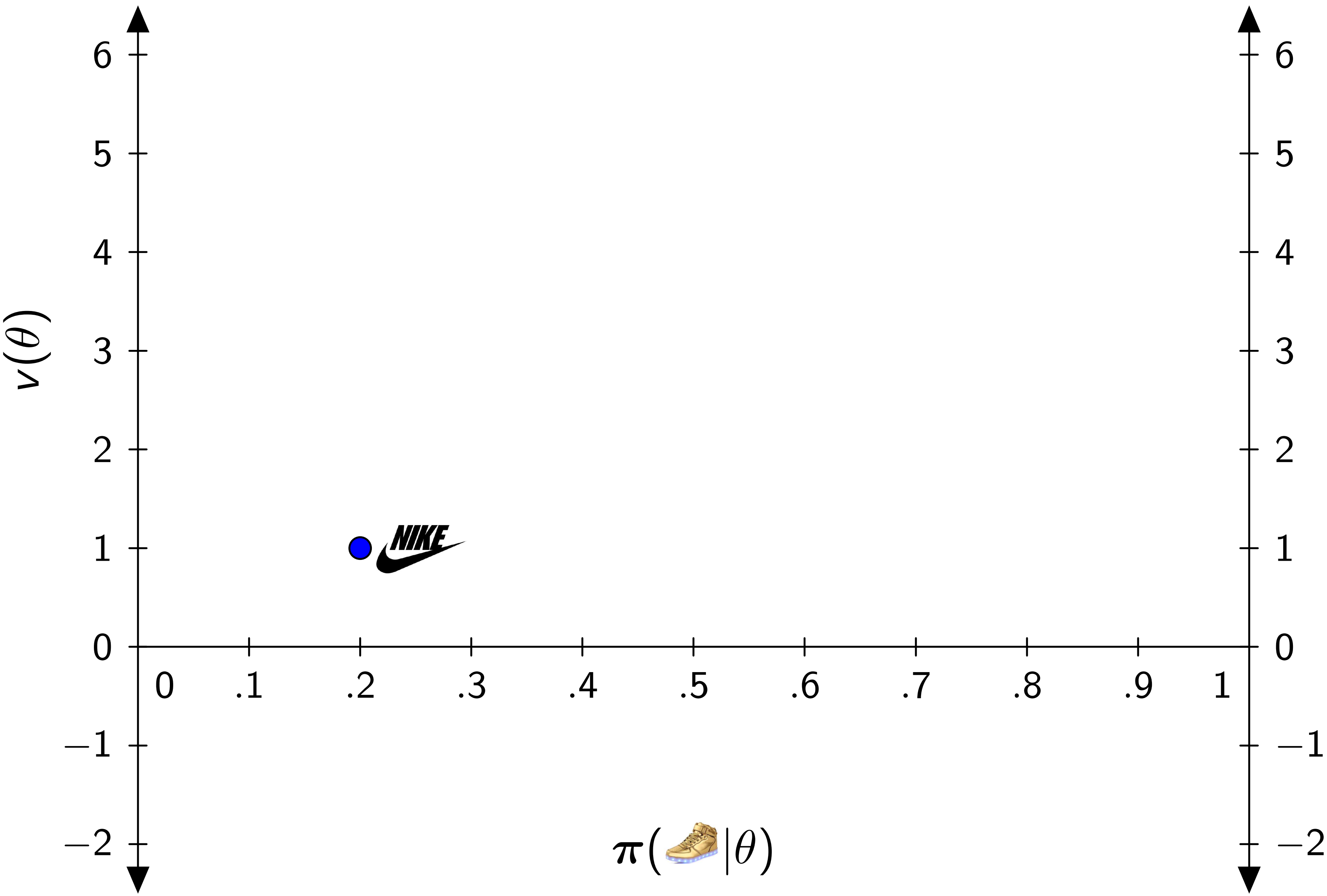
Mechanism Design with Correlated Distributions



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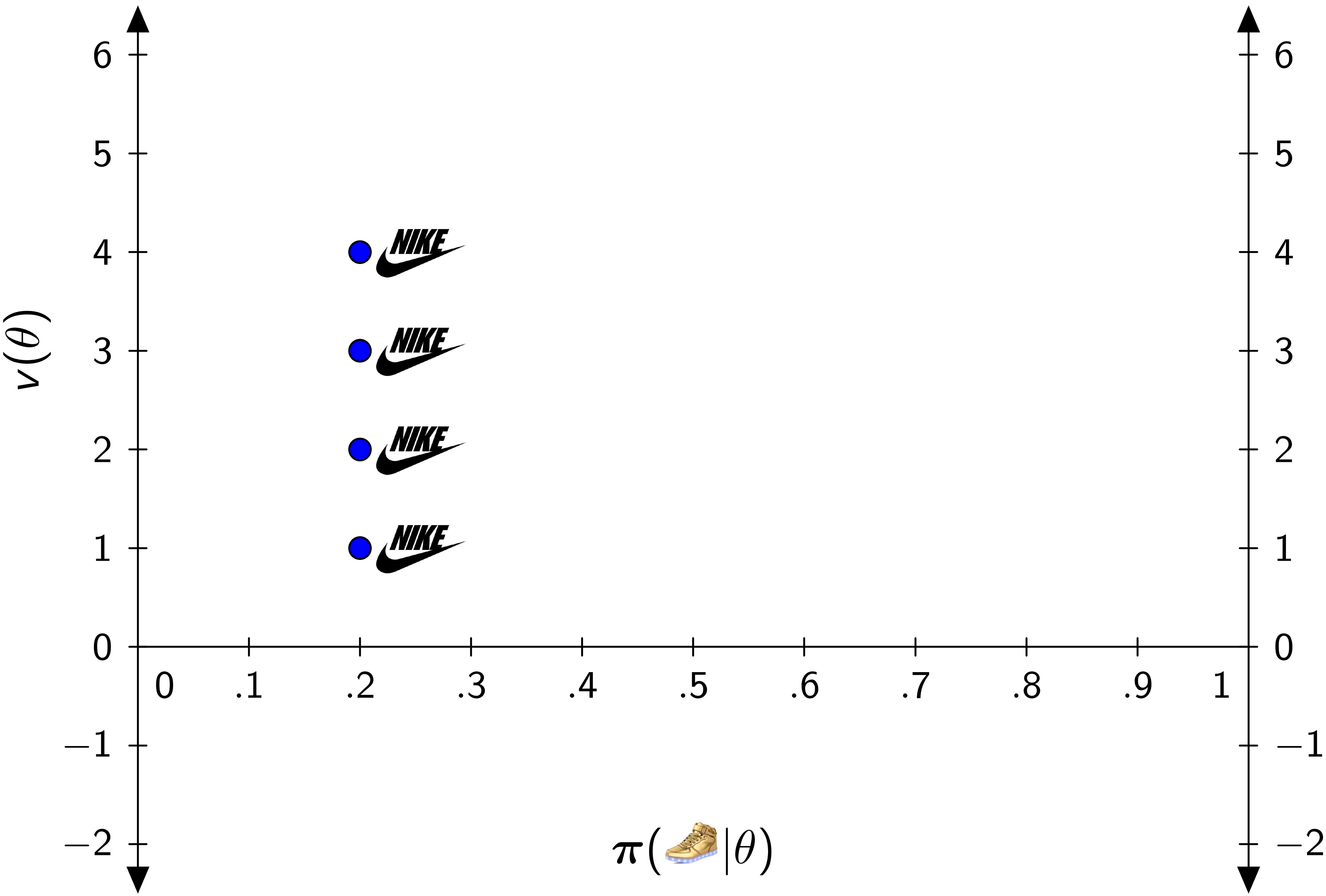

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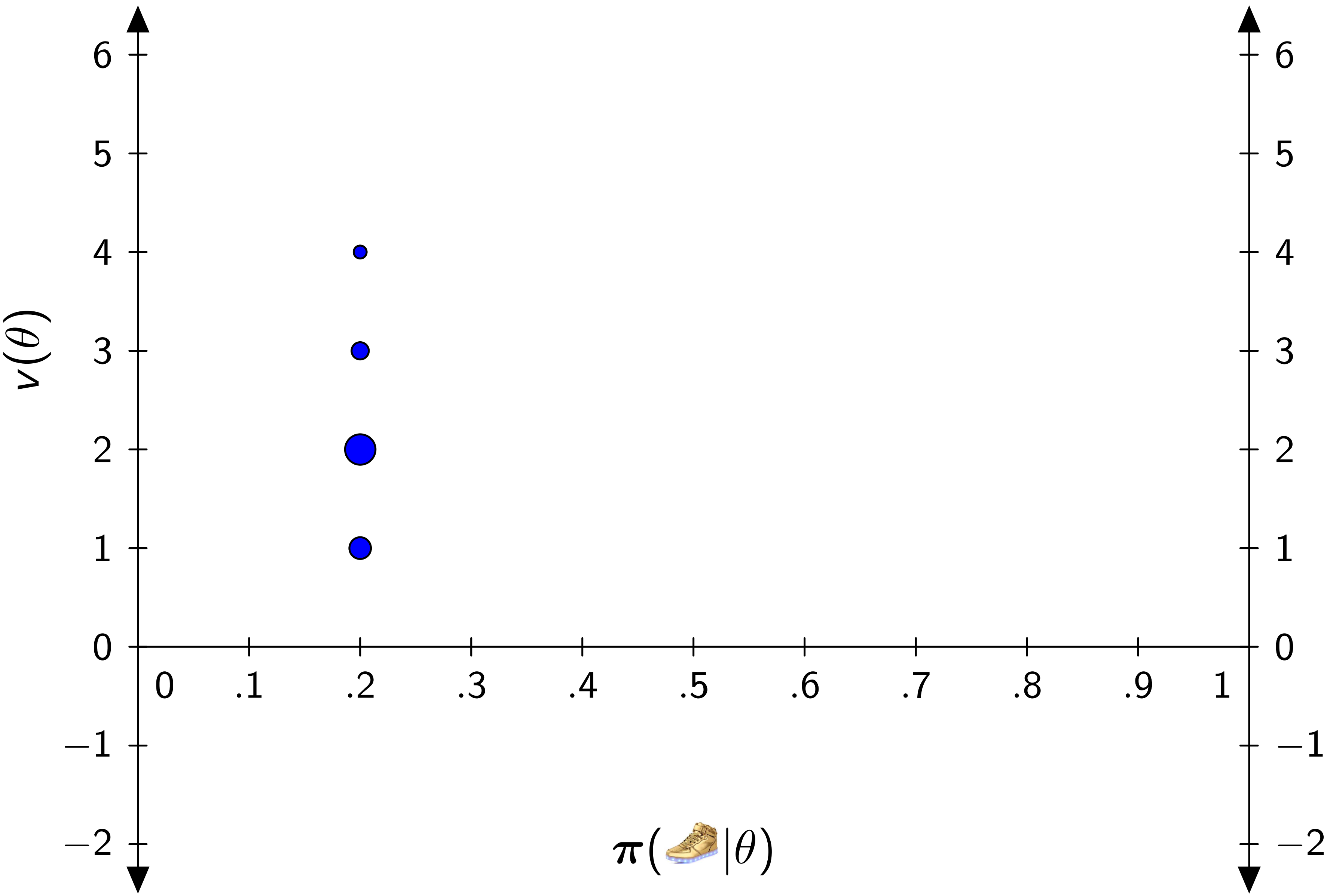

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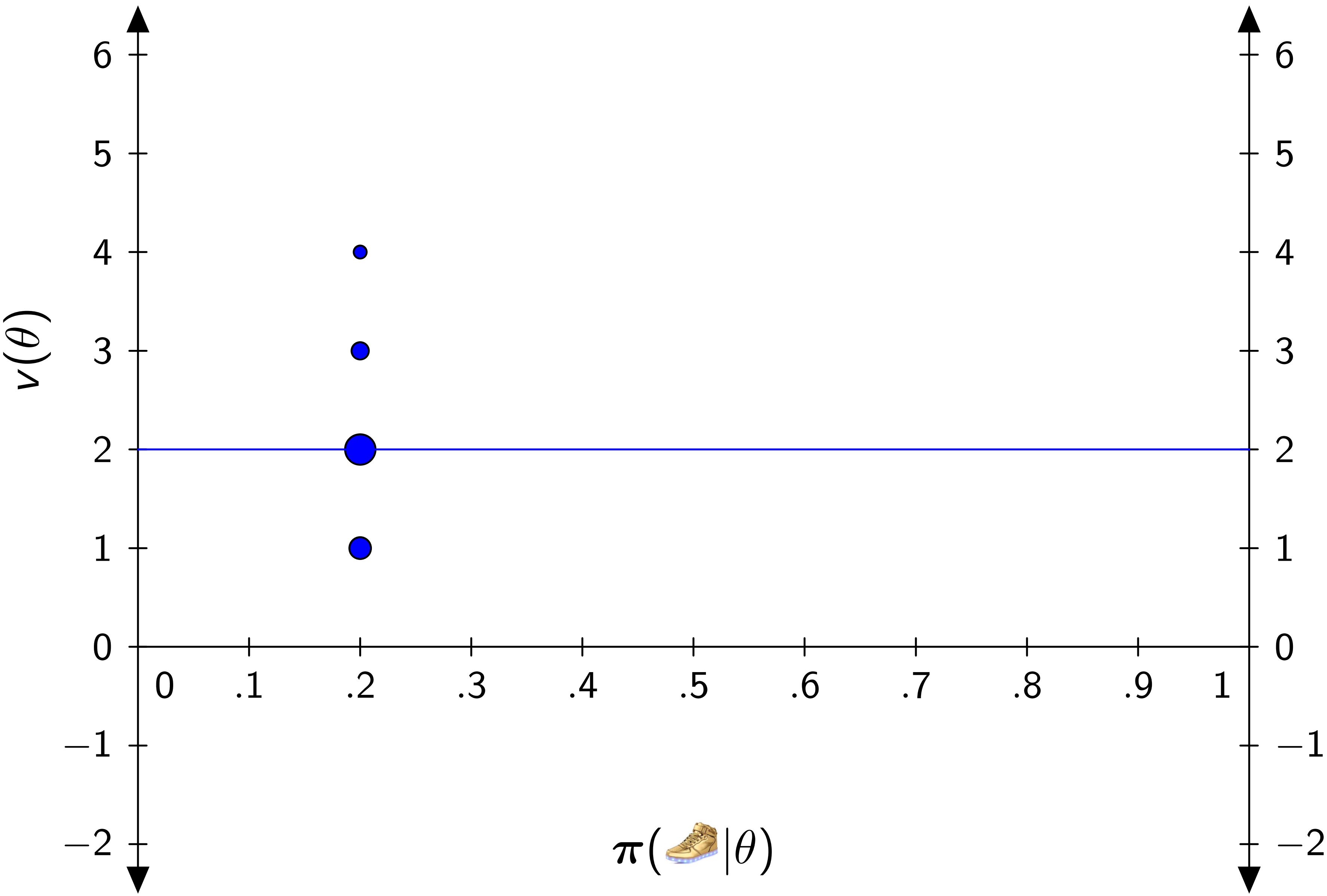

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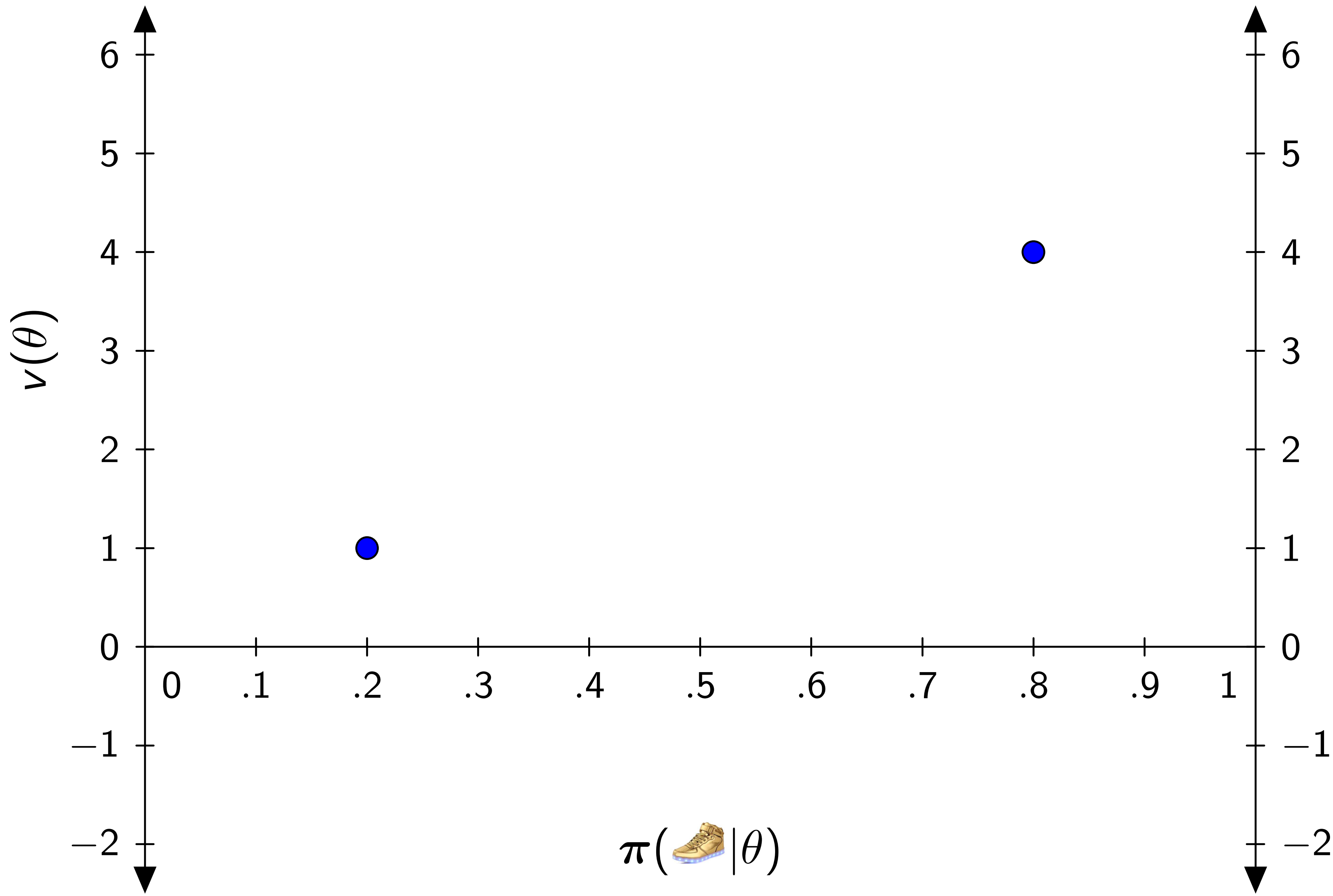

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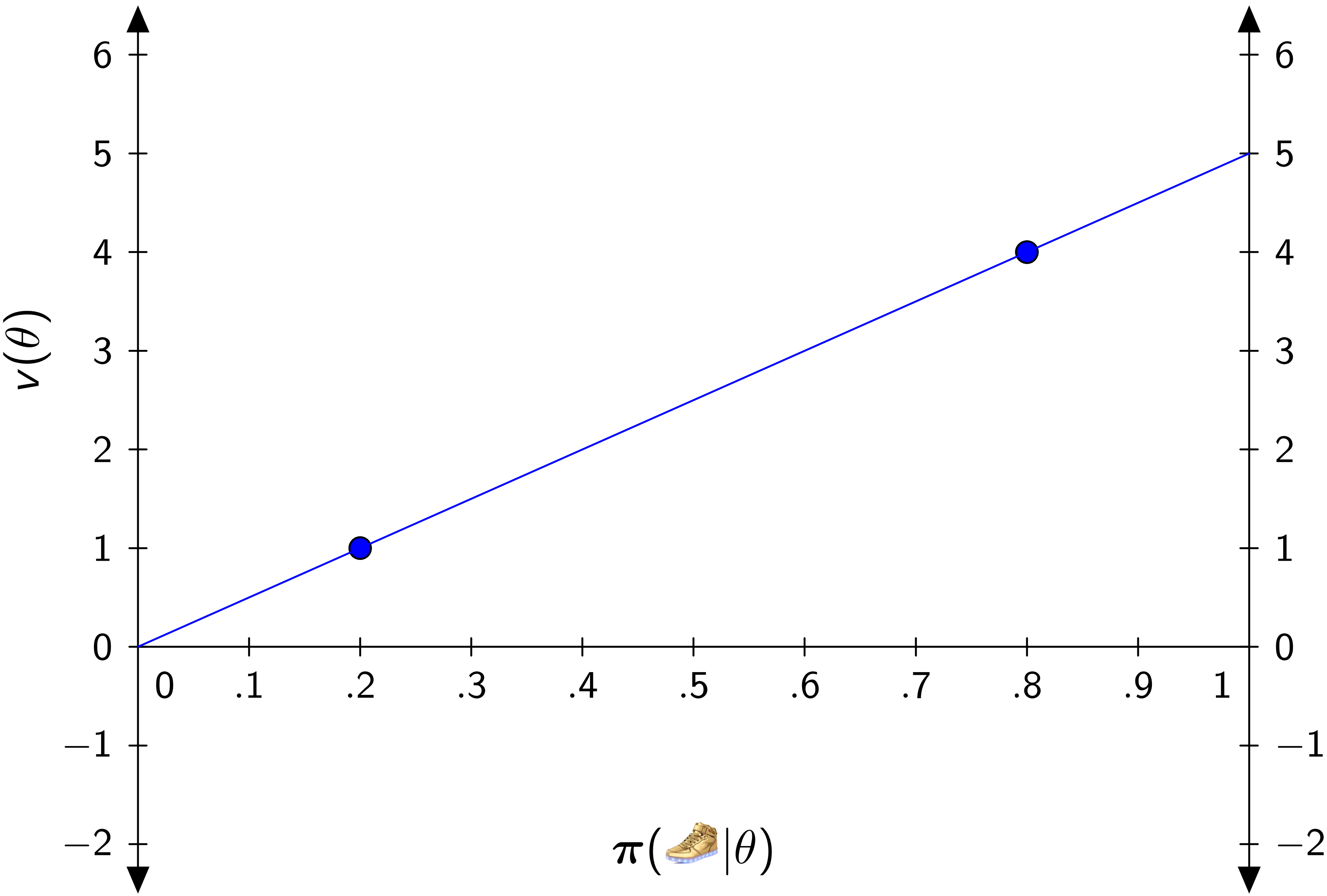

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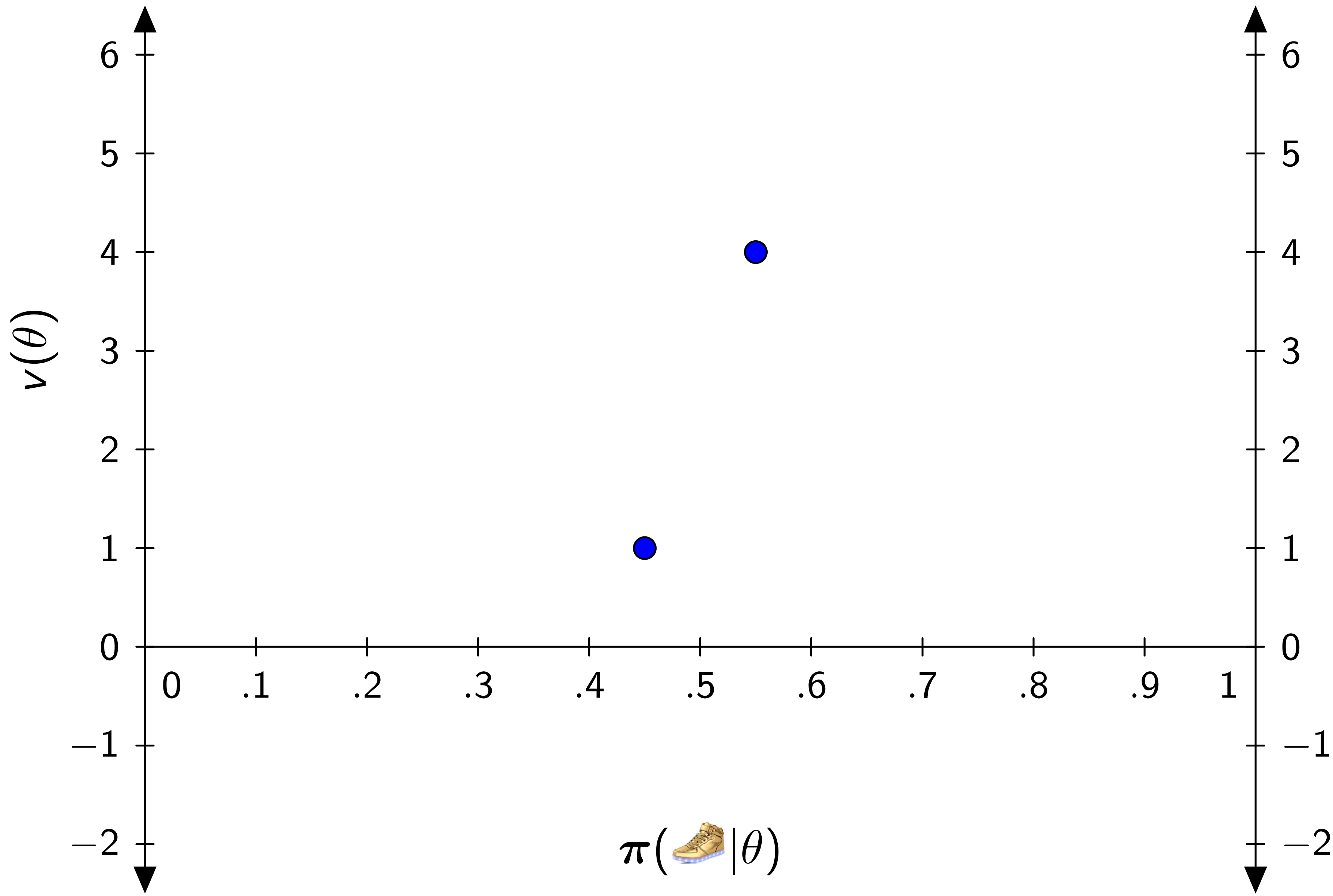

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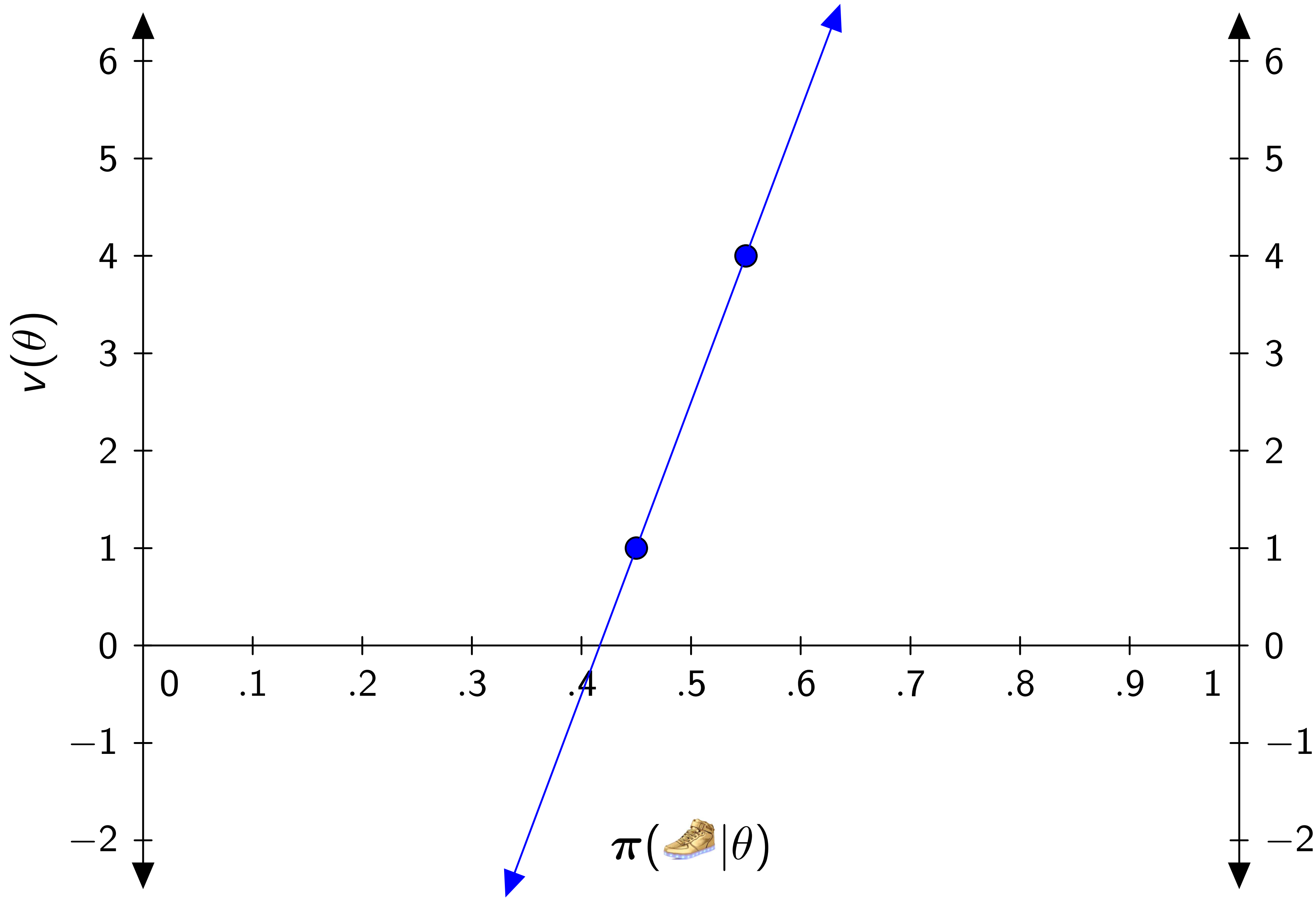
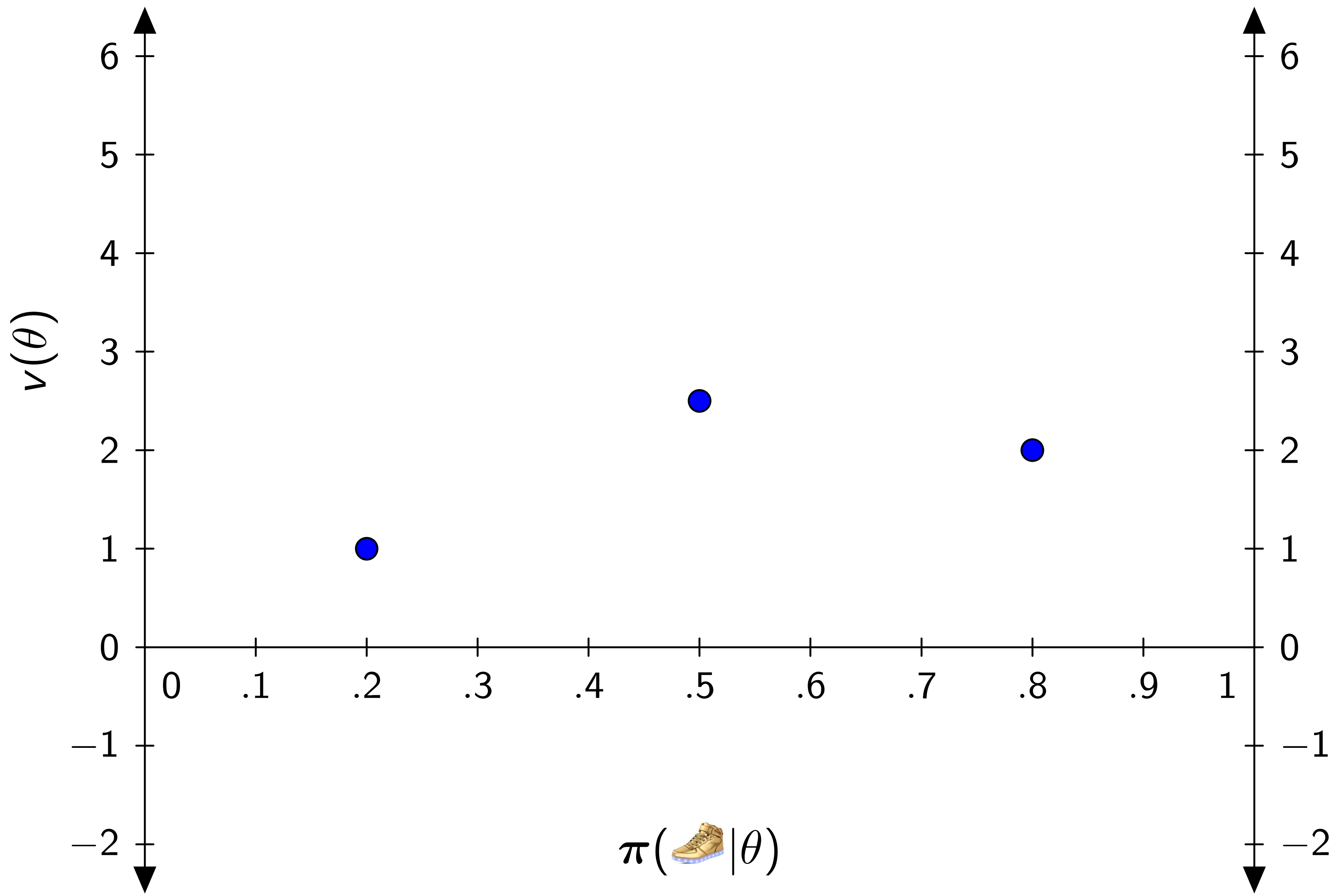


Necessary and Sufficient Condition



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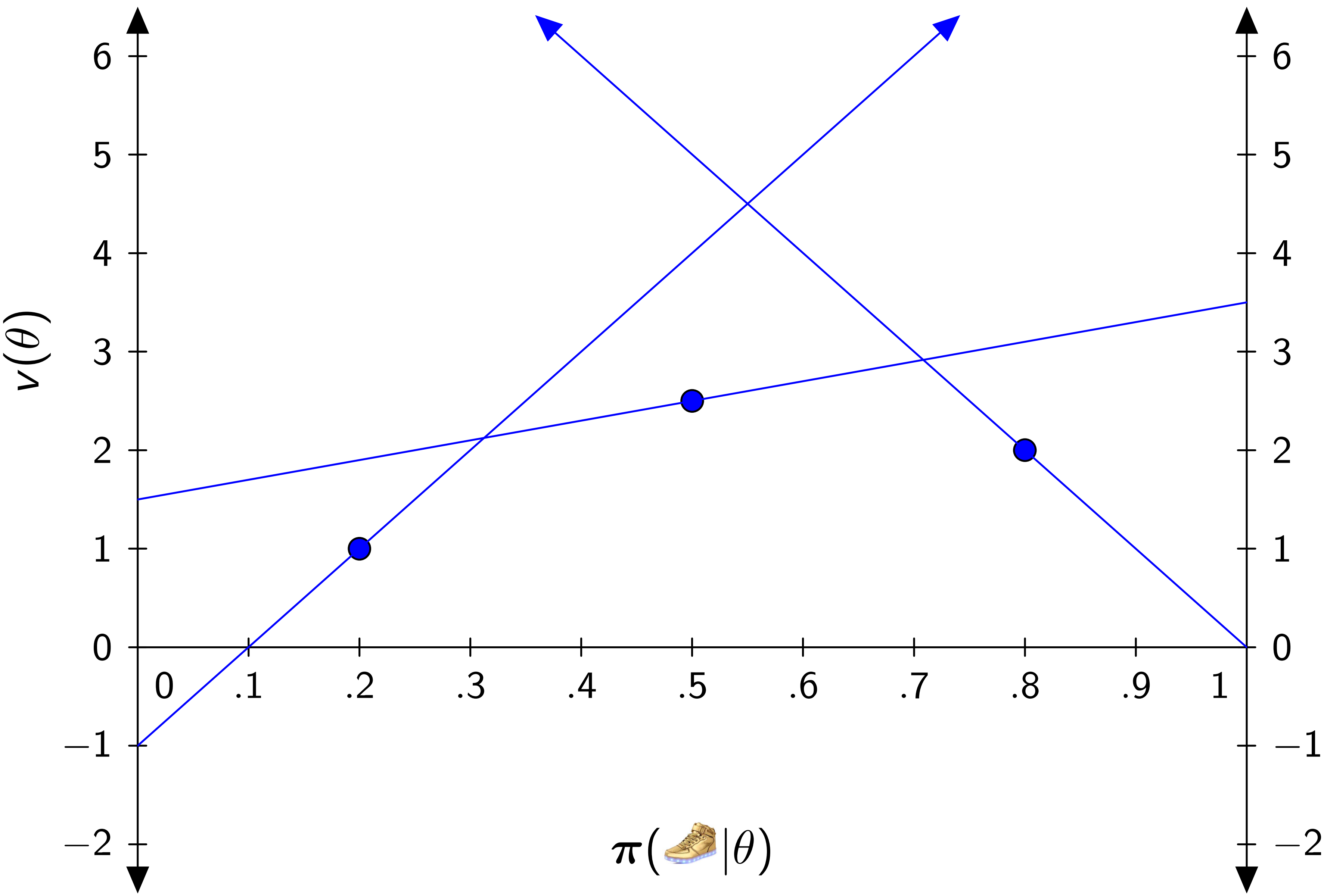

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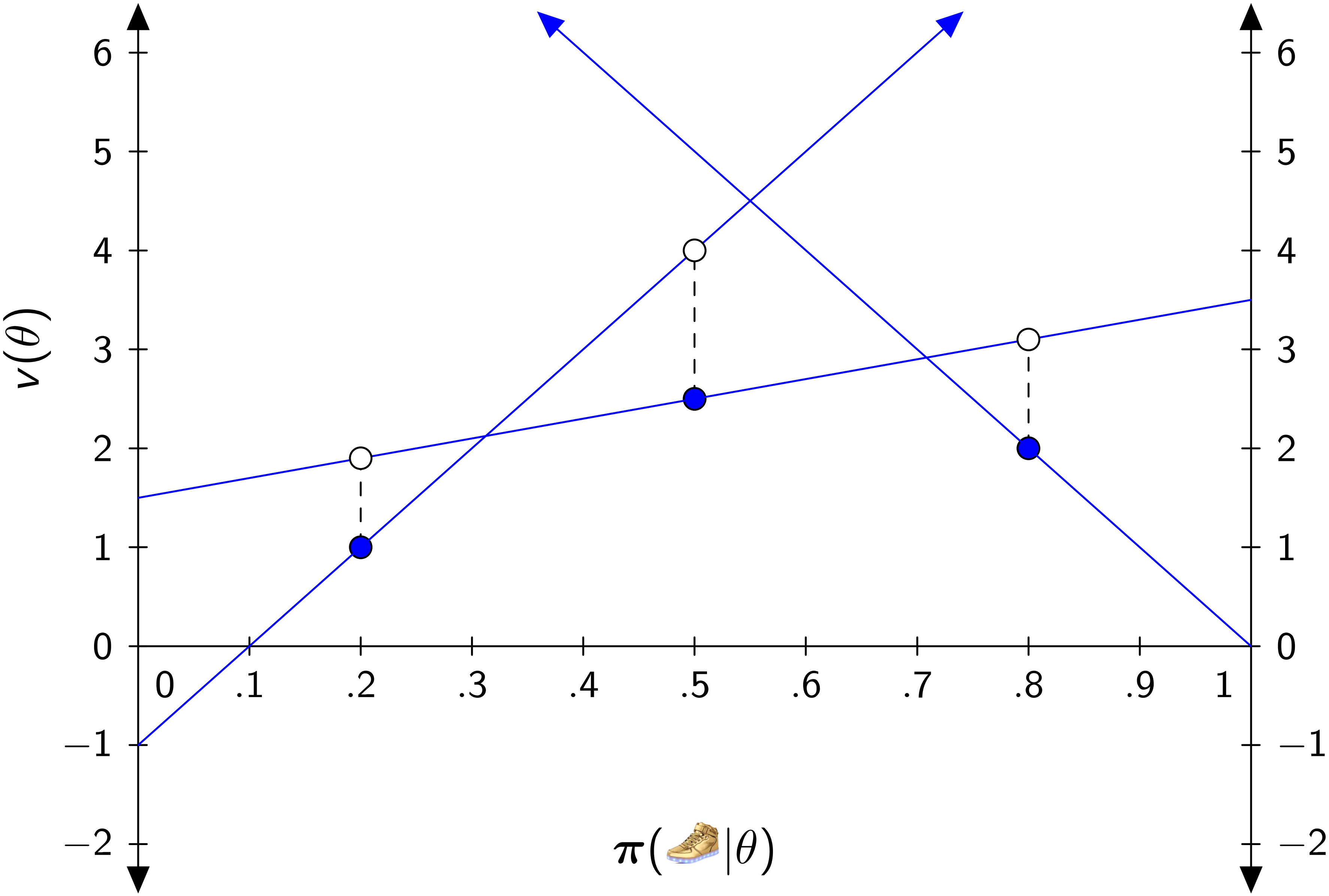

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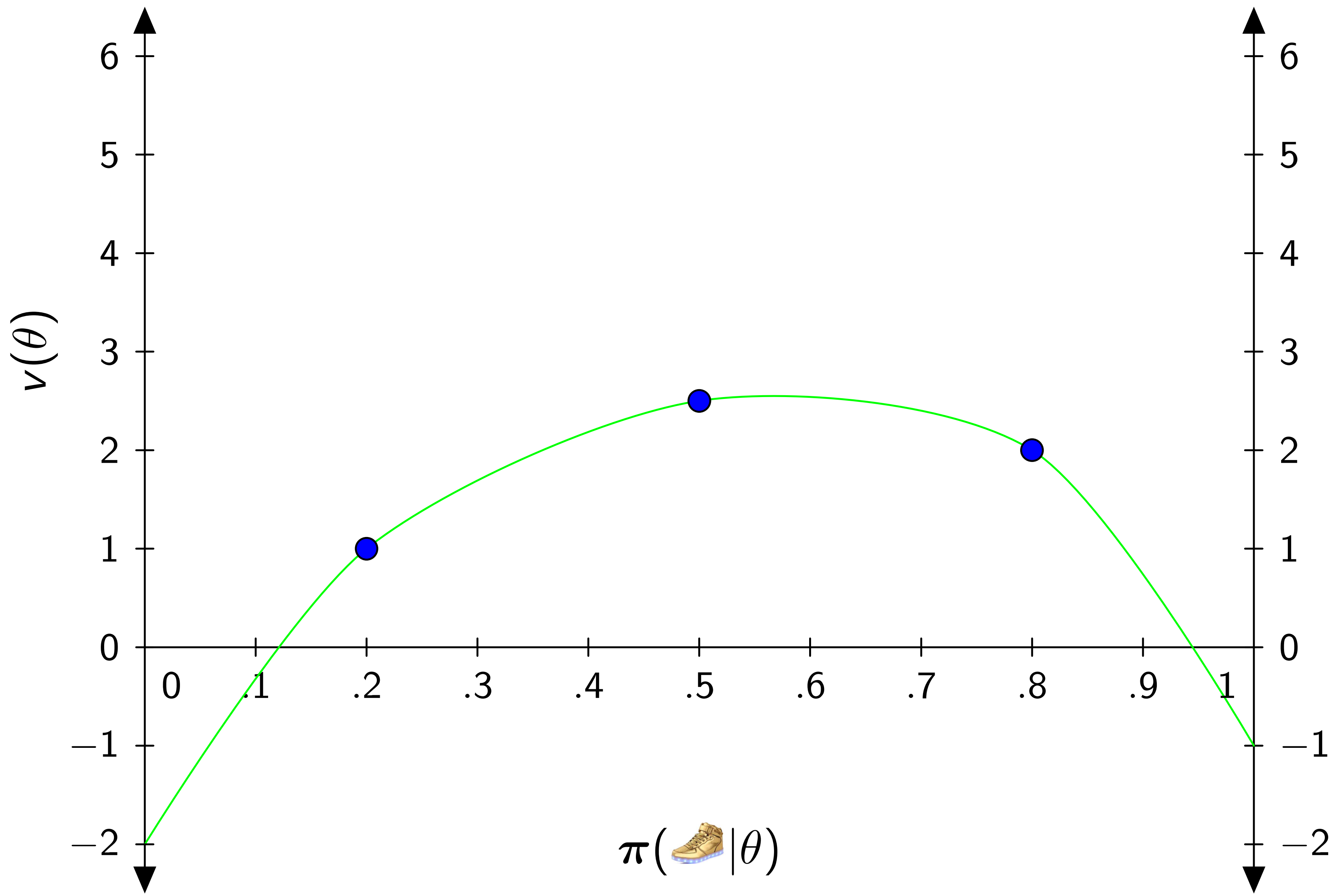

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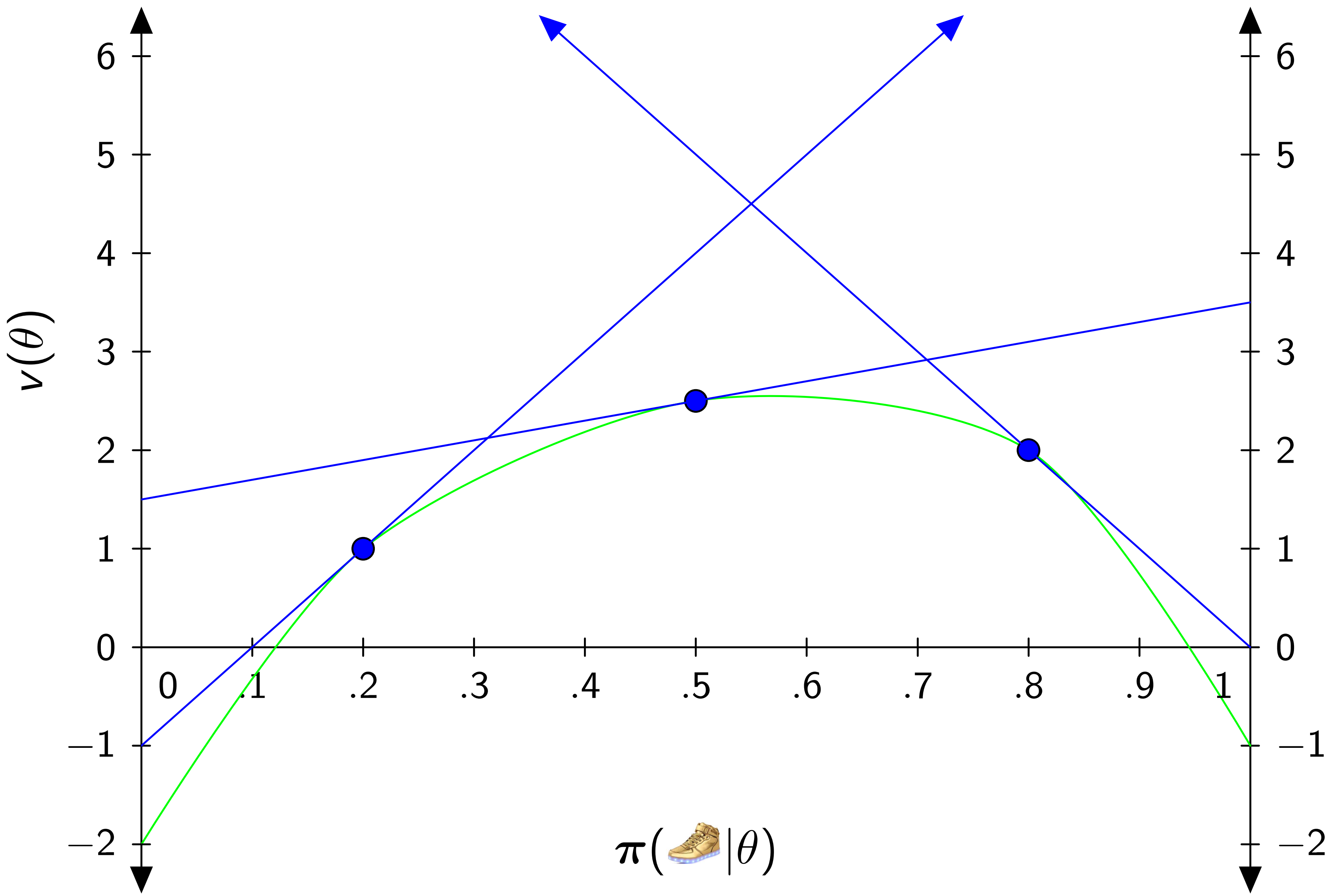

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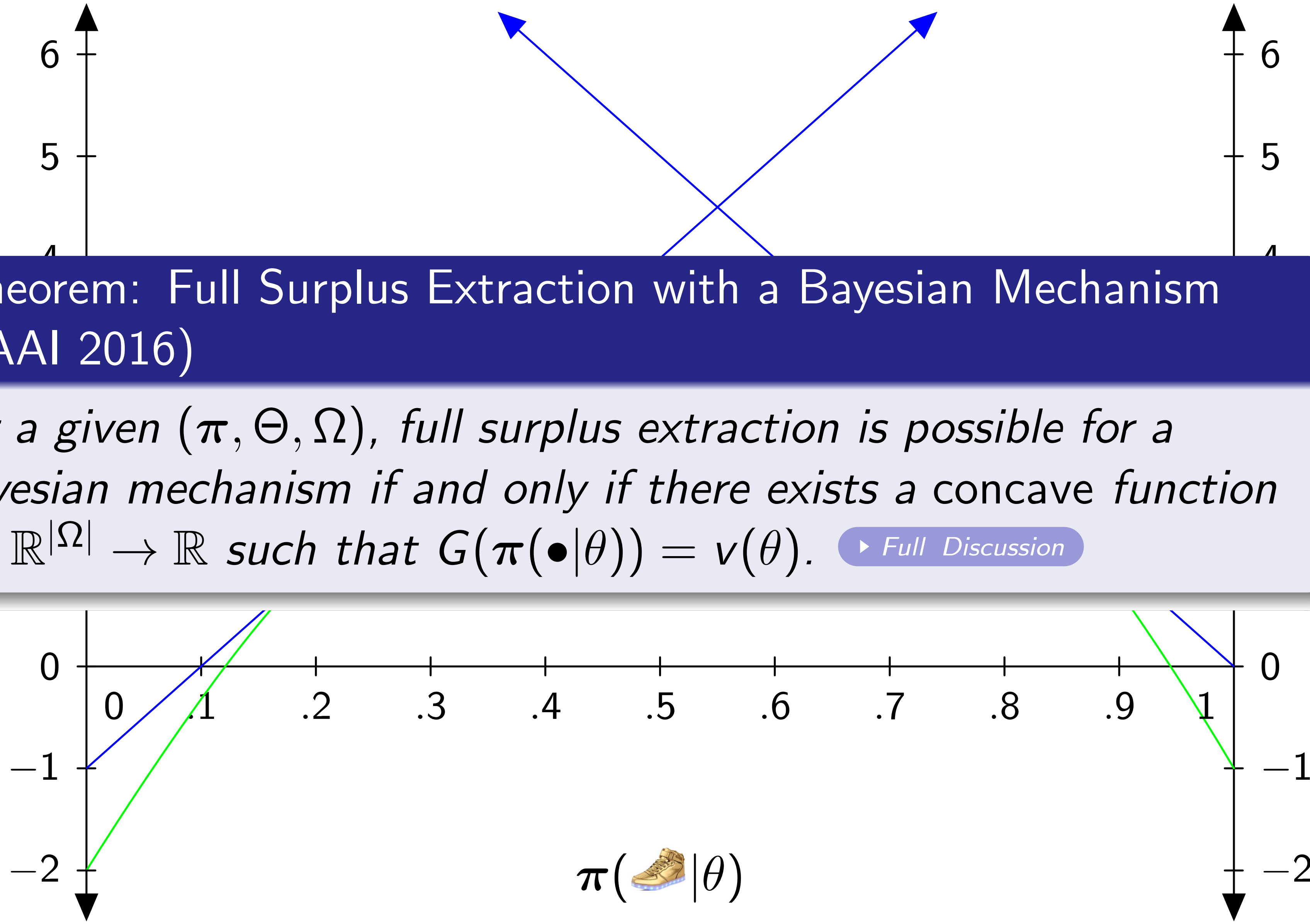

Theorem: Full Surplus Extraction with a Bayesian Mechanism (AAAI 2016)

For a given (π, Θ, Ω), *full surplus extraction is possible for a Bayesian mechanism if and only if there exists a concave function* $G : \mathbb{R}^{|\Omega|} \to \mathbb{R}$ *such that* $G(\pi(\bullet|\theta)) = v(\theta)$. ▸ *Full Discussion*

What if we have access to samples and there is "sufficient" correlation?

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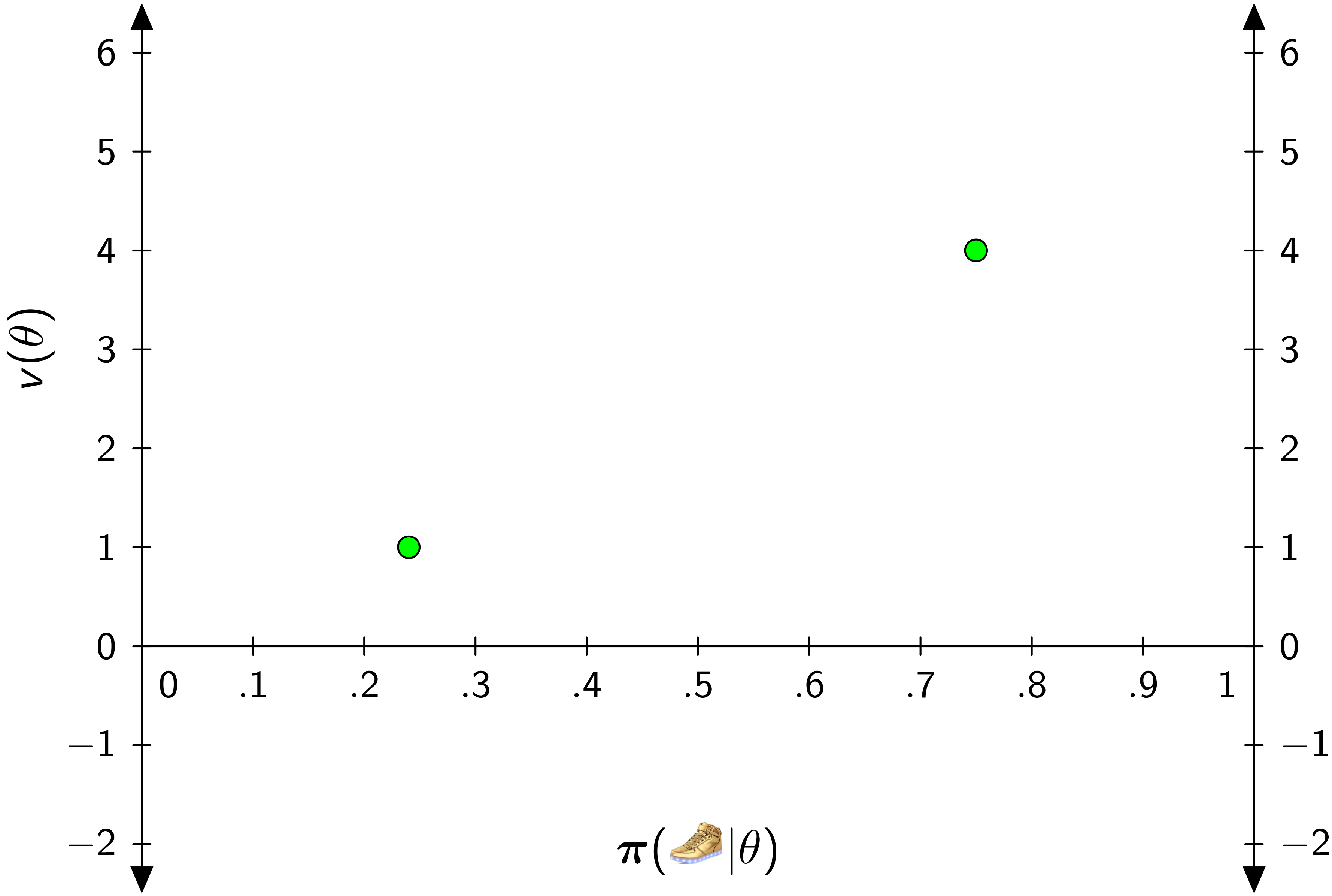
Theorem: Learning is Impossible (AAMAS 17)

For any finite number of samples, there exists a distribution for which the optimal learned mechanism is no better than the ex-post mechanism. Discussion

Consistent Distributions



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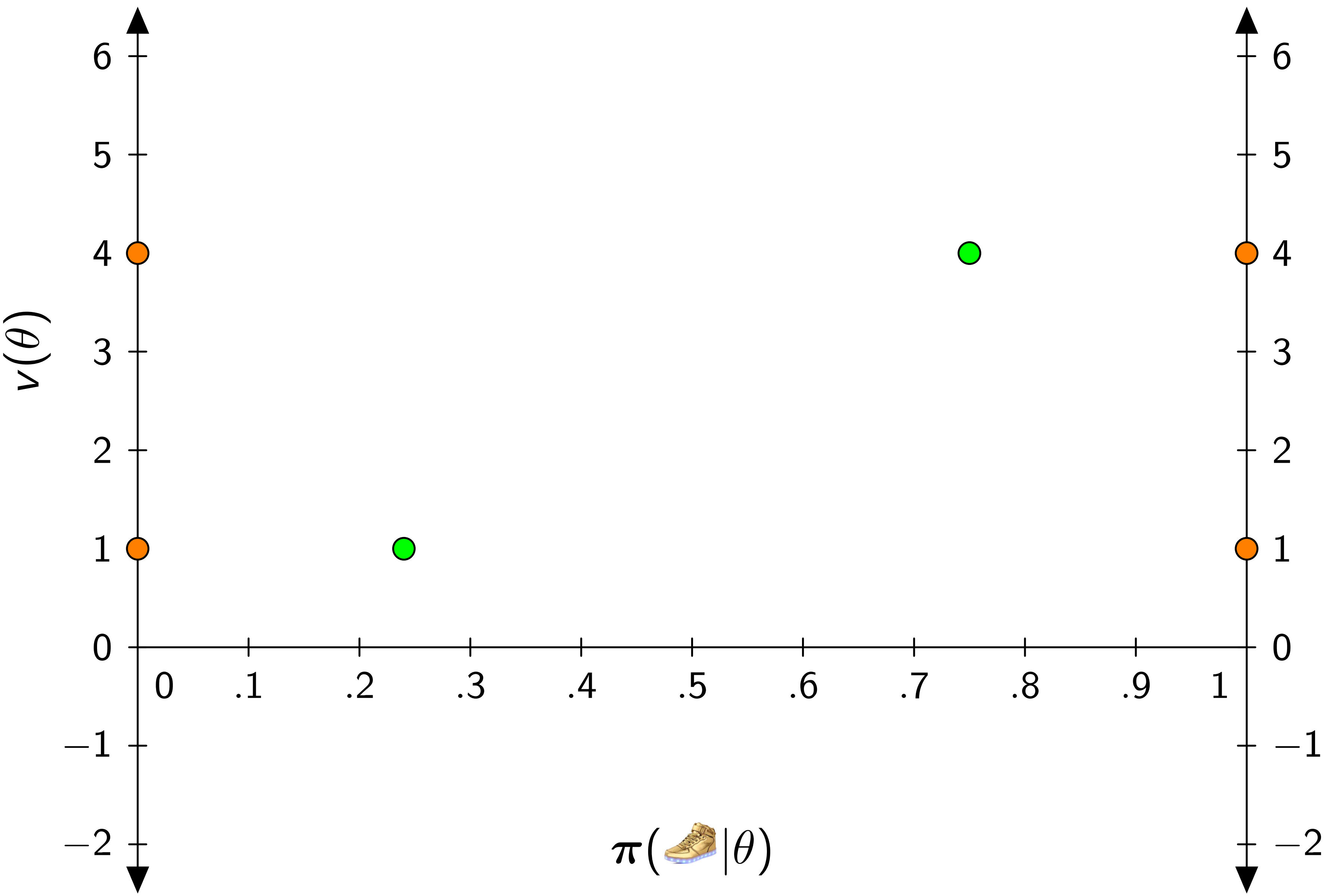

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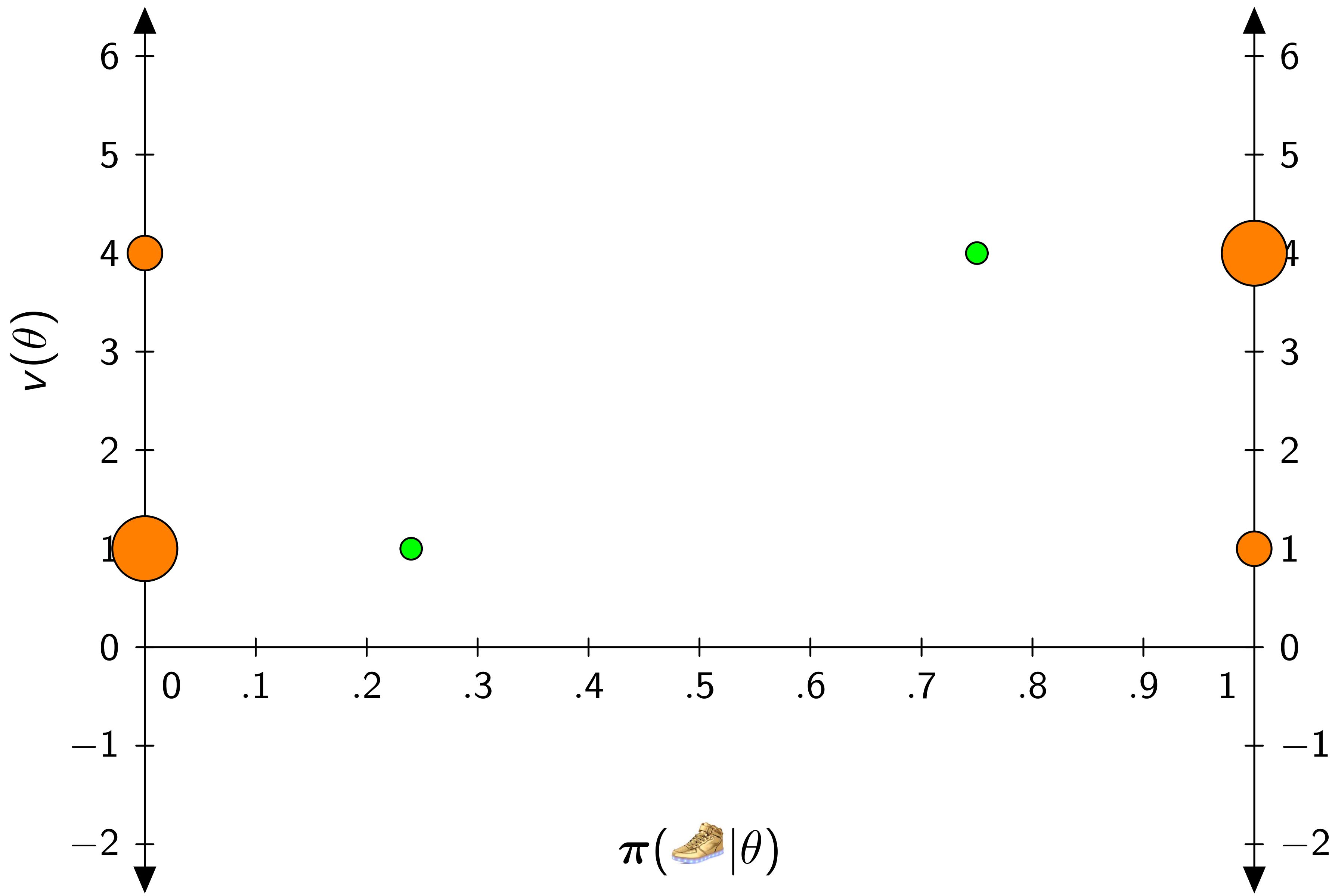

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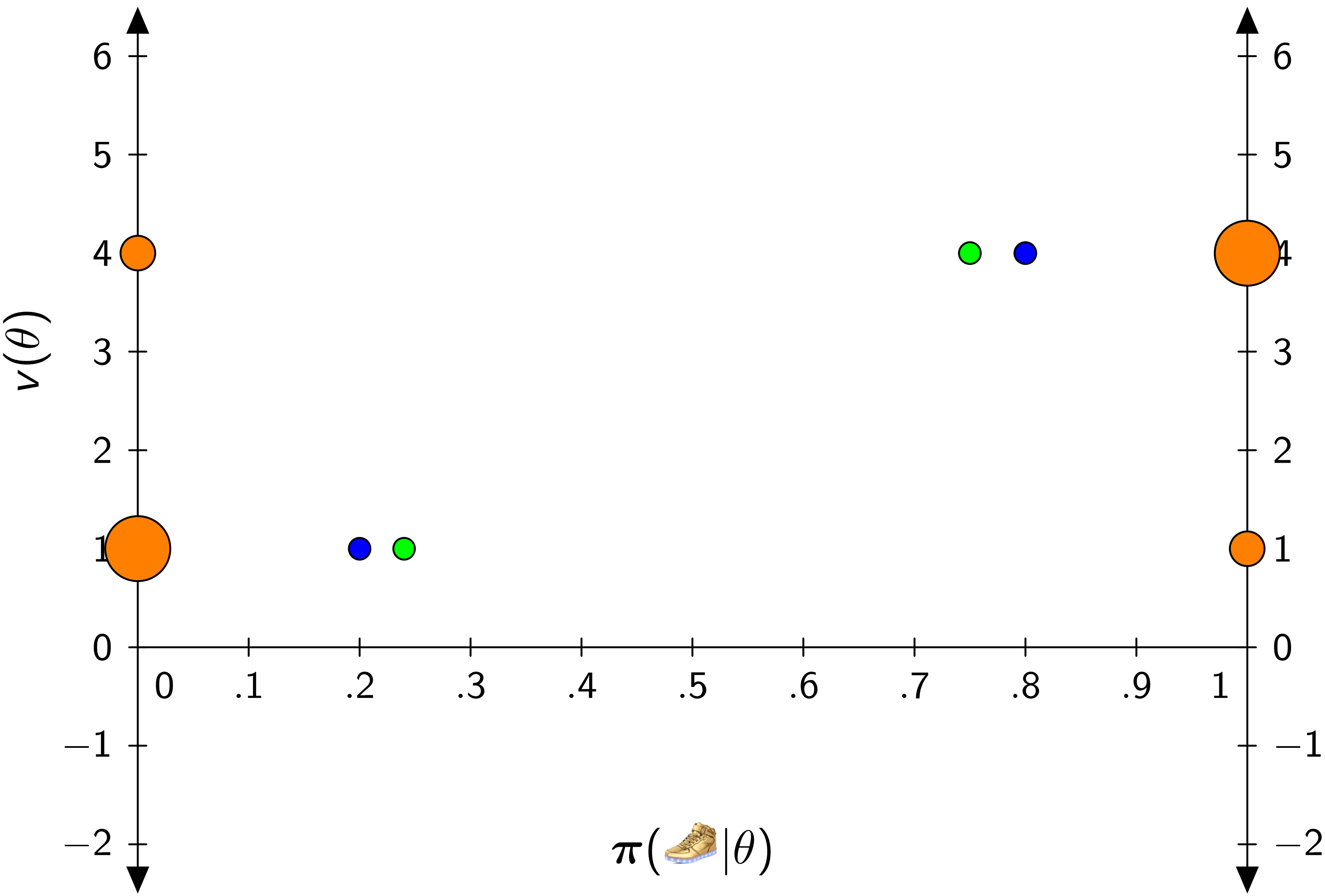

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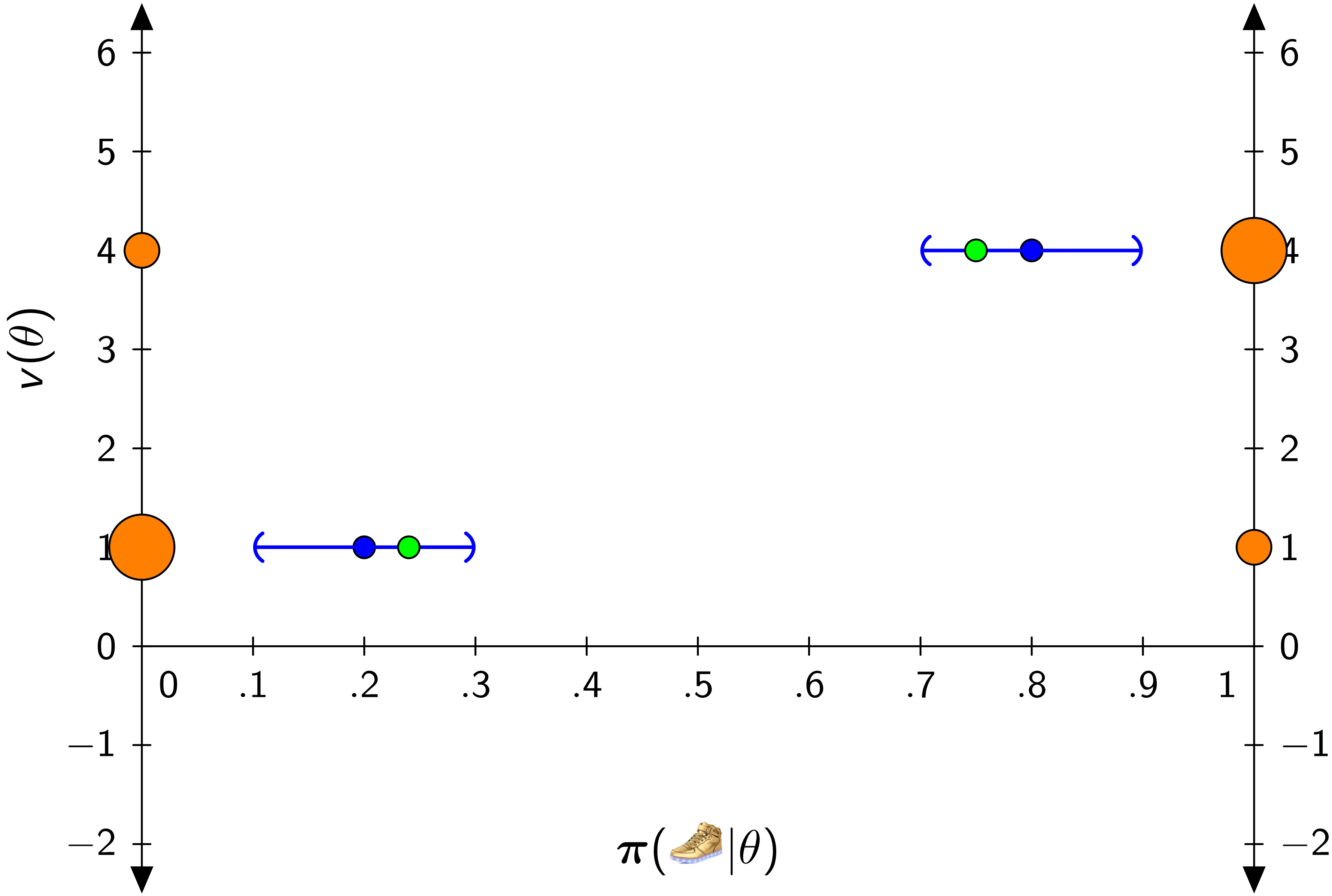

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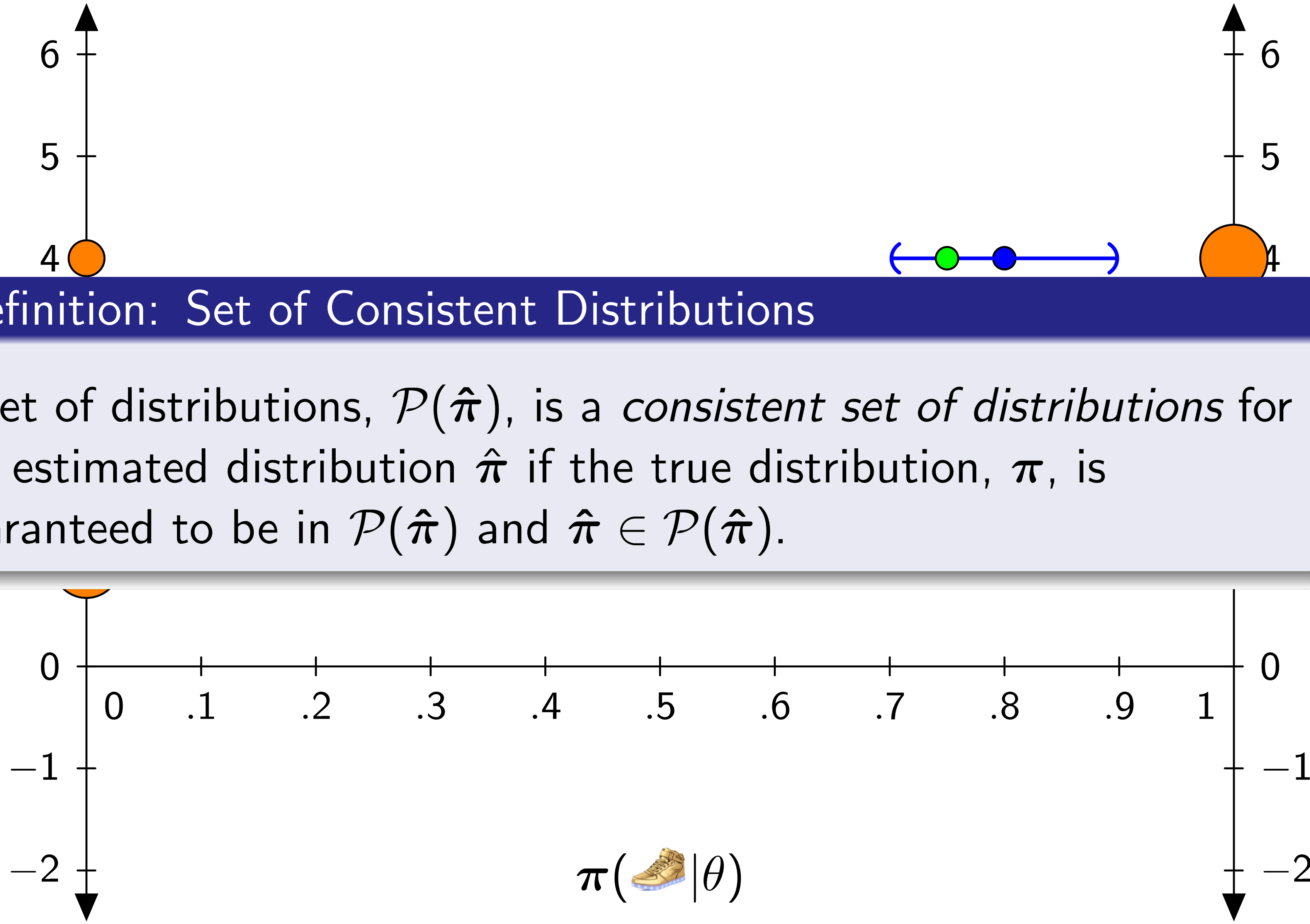
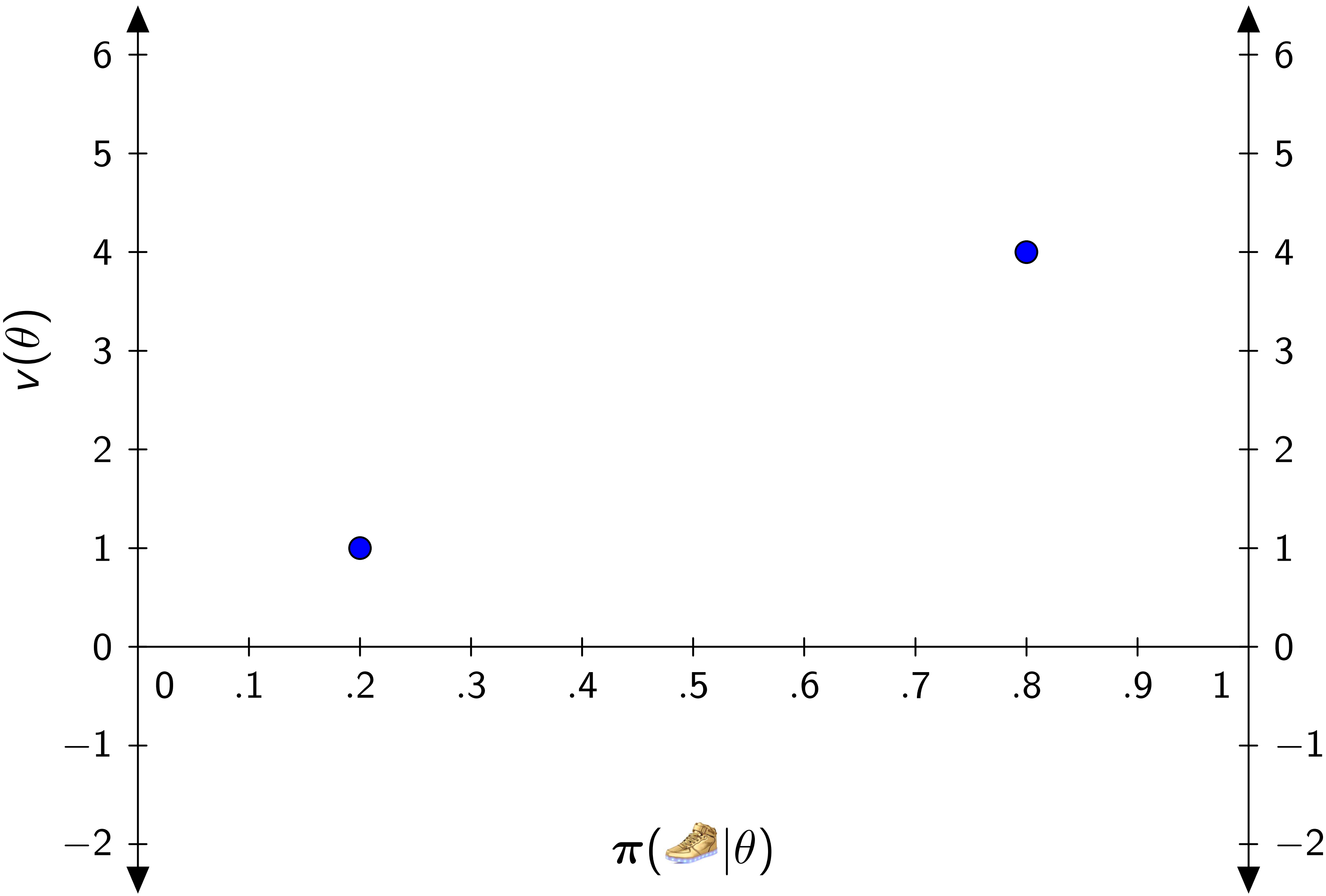


Definition: Set of Consistent Distributions

A set of distributions, $\mathcal{P}(\hat{\pi})$, is a *consistent set of distributions* for the estimated distribution $\hat{\pi}$ if the true distribution, π, is guaranteed to be in $\mathcal{P}(\hat{\pi})$ and $\hat{\pi} \in \mathcal{P}(\hat{\pi})$.

Robust Mechanism Design



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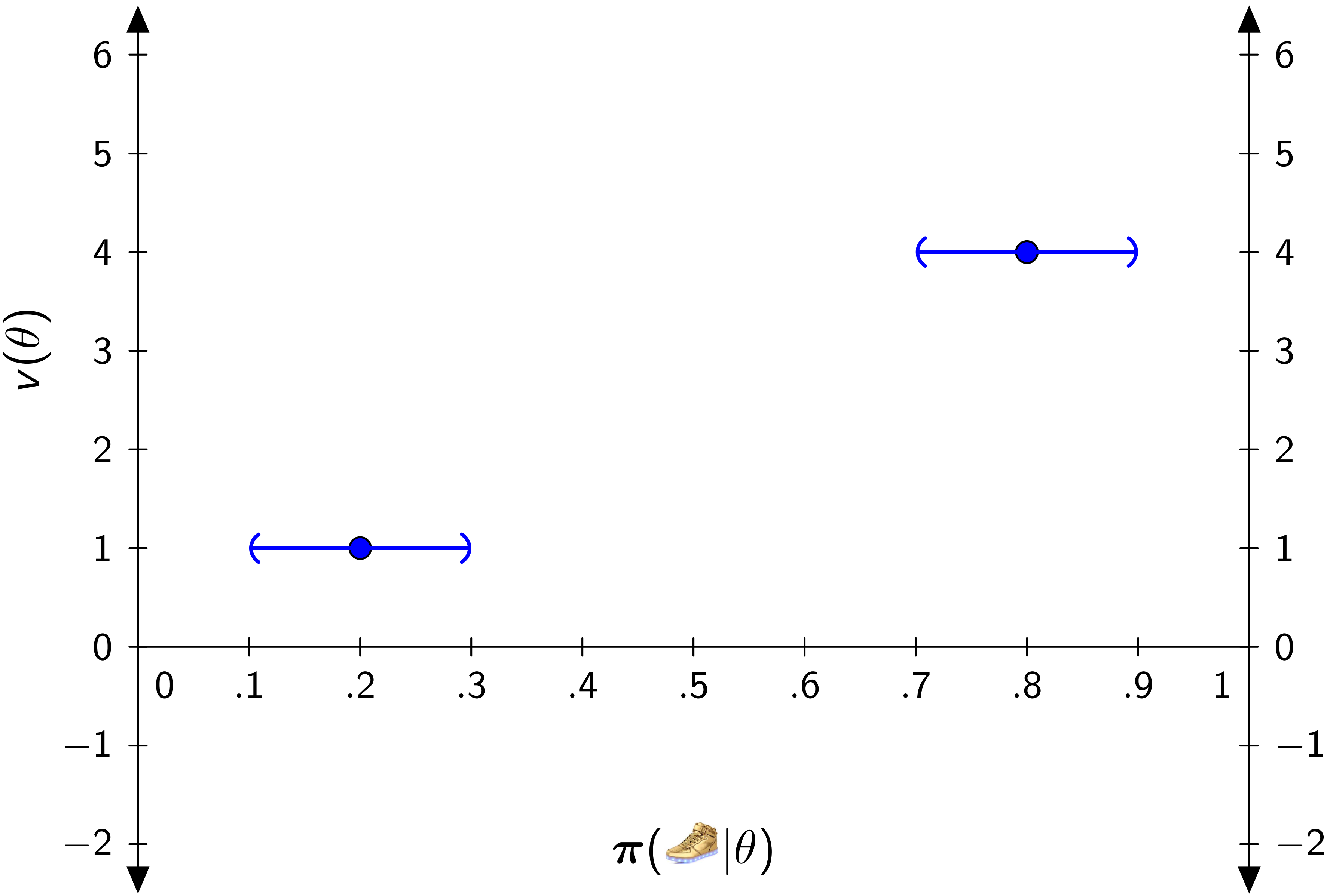

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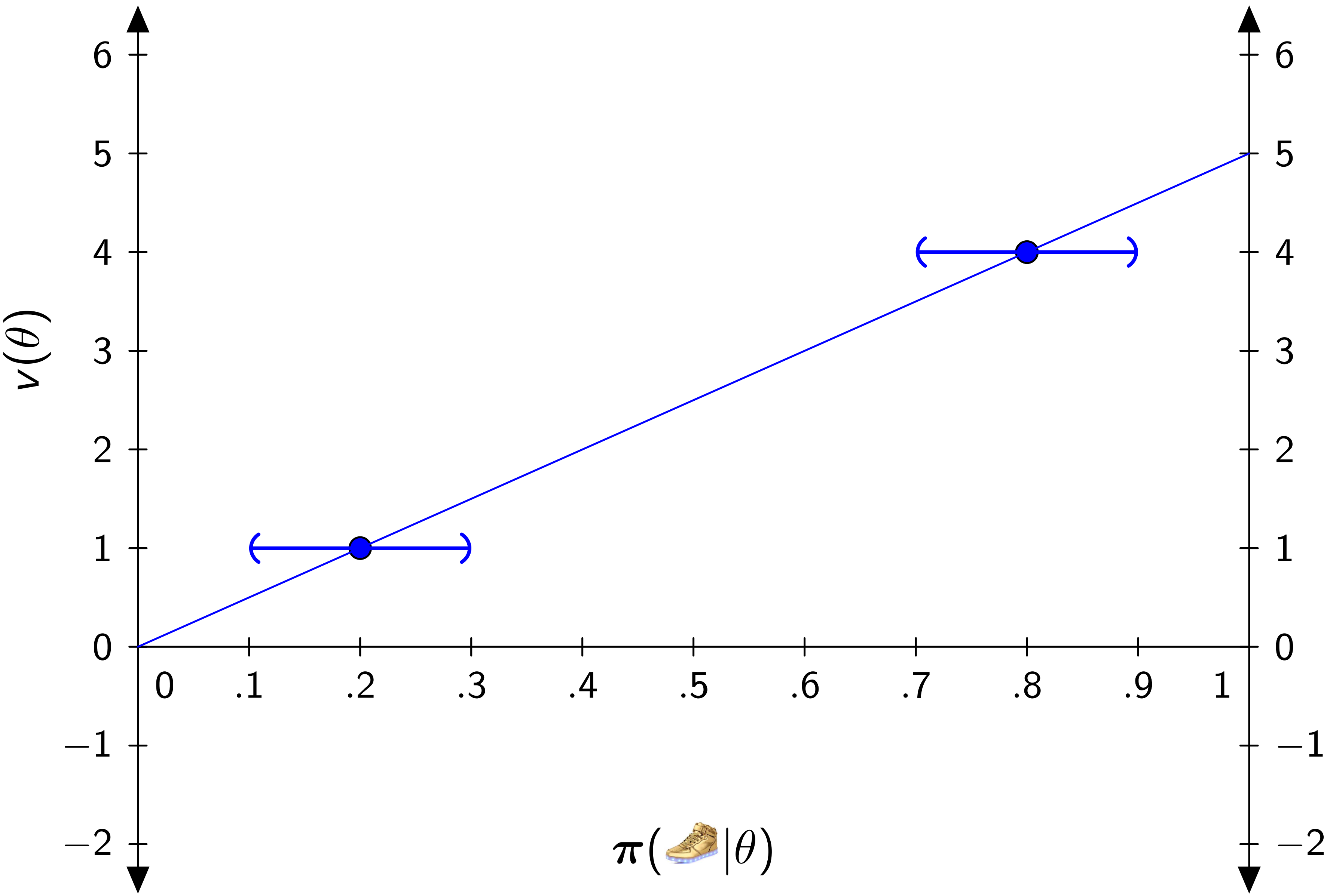

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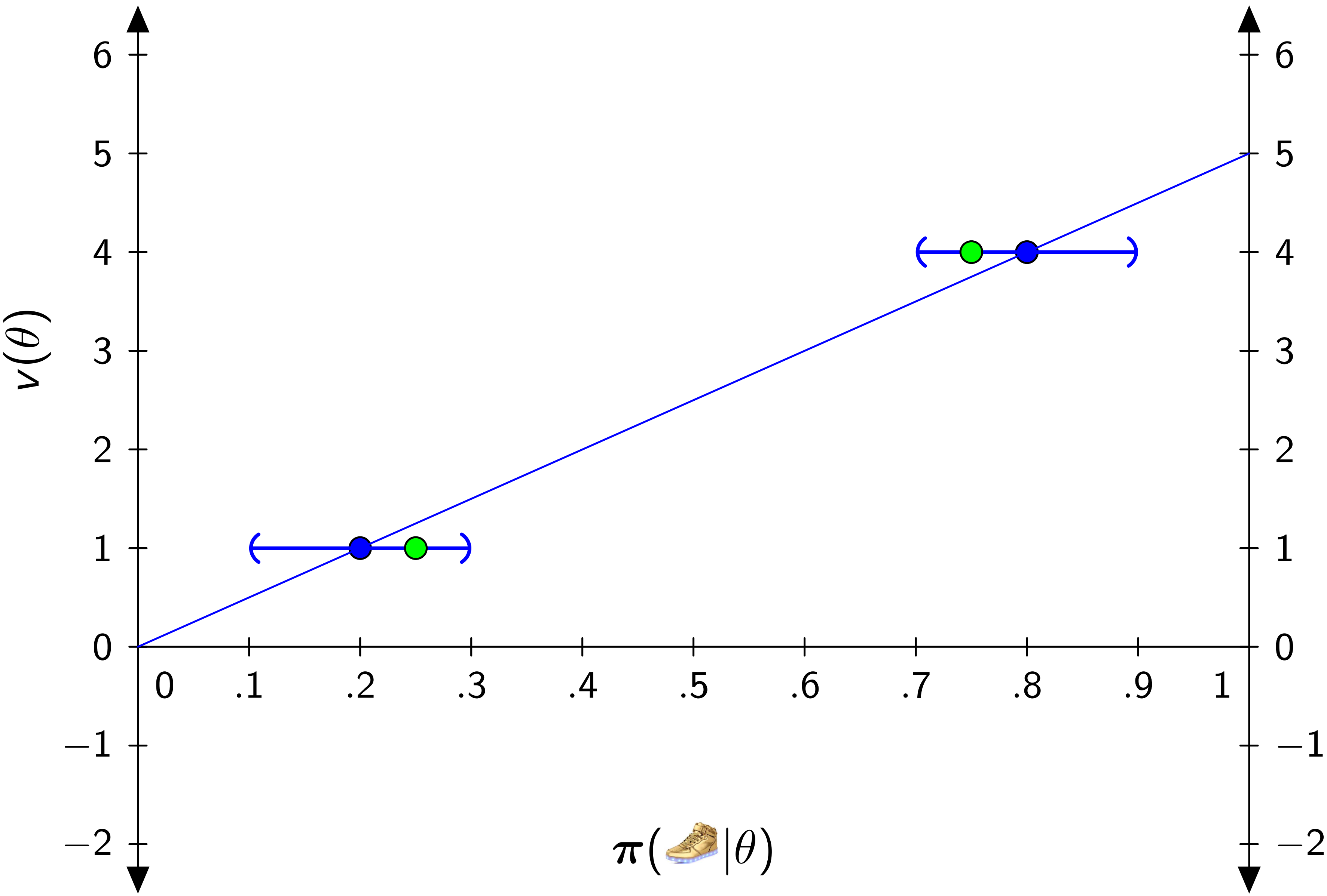

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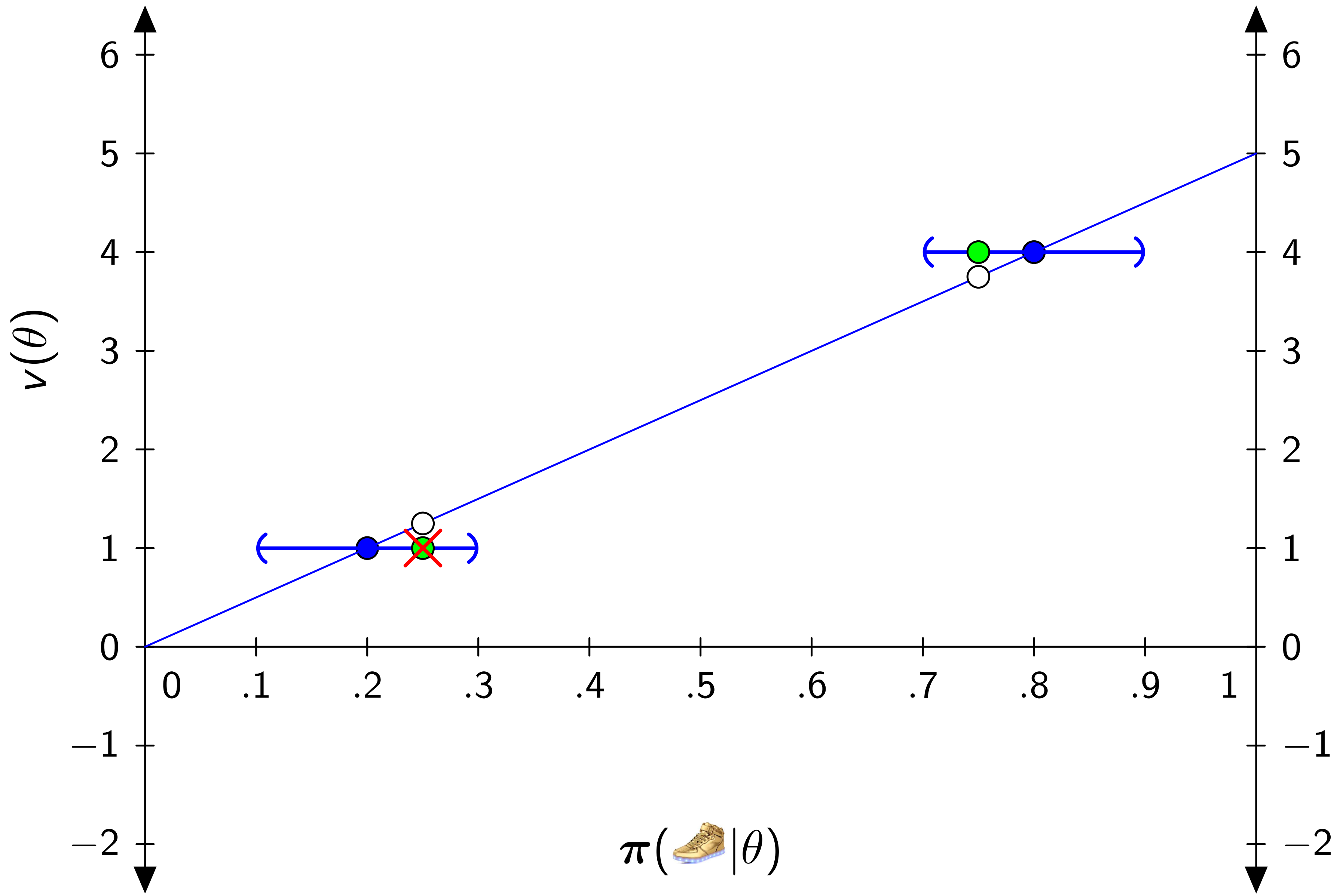

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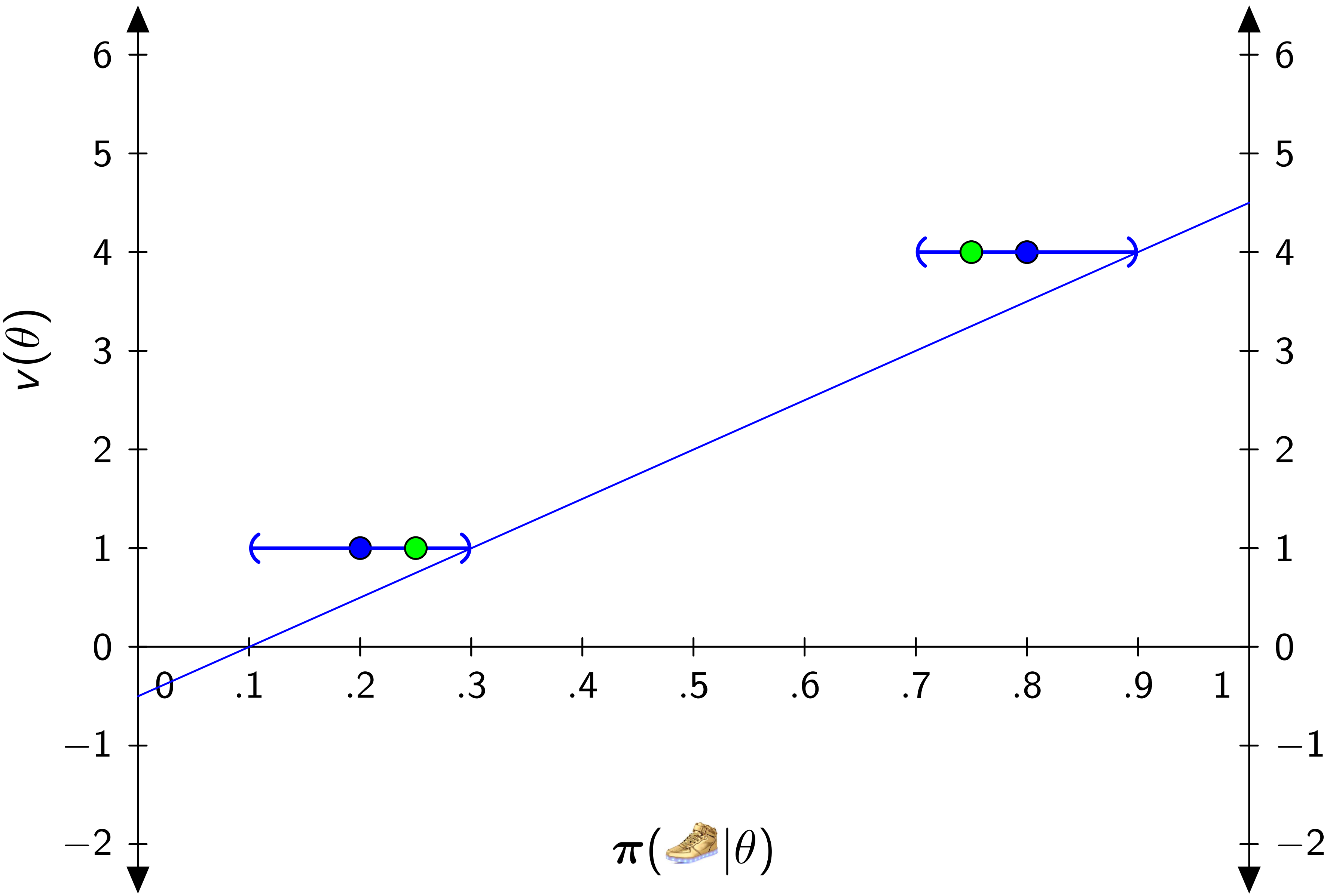

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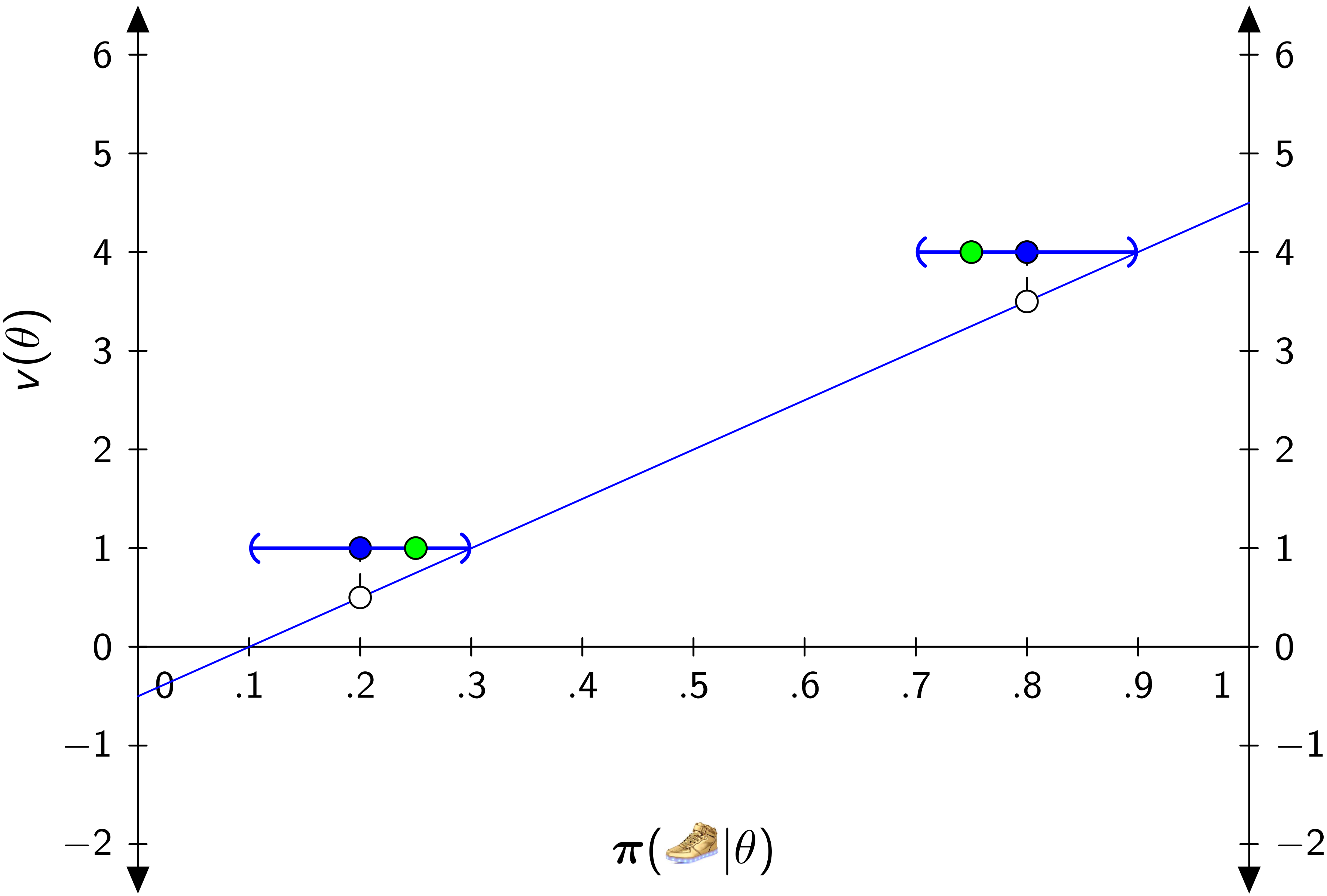

Linear Program for Robust Mechanisms

$$\max_{x(\theta,\omega),\, p(\theta,\omega)} \sum_{\theta,\omega} \hat{\pi}(\theta,\omega) x(\theta,\omega)$$

subject to **Robust Individual Rationality** (**IR**):

$$\sum_{\omega\in\Omega} \boldsymbol{\pi}(\omega|\theta) U(\theta,\theta,\omega) \geq 0 \qquad \forall\, \theta \in \Theta,\ \omega \in \Omega, \boldsymbol{\pi} \in \mathcal{P}(\hat{\boldsymbol{\pi}})$$

and subject to **Robust Incentive Compatibility** (**IC**):

$$\sum_{\omega\in\Omega} \boldsymbol{\pi}(\omega|\theta) U(\theta,\theta,\omega) \geq \sum_{\omega\in\Omega} \boldsymbol{\pi}(\omega|\theta) U(\theta,\theta',\omega)$$

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and subject to an allocation constraint:

$$0 \leq p(\theta,\omega) \leq 1 \qquad \forall\, \theta \in \Theta,\ \omega \in \Omega$$

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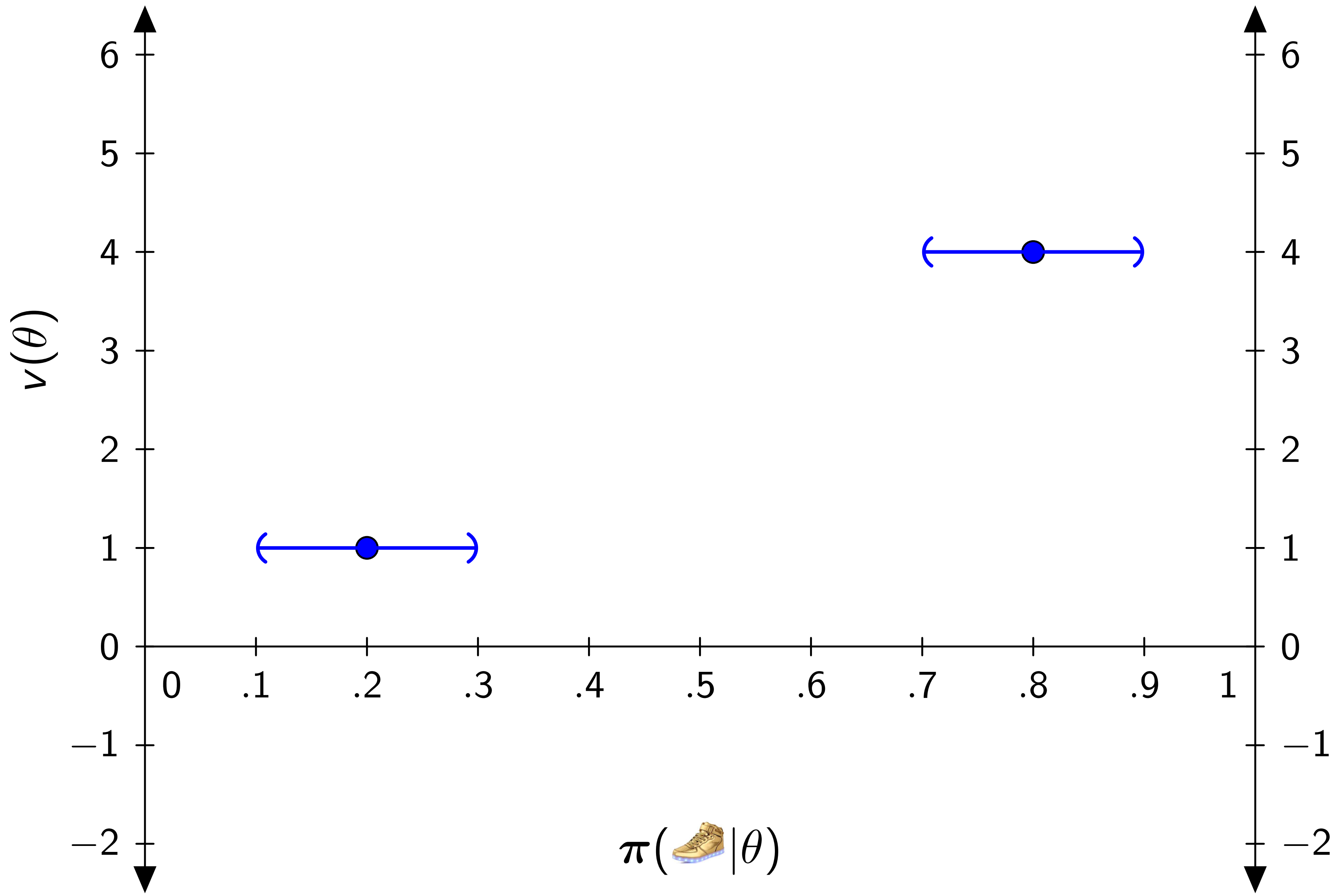

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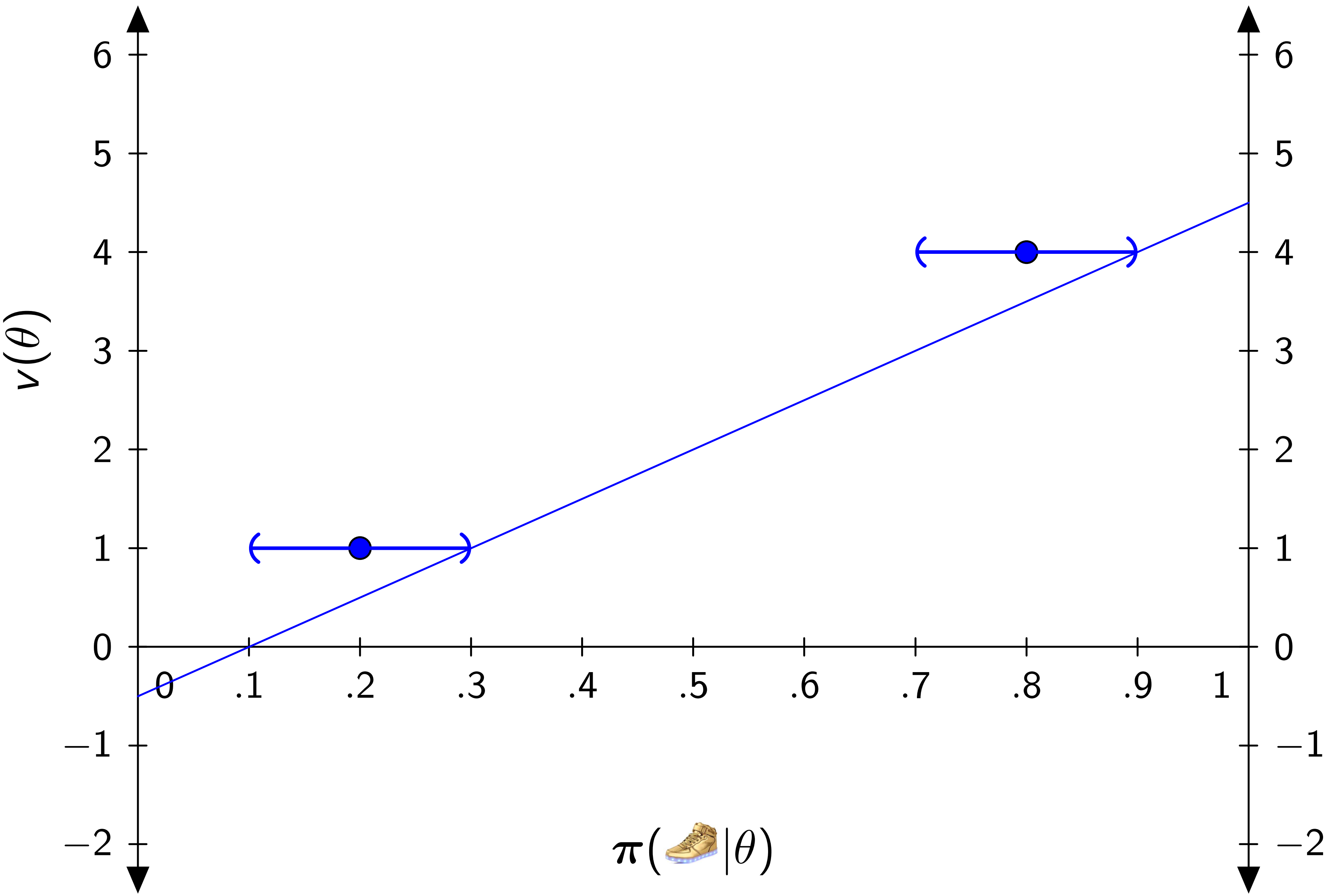

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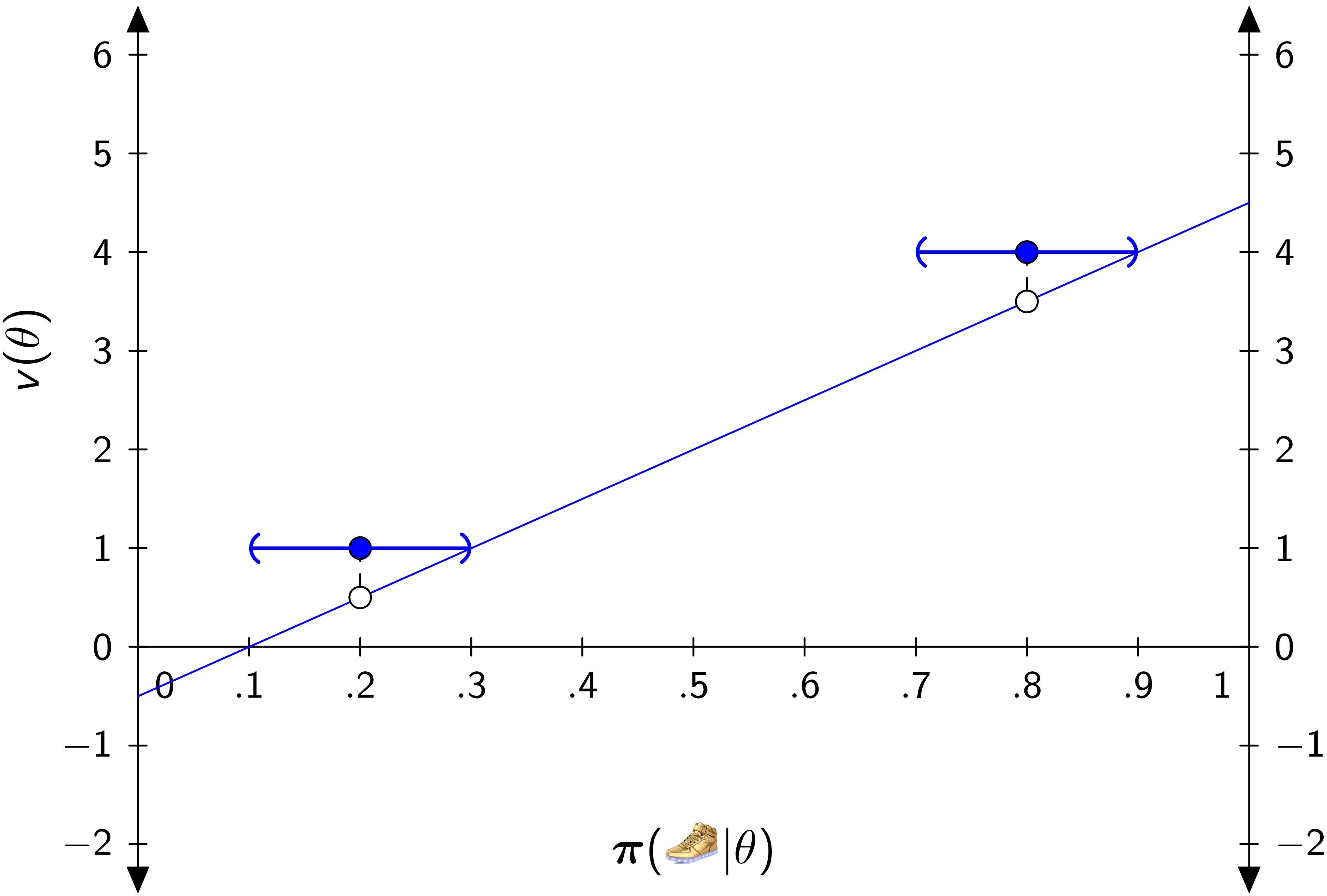

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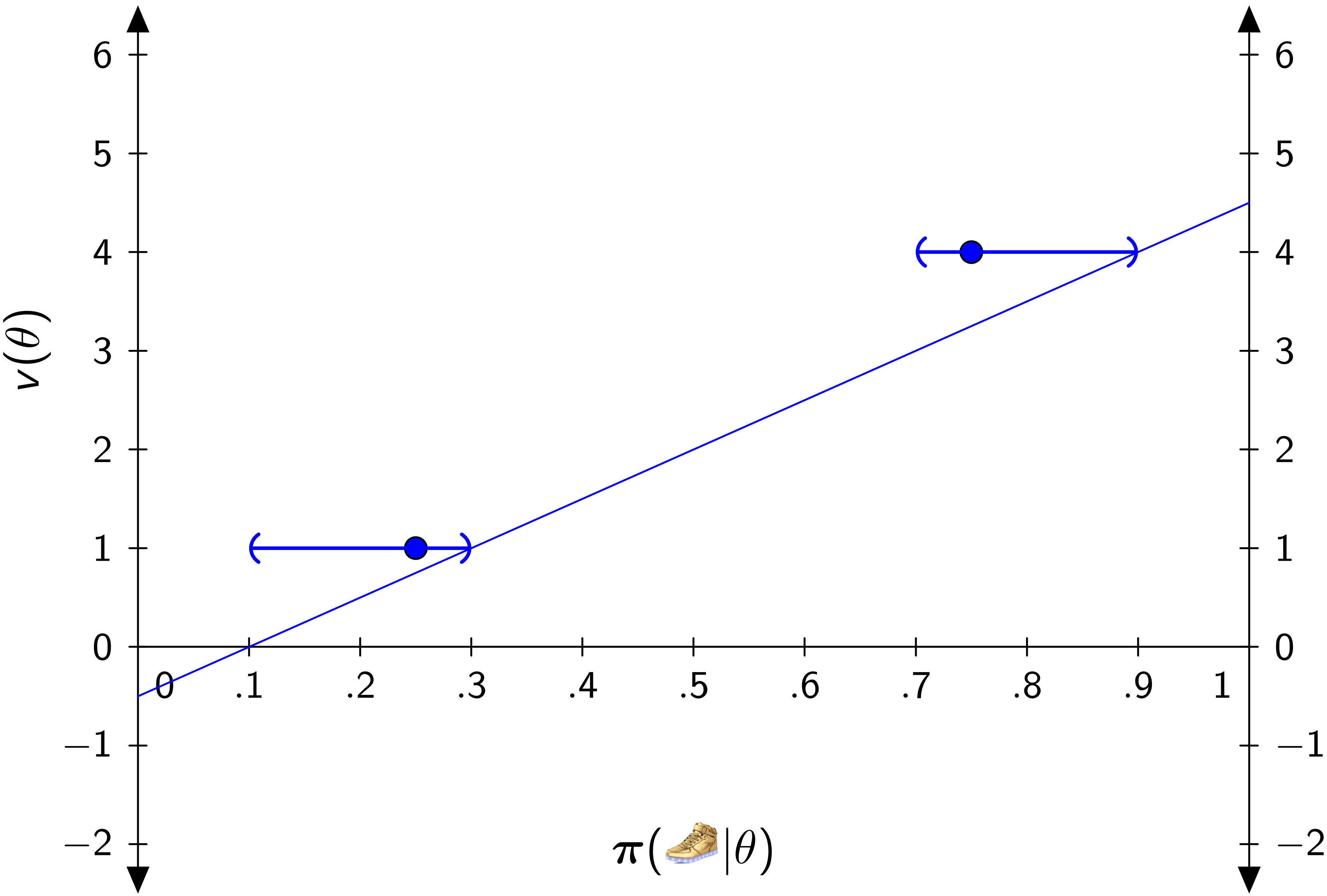

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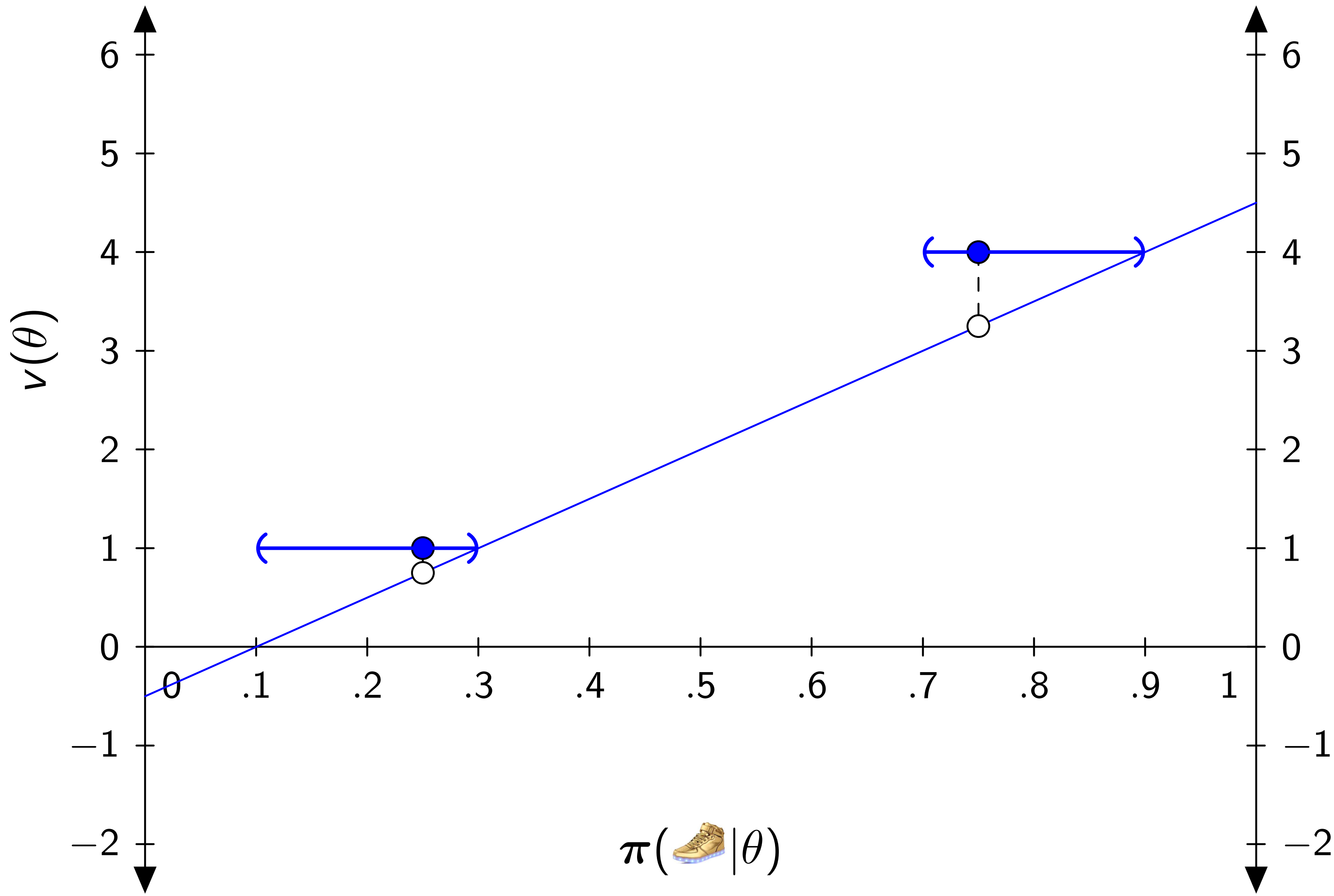

ϵ-Robust Mechanism Design

Robust is not sufficient

- All results and intuition for robust mechanism design carries over to restricted ϵ-robust mechanism design

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A set of distributions, $\mathcal{P}_\epsilon(\hat{\boldsymbol{\pi}})$, is an ϵ*-consistent set of distributions* for the estimated distribution $\hat{\boldsymbol{\pi}}$ if the true distribution, $\boldsymbol{\pi}$, is in $\mathcal{P}_\epsilon(\hat{\boldsymbol{\pi}})$ with probability $1 - \epsilon$ and $\hat{\boldsymbol{\pi}} \in \mathcal{P}_\epsilon(\hat{\boldsymbol{\pi}})$.

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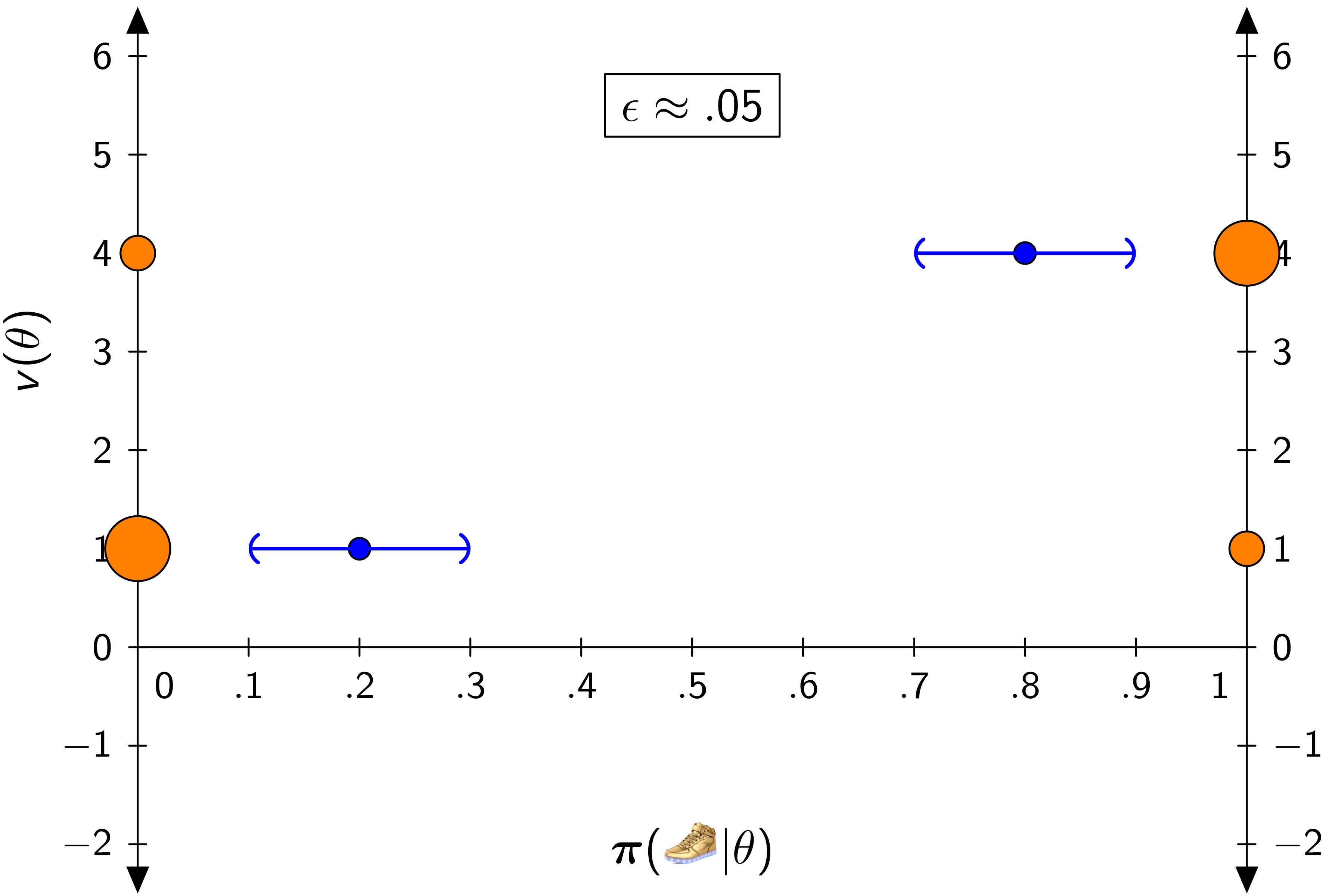
Parameterized Bayesian IC and IR with ϵ



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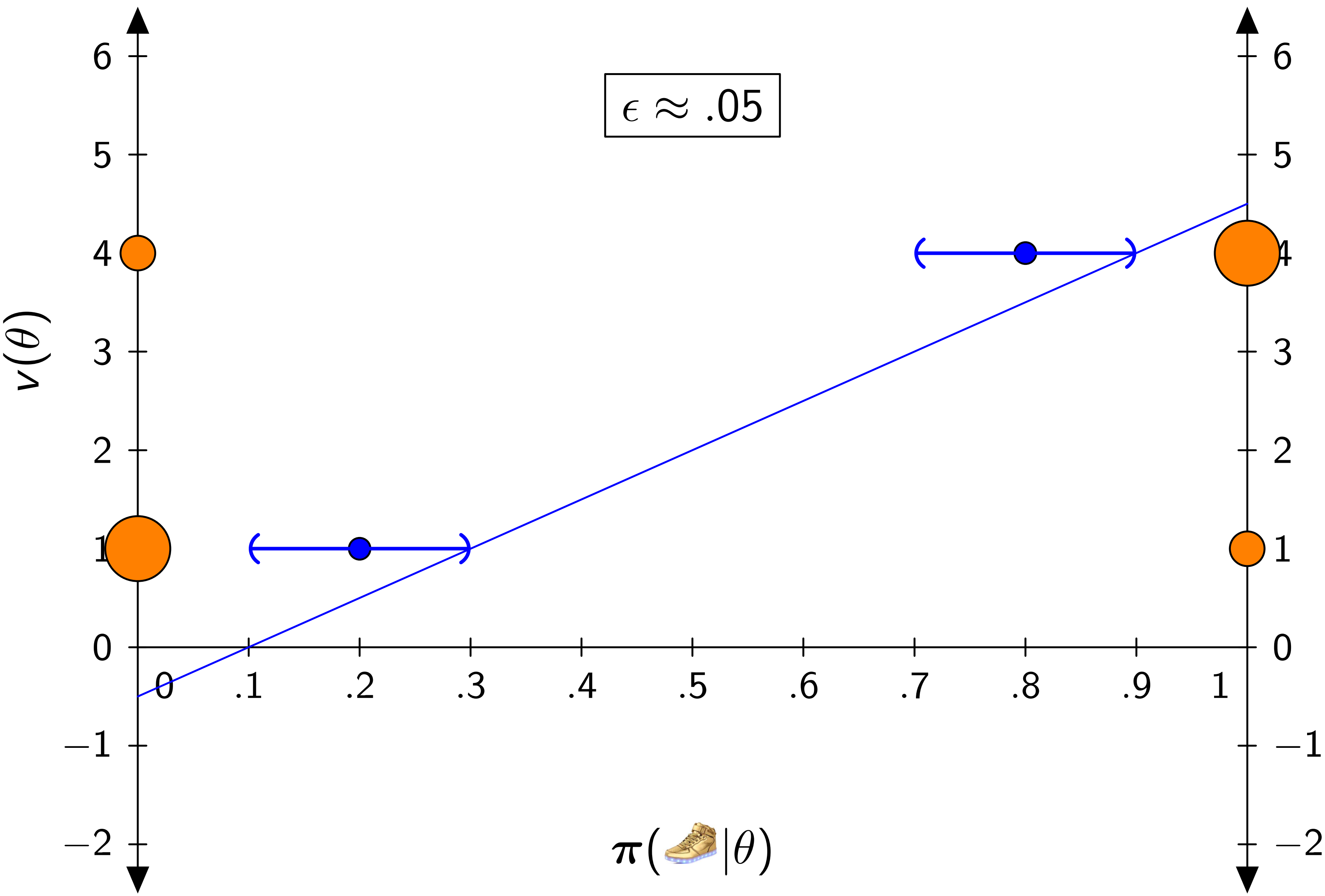

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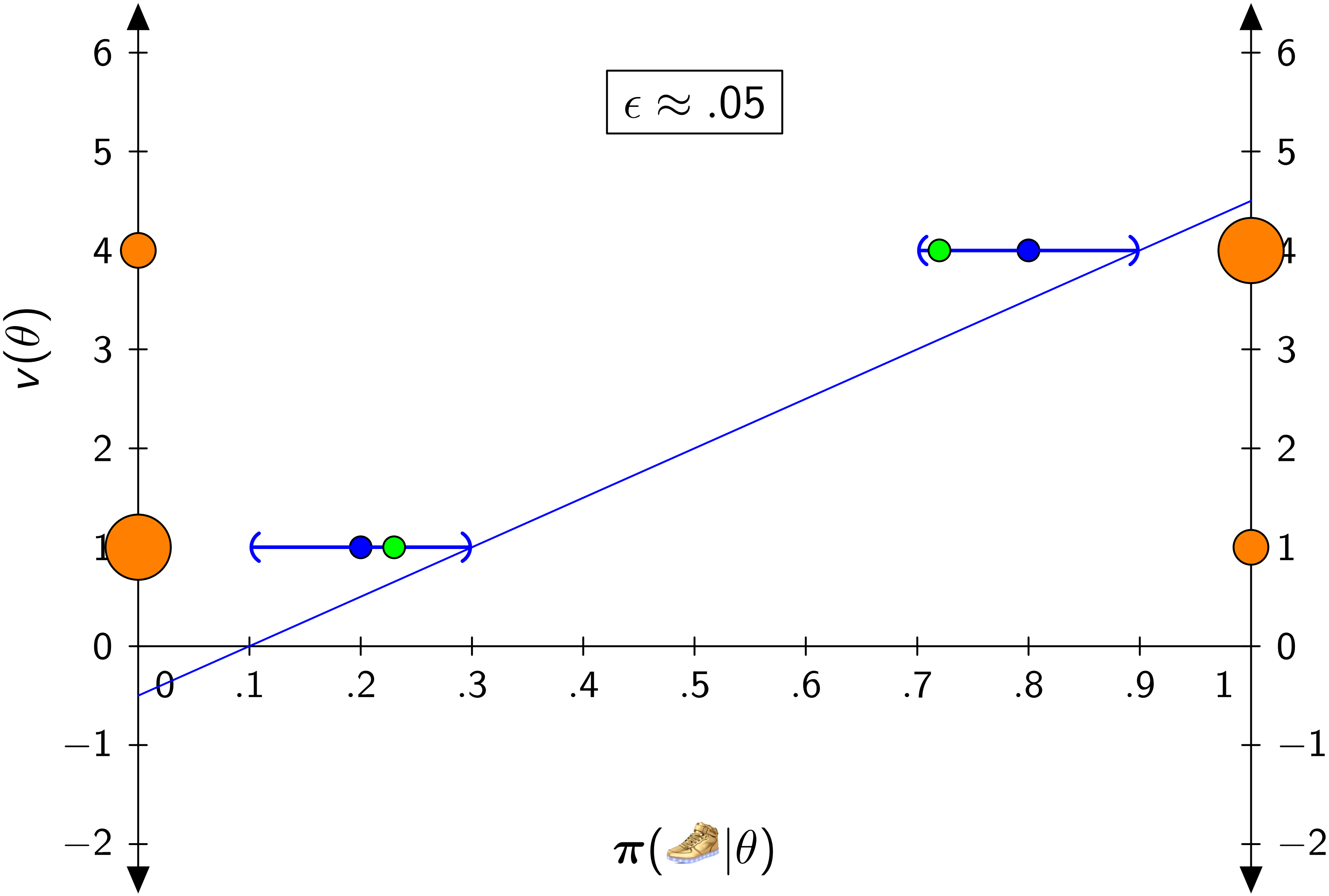

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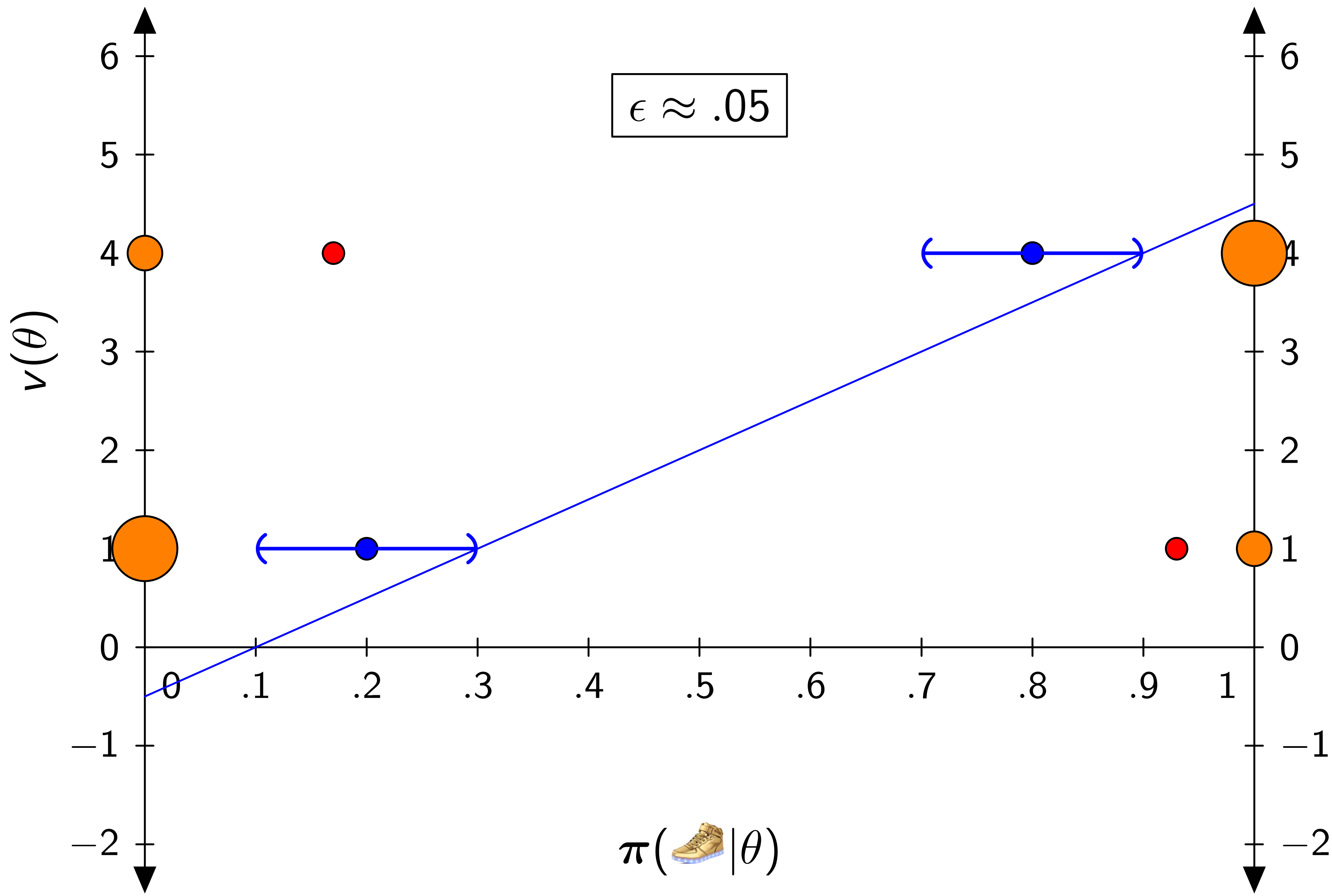

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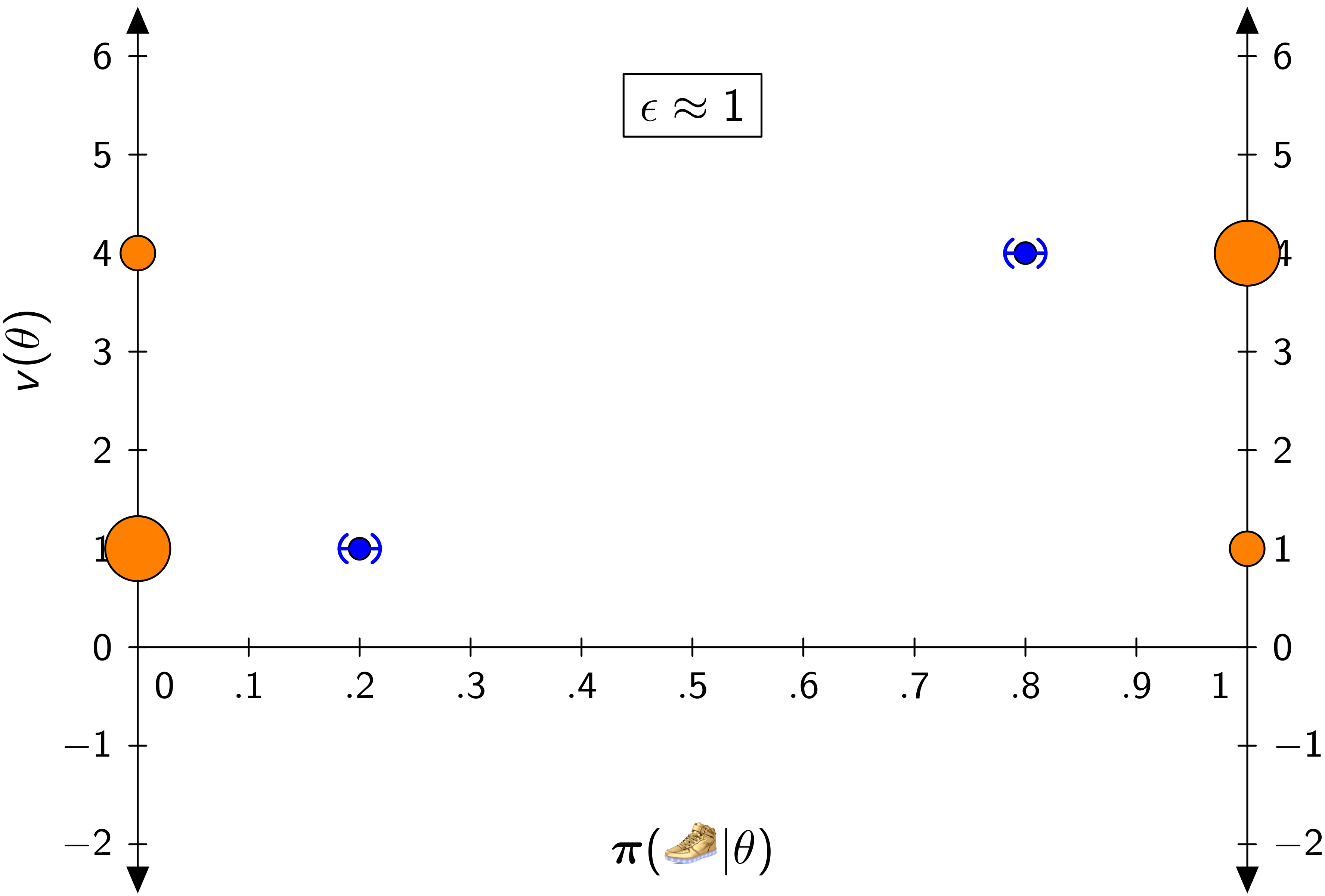

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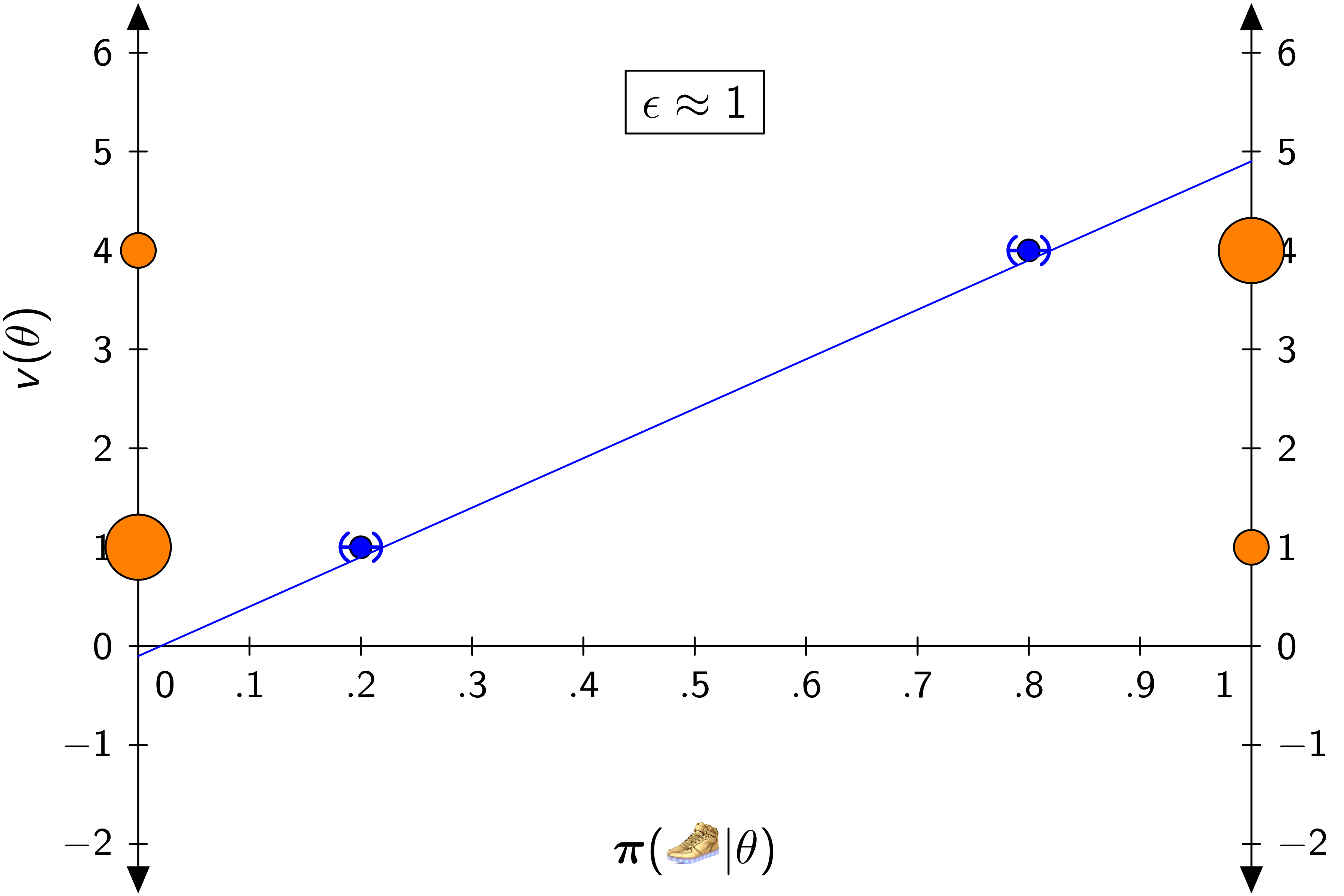

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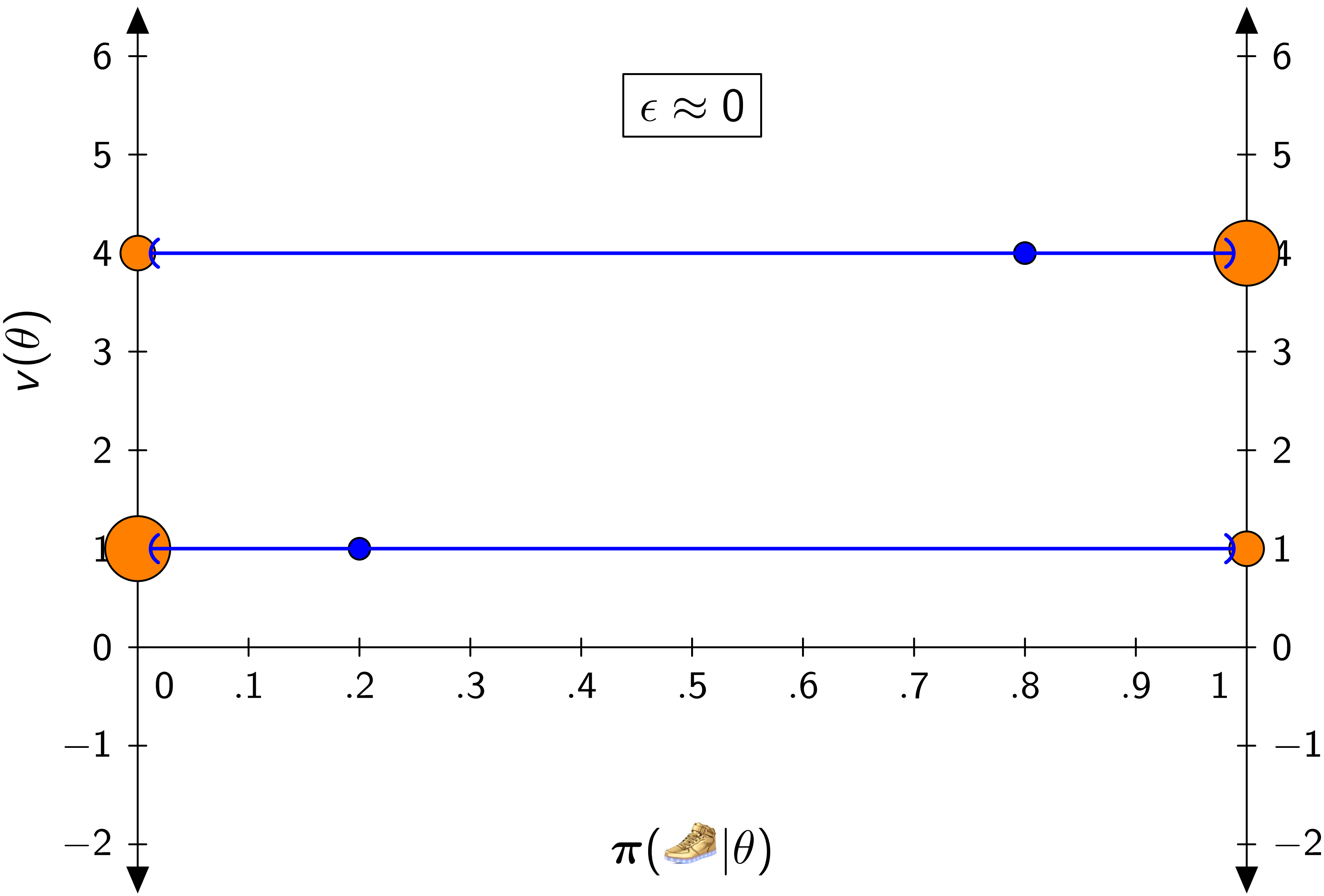

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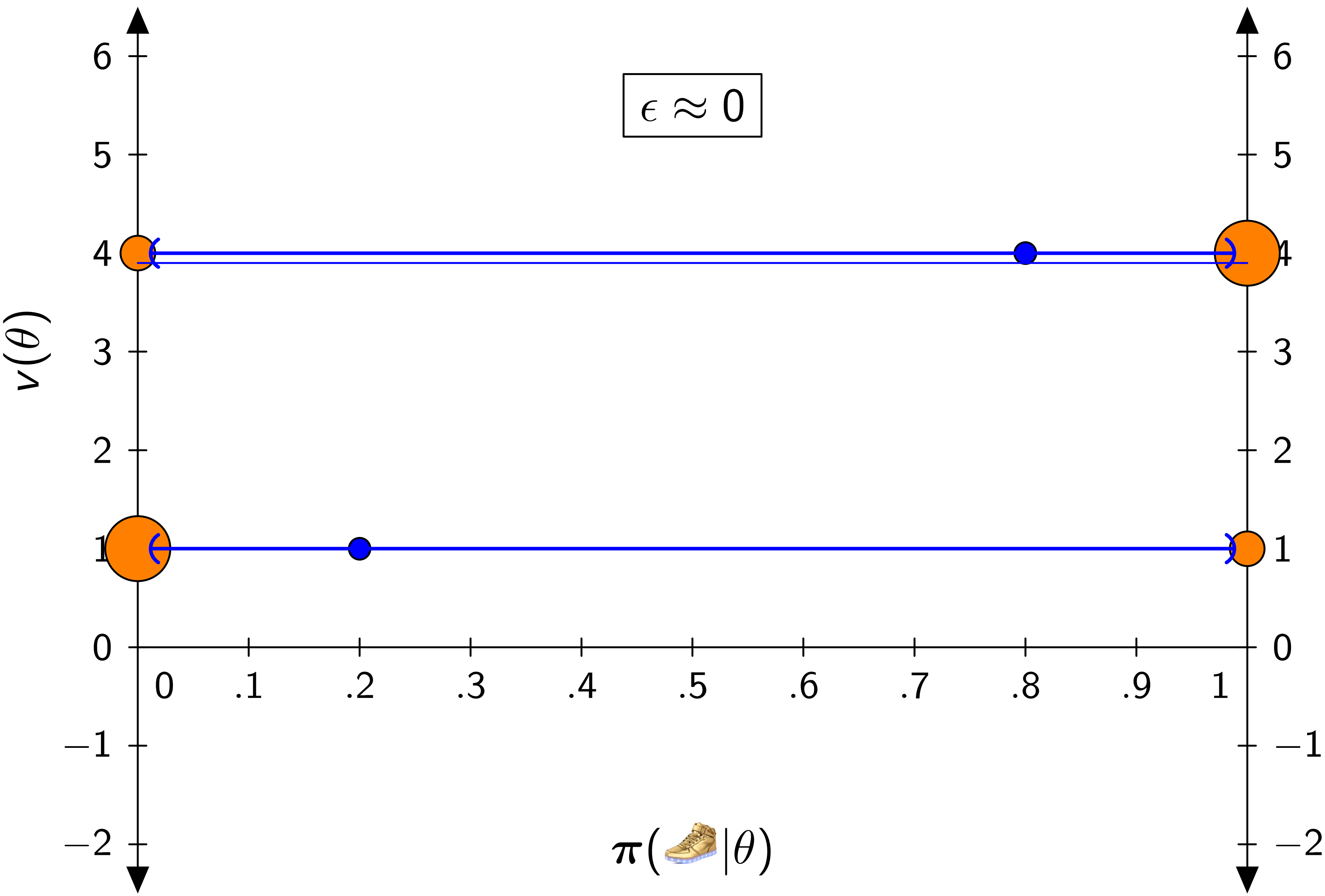

Revenue Guarantee for ϵ-Robust Mechanism Design

How do ϵ-robust mechanisms perform?

- By our impossibility result, may perform arbitrarily badly
- We require that beliefs by γ*-separated*

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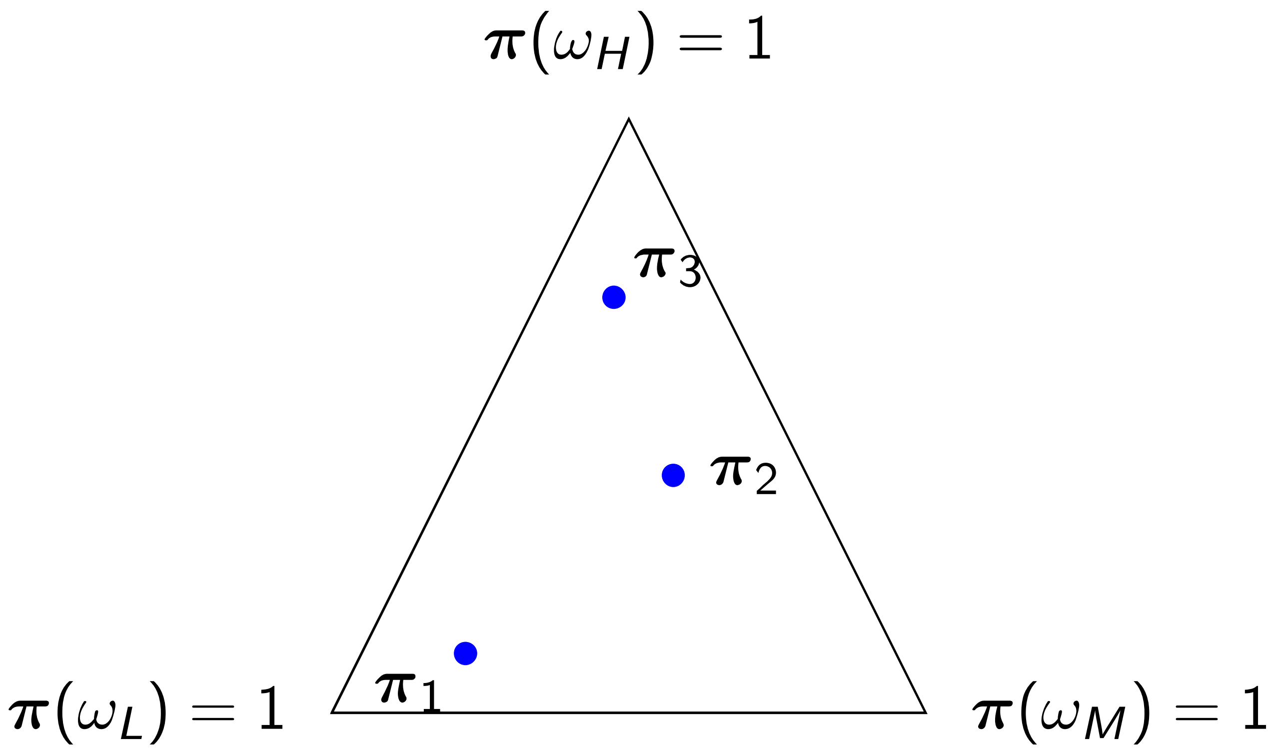

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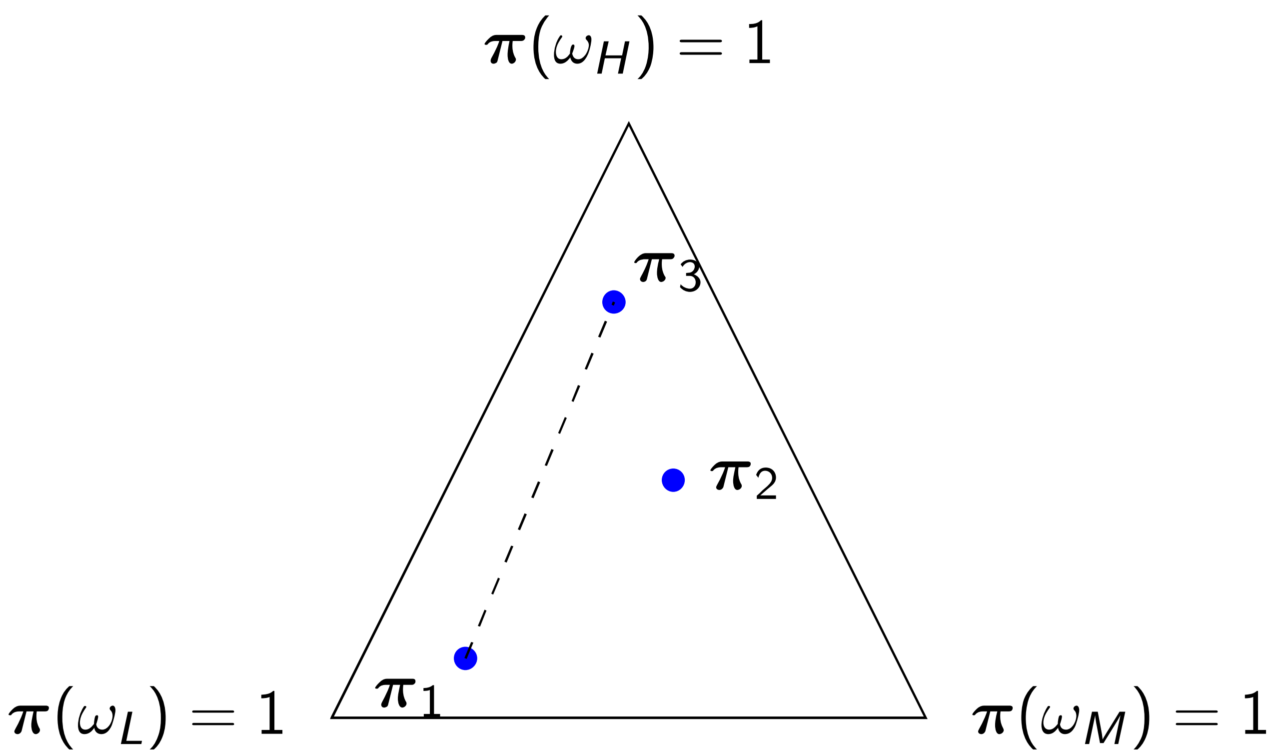

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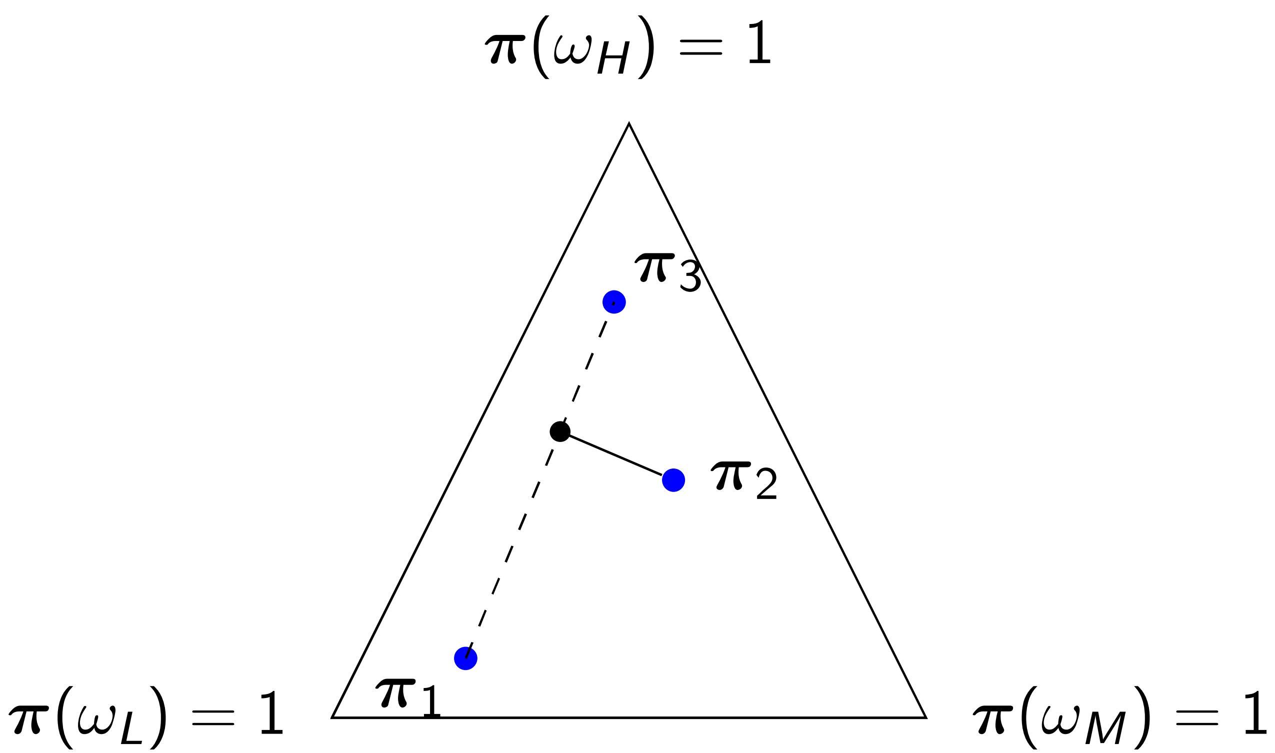

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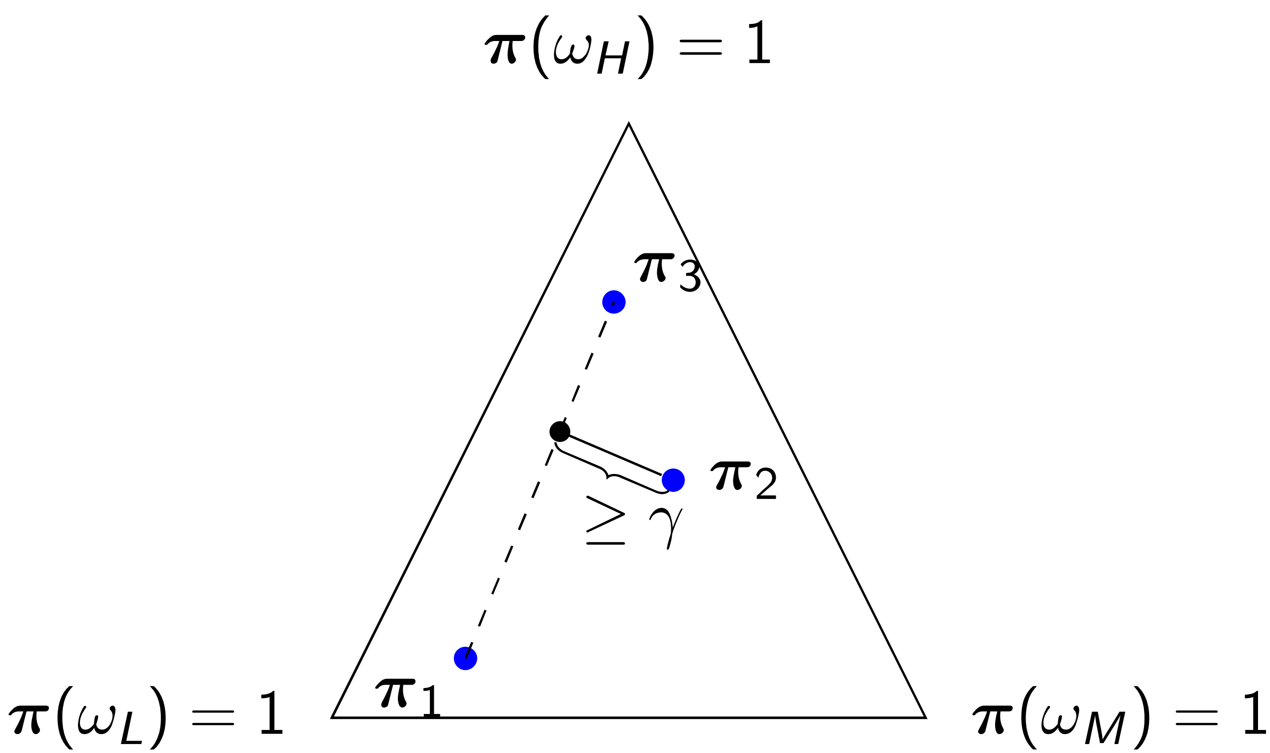

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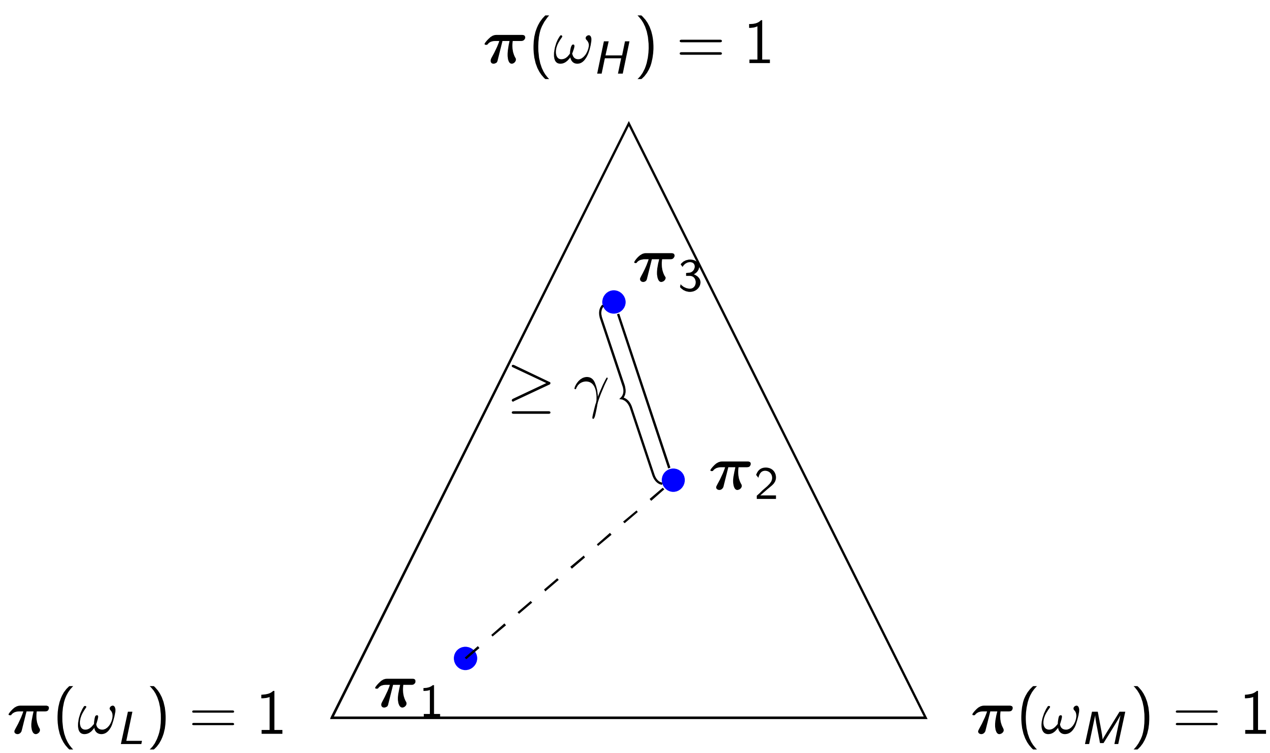

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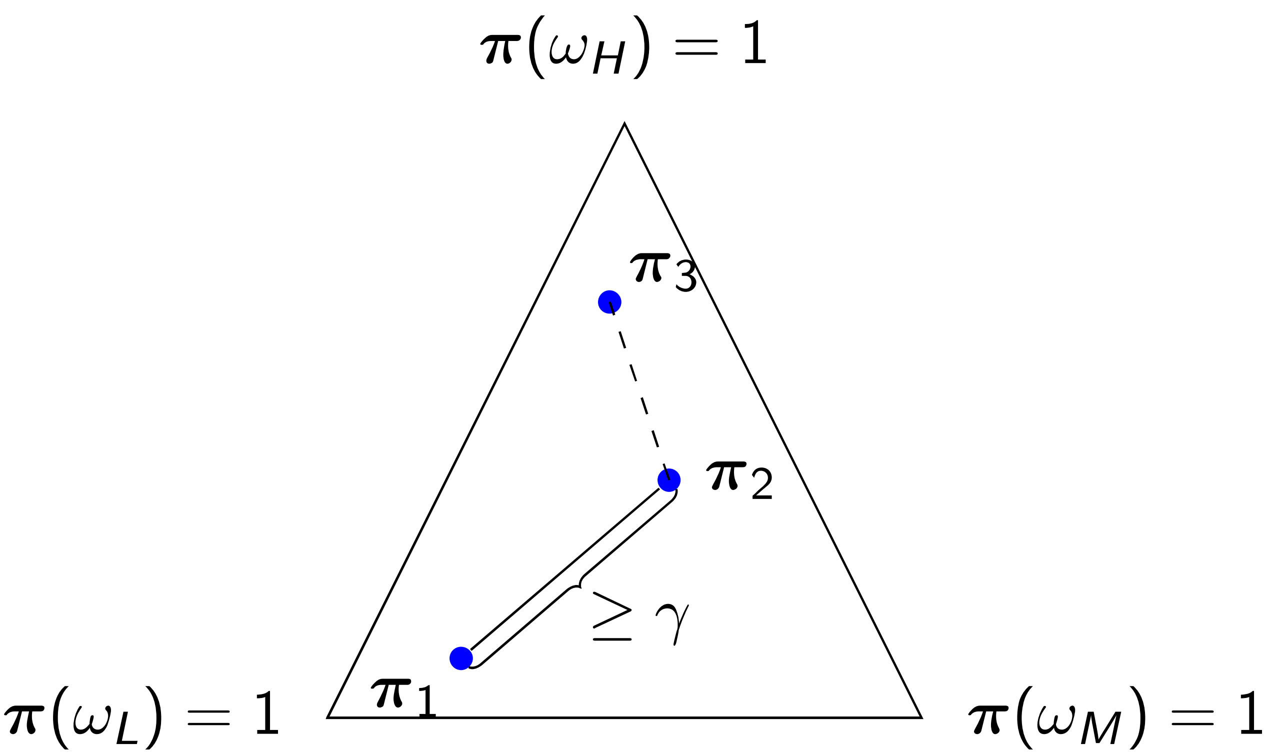

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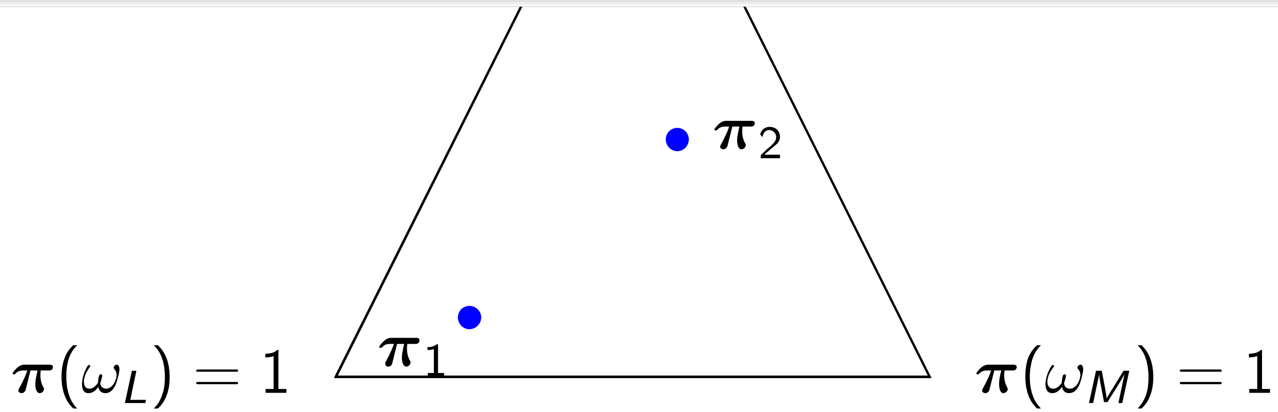
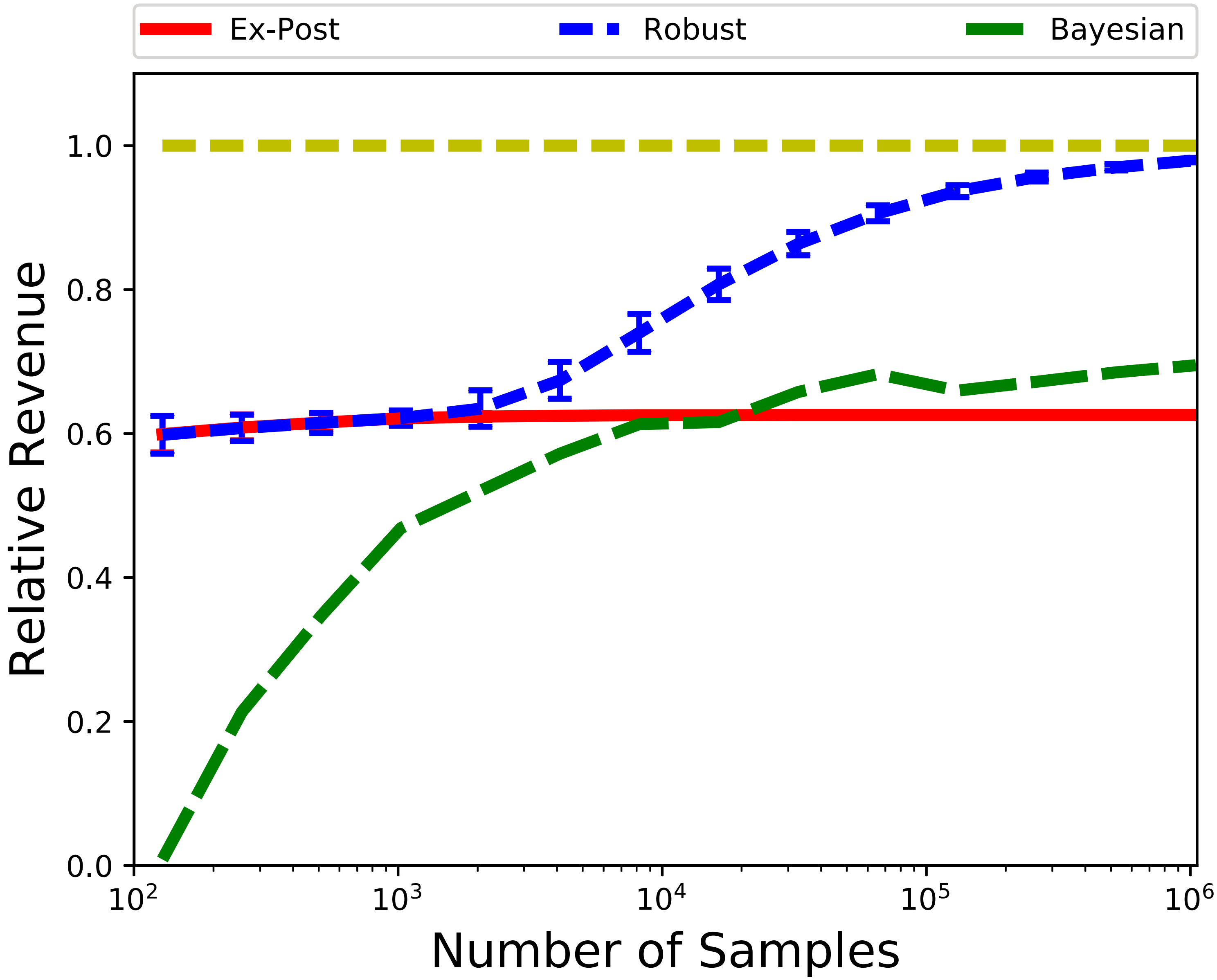
Sample Complexity of ϵ-Robust Mechanism Design

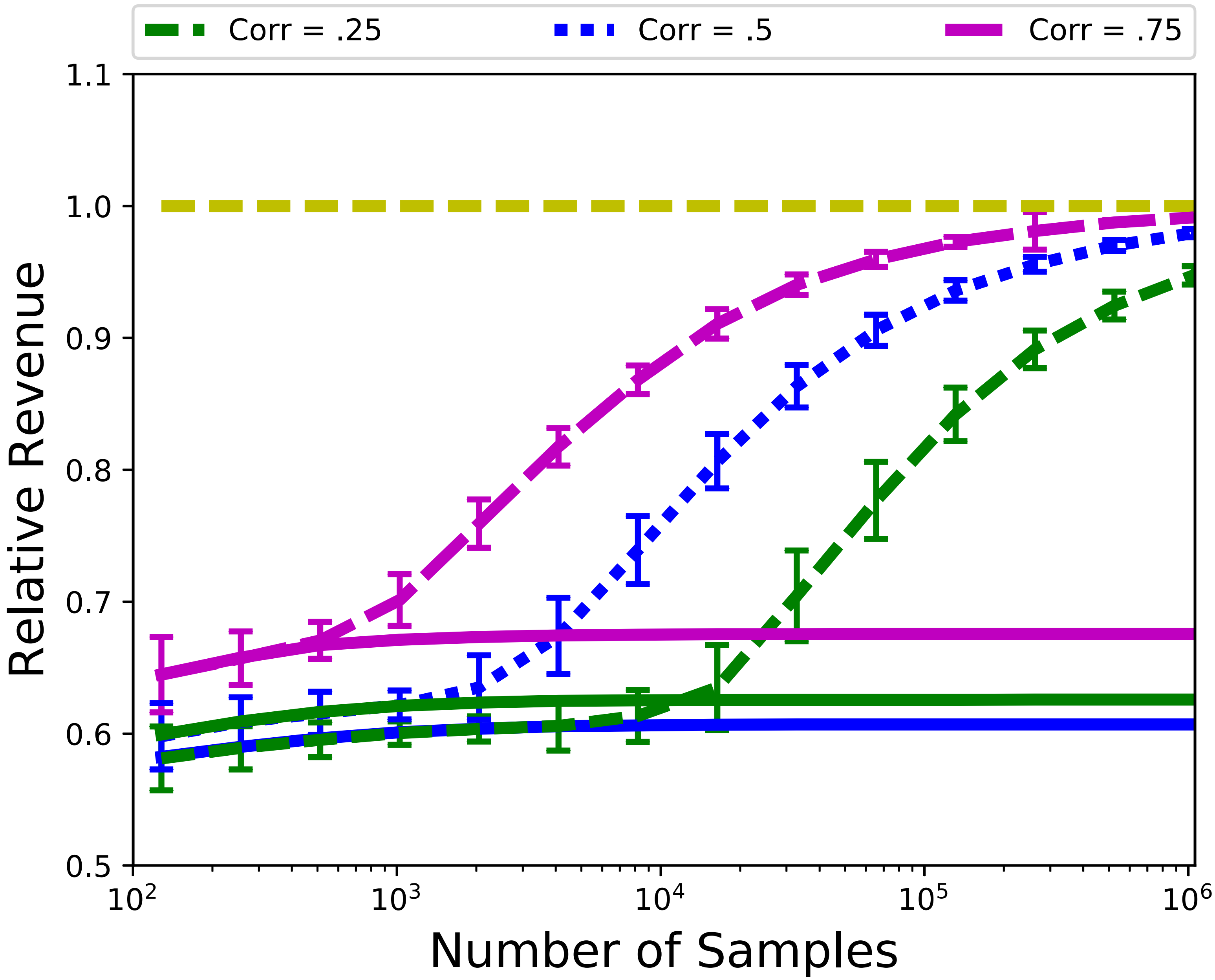
The **sample complexity** *for constructing an ϵ-robust mechanism that is an* **additive** k-**approximation** *to the optimal mechanism is* $\mathbf{O}(\mathbf{poly}(\frac{1}{\mathbf{k}}, \frac{1}{\gamma}, |\Theta|, |\Omega|, \mathbf{v}(|\Theta|)))$. Proof



Simulations

- True distribution is discretized bivariate normal distribution
- Sample from the true distribution N times
- Use Bayesian methods to estimate the distribution
- Calculate empirical confidence intervals for elements of the distribution
- Parameters unless otherwise specified:
  - Correlation $= .5$
  - $\epsilon = .05$
  - $\Theta = \{1, 2, ..., 10\}$
  - $|\Omega| = 10$
  - $v(\theta) = \theta$


Ex-Post
Robust
Bayesian
Relative Revenue
0.0
0.2
0.4
0.6
0.8
1.0
10^2
10^3
10^4
10^5
10^6
Number of Samples


Corr = .25
Corr = .5
Corr = .75
Relative Revenue
1.1
1.0
0.9
0.8
0.7
0.6
0.5
10^2
10^3
10^4
10^5
10^6
Number of Samples