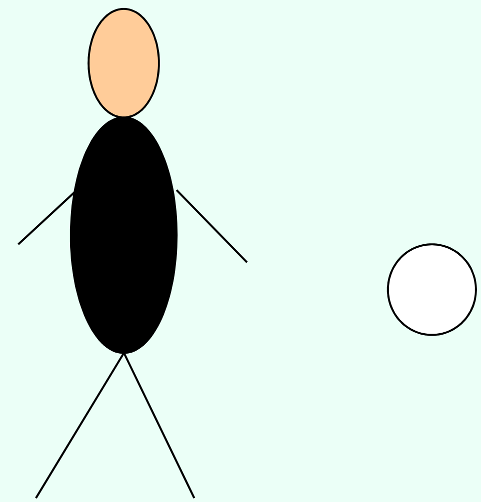


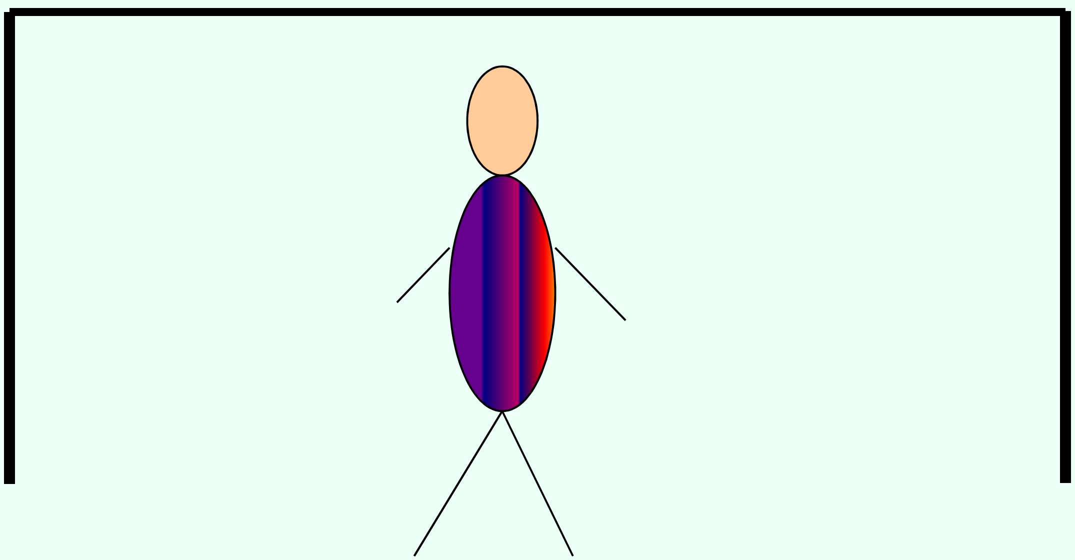


	L	R
L	0, 0	-1, 1
R	-1, 1	0, 0

Penalty kick

(also known as: matching pennies)

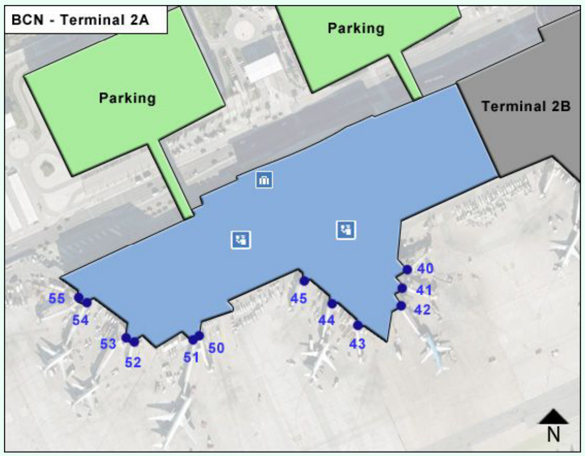


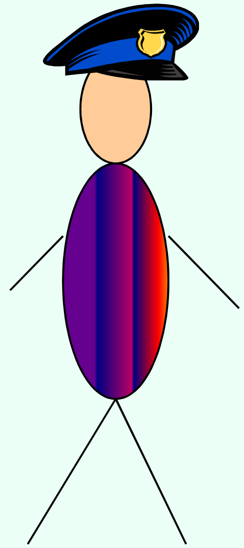


		.5	.5
		L	R
.5	L	0, 0	-1, 1
.5	R	-1, 1	0, 0

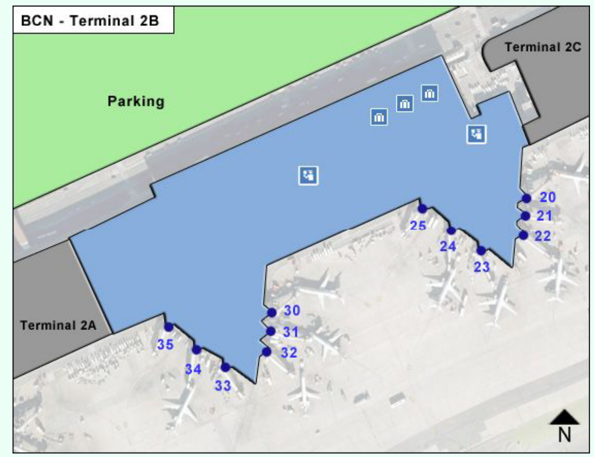
Security example

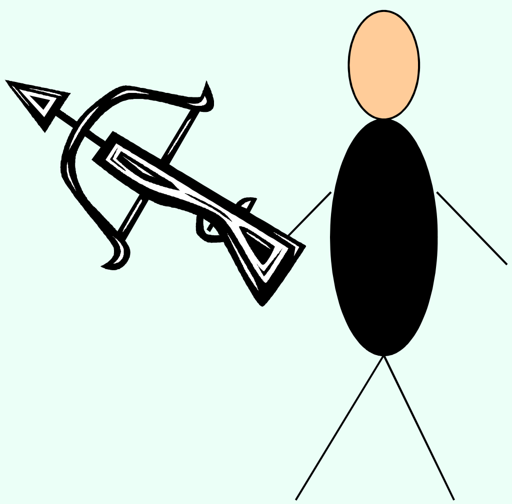
BCN terminal 2A



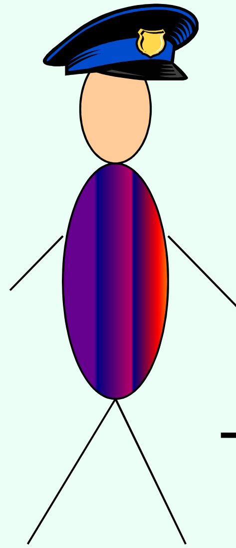


BCN terminal 2B

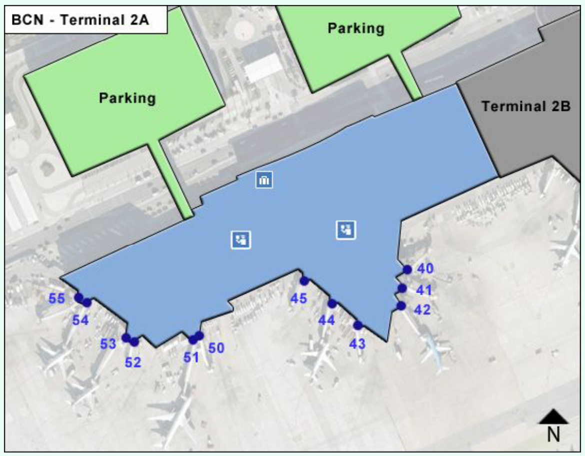




Security example

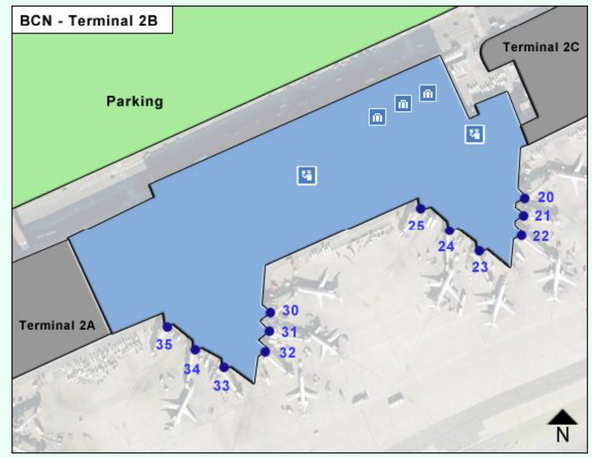


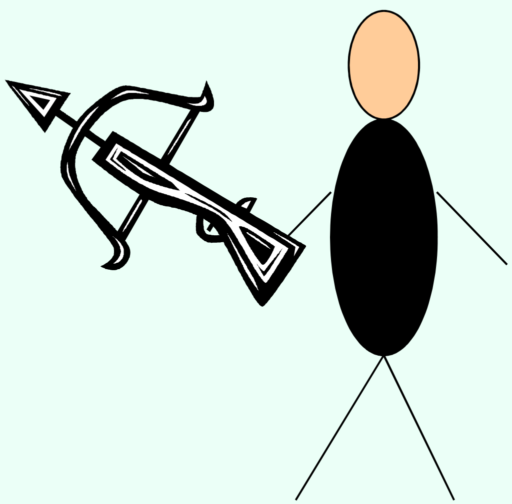
BCN terminal 2A



action

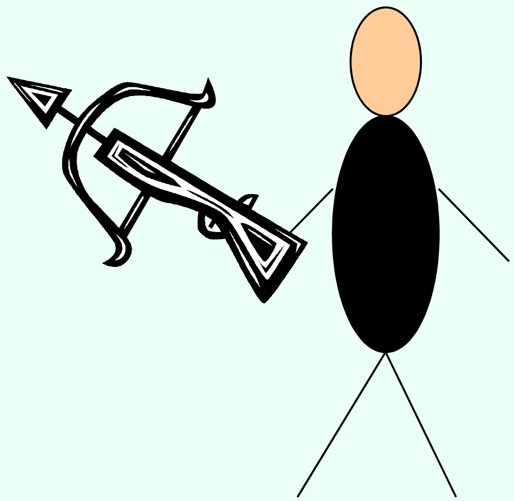
BCN terminal 2B

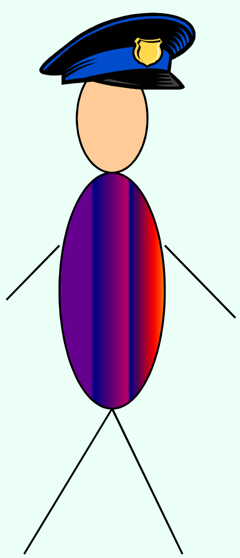




action

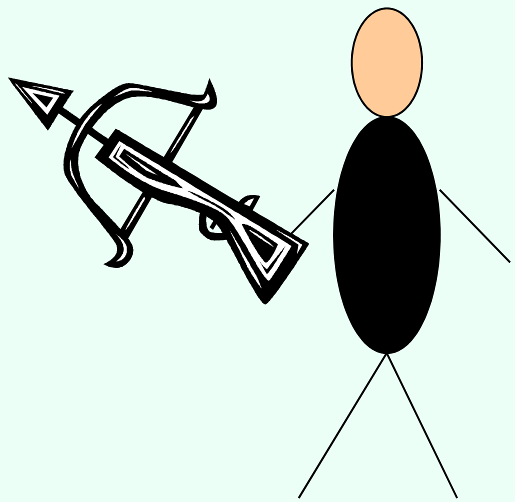
Security game

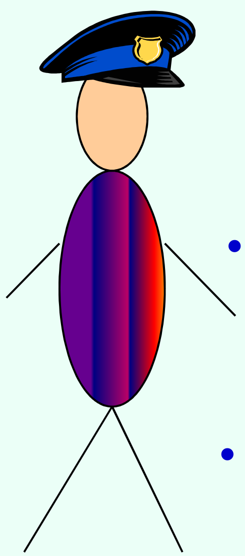




	2A	2B
2A	0, 0	-1, 2
2B	-1, 1	0, 0

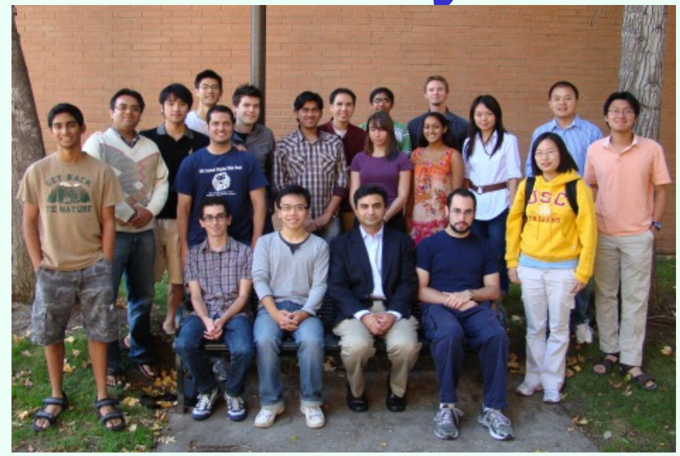
Security game





		2A	2B
.33	2A	0, 0	-1, 2
.67	2B	-1, 1	0, 0

Recent deployments in security



- Tambe's TEAMCORE group at USC
- Airport security
  - Where should checkpoints, canine units, etc. be deployed?
  - Deployed at LAX and another US airport, being evaluated for deployment at all US airports
- Federal Air Marshals
- Coast Guard
- …

“Should I buy an SUV?”

(also known as the Prisoner’s Dilemma)

purchasing + gas cost

cost: 5

cost: 3

“Should I buy an SUV?”

(also known as the Prisoner’s Dilemma)

purchasing + gas cost

cost: 5

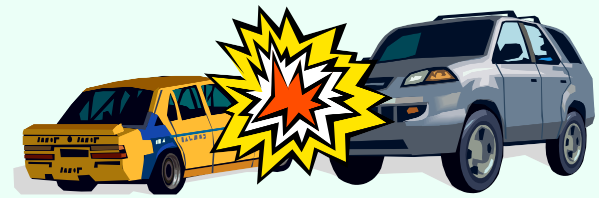
cost: 3

accident cost

cost: 5

cost: 5

cost: 8



cost: 2

cost: 5

cost: 5

“Should I buy an SUV?”

(also known as the Prisoner’s Dilemma)

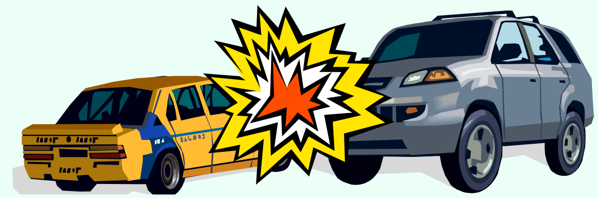
purchasing + gas cost

cost: 5

cost: 3

accident cost

cost: 5  cost: 5

cost: 8  cost: 2

cost: 5  cost: 5

		
	-10, -10	-7, -11
	-11, -7	-8, -8

“Should I buy an SUV?”

(also known as the Prisoner’s Dilemma)

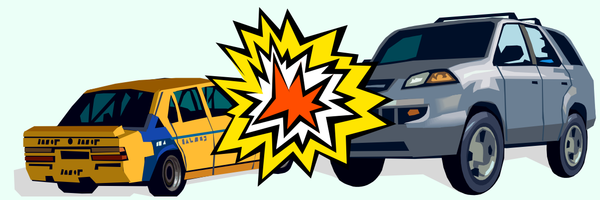
purchasing + gas cost

cost: 5

cost: 3

accident cost

cost: 5  cost: 5

cost: 8  cost: 2

cost: 5  cost: 5

		
	-10, -10	-7, -11
	-11, -7	-8, -8

“Should I buy an SUV?”

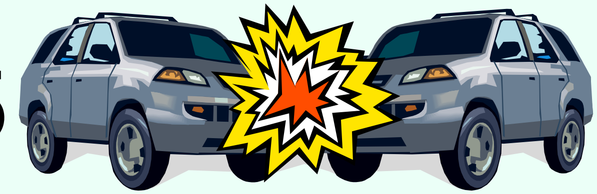
(also known as the Prisoner’s Dilemma)

purchasing + gas cost

cost: 5

cost: 3

accident cost

cost: 5  cost: 5

cost: 8  cost: 2

cost: 5  cost: 5

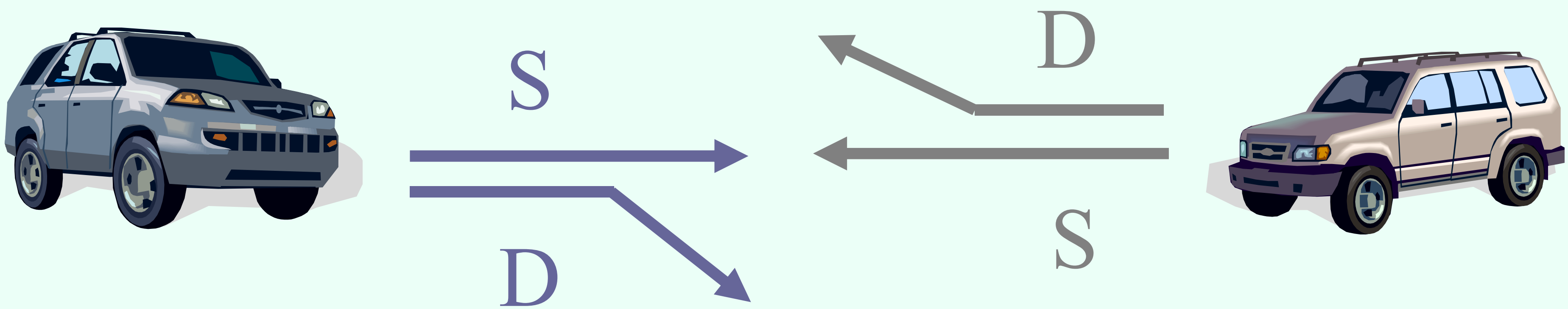
 

SUV	-10, -10	-7, -11
car	-11, -7	-8, -8

Computational aspects of dominance: Gilboa, Kalai, Zemel Math of OR ‘93; C. & Sandholm EC ’05, AAAI’05; Brandt, Brill, Fischer, Harrenstein TOCS ‘11

"Chicken"

- Two players drive cars towards each other
- If one player goes straight, that player wins
- If both go straight, they both die



	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

Nash equilibrium [Nash '50]

Nash equilibrium [Nash '50]

Nash equilibrium [Nash '50]

Nash equilibrium [Nash '50]

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- A profile (= strategy for each player) so that no player wants to deviate

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Nash equilibrium [Nash ‘50]

- A profile (= strategy for each player) so that no player wants to deviate

	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

- This game has another Nash equilibrium in mixed strategies – both play D with 80%

The presentation game

	Pay attention (A)	*Do not pay attention (NA)*
Put effort into presentation (E)	2, 2	-1, 0
Do not put effort into presentation (NE)	-7, -8	0, 0

The presentation game

	Pay attention (A)	*Do not pay attention (NA)*
Put effort into presentation (E)	2, 2	-1, 0
Do not put effort into presentation (NE)	-7, -8	0, 0

- Pure-strategy Nash equilibria: (E, A), (NE, NA)

The presentation game

	Pay attention (A)	Do not pay attention (NA)
Put effort into presentation (E)	2, 2	-1, 0
Do not put effort into presentation (NE)	-7, -8	0, 0

- Pure-strategy Nash equilibria: (E, A), (NE, NA)
- Mixed-strategy Nash equilibrium:

The presentation game

	Pay attention (A)	*Do not pay attention (NA)*
Put effort into presentation (E)	2, 2	-1, 0
Do not put effort into presentation (NE)	-7, -8	0, 0

- Pure-strategy Nash equilibria: (E, A), (NE, NA)
- Mixed-strategy Nash equilibrium:

  ((4/5 E, 1/5 NE), (1/10 A, 9/10 NA))

The presentation game

	Pay attention (A)	Do not pay attention (NA)
Put effort into presentation (E)	2, 2	-1, 0
Do not put effort into presentation (NE)	-7, -8	0, 0

- Pure-strategy Nash equilibria: (E, A), (NE, NA)
- Mixed-strategy Nash equilibrium:

  ((4/5 E, 1/5 NE), (1/10 A, 9/10 NA))

  – Utility -7/10 for presenter, 0 for audience

Modeling and representing games

THIS TALK (unless specified otherwise)

2, 2	-1, 0
-7, -8	0, 0

normal-form games

Modeling and representing games

THIS TALK (unless specified otherwise)

2, 2	-1, 0
-7, -8	0, 0

normal-form games

row player type 1 (prob. 0.5)

	L	R
U	4	6
D	2	4

column player type 1 (prob. 0.5)

	L	R
U	4	6
D	4	6

row player type 2 (prob. 0.5)

	L	R
U	2	4
D	4	2

column player type 2 (prob. 0.5)

	L	R
U	2	2
D	4	2

Bayesian games

Modeling and representing games

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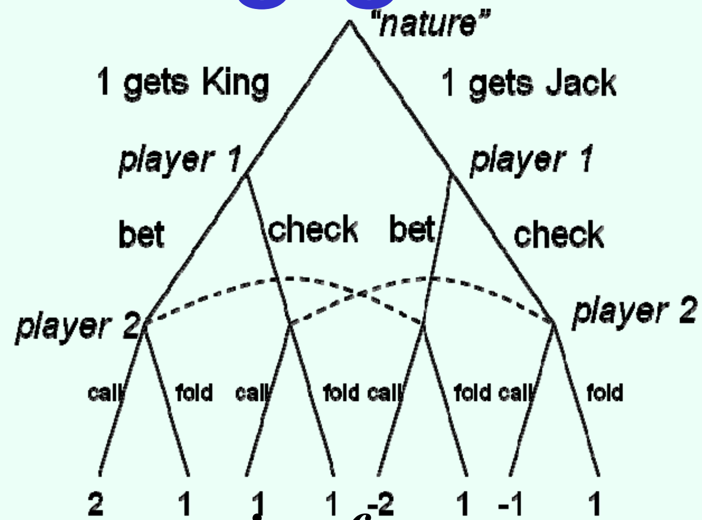
Bayesian games



extensive-form games

Modeling and representing games

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D	4	6

row player type 2 (prob. 0.5)

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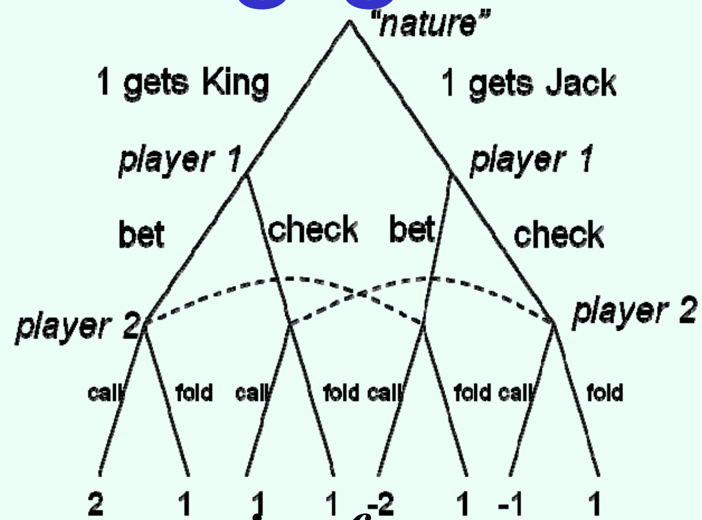
D	4	2

column player type 2 (prob. 0.5)

	L	R
U	2	2
D	4	2

Bayesian games



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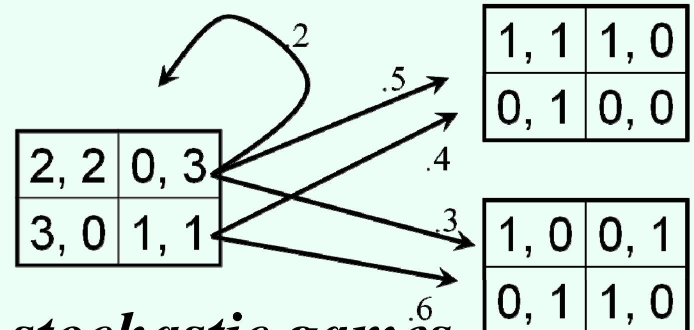

stochastic games

Modeling and representing games

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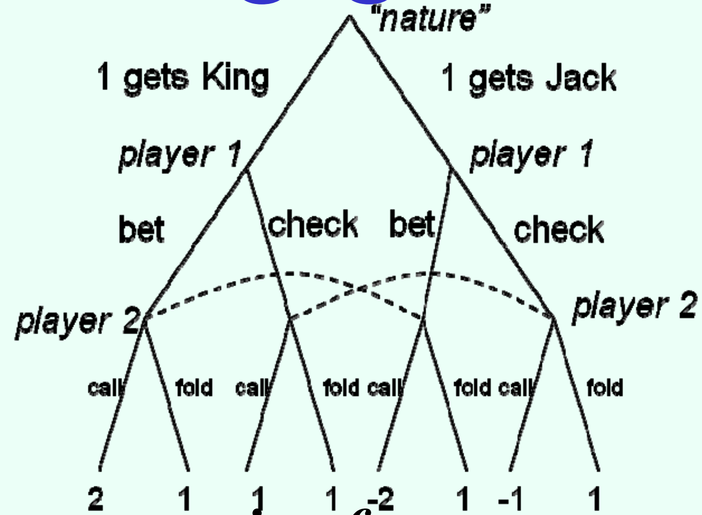
normal-form games



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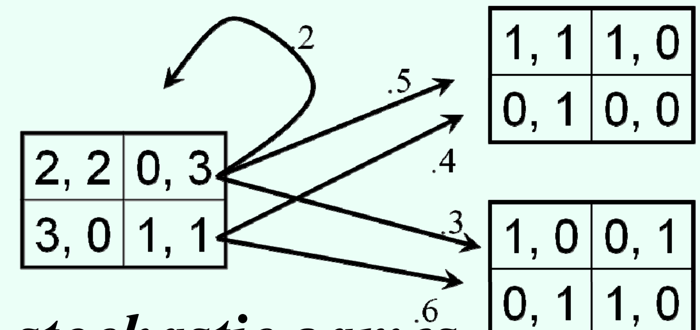
D	4	2

Bayesian games



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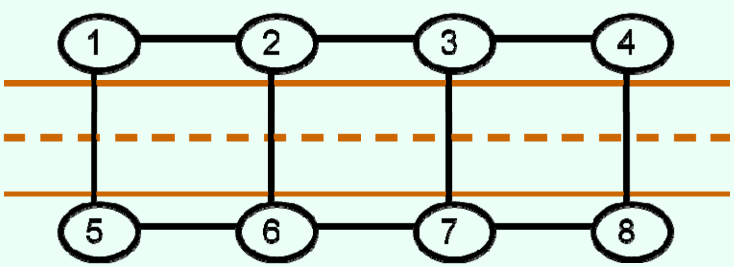

graphical games

[Kearns, Littman, Singh UAI'01]

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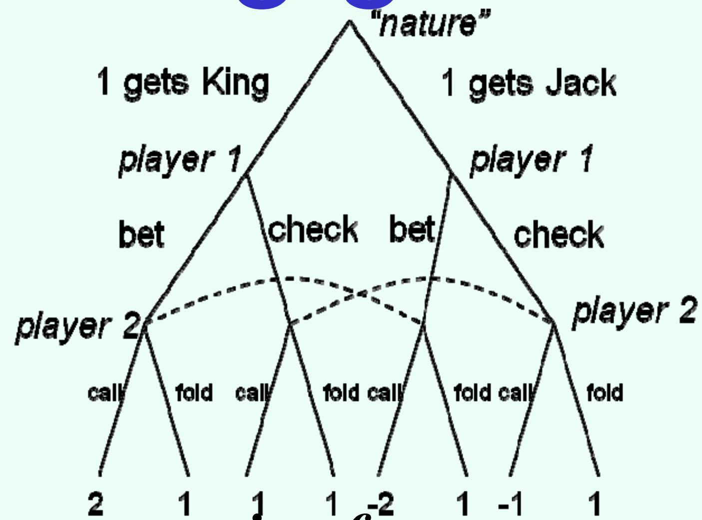
Bayesian games



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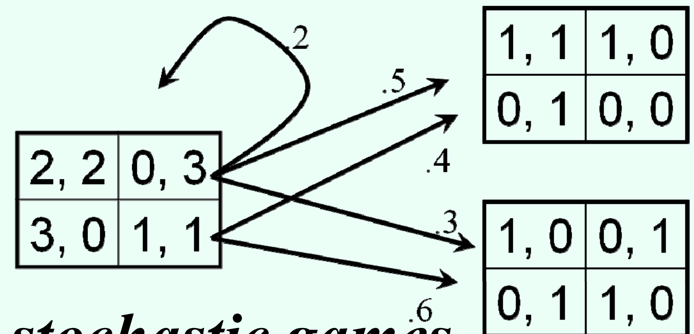

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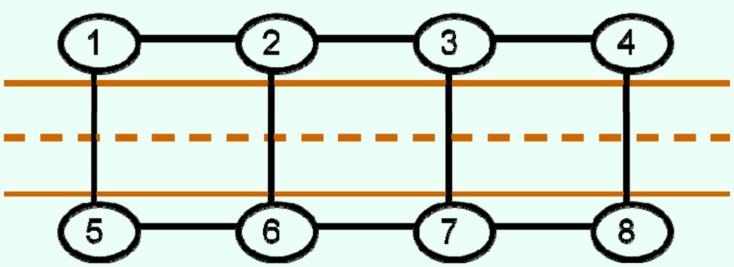

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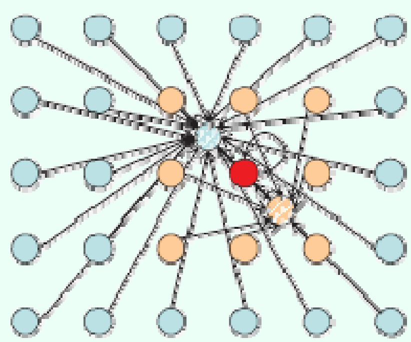

action-graph games

[Leyton-Brown & Tennenholtz IJCAI'03

[Bhat & Leyton-Brown, UAI'04]

[Jiang, Leyton-Brown, Bhat GEB'11]

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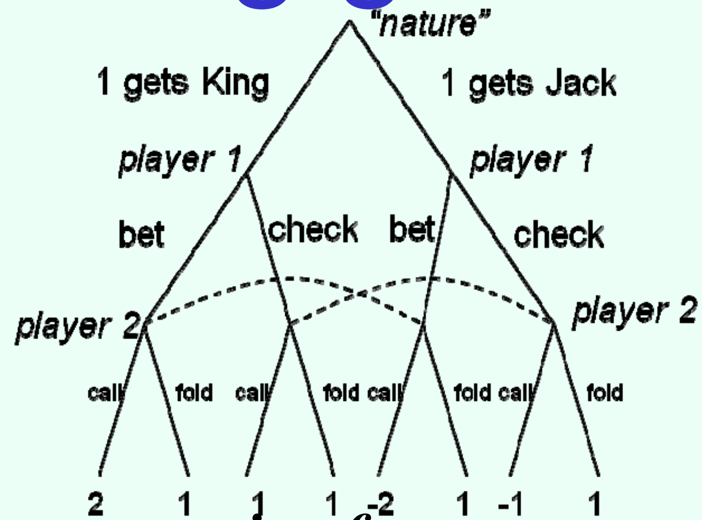

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U	4	6
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	L	R
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D	4	2

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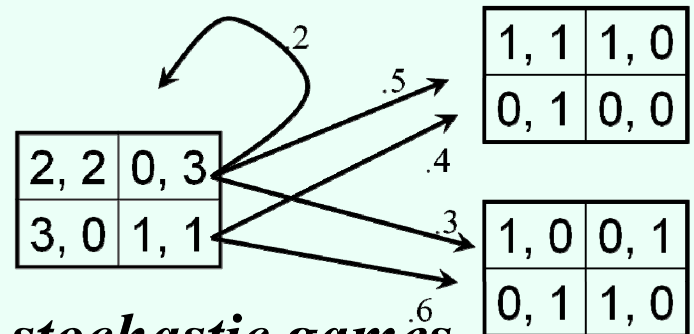
	L	R
U	2	2
D	4	2

Bayesian games



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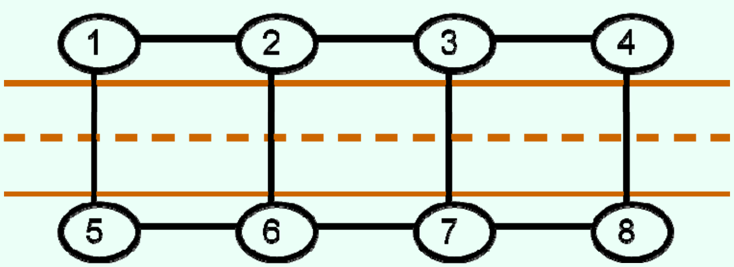

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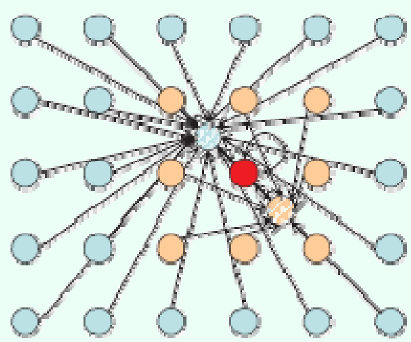

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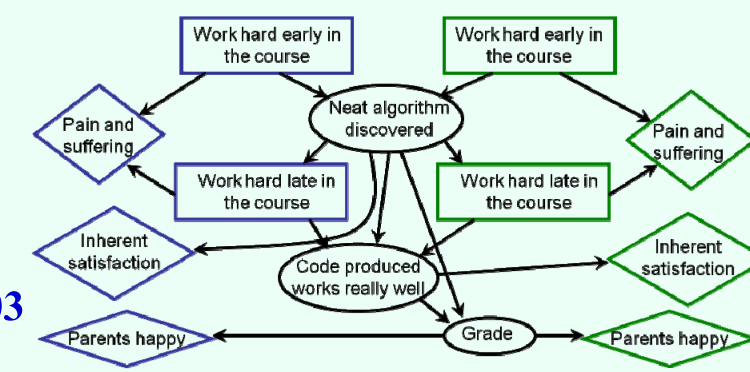

MAIDs

[Koller & Milch. IJCAI'01/GEB'03]

Computing a single Nash equilibrium

Christos Papadimitriou,
STOC'01

"Together with factoring, *the complexity of finding a Nash equilibrium is in my opinion the most important concrete open question on the boundary of P today.*"

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- Is one Nash equilibrium all we need to know?

A useful reduction (SAT → game)

[C. & Sandholm IJCAI'03, Games and Economic Behavior '08]

(Earlier reduction with weaker implications: Gilboa & Zemel GEB '89)

Formula: **(x_1 or -x_2) and (-x_1 or x_2)**

Solutions:

x_1=true,x_2=true

x_1=false,x_2=false

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Formula: **(x_1 or $-x_2$) and ($-x_1$ or x_2)**

Solutions:
x_1=true,x_2=true
x_1=false,x_2=false

Game:

	x_1	x_2	$+x_1$	$-x_1$	$+x_2$	$-x_2$	(x_1 or $-x_2$)	($-x_1$ or x_2)	default
x_1	-2,-2	-2,-2	0,-2	0,-2	2,-2	2,-2	-2,-2	-2,-2	0,1
x_2	-2,-2	-2,-2	2,-2	2,-2	0,-2	0,-2	-2,-2	-2,-2	0,1
$+x_1$	-2,0	-2,2	1,1	-2,-2	1,1	1,1	-2,0	-2,2	0,1
$-x_1$	-2,0	-2,2	-2,-2	1,1	1,1	1,1	-2,2	-2,0	0,1
$+x_2$	-2,2	-2,0	1,1	1,1	1,1	-2,-2	-2,2	-2,0	0,1
$-x_2$	-2,2	-2,0	1,1	1,1	-2,-2	1,1	-2,0	-2,2	0,1
(x_1 or $-x_2$)	-2,-2	-2,-2	0,-2	2,-2	2,-2	0,-2	-2,-2	-2,-2	0,1
($-x_1$ or x_2)	-2,-2	-2,-2	2,-2	0,-2	0,-2	2,-2	-2,-2	-2,-2	0,1
default	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	ε, ε

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Game:

	x_1	**x_2**	**+x_1**	**-x_1**	**+x_2**	**-x_2**	**(x_1 or -x_2)**	**(-x_1 or x_2)**	**default**
x_1	-2,-2	-2,-2	0,-2	0,-2	2,-2	2,-2	-2,-2	-2,-2	0,1
x_2	-2,-2	-2,-2	2,-2	2,-2	0,-2	0,-2	-2,-2	-2,-2	0,1
+x_1	-2,0	-2,2	1,1	-2,-2	1,1	1,1	-2,0	-2,2	0,1
-x_1	-2,0	-2,2	-2,-2	1,1	1,1	1,1	-2,2	-2,0	0,1
+x_2	-2,2	-2,0	1,1	1,1	1,1	-2,-2	-2,2	-2,0	0,1
-x_2	-2,2	-2,0	1,1	1,1	-2,-2	1,1	-2,0	-2,2	0,1
(x_1 or -x_2)	-2,-2	-2,-2	0,-2	2,-2	2,-2	0,-2	-2,-2	-2,-2	0,1
(-x_1 or x_2)	-2,-2	-2,-2	2,-2	0,-2	0,-2	2,-2	-2,-2	-2,-2	0,1
default	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	ε, ε

- Every satisfying assignment (if there are any) corresponds to an equilibrium with utilities 1, 1
- Exactly one additional equilibrium with utilities **ε**, **ε** that always exists

Some algorithm families for computing Nash equilibria of 2-player normal-form games

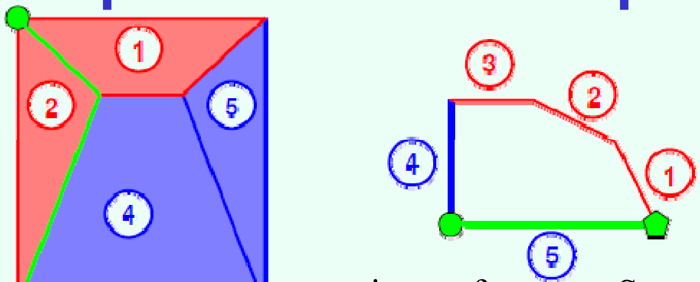

image from von Stengel

Lemke-Howson [J. SIAM '64]

Exponential time due to Savani & von Stengel [FOCS'04 / Econometrica'06]

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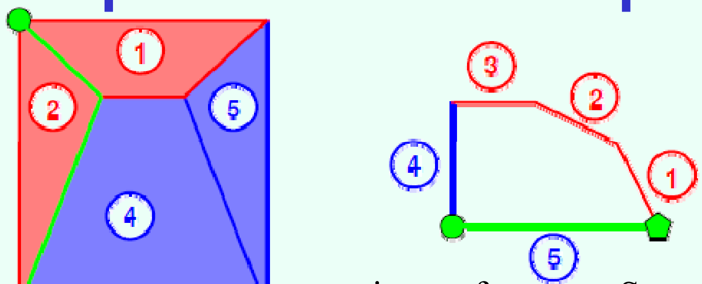

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– for both i, for any $s_i \in S_i - X_i$, $\mathbf{p_i(s_i)} = 0$

– for both i, for any $s_i \in X_i$, $\Sigma \mathbf{p_{-i}(s_{-i})} u_i(s_i, s_{-i}) = \mathbf{u_i}$

– for both i, for any $s_i \in S_i - X_i$, $\Sigma \mathbf{p_{-i}(s_{-i})} u_i(s_i, s_{-i}) \leq \mathbf{u_i}$

Search over supports / MIP

[Dickhaut & Kaplan, Mathematica J. '91]

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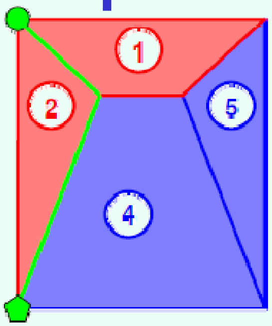

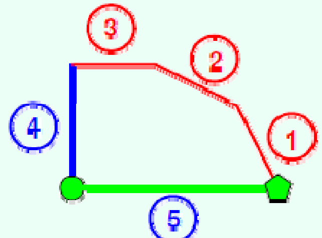

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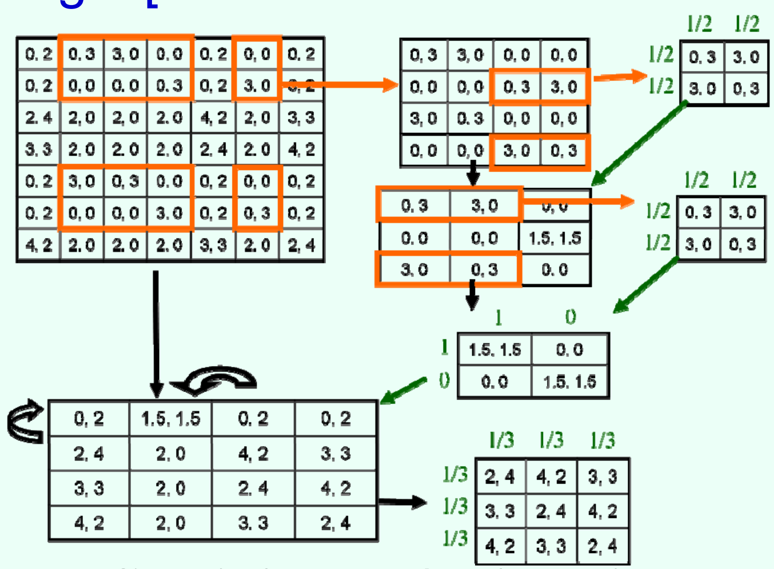

Special cases / subroutines

[C. & Sandholm AAAI'05, AAMAS'06; Benisch, Davis, Sandholm AAAI'06 / JAIR'10; Kontogiannis & Spirakis APPROX'11; Adsul, Garg, Mehta, Sohoni STOC'11; …]

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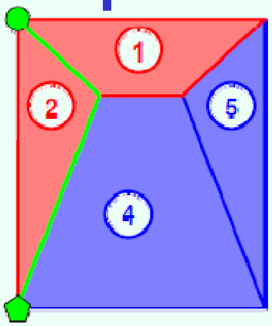

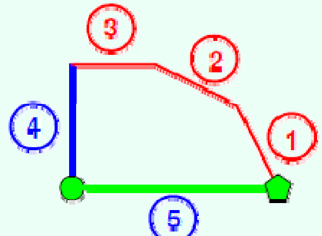

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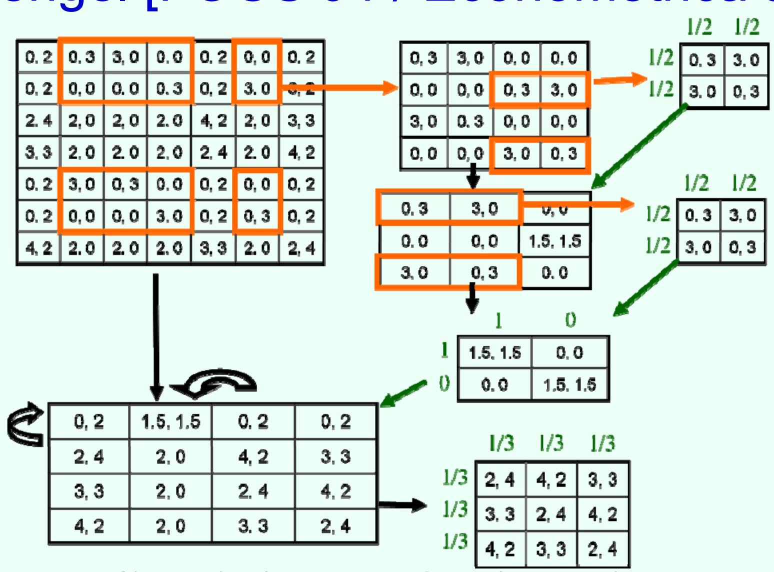

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[Porter, Nudelman, Shoham AAAI'04 / GEB'08]
[Sandholm, Gilpin, C. AAAI'05]

	0, 1	0, 1	½, ½	½, ½	½, ½
	1, 0	1, 0	0, 1	0, 1	0, 1
	1, 0	1, 0	0, 1	0, 1	0, 1
	½, ½	½, ½	1, 0	1, 0	1, 0
	½, ½	½, ½	1, 0	1, 0	1, 0

Approximate equilibria

[Brown '51 / C. '09 / Goldberg, Savani, Sørensen, Ventre '11; Althöfer '94, Lipton, Markakis, Mehta '03, Daskalakis, Mehta, Papadimitriou '06, '07, Feder, Nazerzadeh, Saberi '07, Tsaknakis & Spirakis '07, Spirakis '08, Bosse, Byrka, Markakis '07, …]

Sidestepping the problems

Sidestepping the problems

(one *solution concept* is not enough…?)

Nash is not optimal if one player can commit

1, 1	3, 0
0, 0	2, 1

von Stackelberg

Nash is not optimal if one player can commit

1, 1	3, 0
0, 0	2, 1

Unique Nash equilibrium

von Stackelberg

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- Suppose the game is played as follows:

Nash is not optimal if one player can commit

Unique Nash equilibrium

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- Suppose the game is played as follows:
  - Player 1 commits to playing one of the rows,

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- Suppose the game is played as follows:
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1, 1	3, 0
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Unique Nash equilibrium

von Stackelberg

- Suppose the game is played as follows:
  - Player 1 commits to playing one of the rows,
  - Player 2 observes the commitment and then chooses a column
- Optimal strategy for player 1: commit to Down

Commitment to mixed strategies

1, 1	3, 0
0, 0	2, 1

Commitment to mixed strategies

.49	1, 1	3, 0
.51	0, 0	2, 1

Commitment to mixed strategies

	0	1
.49	1, 1	3, 0
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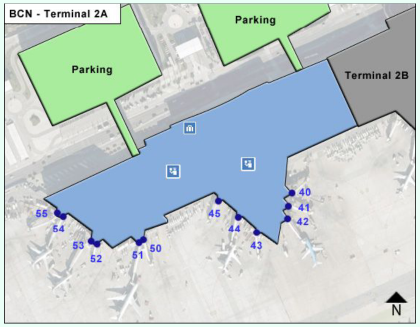
Commitment to mixed strategies

	0	1
.49	1, 1	3, 0
.51	0, 0	2, 1

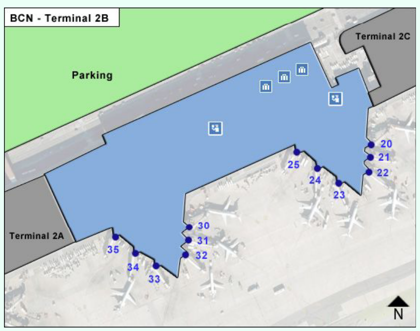
– Sometimes also called a Stackelberg (mixed) strategy

Observing the defender's distribution in security

BCN terminal 2A



BCN terminal 2B



observe

Mo Tu We Th Fr Sa

Observing the defender's distribution in security

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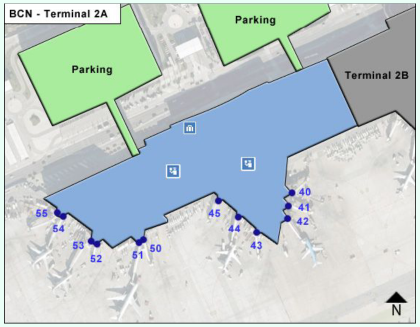

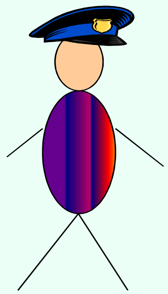

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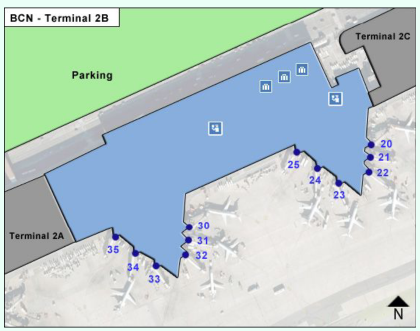

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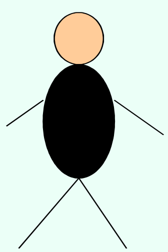

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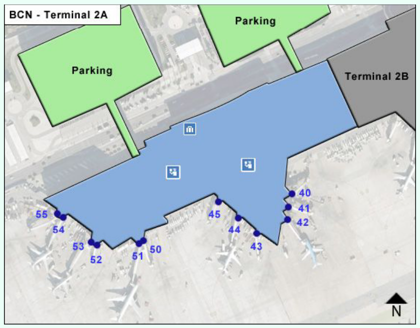

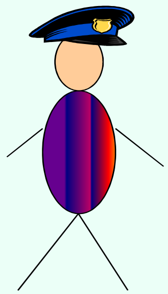

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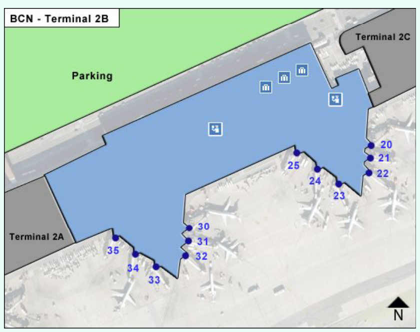

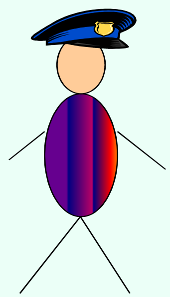

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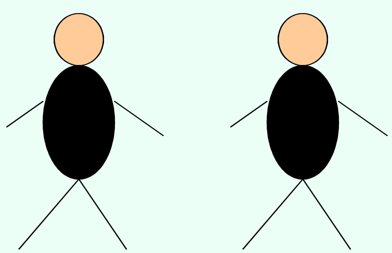

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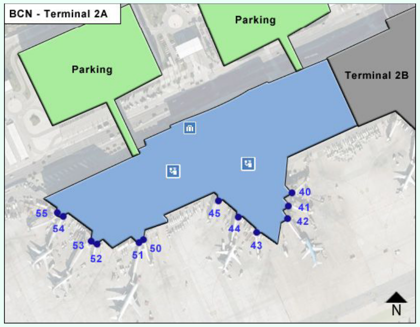
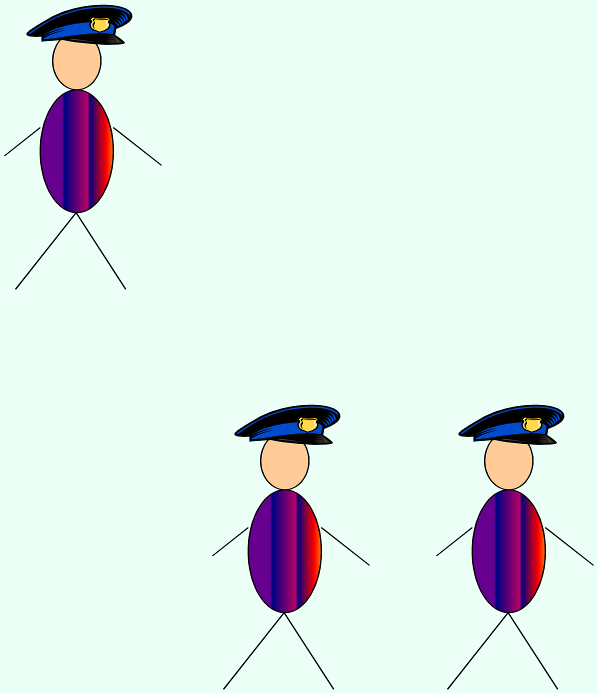
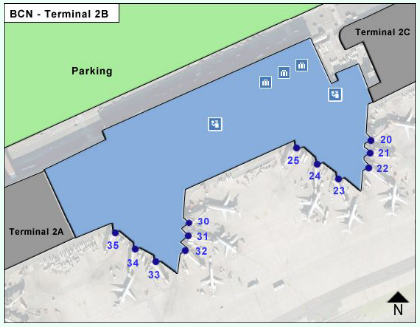
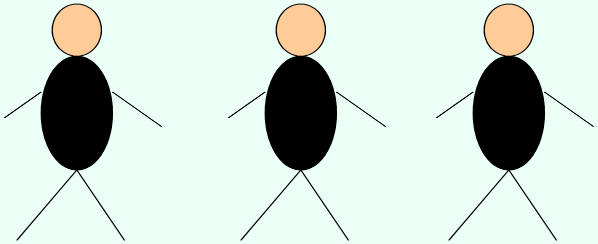




BCN terminal 2B



observe



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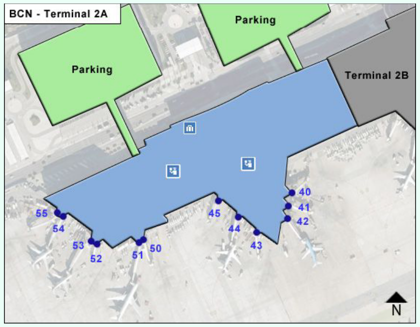
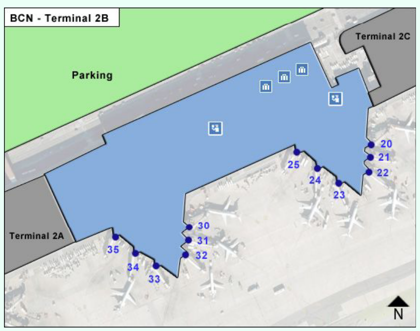
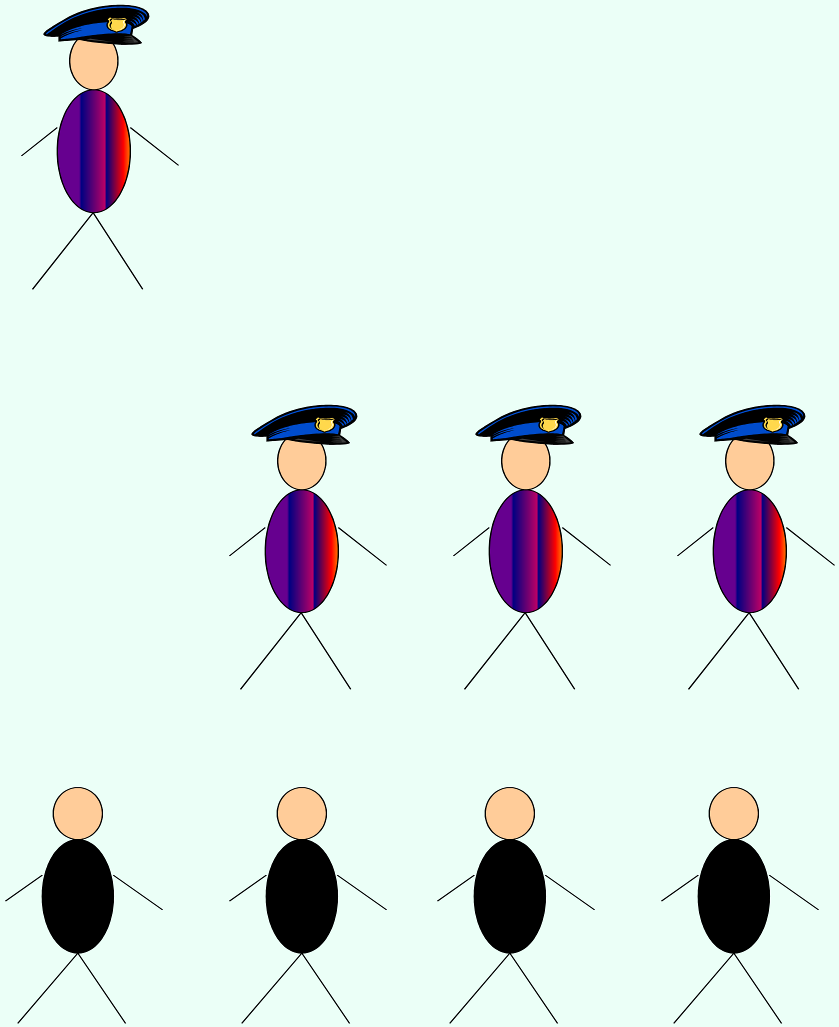


BCN terminal 2B





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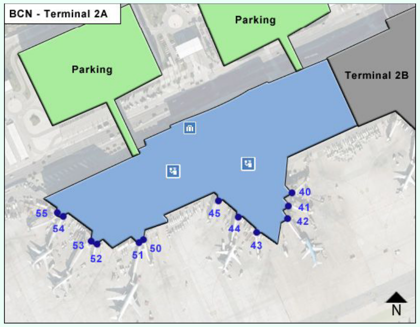

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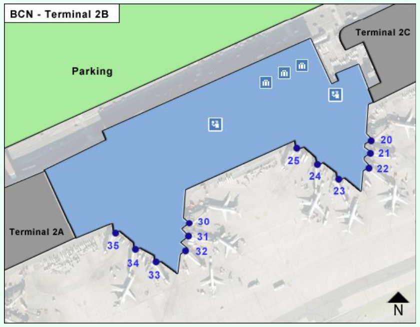

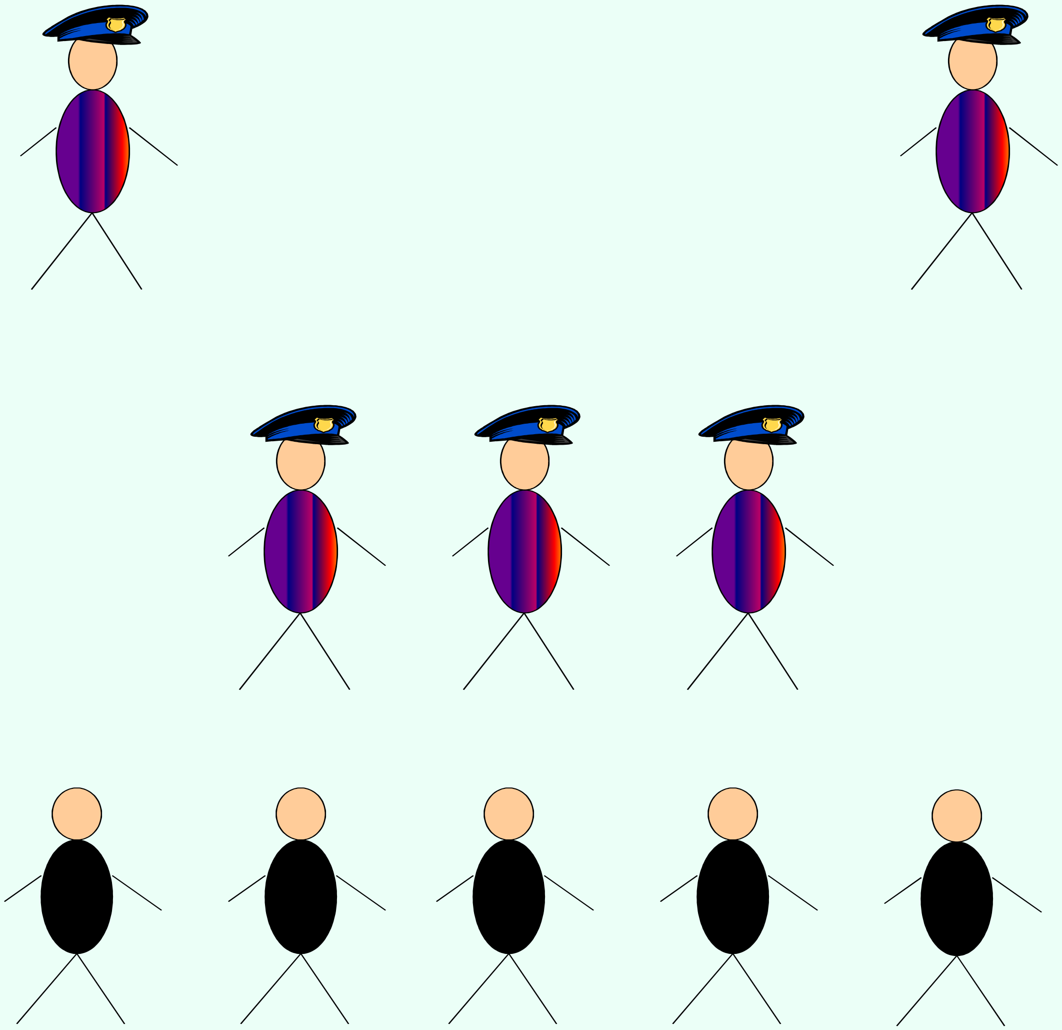

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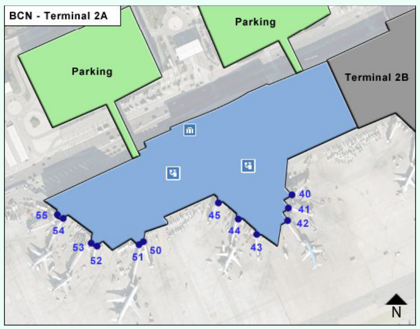
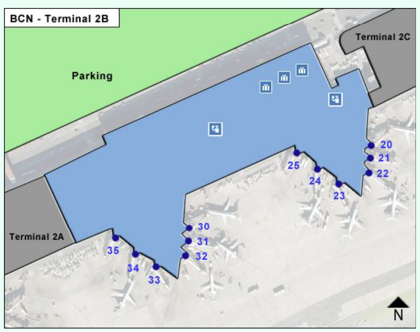
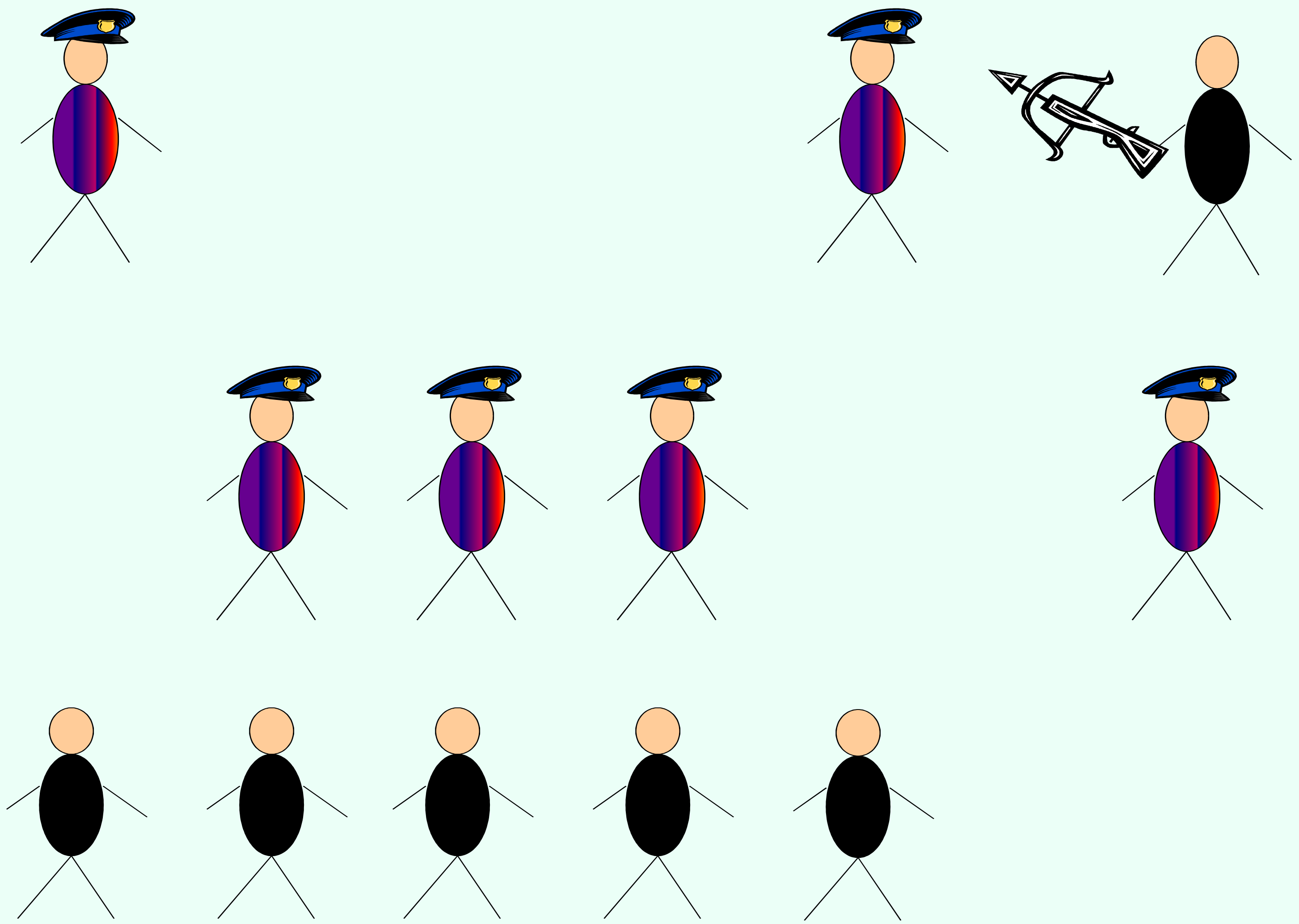


BCN terminal 2B





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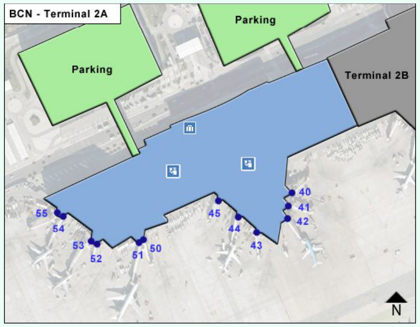

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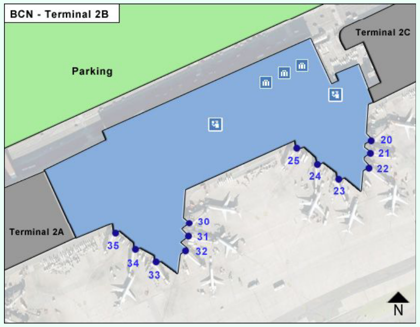

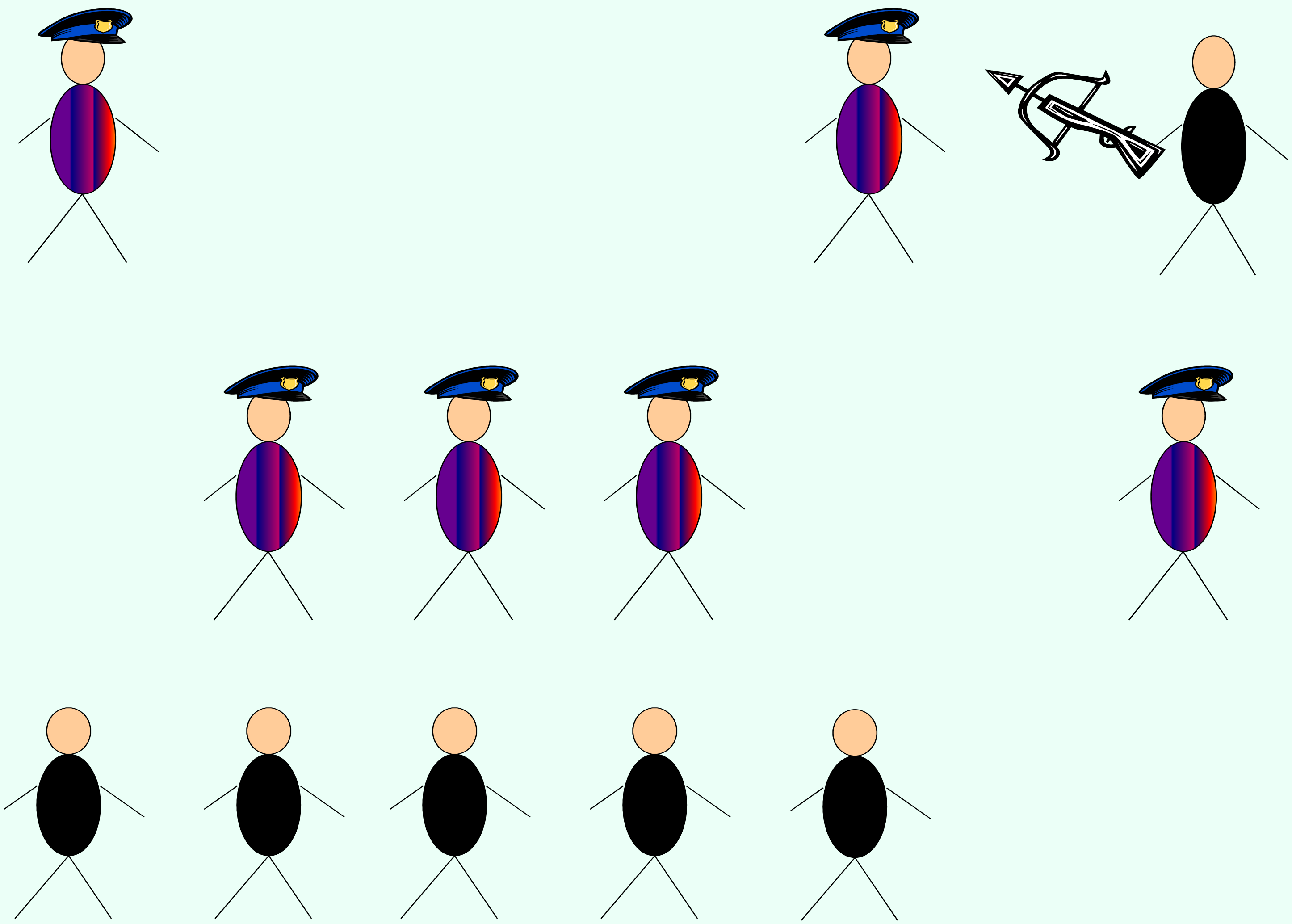

observe

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This argument is not uncontroversial… [Pita, Jain, Tambe, Ordóñez, Kraus AIJ'10; Korzhyk, Yin, Kiekintveld, C., Tambe JAIR'11; Korzhyk, C., Parr AAMAS'11]

Computing the optimal mixed strategy to commit to

[C. & Sandholm EC'06, von Stengel & Zamir GEB'10]

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- Separate LP for every column c^*:

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- Separate LP for every column c^*:

maximize $\Sigma_r\, p_r\, u_R(r, c^*)$

subject to

for all c, $\Sigma_r\, p_r\, u_C(r, c) \leq \Sigma_r\, p_r\, u_C(r, c^*)$

$\Sigma_r\, p_r = 1$

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subject to

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Other nice properties of commitment to mixed strategies

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- Agrees w. Nash in zero-sum games

0, 0	-1, 1
-1, 1	0, 0

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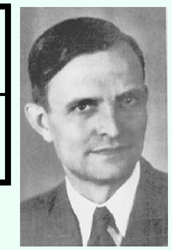

- Leader's payoff at least as good as any Nash eq. or even correlated eq. (von Stengel & Zamir [GEB '10]; see also C. & Korzhyk [AAAI '11], Letchford & C. [draft])

≥

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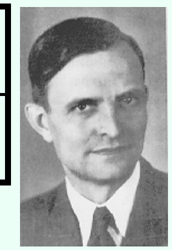

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$\geq$

- No equilibrium selection problem

0, 0	-1, 1
1, -1	-5, -5

Some other work on commitment in unrestricted games

2, 2	-1, 0
-7, -8	0, 0

normal-form games

learning to commit [Letchford, C., Munagala SAGT'09]

uncertain observability [Korzhyk, C., Parr AAMAS'11]

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row player type 1 (prob. 0.5)

	L	R
U	4	6
D	2	4

column player type 1 (prob. 0.5)

	L	R
U	4	6
D	4	6

row player type 2 (prob. 0.5)

	L	R
U	2	4
D	4	2

column player type 2 (prob. 0.5)

	L	R
U	2	2
D	4	2

commitment in Bayesian games

[C. & Sandholm EC'06; Paruchuri, Pearce, Marecki, Tambe, Ordóñez, Kraus AAMAS'08; Letchford, C., Munagala SAGT'09; Pita, Jain, Tambe, Ordóñez, Kraus AIJ'10; Jain, Kiekintveld, Tambe AAMAS'11]

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	L	R
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D	4	2

column player type 2 (prob. 0.5)

	L	R
U	2	2
D	4	2

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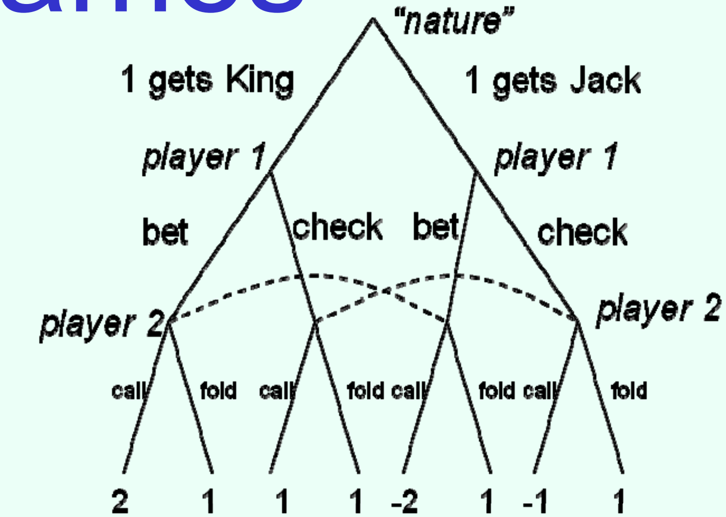

extensive-form games

[Letchford & C., EC'10]

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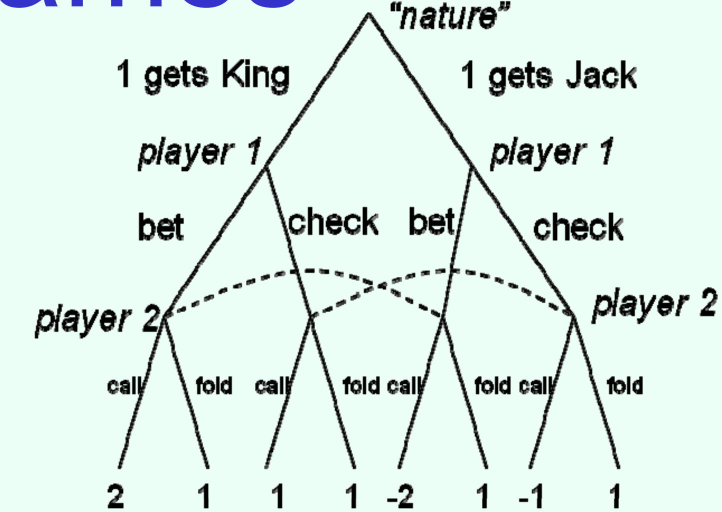

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U	4	6
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column player type 1 (prob. 0.5)

	L	R
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D	4	6

row player type 2 (prob. 0.5)

	L	R
U	2	4
D	4	2

column player type 2 (prob. 0.5)

	L	R
U	2	2
D	4	2

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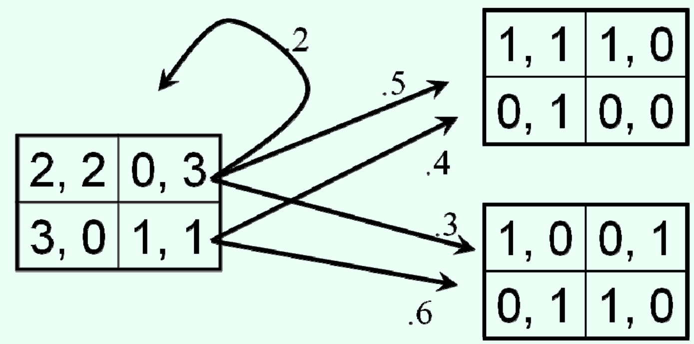

stochastic games

ongoing work with Korzhyk, Letchford, Parr

Security resource allocation games

[Kiekintveld, Jain, Tsai, Pita, Ordóñez, Tambe AAMAS'09]

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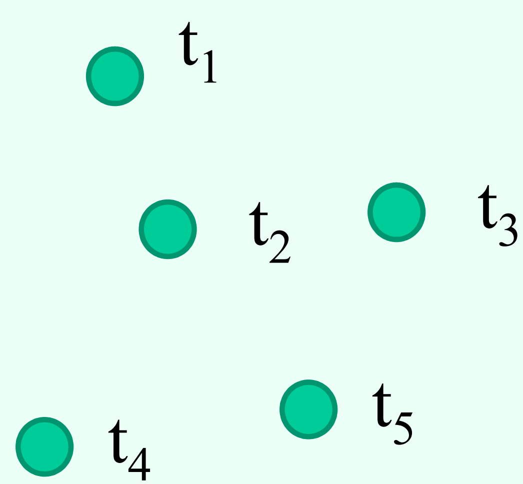
- Set of targets *T*



Security resource allocation games

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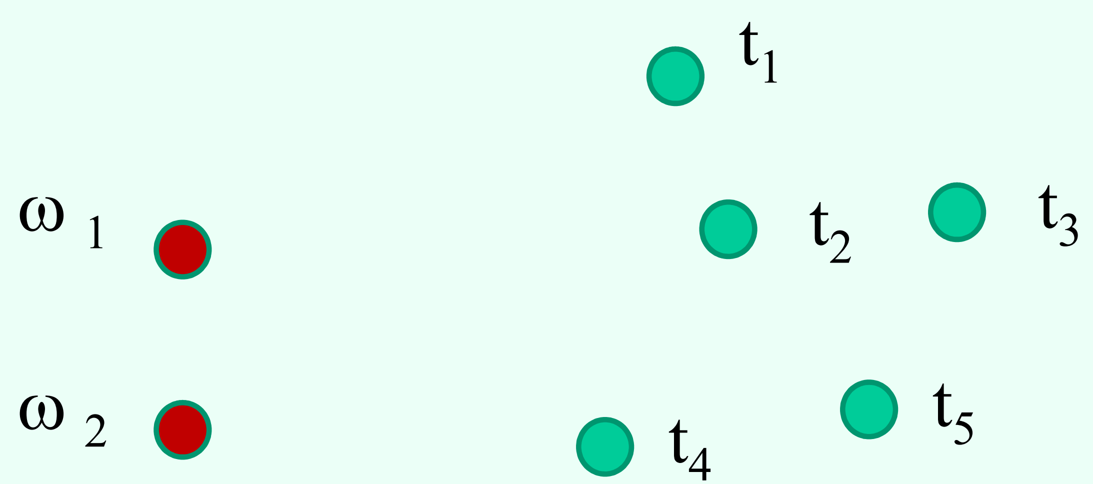

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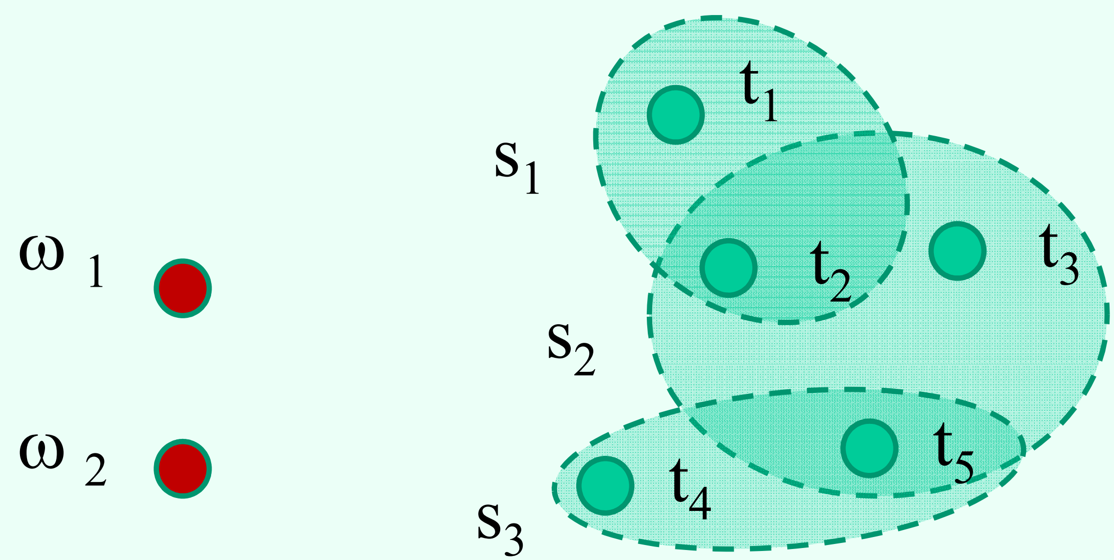

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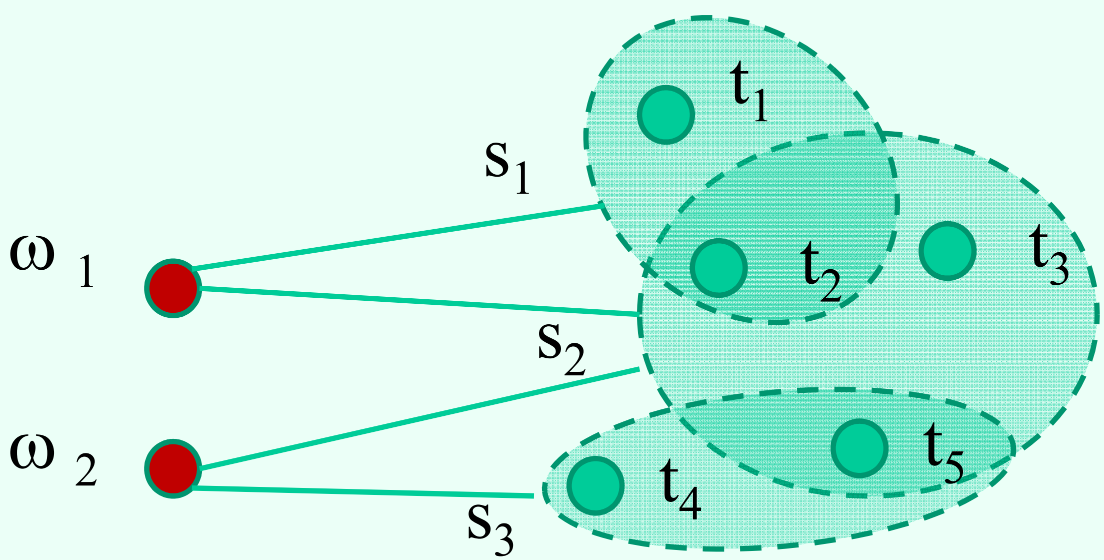

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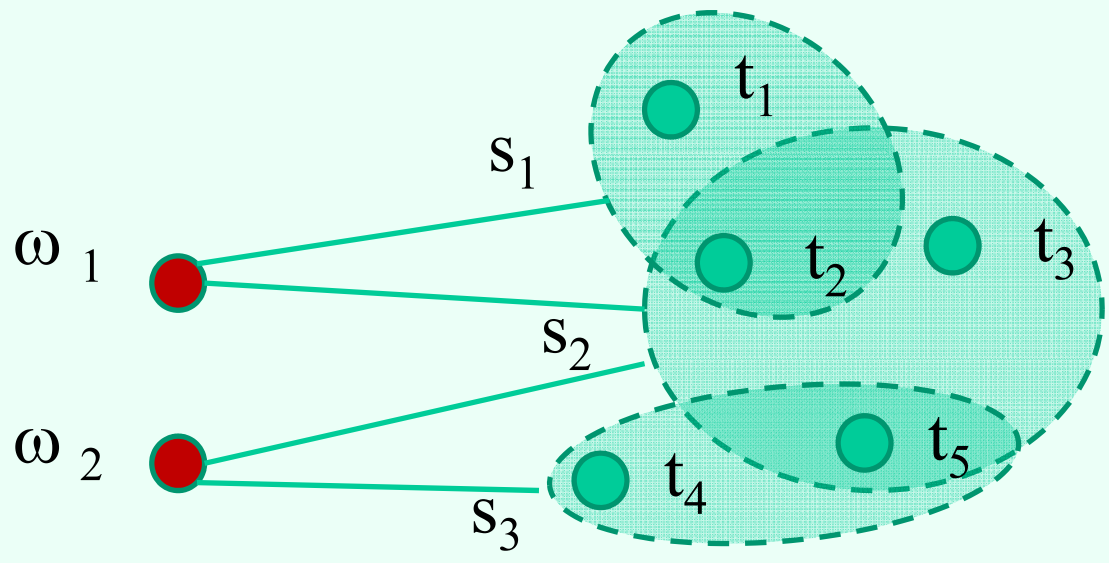

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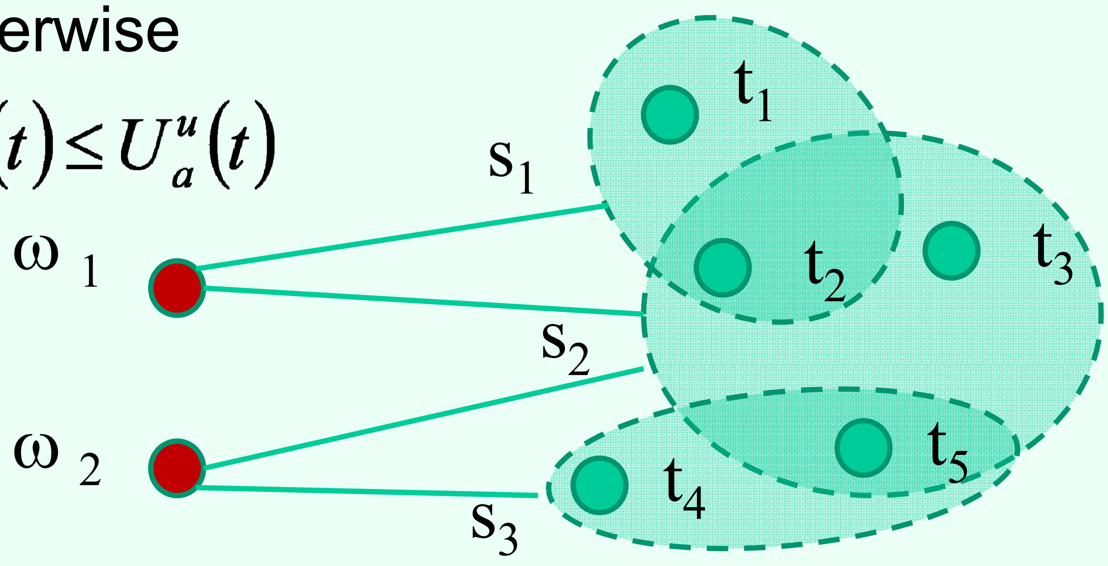
- Attacker (follower) chooses one target to attack
- Utilities: $U_d^c(t), U_a^c(t)$ if the attacked target is defended, $U_d^u(t), U_a^u(t)$ otherwise
- $U_d^c(t) \geq U_d^u(t); U_a^c(t) \leq U_a^u(t)$



Game-theoretic properties of security resource allocation games [Korzhyk, Yin, Kiekintveld, C., Tambe JAIR'11]

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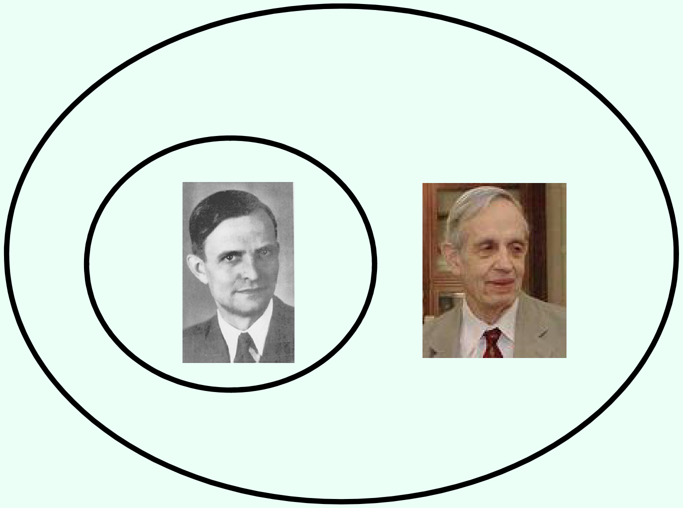

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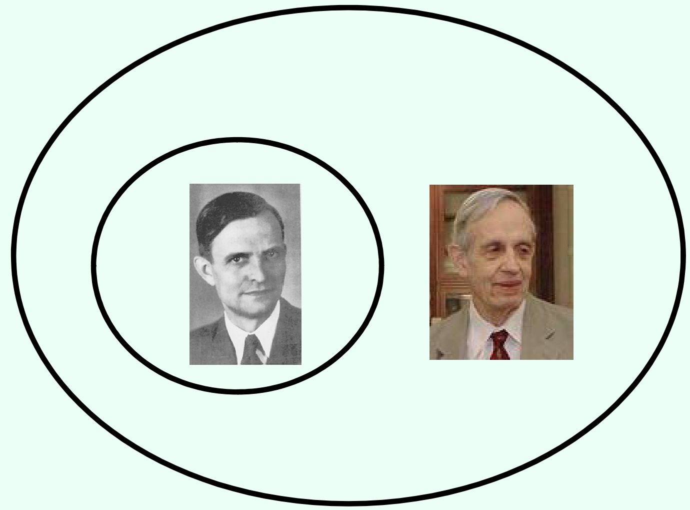
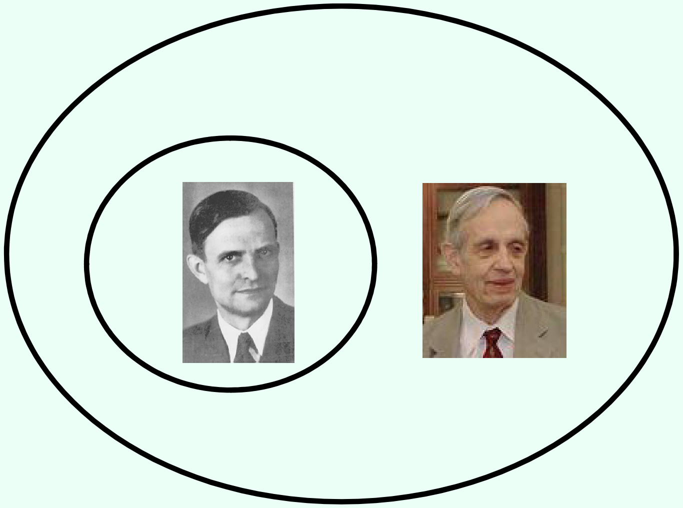


- Interchangeability property for Nash equilibria ("solvable")
  - no equilibrium selection problem
  - still true with multiple attacks [Korzhyk, C., Parr IJCAI'11 – poster W. 3:30pm, talk F. 10:30am]

1, 2	1, 0	2, 2
1, 1	1, 0	2, 1
0, 1	0, 0	0, 1

Game-theoretic properties of security resource allocation games [Korzhyk, Yin, Kiekintveld, C., Tambe JAIR’11]

- For the defender: Stackelberg strategies are also Nash strategies
  - minor assumption needed
  - not true with multiple attacks



- Interchangeability property for Nash equilibria (“solvable”)
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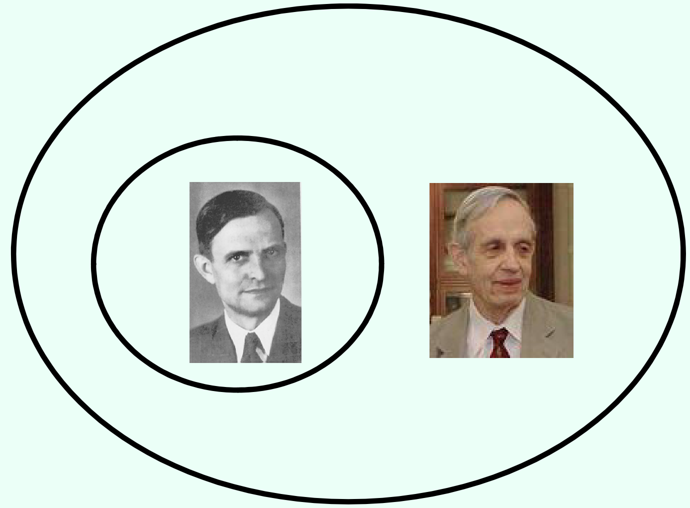

			
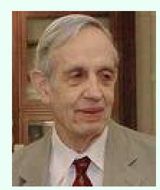	1, 2	1, 0	2, 2
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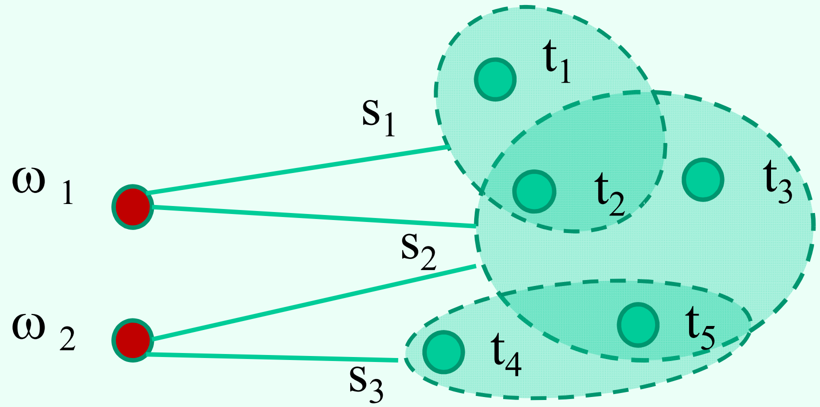
Scalability in security games



basic model

[Kiekintveld, Jain, Tsai, Pita, Ordóñez, Tambe AAMAS'09; Korzhyk, C., Parr, AAAI'10; Jain, Kardeş, Kiekintveld, Ordóñez, Tambe AAAI'10; Korzhyk, C., Parr, IJCAI'11]

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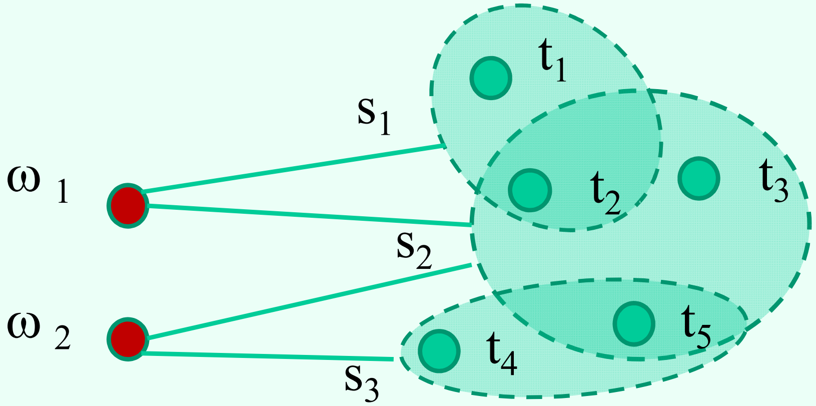

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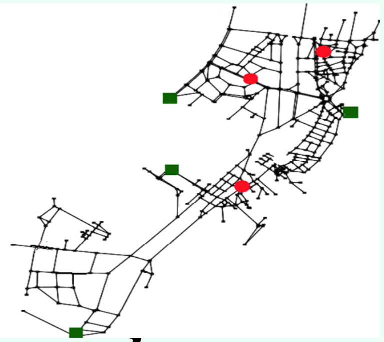

games on graphs (usually zero-sum)

[Halvorson, C., Parr IJCAI'09; Tsai, Yin, Kwak, Kempe, Kiekintveld, Tambe AAAI'10; Jain, Korzhyk, Vaněk, C., Pěchouček, Tambe AAMAS'11]; ongoing work with Letchford, Vorobeychik

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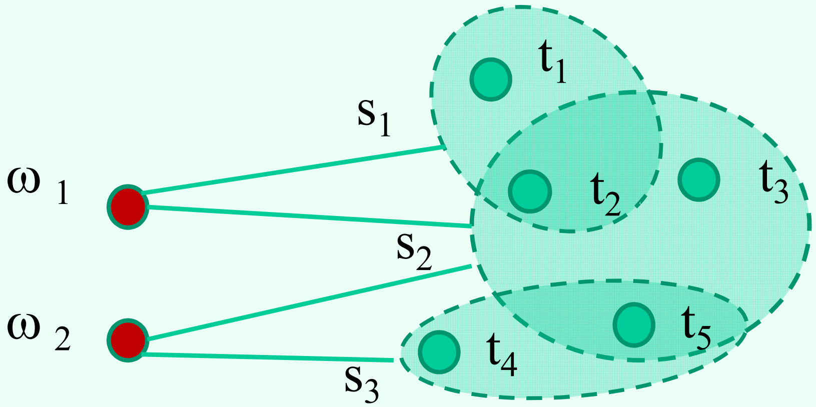

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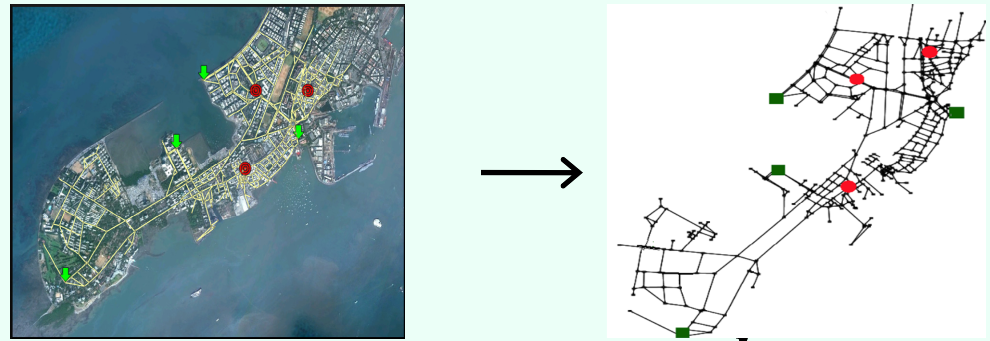

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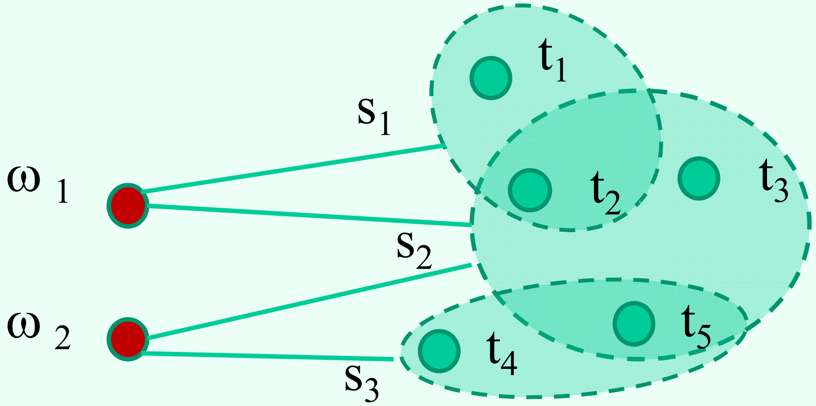
Techniques:

Scalability in security games



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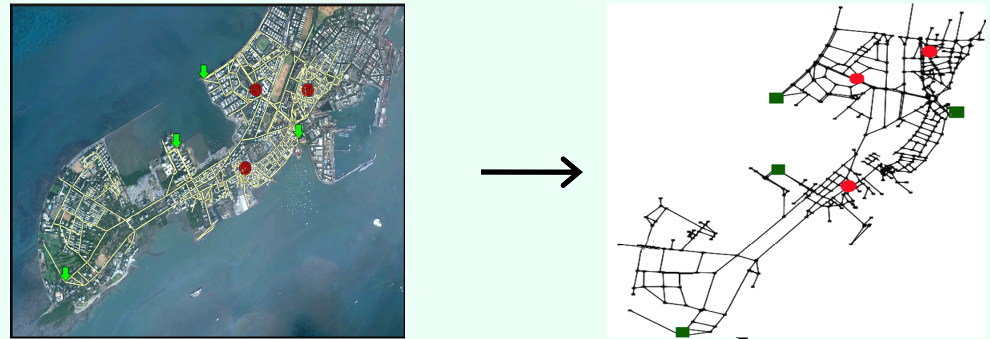

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Techniques:

compact linear/integer programs

Maximize $U_d^c(t^*)\sum_{\omega}\sum_{s:t^*\in s} c_{\omega,s} + U_d^u(t^*)\left(1-\sum_{\omega}\sum_{s:t^*\in s} c_{\omega,s}\right)$ — Defender utility

Marginal probability of t^* being defended (?)

Subject to

$\forall \omega: \sum_{s} c_{\omega,s} \le 1$

$\forall t: \sum_{\omega}\sum_{s:t\in s} c_{\omega,s} \le 1$

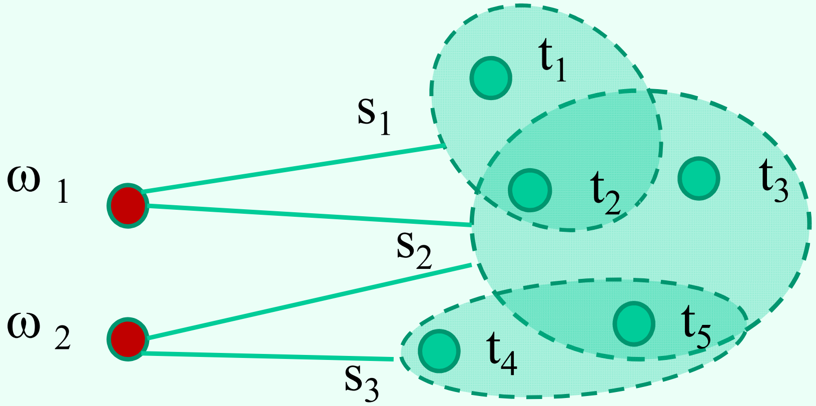
— Distributional constraints

$\forall t: U_a^c(t)\sum_{\omega}\sum_{s:t\in s} c_{\omega,s} + U_a^u(t)\left(1-\sum_{\omega}\sum_{s:t\in s} c_{\omega,s}\right)$

$\le U_a^c(t^*)\sum_{\omega}\sum_{s:t^*\in s} c_{\omega,s} + U_a^u(t^*)\left(1-\sum_{\omega}\sum_{s:t^*\in s} c_{\omega,s}\right)$

— Attacker optimality

Scalability in security games



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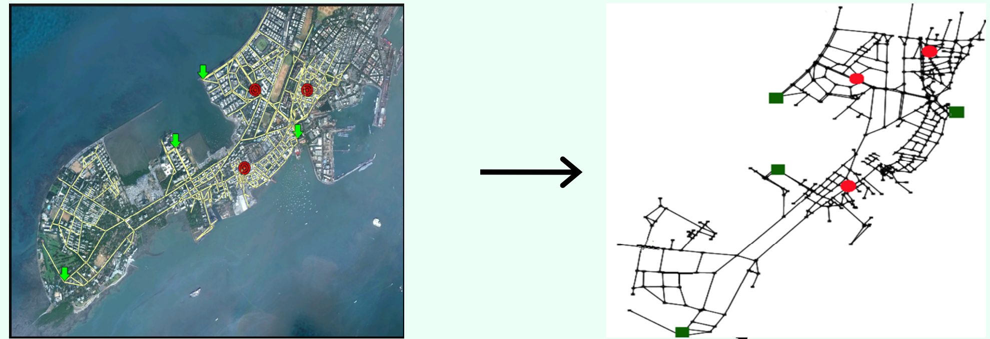

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$$\le U_a^c(t^*)\sum_{\omega}\sum_{s:t^*\in s} c_{\omega,s} + U_a^u(t^*)\left(1-\sum_{\omega}\sum_{s:t^*\in s} c_{\omega,s}\right)$$

Attacker optimality

strategy generation

min u

subject to

$$\sigma_h(s_{h_0}) + \dots \sigma_h(s_{h_k}) = 1$$

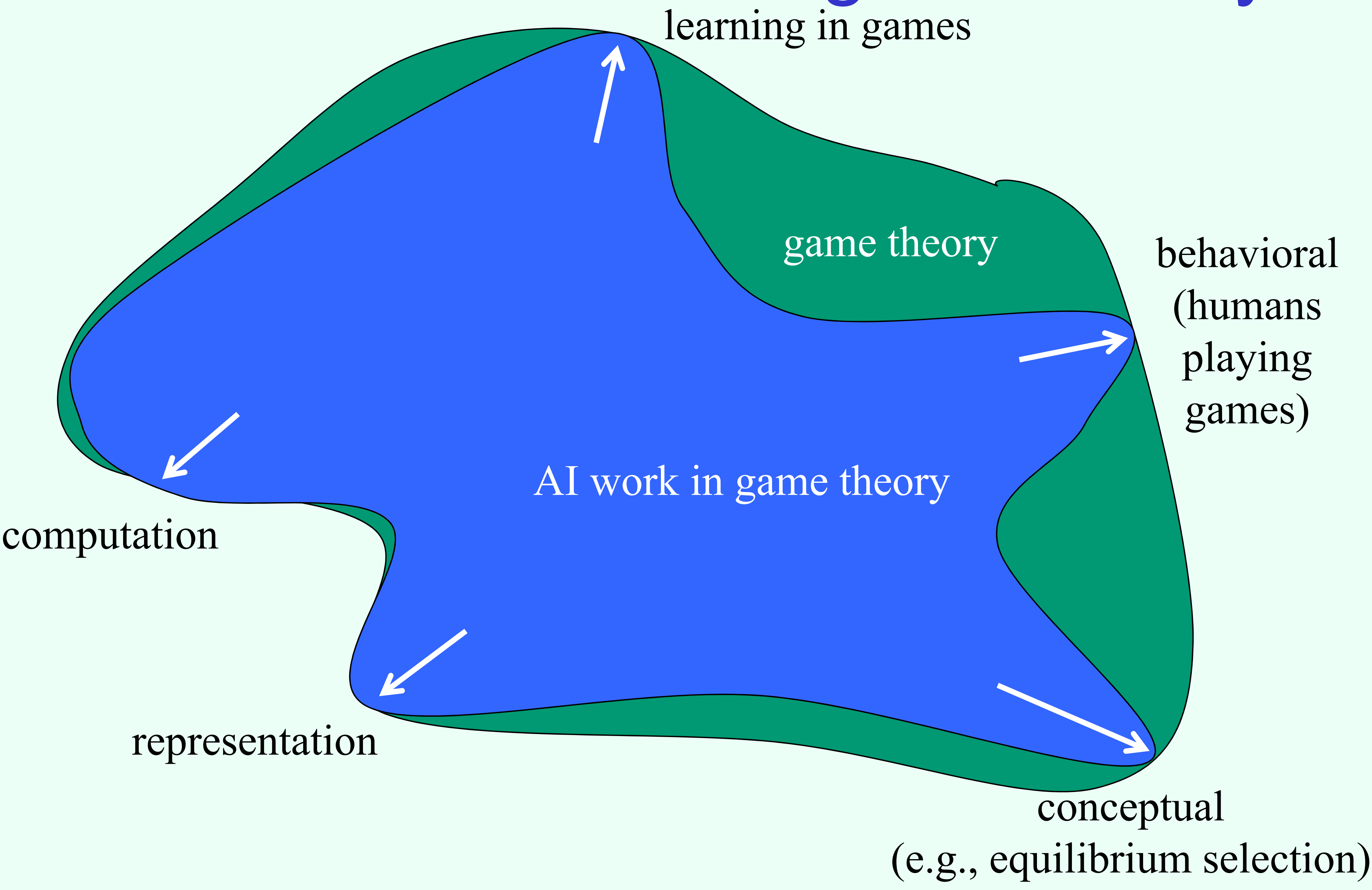
$$u \ge \sigma_h(s_{h_0})\cdot u(s_{s_0}, s_{h_0}) + \dots \sigma_h(s_{h_2})\cdot u(s_{s_0}, s_{h_2})$$

$$u \ge \sigma_h(s_{h_0})\cdot u(s_{s_1}, s_{h_0}) + \dots \sigma_h(s_{h_2})\cdot u(s_{s_1}, s_{h_2})$$

$$\vdots \qquad \ddots \qquad \vdots$$

$$u \ge \sigma_h(s_{h_0})\cdot u(s_{s_k}, s_{h_0}) + \dots \sigma_h(s_{h_k})\cdot u(s_{s_k}, s_{h_k})$$

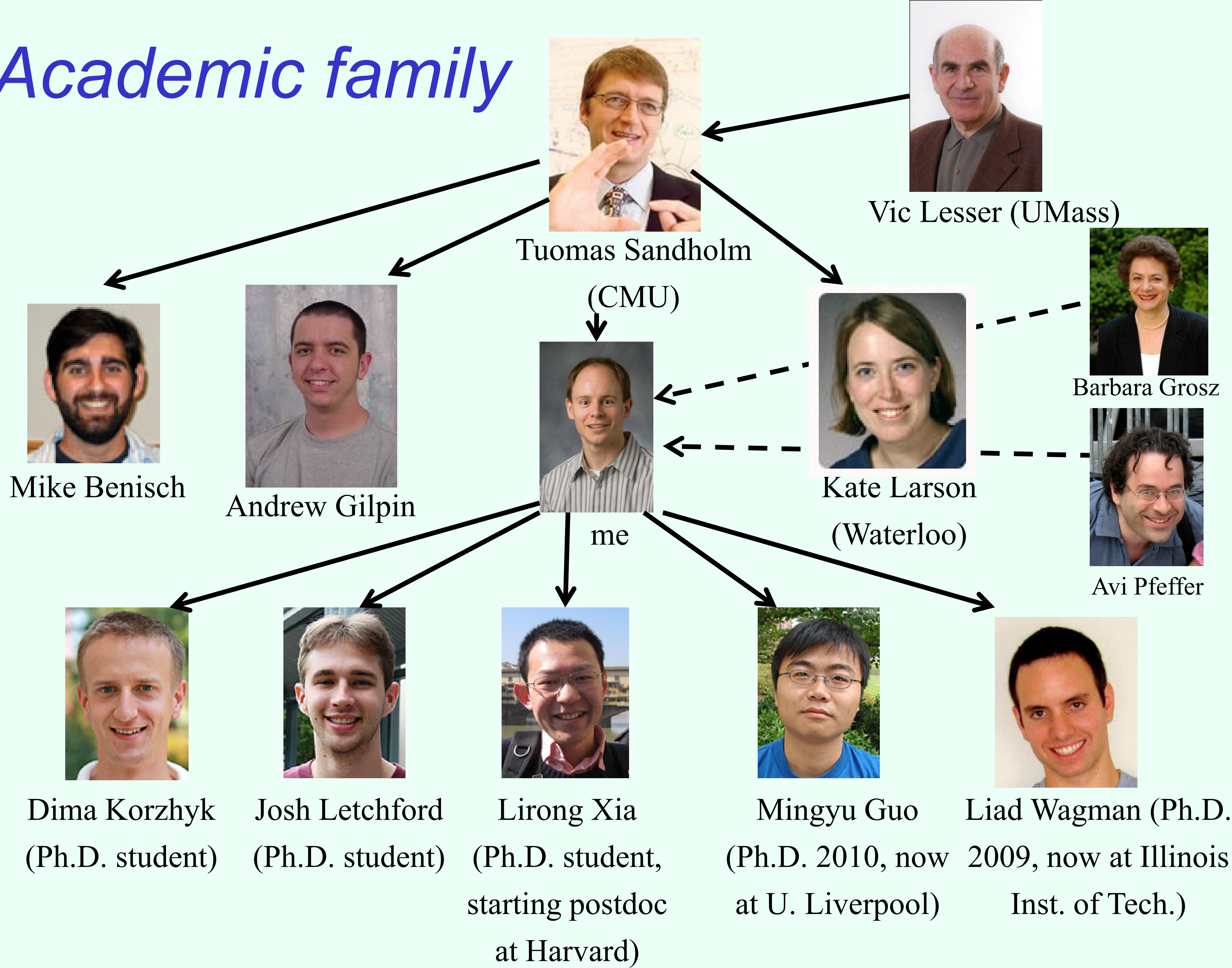
In summary: AI pushing at some of the boundaries of game theory



Funding

Any opinions, conclusions or recommendations are mine and do not necessarily reflect the views of the funding agencies

Academic family



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