Introduction to Voting

4x😐 : 🎅>👷>👮

3x😐 : 👮>👷>🎅

2x😐 : 👷>👮>🎅

Introduction to Voting

4x😐 : 🎅>👷>👮

3x😐 : 👮>👷>🎅

2x😐 : 👷>👮>🎅

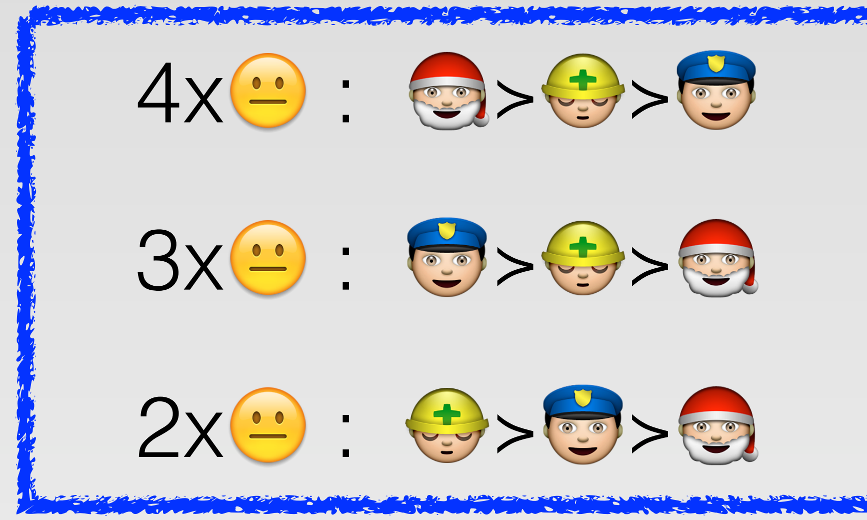
Pairwise comparisons:

👷>👮

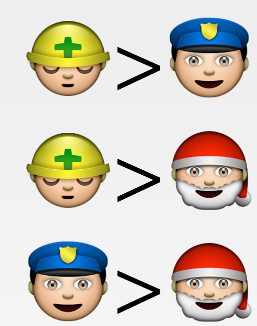
👷>🎅

👮>🎅

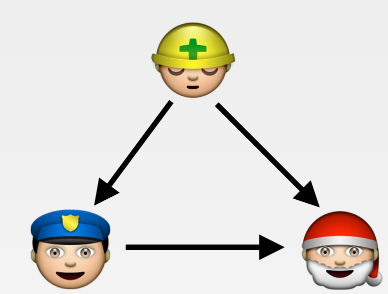
Introduction to Voting



Pairwise comparisons:



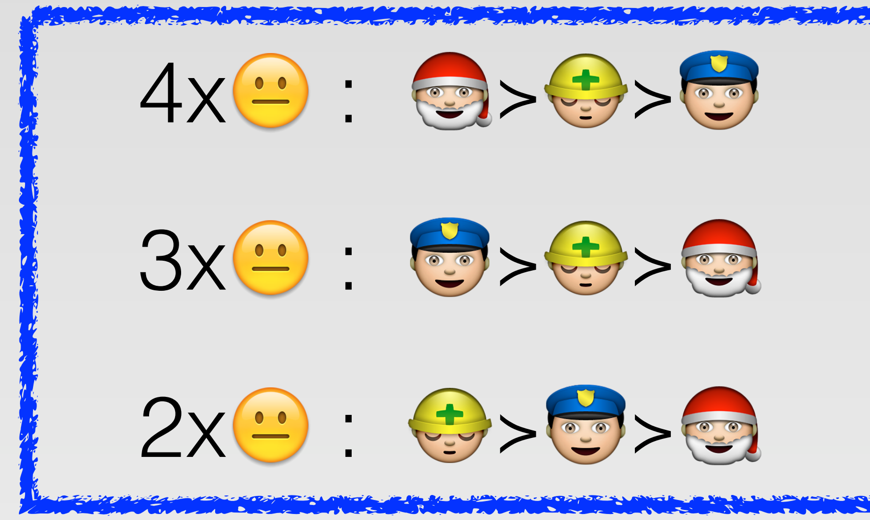
Majority tournament:



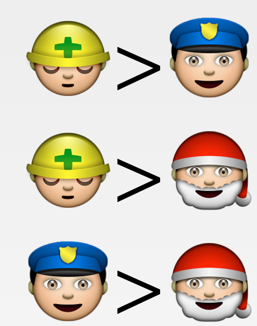
Duke UNIVERSITY

Introduction to Voting

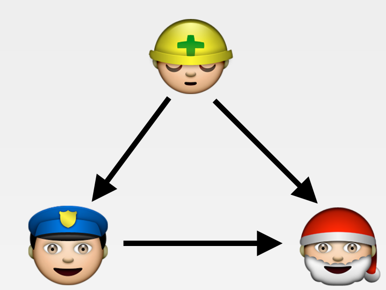
- A candidate is a **Condorcet winner** if he wins all pairwise comparisons



Pairwise comparisons:

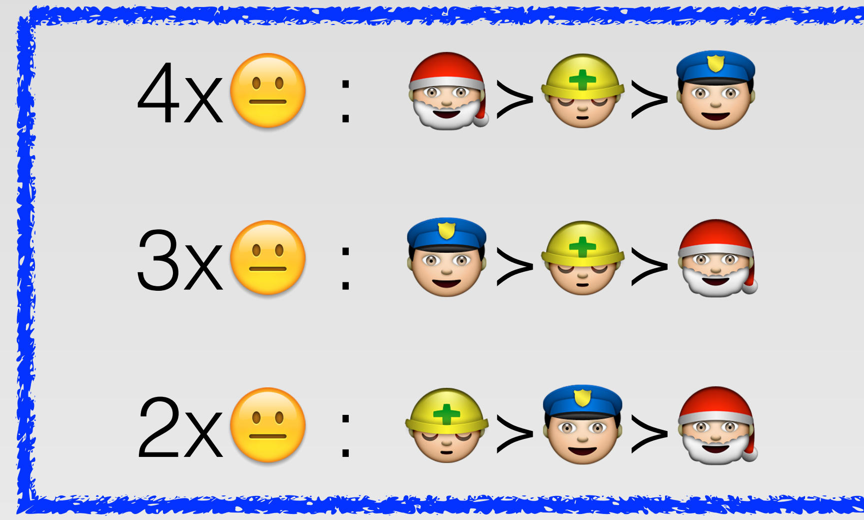


Majority tournament:

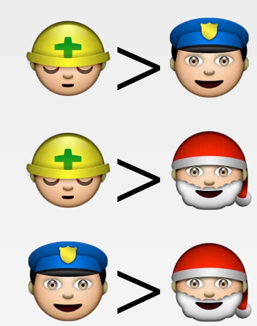


Introduction to Voting

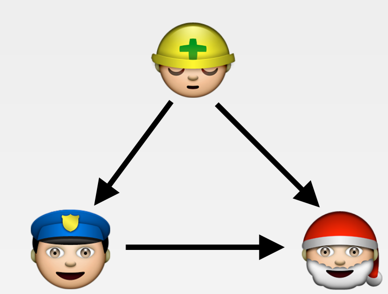
- A candidate is a **Condorcet winner** if he wins all pairwise comparisons
  - a rule is **Condorcet-consistent** if it selects a Condorcet winner whenever one exists



Pairwise comparisons:

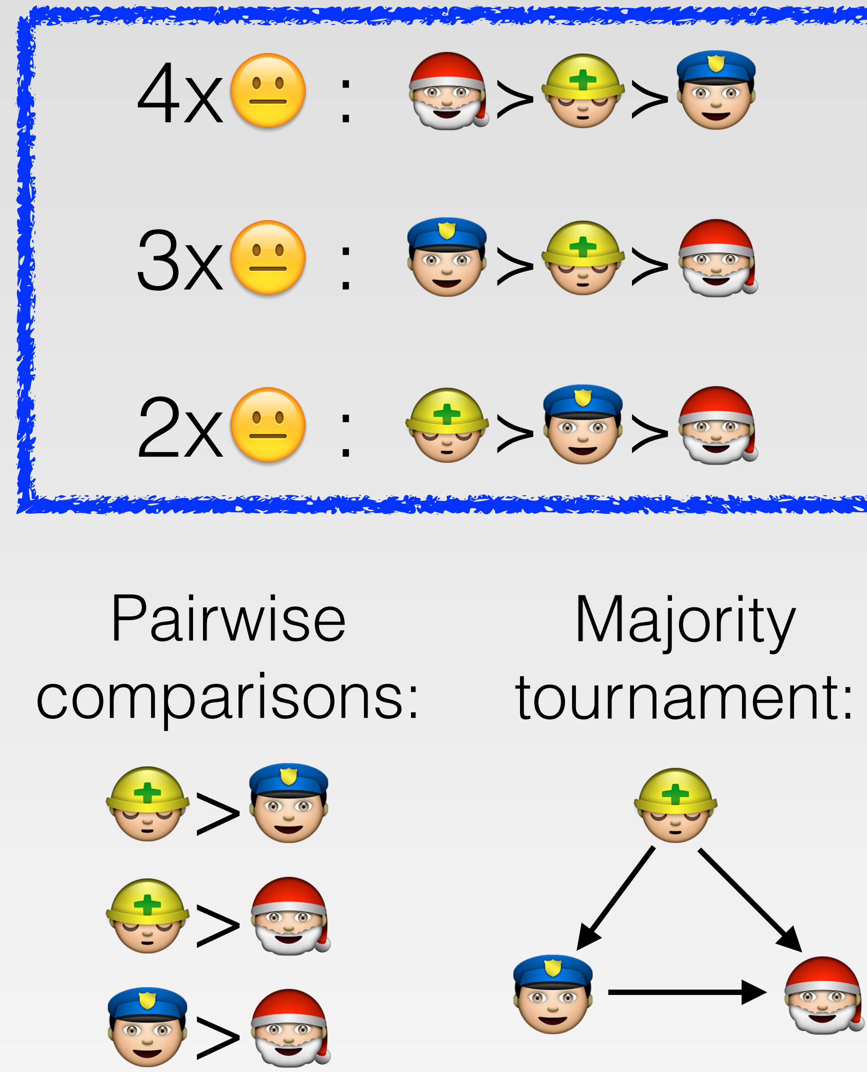


Majority tournament:



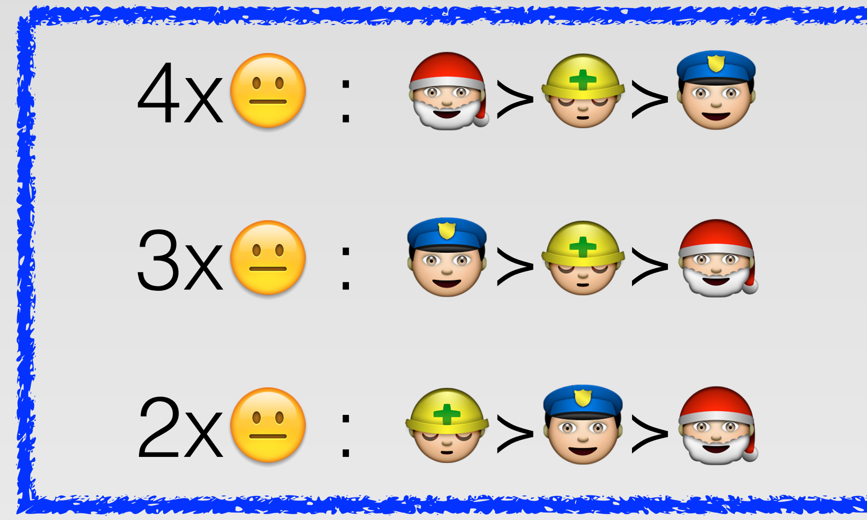
Introduction to Voting

- A candidate is a **Condorcet winner** if he wins all pairwise comparisons
  - a rule is **Condorcet-consistent** if it selects a Condorcet winner whenever one exists
- A candidate is a **majority winner** if he is ranked first by a majority of voters

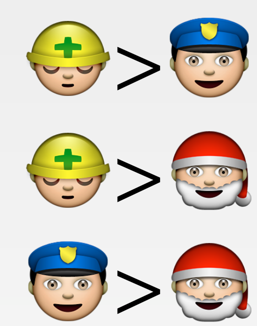


Introduction to Voting

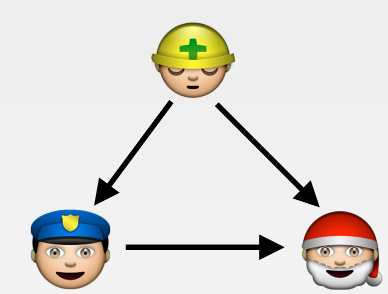
- A candidate is a **Condorcet winner** if he wins all pairwise comparisons
  - a rule is **Condorcet-consistent** if it selects a Condorcet winner whenever one exists
- A candidate is a **majority winner** if he is ranked first by a majority of voters
  - a rule is **majority-consistent** if it selects a majority winner whenever one exists



Pairwise comparisons:

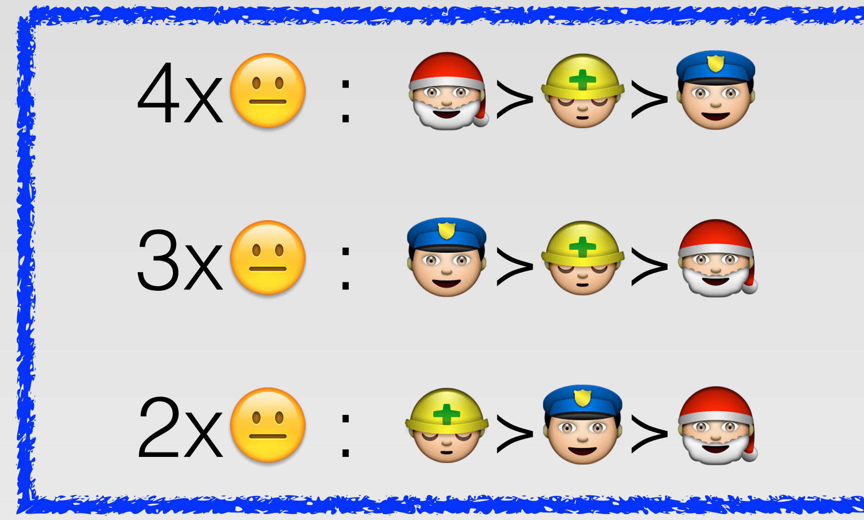


Majority tournament:

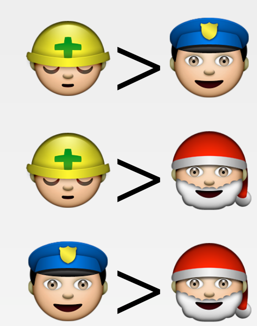


Introduction to Voting

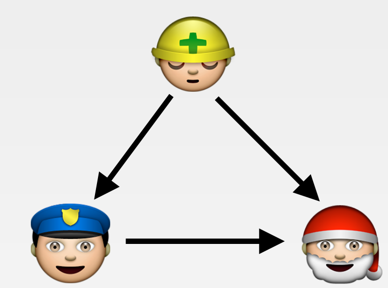
- A candidate is a **Condorcet winner** if he wins all pairwise comparisons
  - a rule is **Condorcet-consistent** if it selects a Condorcet winner whenever one exists
- A candidate is a **majority winner** if he is ranked first by a majority of voters
  - a rule is **majority-consistent** if it selects a majority winner whenever one exists
- Condorcet-consistency implies majority-consistency



Pairwise comparisons:



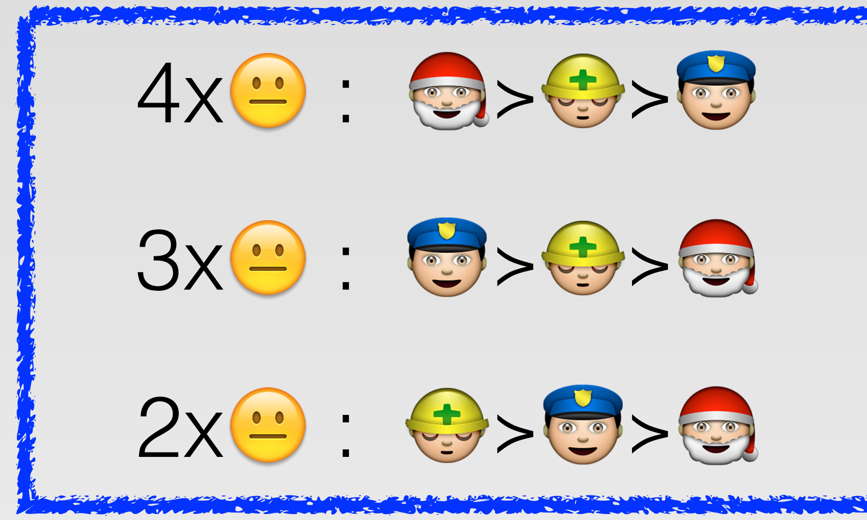
Majority tournament:



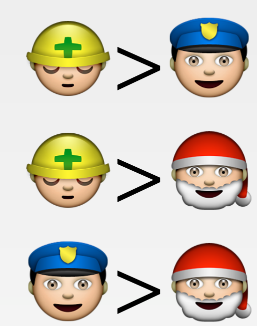
Duke UNIVERSITY

Introduction to Voting

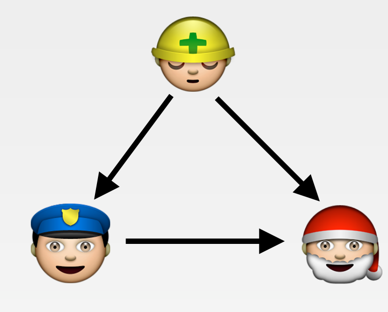
- A candidate is a **Condorcet winner** if he wins all pairwise comparisons
  - a rule is **Condorcet-consistent** if it selects a Condorcet winner whenever one exists
- A candidate is a **majority winner** if he is ranked first by a majority of voters
  - a rule is **majority-consistent** if it selects a majority winner whenever one exists
- Condorcet-consistency implies majority-consistency
  - a majority winner is also a Condorcet winner



Pairwise comparisons:



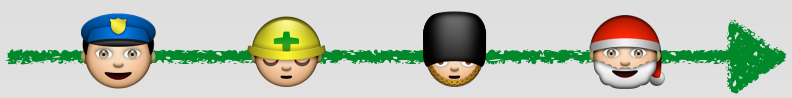
Majority tournament:



Duke UNIVERSITY

Single-Peaked Preferences

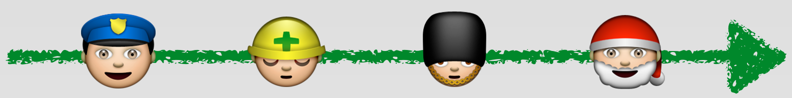
Single-Peaked Preferences

- Let **L** be a linear ordering of the candidates 

Duke UNIVERSITY

Single-Peaked Preferences

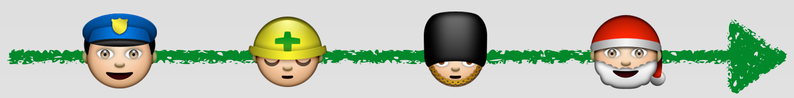
- Let **L** be a linear ordering of the candidates

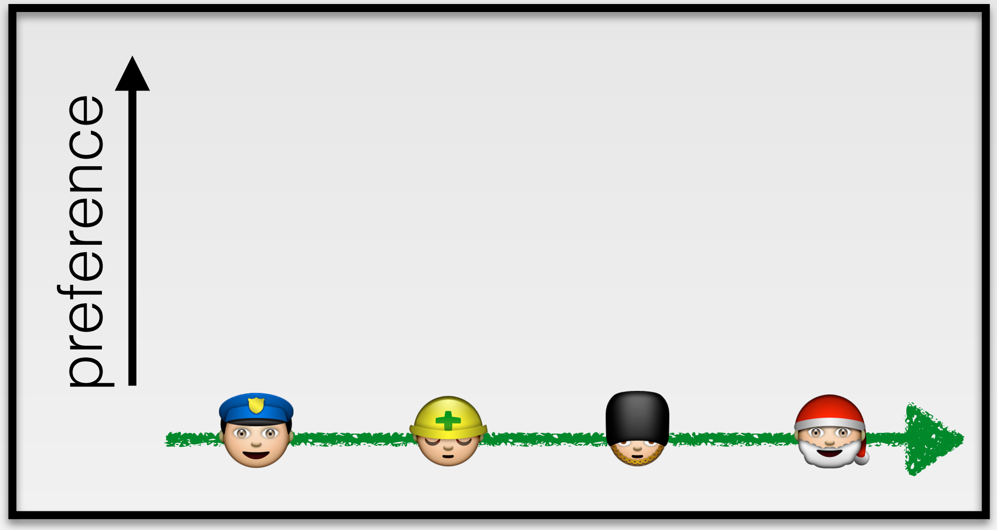


- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak

Duke UNIVERSITY

Single-Peaked Preferences

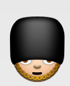
- Let **L** be a linear ordering of the candidates 
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak

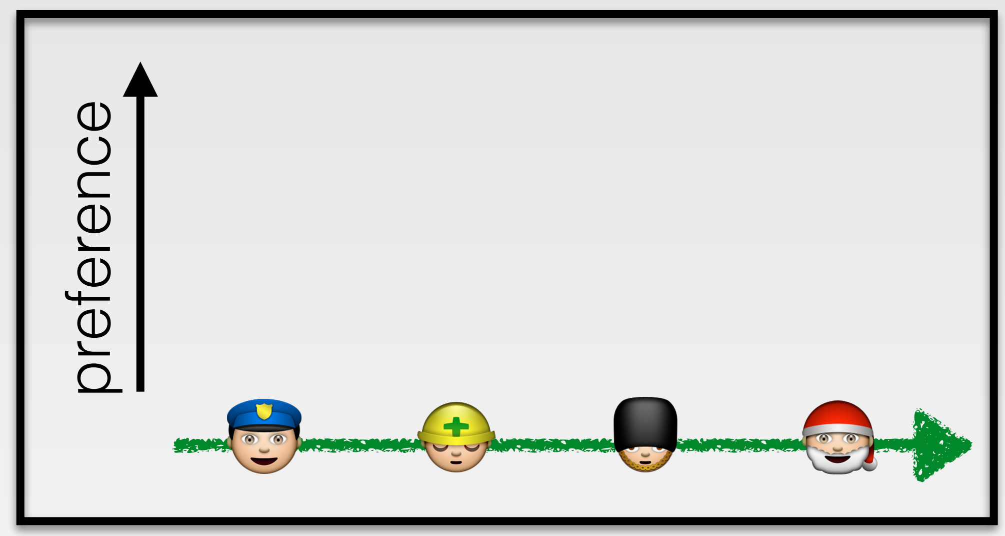


Duke UNIVERSITY

Single-Peaked Preferences

- Let **L** be a linear ordering of the candidates

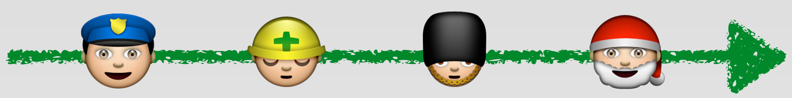
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak
  -  >  >  >  is **single-peaked** w.r.t. **L**



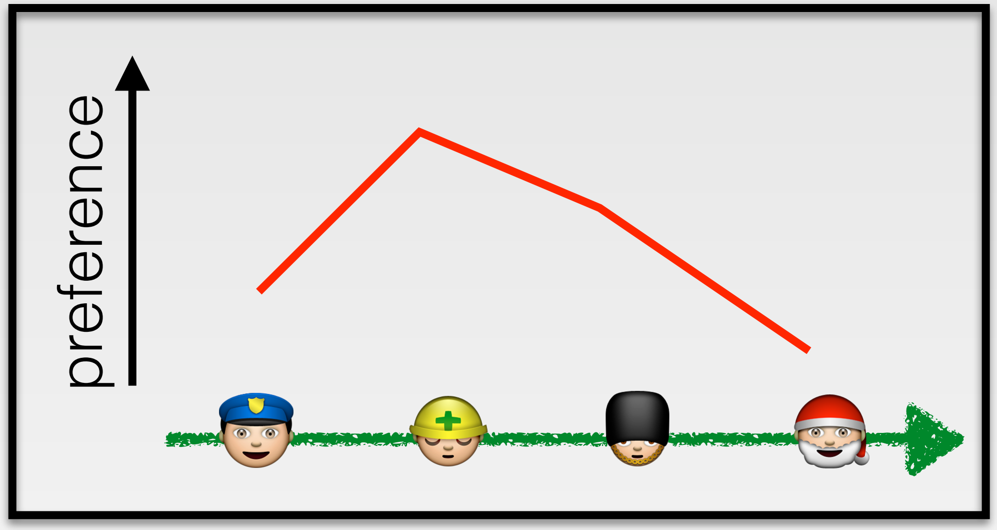
Duke UNIVERSITY

Single-Peaked Preferences

- Let **L** be a linear ordering of the candidates

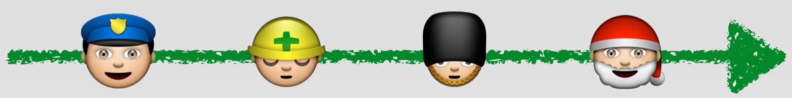


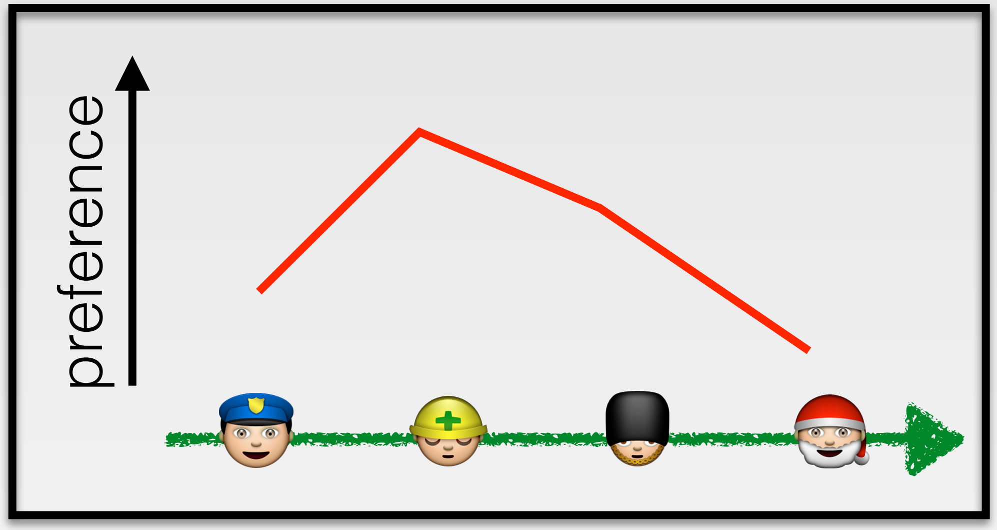
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak
  - 👷 > 💂 > 👮 > 🎅 is **single-peaked** w.r.t. **L**



Duke UNIVERSITY

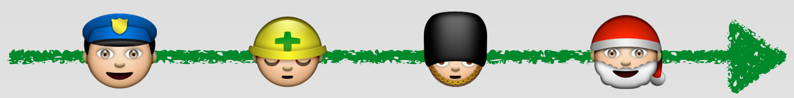
Single-Peaked Preferences

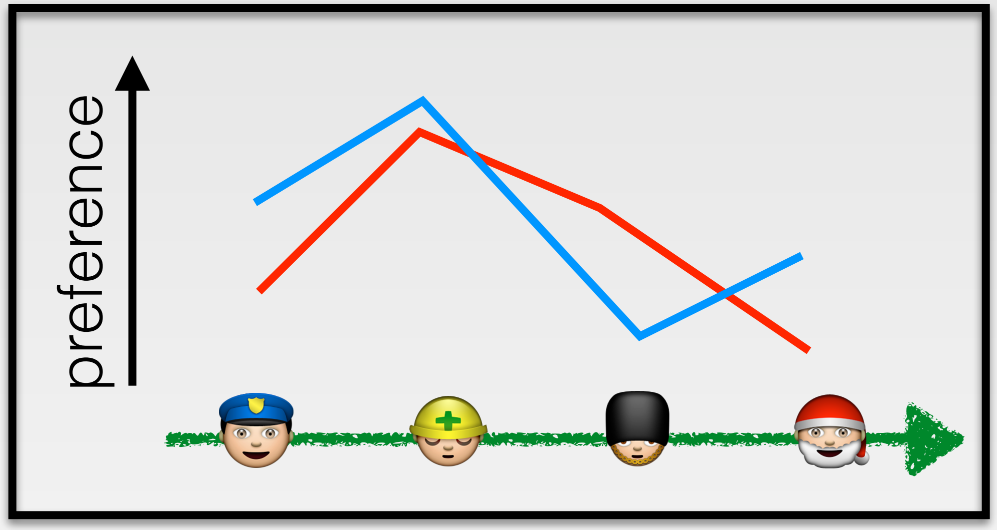
- Let **L** be a linear ordering of the candidates 
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak
  - 👷 > 💂 > 👮 > 🎅 is **single-peaked** w.r.t. **L**
  - 👷 > 👮 > 🎅 > 💂 is **not single-peaked** w.r.t. **L**



Duke UNIVERSITY

Single-Peaked Preferences

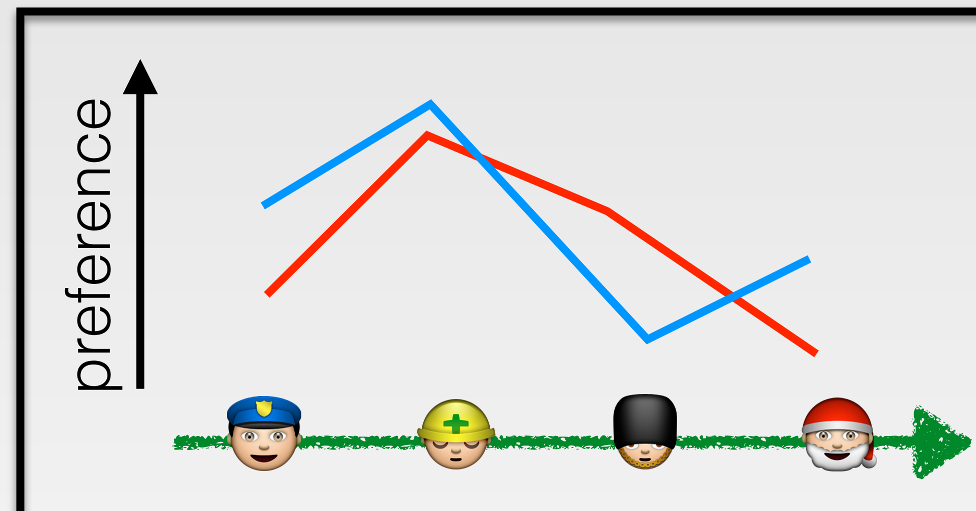
- Let **L** be a linear ordering of the candidates 
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak
  - 👷 > 💂 > 👮 > 🎅 is **single-peaked** w.r.t. **L**
  - 👷 > 👮 > 🎅 > 💂 is **not single-peaked** w.r.t. **L**



Duke UNIVERSITY

Single-Peaked Preferences

- Let **L** be a linear ordering of the candidates 
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak
  -  is **single-peaked** w.r.t. **L**
  -  is **not single-peaked** w.r.t. **L**



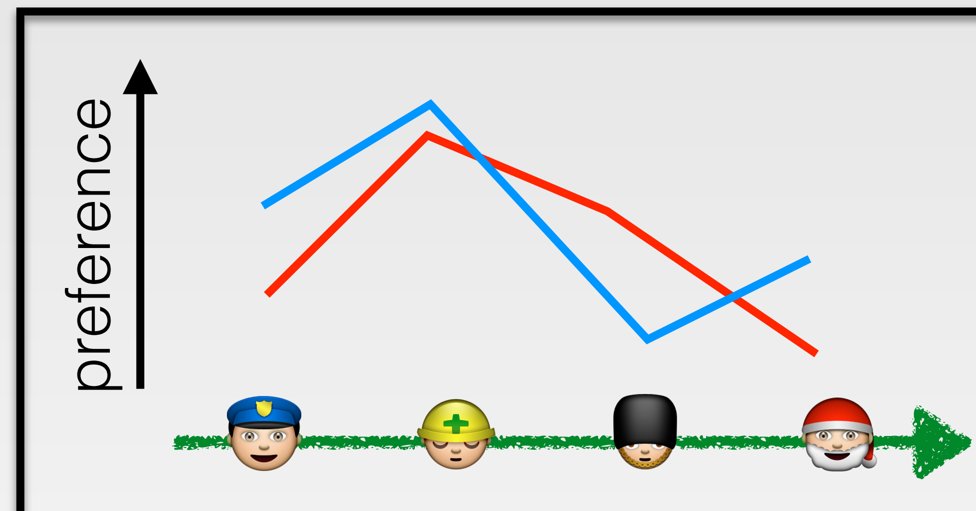
- Every single-peaked preference profile has a Condorcet winner

Duke UNIVERSITY

Single-Peaked Preferences

- Let **L** be a linear ordering of the candidates

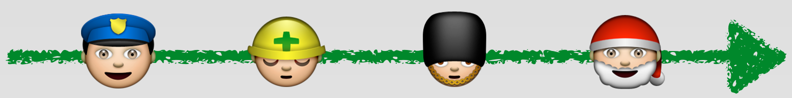
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak
  - 👷 > 💂 > 👮 > 🎅 is **single-peaked** w.r.t. **L**
  - 👷 > 👮 > 🎅 > 💂 is **not single-peaked** w.r.t. **L**

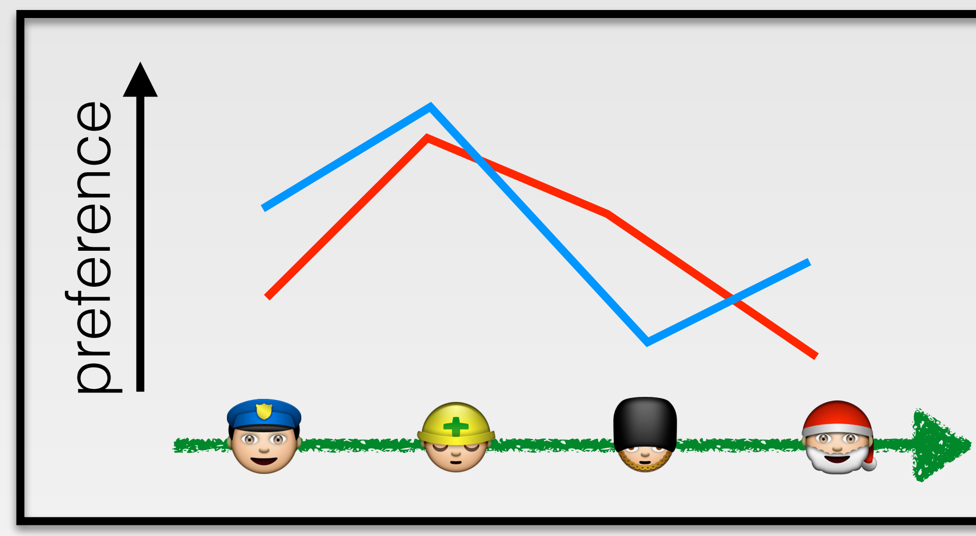


- Every single-peaked preference profile has a Condorcet winner
  - Condorcet winner is the most preferred alternative of the **median voter**

Duke UNIVERSITY

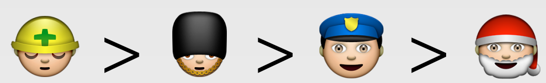
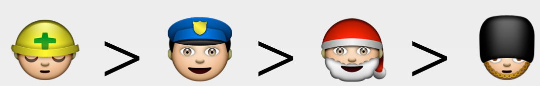
Single-Peaked Preferences

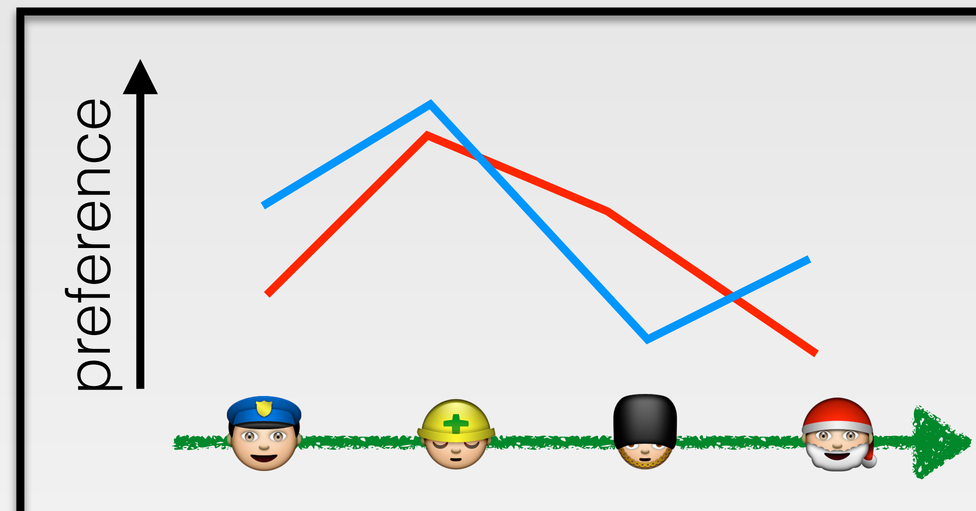
- Let **L** be a linear ordering of the candidates 
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak
  -  is **single-peaked** w.r.t. **L**
  -  is **not single-peaked** w.r.t. **L**



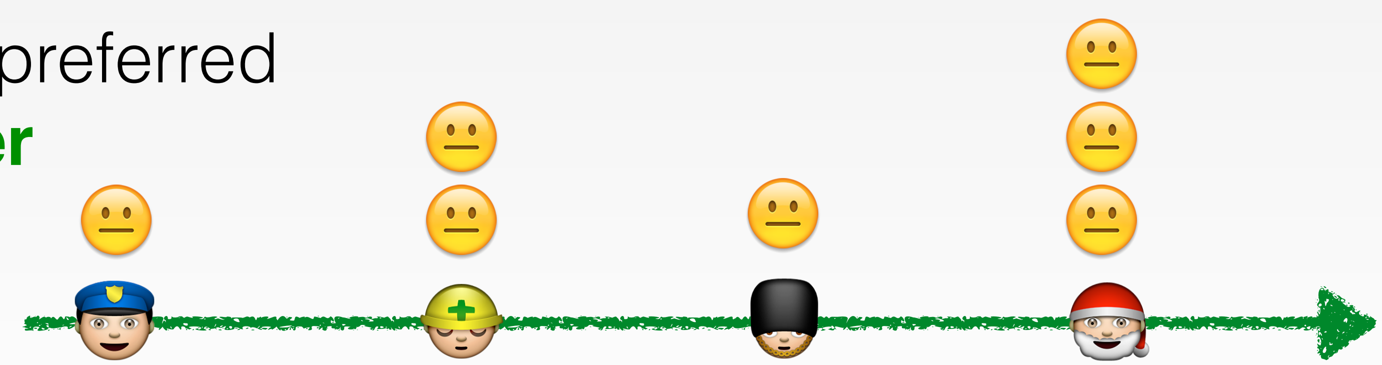
- Every single-peaked preference profile has a Condorcet winner
  - Condorcet winner is the most preferred alternative of the **median voter**

Single-Peaked Preferences

- Let **L** be a linear ordering of the candidates 
- A ranking is **single-peaked w.r.t. L** if its corresponding preference curve has a unique peak, and preference is declining if we move away from the peak
  -  is **single-peaked** w.r.t. **L**
  -  is **not single-peaked** w.r.t. **L**



- Every single-peaked preference profile has a Condorcet winner
  - Condorcet winner is the most preferred alternative of the **median voter**



Duke UNIVERSITY

Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**

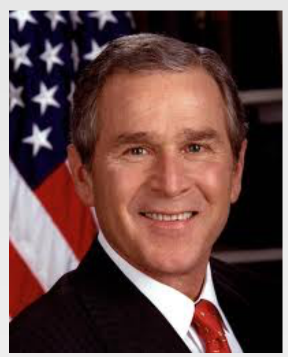
Duke UNIVERSITY

Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**
- This ignores **candidates' incentives** to enter/leave an election

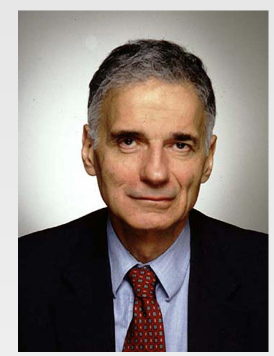
Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**
- This ignores **candidates' incentives** to enter/leave an election
  - 2000 US presidential election: Bush vs. Gore vs. Nader



~48%

~48%



~4%

Duke UNIVERSITY

Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**
- This ignores **candidates' incentives** to enter/leave an election
  - 2000 US presidential election: Bush vs. Gore vs. Nader

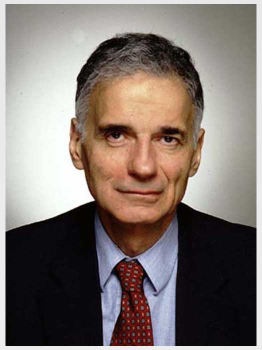
Duke UNIVERSITY

Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**
- This ignores **candidates' incentives** to enter/leave an election
  - 2000 US presidential election: Bush vs. Gore vs. Nader

~48%

~48%



~4%

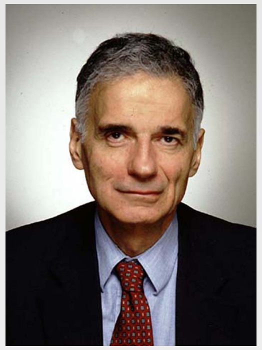
Duke UNIVERSITY

Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**
- This ignores **candidates' incentives** to enter/leave an election
  - 2000 US presidential election: Bush vs. Gore vs. Nader

~48%

~48%



~4%

- Dutta, Jackson, & Le Breton [Econometrica 2001]: **strategic candidacy**

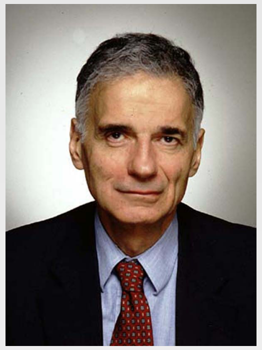
Duke UNIVERSITY

Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**
- This ignores **candidates' incentives** to enter/leave an election
  - 2000 US presidential election: Bush vs. Gore vs. Nader

~48%

~48%



~4%

- Dutta, Jackson, & Le Breton [Econometrica 2001]: **strategic candidacy**
- Most papers on strategic candidacy assume **truthful voting**

Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**
- This ignores **candidates' incentives** to enter/leave an election
  - 2000 US presidential election: Bush vs. Gore vs. Nader

~48% ~48% ~4%

- Dutta, Jackson, & Le Breton [Econometrica 2001]: **strategic candidacy**
- Most papers on strategic candidacy assume **truthful voting**
- Question: What if **both** voters and candidates act strategically?

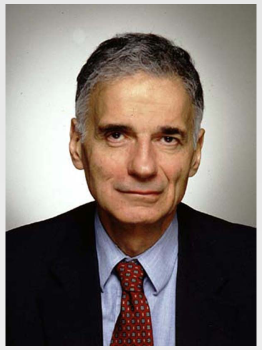
Duke UNIVERSITY

Strategic Voting & Strategic Candidacy

- Standard assumption in voting theory: set of candidates is **fixed**
- This ignores **candidates' incentives** to enter/leave an election
  - 2000 US presidential election: Bush vs. Gore vs. Nader

~48%

~48%



~4%

- Dutta, Jackson, & Le Breton [Econometrica 2001]: **strategic candidacy**
- Most papers on strategic candidacy assume **truthful voting**
- Question: What if **both** voters and candidates act strategically?
  - will this lead to "better" voting outcomes?

The Candidacy Game

- Finite set of candidates C = {👮,👷,💂,🎅, …}

The Candidacy Game

- Finite set of candidates C = {👮,👷,💂,🎅, …}
- Finite set of voters V = {😐,😐,😐, …}
  - we assume that |V| is odd

The Candidacy Game

- Finite set of candidates C = {👮,👷,💂,🎅, …}
- Finite set of voters V = {😐,😐,😐, …}
  - we assume that |V| is odd
- Both voters and candidates have **preferences over candidates**

😐 : 👮>👷>💂>🎅

👷 : 👷>🎅>👮>💂

The Candidacy Game

- Finite set of candidates C = {👮,👷,💂,🎅, …}
- Finite set of voters V = {😐,😐,😐, …}
  - we assume that |V| is odd
- Both voters and candidates have **preferences over candidates**

😐 : 👮>👷>💂>🎅   👷 : 👷>🎅>👮>💂

- Two-stage **game**
  - stage 1: candidates decide to run or not
  - stage 2: voters submit ranking of running candidates

The Candidacy Game

- Finite set of candidates C = {👮,👷,💂,🎅, …}
- Finite set of voters V = {😐,😐,😐, …}
  - we assume that |V| is odd
- Both voters and candidates have **preferences over candidates**

  😐 : 👮>👷>💂>🎅   👷 : 👷>🎅>👮>💂

- Two-stage **game**
  - stage 1: candidates decide to run or not
  - stage 2: voters submit ranking of running candidates
- What are the **equilibrium outcomes** of this game?
  - setting 1: single-peaked preferences and majority-consistent voting rules

Related Work

- Dutta, Jackson, & Le Breton [Econometrica 2001]: **impossibility result**
  - alternative proofs and extensions: Ehlers & Weymark [ET 2003], Eraslan & McLennan [JET 2004], Samejima [Econ Lett 2005], Rodríguez-Álvarez [ET 2006, SCW 2006]
- Dutta, Jackson, & Le Breton [JET 2002]: binary voting rules
  - characterization of equilibrium outcomes for **successive elimination**
- Samejima [Jap Econ Rev 2007]: **single-peaked preferences**
  - characterization of voting rules that never give candidates incentives not to run
- Lang, Maudet, & Polukarov [SAGT 2013]: existence of **pure equilibria**

Duke UNIVERSITY

Setting

- Preferences of voters and candidates are **single-peaked**
  - there exists a Condorcet winner, denoted 🎅

Setting

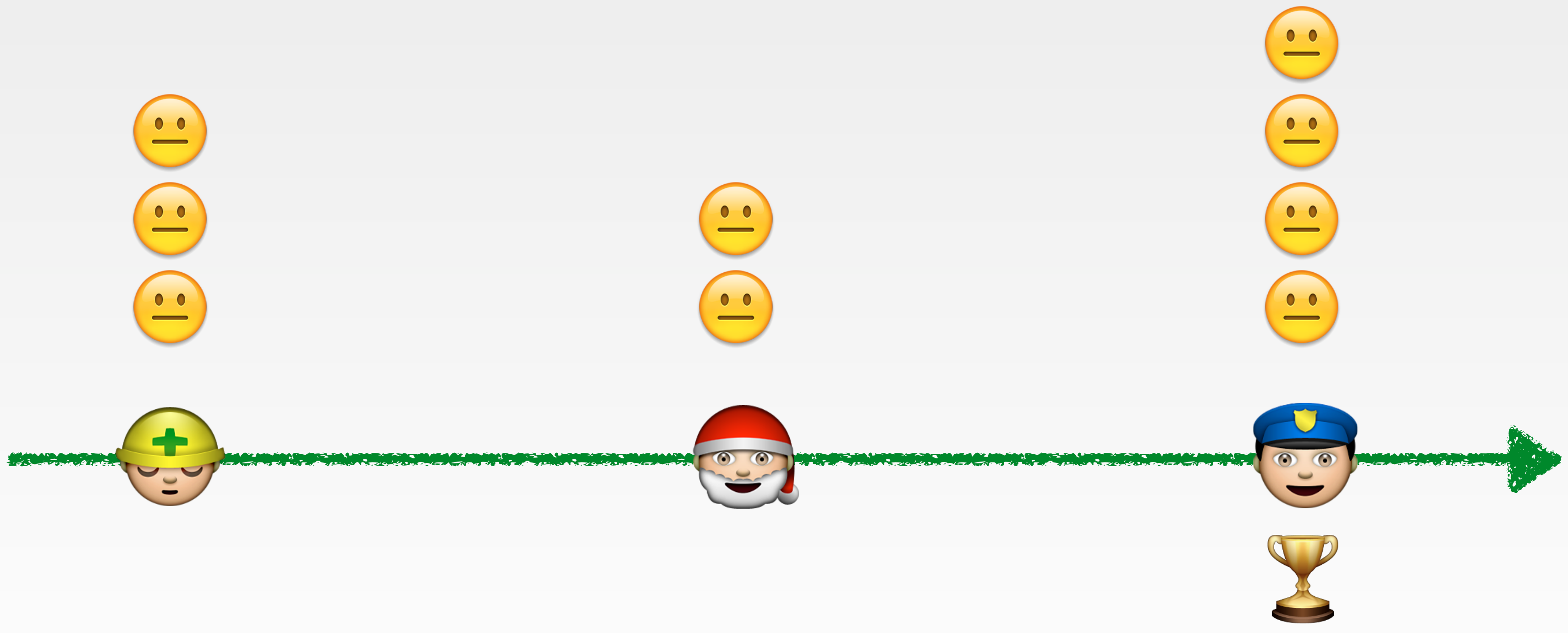
- Preferences of voters and candidates are **single-peaked**
  - there exists a Condorcet winner, denoted 🎅

Setting

- Preferences of voters and candidates are **single-peaked**
  - there exists a Condorcet winner, denoted 🎅
- **Majority-consistent** voting rules
  - examples: **plurality**, plurality with runoff, instant runoff (STV)

Setting

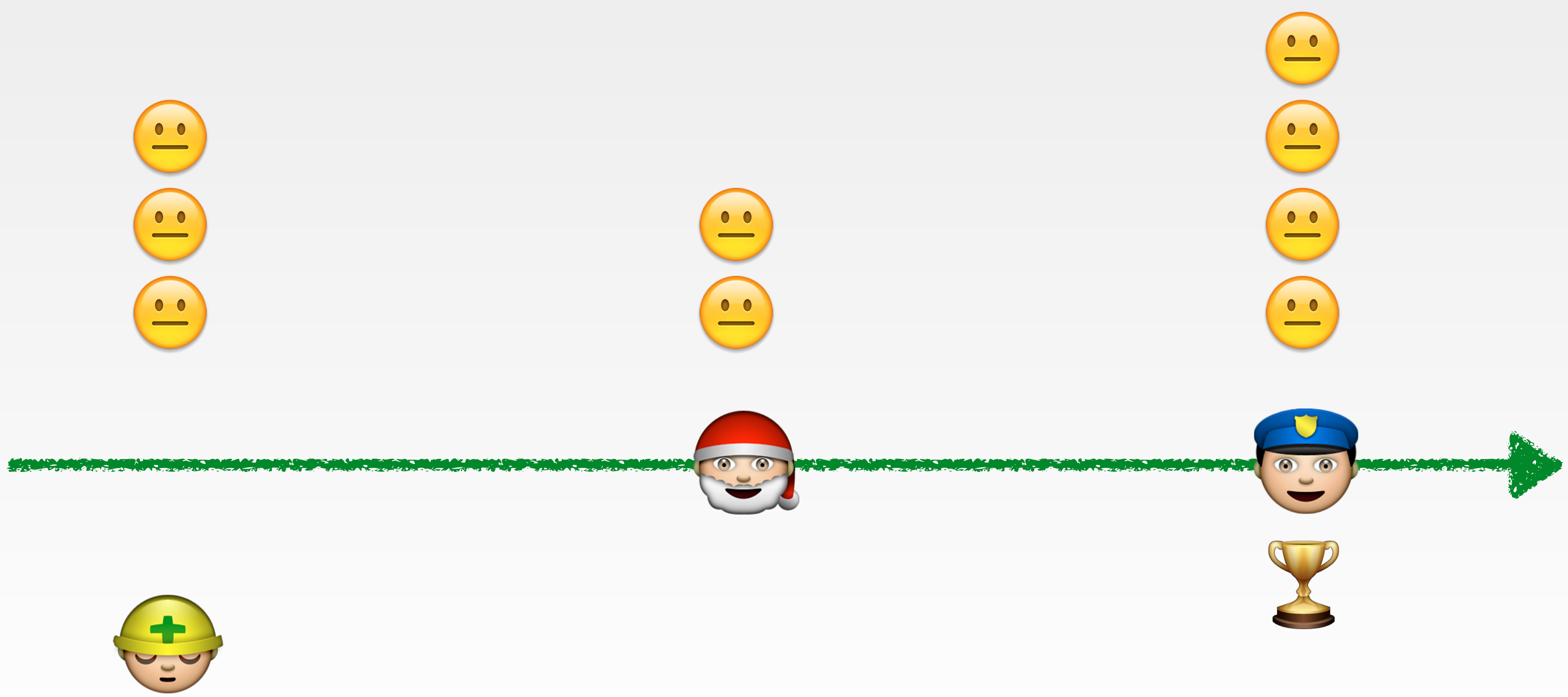
- Preferences of voters and candidates are **single-peaked**
  - there exists a Condorcet winner, denoted 🎅
- **Majority-consistent** voting rules
  - examples: **plurality**, plurality with runoff, instant runoff (STV)



Duke UNIVERSITY

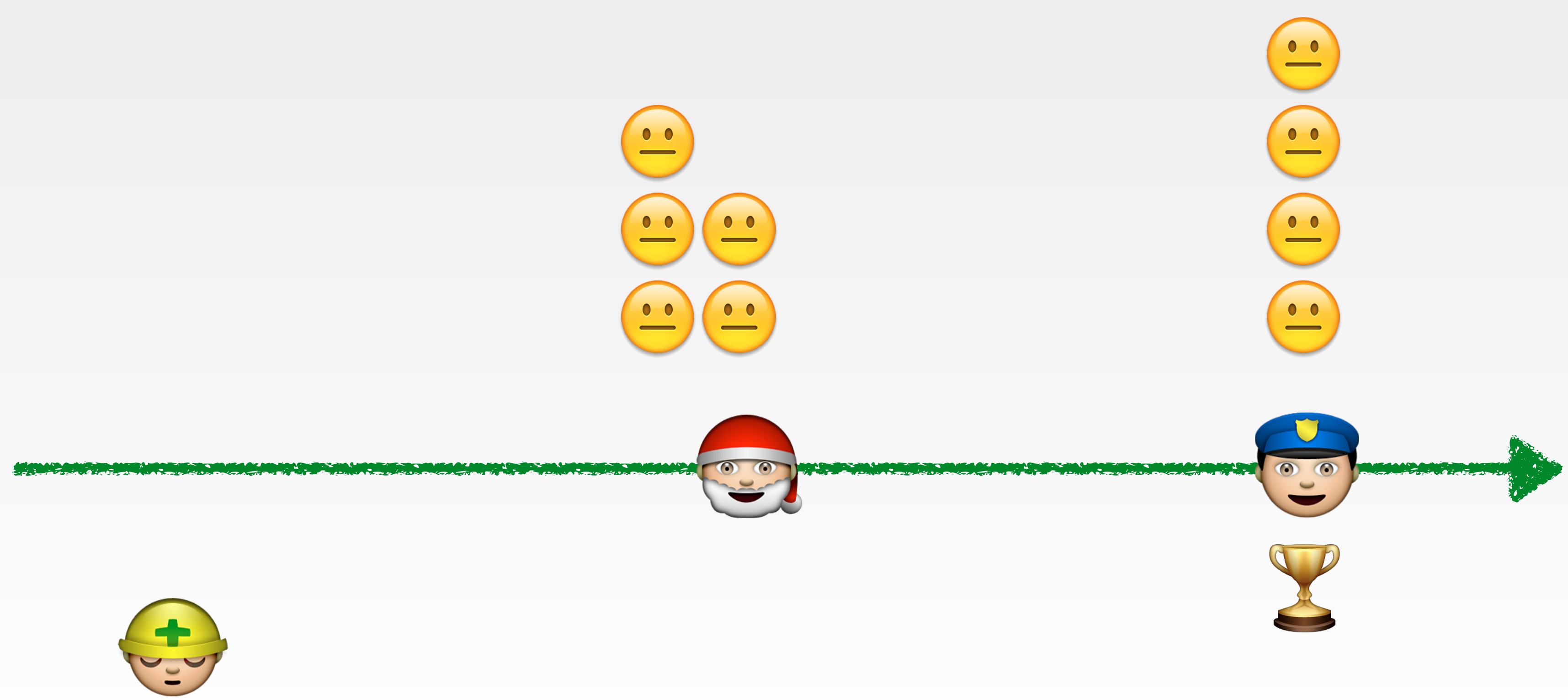
Setting

- Preferences of voters and candidates are **single-peaked**
  - there exists a Condorcet winner, denoted 🎅
- **Majority-consistent** voting rules
  - examples: **plurality**, plurality with runoff, instant runoff (STV)



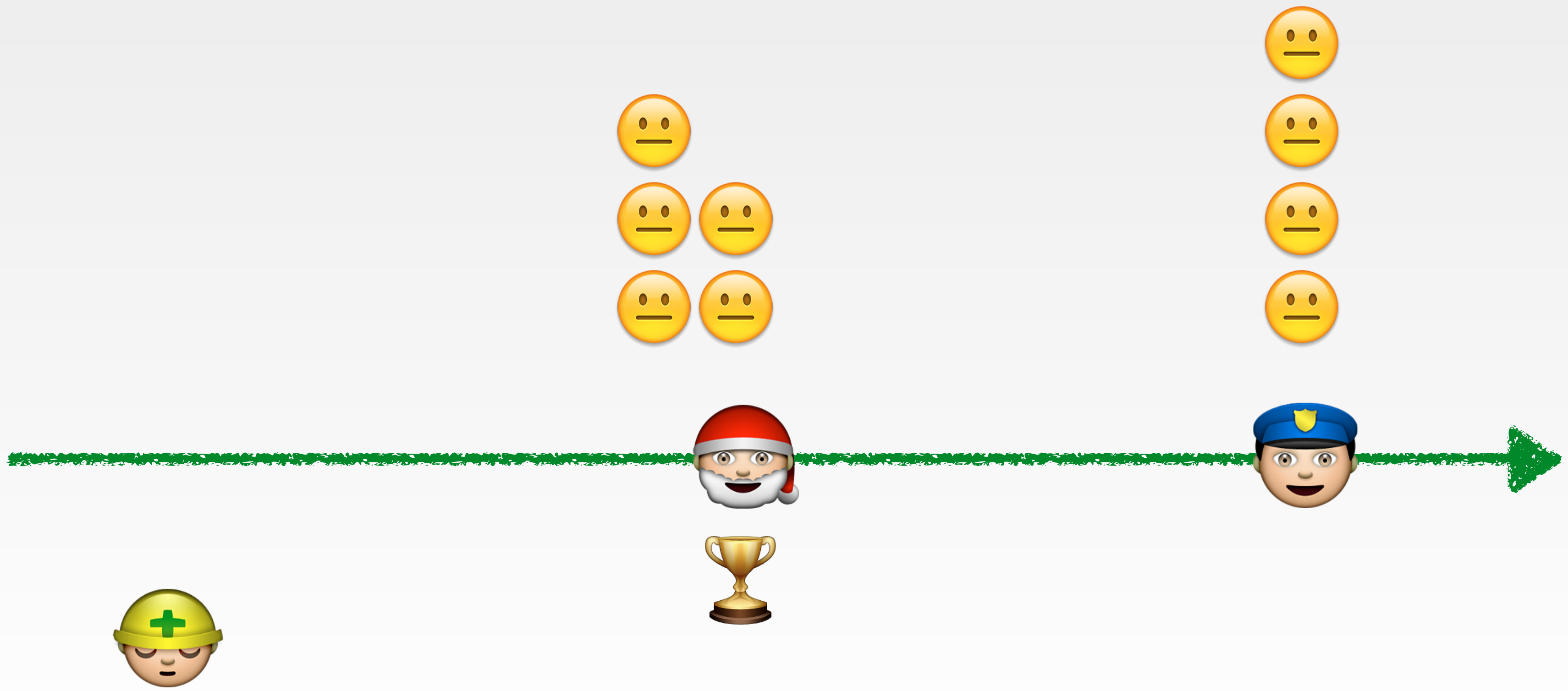
Setting

- Preferences of voters and candidates are **single-peaked**
  - there exists a Condorcet winner, denoted 🎅
- **Majority-consistent** voting rules
  - examples: **plurality**, plurality with runoff, instant runoff (STV)



Setting

- Preferences of voters and candidates are **single-peaked**
  - there exists a Condorcet winner, denoted 🎅
- **Majority-consistent** voting rules
  - examples: **plurality**, plurality with runoff, instant runoff (STV)



Duke UNIVERSITY

Equilibrium Concepts

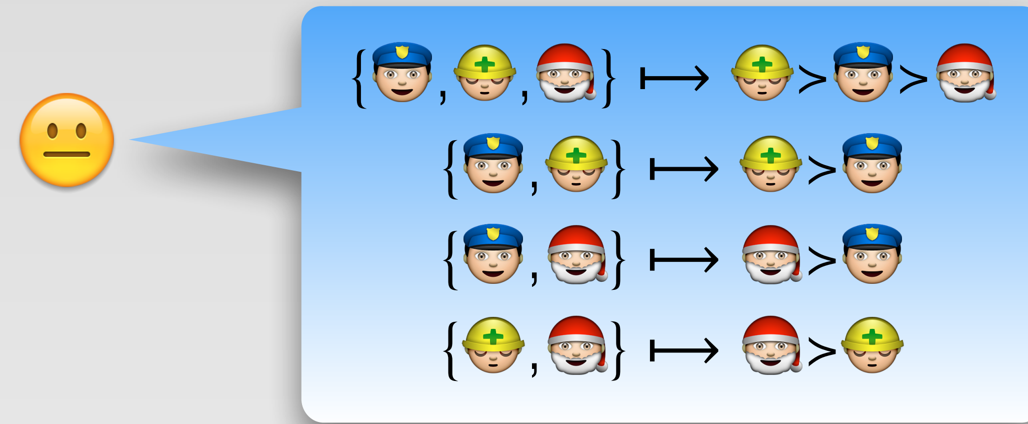
- **Strategies** in the two-stage game

Equilibrium Concepts

- **Strategies** in the two-stage game
  - candidates: “run” or “not run”

Equilibrium Concepts

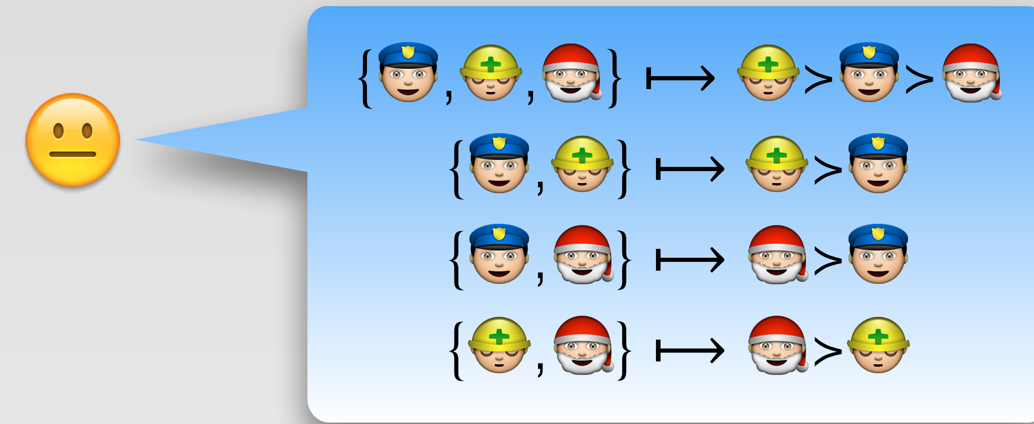
- **Strategies** in the two-stage game
  - candidates: “run” or “not run”
  - voters: for each B⊆C, a ranking of B



Duke UNIVERSITY

Equilibrium Concepts

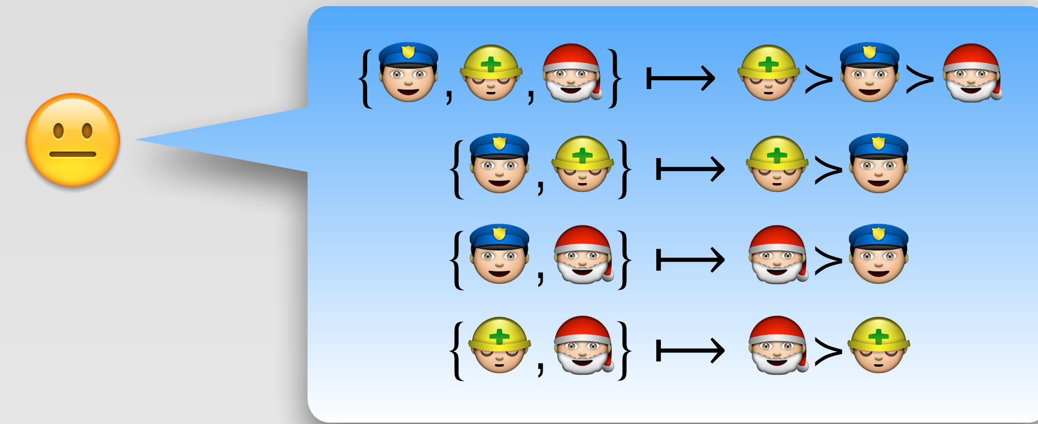
- **Strategies** in the two-stage game
  - candidates: “run” or “not run”
  - voters: for each B⊆C, a ranking of B



- A strategy profile s = (s_C,s_V) is a

Equilibrium Concepts

- **Strategies** in the two-stage game
  - candidates: “run” or “not run”
  - voters: for each $B \subseteq C$, a ranking of B

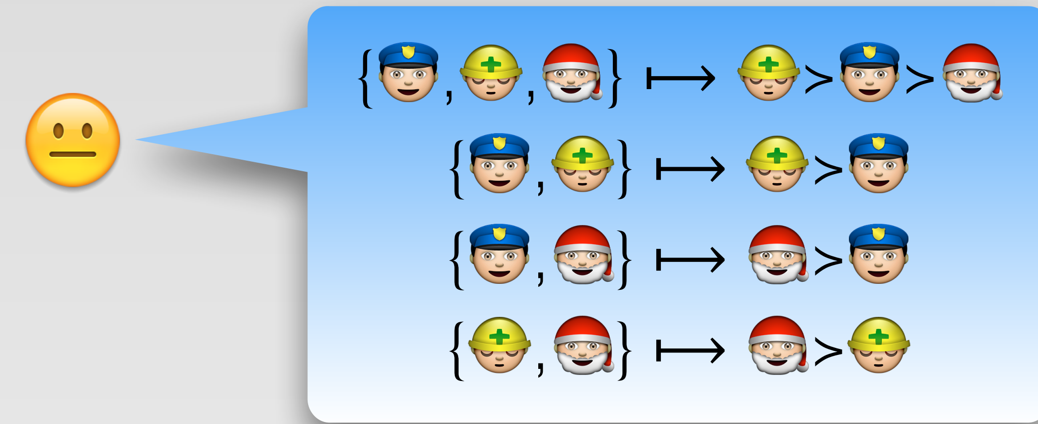


- A strategy profile $s = (s_C, s_V)$ is a
  - **C-equilibrium** if no candidate wants to deviate
  - **strong C-equilibrium** if no coalition of candidates wants to deviate

Duke UNIVERSITY

Equilibrium Concepts

- **Strategies** in the two-stage game
  - candidates: “run” or “not run”
  - voters: for each $B \subseteq C$, a ranking of B

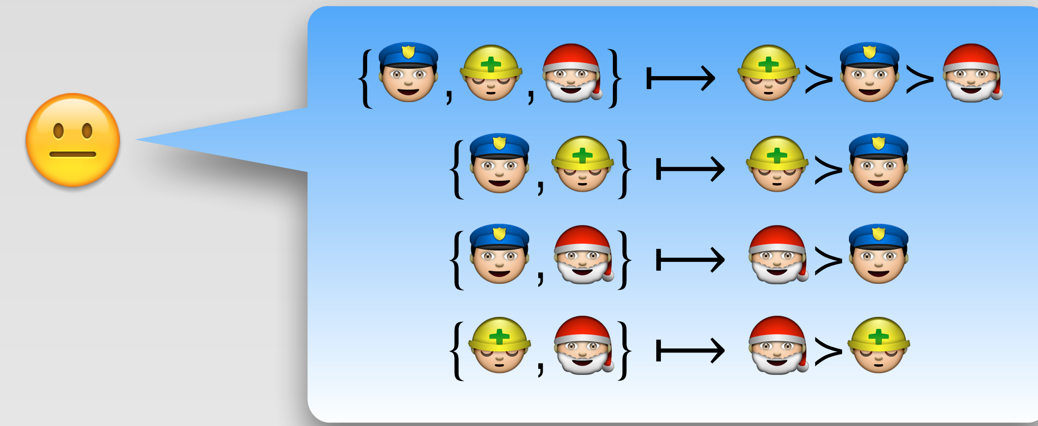


- A strategy profile $s = (s_C, s_V)$ is a
  - **C-equilibrium** if no candidate wants to deviate
  - **strong C-equilibrium** if no coalition of candidates wants to deviate
  - **V-equilibrium** if, for all $B \subseteq C$, no voter wants to deviate
  - **strong V-equilibrium** if, for all $B \subseteq C$, no coalition of voters wants to deviate

Duke UNIVERSITY

Equilibrium Concepts

- **Strategies** in the two-stage game
  - candidates: “run” or “not run”
  - voters: for each B⊆C, a ranking of B



- A strategy profile $s = (s_C, s_V)$ is a
  - **C-equilibrium** if no candidate wants to deviate
  - **strong C-equilibrium** if no coalition of candidates wants to deviate
  - **V-equilibrium** if, for all B⊆C, no voter wants to deviate
  - **strong V-equilibrium** if, for all B⊆C, no coalition of voters wants to deviate
- Relationships
  - (C-eq. ∧ V-eq.) ⇔ subgame-perfect equilibrium
  - (strong C-eq. ∧ strong V-eq.) ⇐ subgame-perfect strong equilibrium

Results

Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

Results

Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

voters / **candidates**	strong V-eq.	V-eq.	truthful voting
strong C-eq.			
C-eq.			
naive candidacy			

Duke UNIVERSITY

Results

Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

candidates \ **voters**	strong V-eq.	V-eq.	truthful voting
strong C-eq.			
C-eq.			
naive candidacy			✗

Results

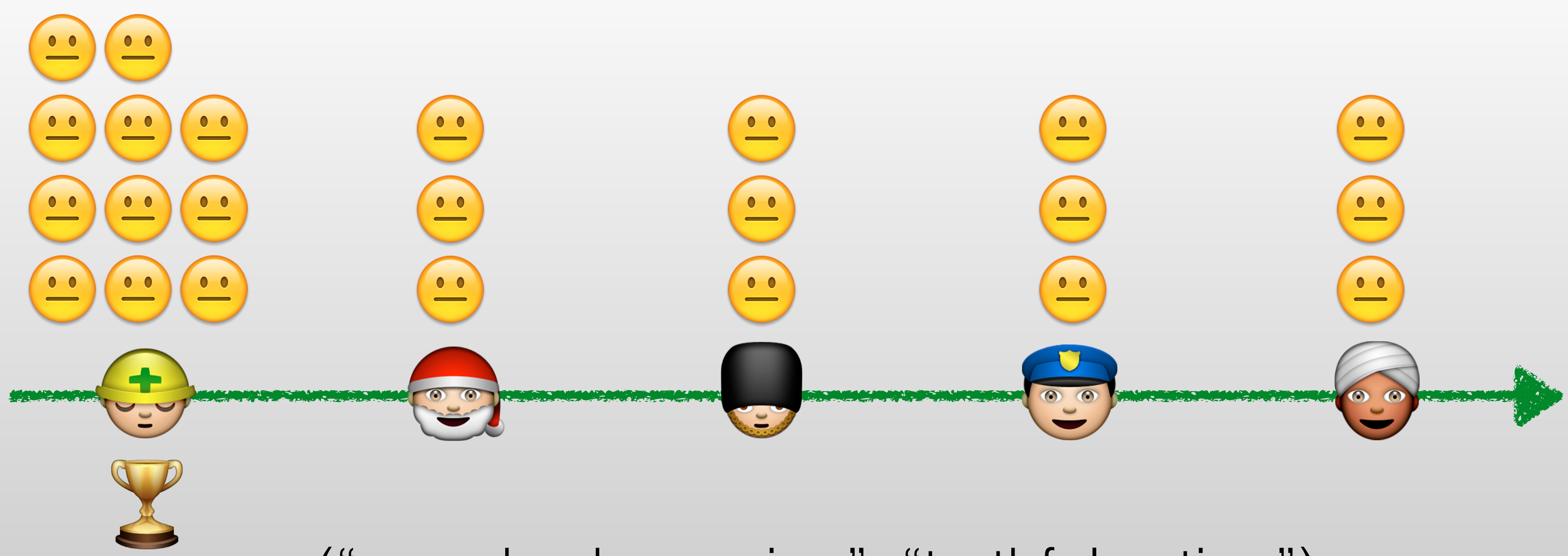
Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

candidates \ voters	strong V-eq.	V-eq.	truthful voting
strong C-eq.			
C-eq.		✗	
naive candidacy			✗

Duke UNIVERSITY

Results



("everybody running", "truthful voting")
is a C-equilibrium and a V-equilibrium

voting rule

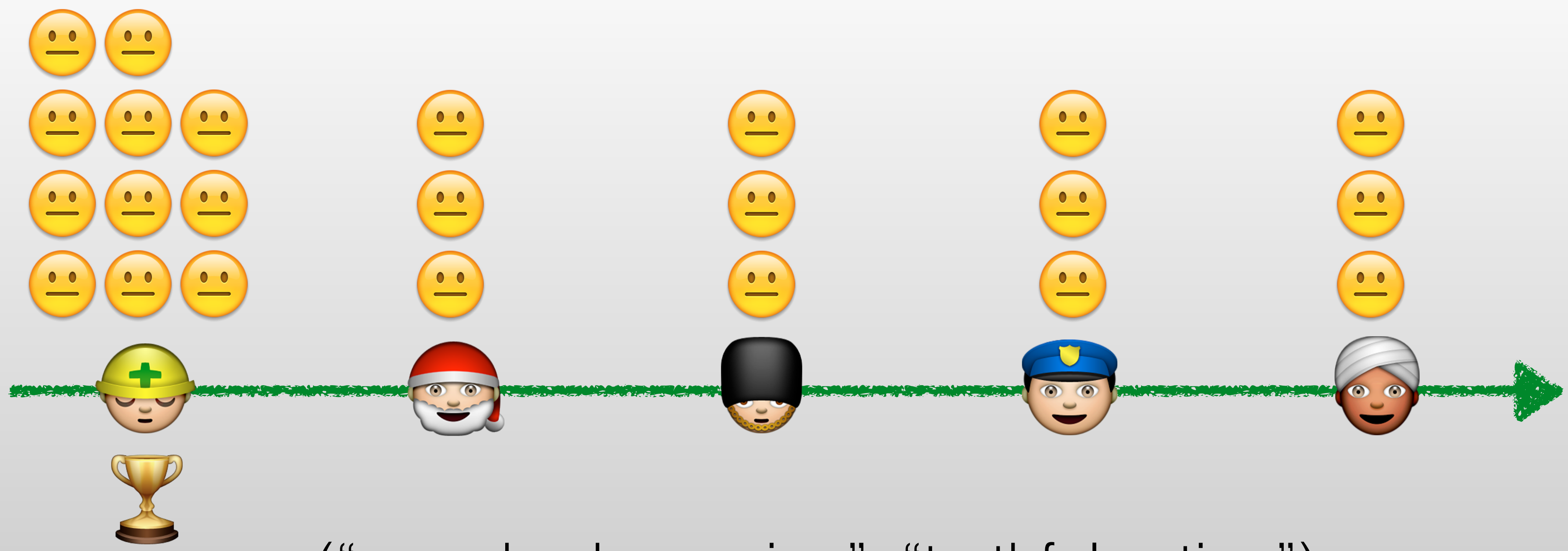
notions

d?

C-eq.		✗	
naive candidacy			✗

Duke UNIVERSITY

Results



("everybody running", "truthful voting")
is a C-equilibrium and a V-equilibrium

voting rule

notions

d?

C-eq.		✗	✗
naive candidacy		✗	✗

Duke UNIVERSITY

Results

Assumptions: single-peaked preferences, majority-consistent voting rule

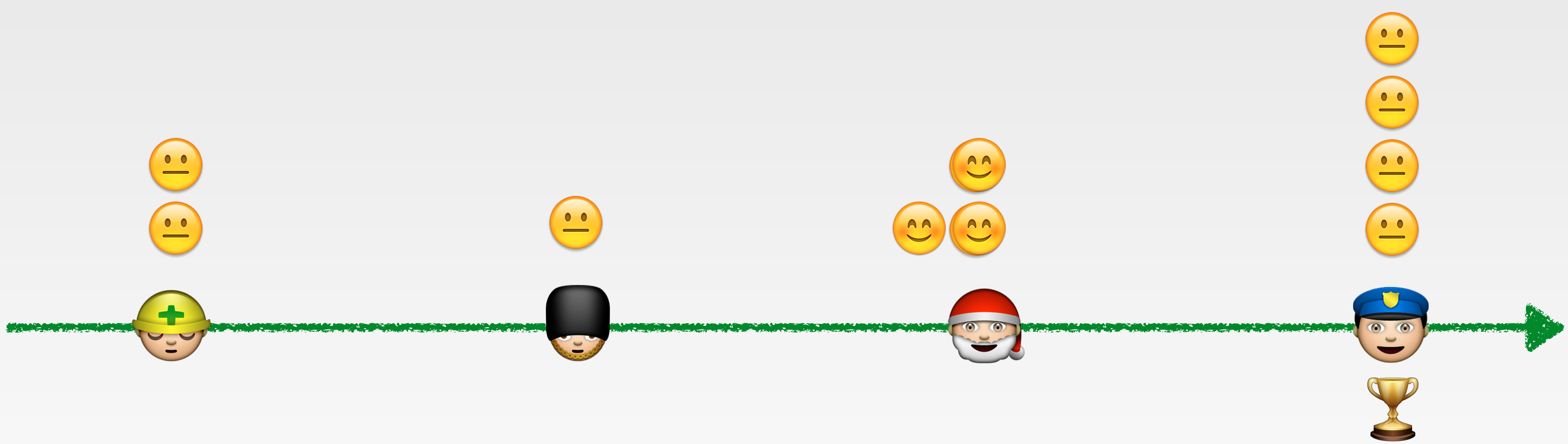
Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

candidates \ **voters**	strong V-eq.	V-eq.	truthful voting
strong C-eq.			
C-eq.		✘	✘
naive candidacy		✘	✘

Duke UNIVERSITY

Strong V-equilibria

- Consider a single-peaked preference profile with Condorcet winner 🎅 and a majority-consistent voting rule.
- **Theorem:** (i) There exists a subgame-perfect strong equilibrium. (ii) In every strong V-equilibrium in which 🎅 runs, 🎅 wins.



- **Corollary:** In every strong V-equilibrium that is also a C-equilibrium (strong or not), 🎅 wins.

Results

Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

voters / candidates	strong V-eq.	V-eq.	truthful voting
strong C-eq.	✔		
C-eq.	✔	✘	✘
naive candidacy	✔	✘	✘

Duke UNIVERSITY

Results

Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

candidates \ voters	strong V-eq.	V-eq.	truthful voting
strong C-eq.	✔		
C-eq.	✔	✘	✘
naive candidacy	✔	✘	✘

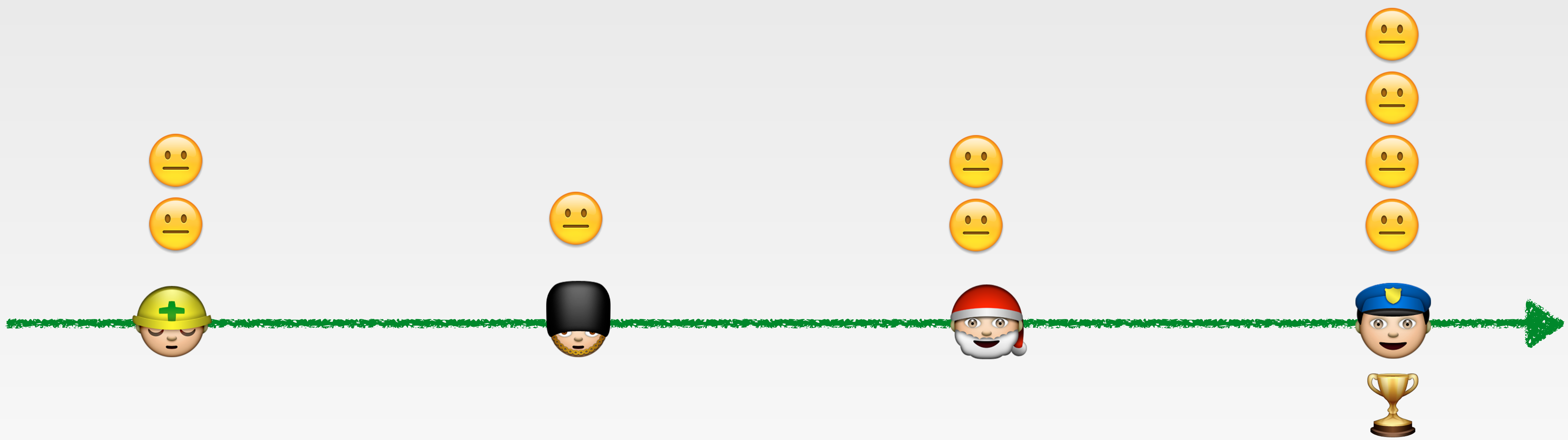
Duke UNIVERSITY

Strong C-equilibria & Truthful Voting

- **Theorem:**
  (i) There exists a strong C-eq. where all voters vote truthfully.
  (ii) In every strong C-eq. where all voters vote truthfully, 🎅 wins.

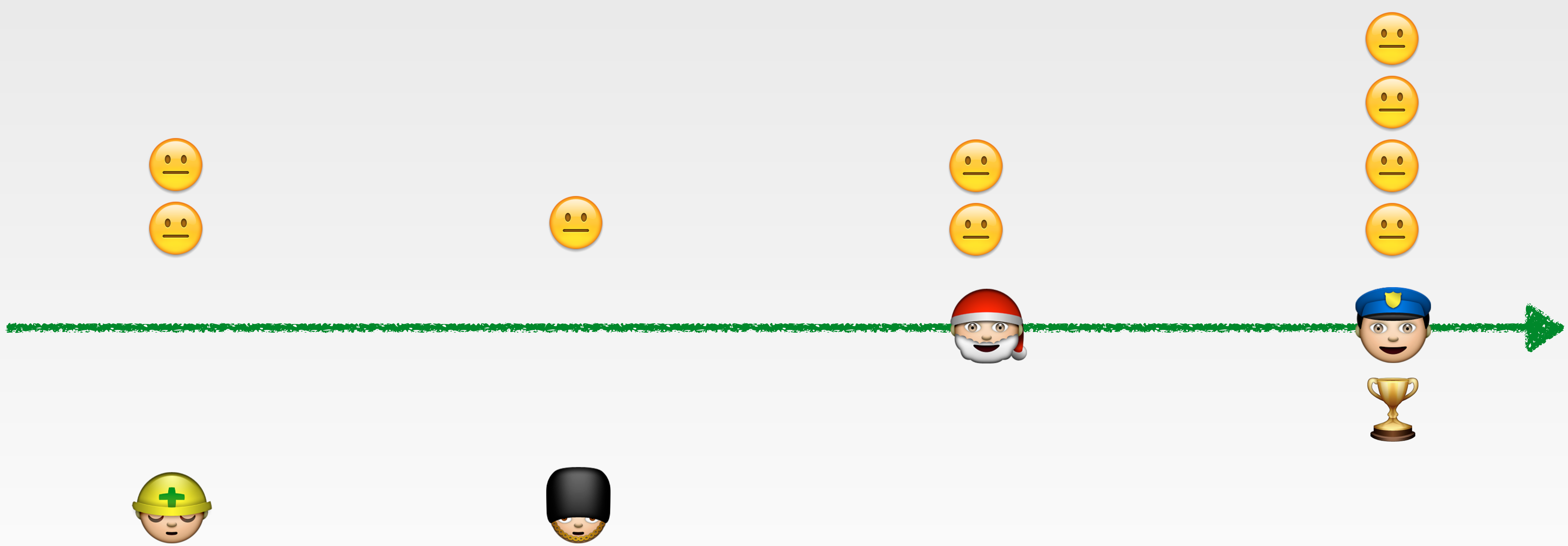
Strong C-equilibria & Truthful Voting

- **Theorem:**
  (i) There exists a strong C-eq. where all voters vote truthfully.
  (ii) In every strong C-eq. where all voters vote truthfully, 🎅 wins.



Strong C-equilibria & Truthful Voting

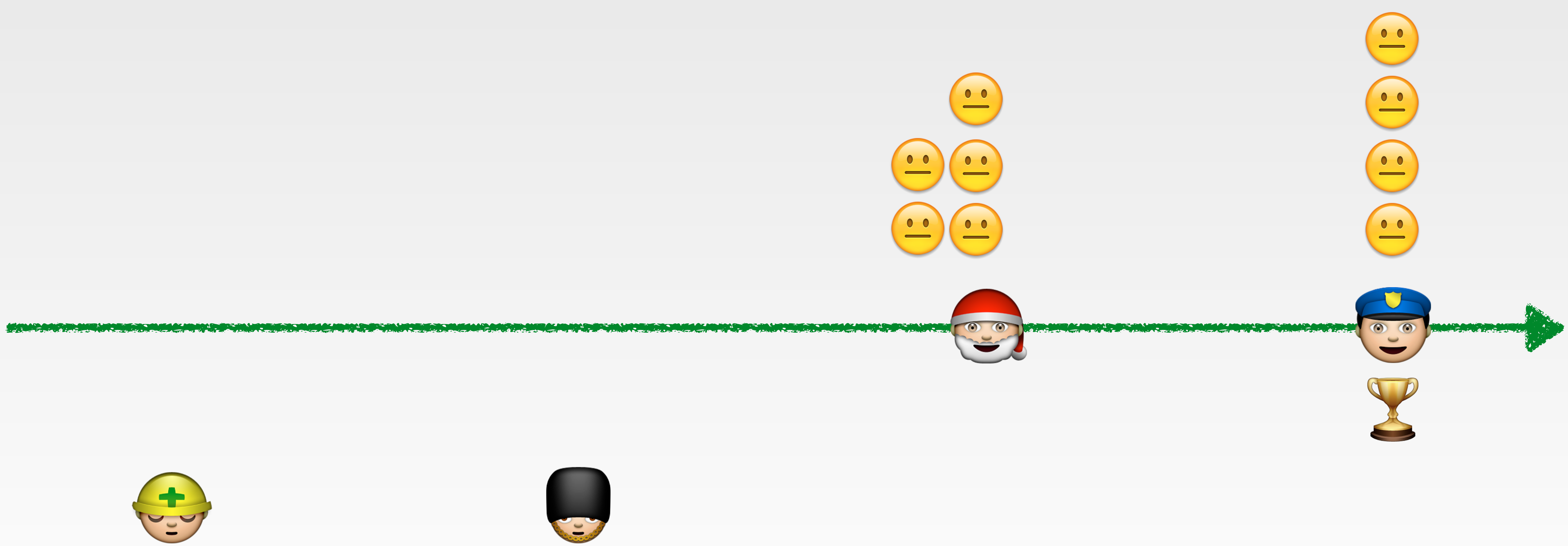
- **Theorem:**
  (i) There exists a strong C-eq. where all voters vote truthfully.
  (ii) In every strong C-eq. where all voters vote truthfully, 🎅 wins.



Duke UNIVERSITY

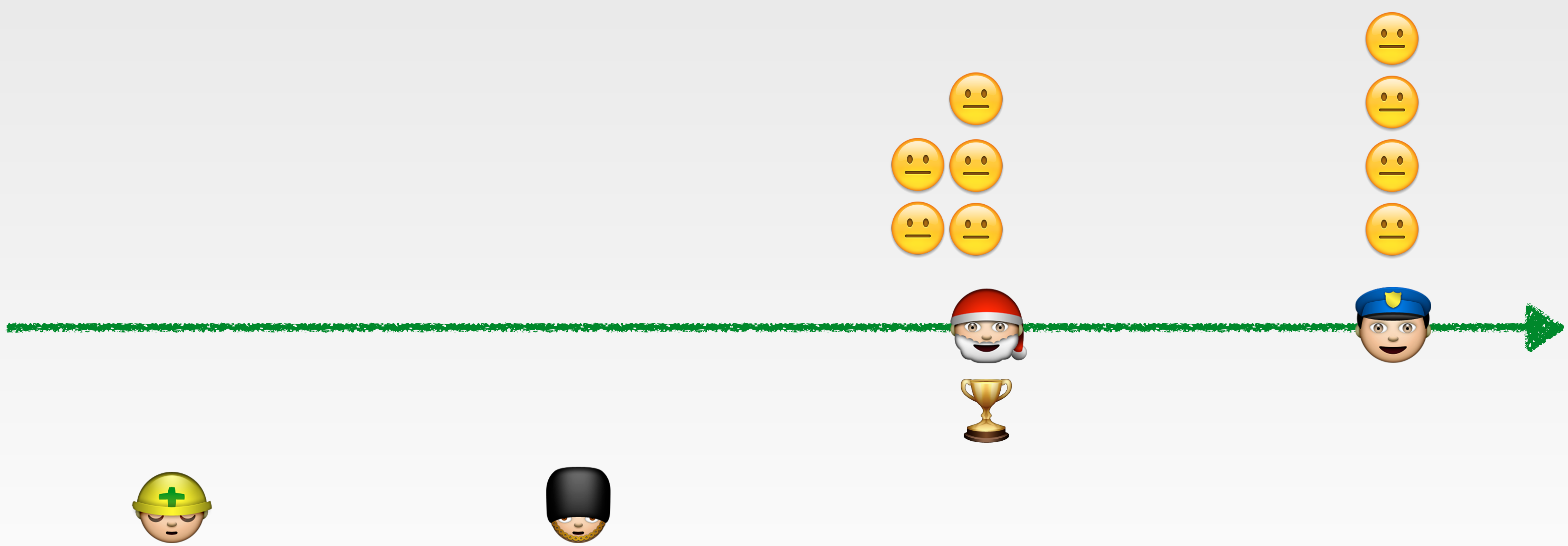
Strong C-equilibria & Truthful Voting

- **Theorem:**
  (i) There exists a strong C-eq. where all voters vote truthfully.
  (ii) In every strong C-eq. where all voters vote truthfully, 🎅 wins.



Strong C-equilibria & Truthful Voting

- **Theorem:**
  (i) There exists a strong C-eq. where all voters vote truthfully.
  (ii) In every strong C-eq. where all voters vote truthfully, 🎅 wins.



Results

Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

candidates \ voters	strong V-eq.	V-eq.	truthful voting
strong C-eq.	✔		✔
C-eq.	✔	✘	✘
naive candidacy	✔	✘	✘

Duke UNIVERSITY

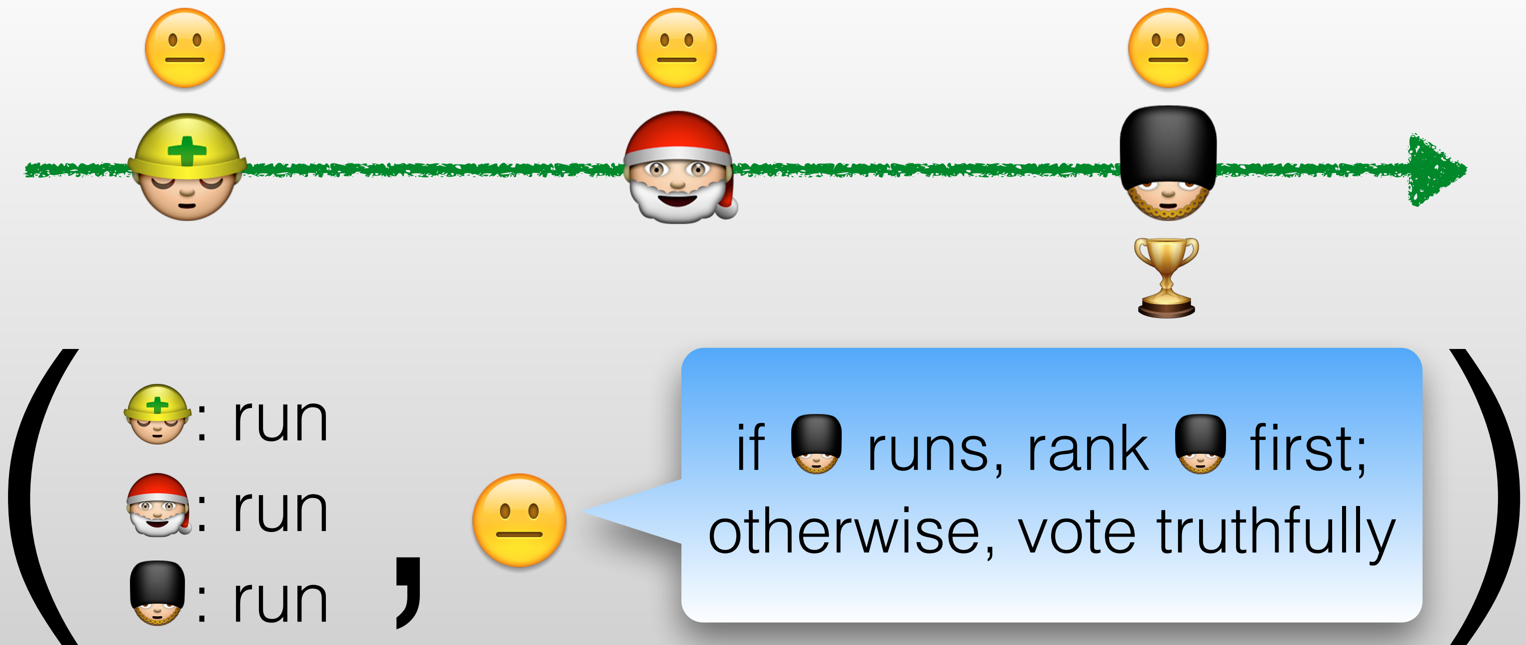
Results

Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

candidates \ voters	strong V-eq.	V-eq.	truthful voting
strong C-eq.	✔	✘	✔
C-eq.	✔	✘	✘
naive candidacy	✔	✘	✘

Duke UNIVERSITY



is a strong C-equilibrium and a V-equilibrium

...nsistent voting rule

...brium notions
...elected?

			truthful voting
strong C-eq.	✔	✘	✔
C-eq.	✔	✘	✘
naive candidacy	✔	✘	✘

Duke UNIVERSITY

Results

Assumptions: single-peaked preferences, majority-consistent voting rule

Question: Which combinations of equilibrium notions guarantee that the Condorcet winner is selected?

voters / **candidates**	strong V-eq.	V-eq.	truthful voting
strong C-eq.	✔	✘	✔
C-eq.	✔	✘	✘
naive candidacy	✔	✘	✘

Duke UNIVERSITY

- (strong C-eq. ∧ strong V-eq.), but not subgame-perfect strong equilibrium

Example 1. *Consider a preference profile with candidates* a, b, c *and a single voter with preferences* $a \succ b \succ c$*. The preferences of candidate* b *are given by* $b \succ_b c \succ_b a$*. The voting rule* f *selects the candidate ranked first by the voter whenever all three candidates run; if, however, at most two candidates run, the lexicographically last one is chosen, ignoring the voter's vote. Let* s *be the strategy profile in which* a *and* c *run and the voter votes truthfully. The outcome of* s *under* f *is* $o_f(s) = c$*. We claim that* s *is (1) a strong* C*-equilibrium and (2) a strong* V*-equilibrium, but (3) not a subgame-perfect strong equilibrium (in fact not even a strong equilibrium).*

For (1), observe that c *has no incentive to participate in any deviation. The same holds for* a*, because the outcome will still be* c *if* a *deviates (whether* b *runs or not). And when all three candidates run, the outcome is* a*, making candidate* b*—the only deviator—worse off. For (2),* s *is a strong* V*-equilibrium because the voter makes his favorite candidate win in the only case where his vote has any influence. For (3), consider the following deviation. Candidate* b *deviates to running and the voter deviates to ranking* b *first whenever* b *runs. The outcome will change to* b*, and both deviators (candidate* b *and the voter) prefer* b *to* c*.*

strong V-equilibria (1)

Example 4. *Let R be a single-peaked preference profile with candidates $a \lhd b \lhd c$ and peak distribution $(5, 0, 4)$. If f is Borda's rule, there does not exist a strong V-equilibrium (and hence no subgame-perfect strong equilibrium). To see this, consider the case where all candidates run. Observe that in any strong V-equilibrium, the outcome would have to be a. (Suppose the outcome is not a. Then, the five voters in $V_R(a)$ can jointly deviate and change the outcome to a. They can do this by having one voter voting $a \succ b \succ c$, and the remaining four voters voting exactly the opposite rankings of the voters in $V_R(c)$.) However, there is no strong V-equilibrium that yields outcome a. This is because the voters in $V_R(c)$ prefer both other alternatives to a, and—no matter how the voters in $V_R(a)$ vote—the voters in $V_R(c)$ can jointly deviate and achieve an outcome other than a. (One of b and c will obtain a score of at least 3 from the voters in $V_R(a)$. Without loss of generality, suppose it is b. Then the voters in $V_R(c)$ can all vote $b \succ c \succ a$, making b the winner.)*

Duke
UNIVERSITY

strong V-equilibria (2)

Example 5. *Let R be a single-peaked preference profile with candidates $a \lhd b \lhd c$ and five voters: three voters have preferences $a \succ b \succ c$ and two voters have preferences $b \succ c \succ a$. The Condorcet winner is a. Let f be the voting rule veto*[8] *and let s be the strategy profile where all candidates run and all voters vote truthfully. Then, $o_f(s) = b$. Moreover, s is a strong C-equilibrium and a strong V-equilibrium. The former holds because any deviation involving a does not change the outcome (provided b still runs), and c can only change the outcome to the less preferred alternative a. For the latter, the only interesting case is when all three candidates run. In this case, the two voters in $V_R(b)$ have no incentive to deviate from truthful voting (their favorite candidate is winning) and there is no way for the three voters in $V_R(a)$ to jointly deviate and achieve outcome a. (They can change the outcome to c by voting $a \succ c \succ b$, but they prefer b to c.) It can furthermore be shown that, when all candidates run,* every *strong V-equilibrium yields outcome b.*

Duke UNIVERSITY

strong C-eq., truthful voting (1)

Example 6. *Consider a single-peaked preference profile with candidates $a \lhd b \lhd c$ and five voters: three voters have preferences $a \succ b \succ c$ and two voters have preferences $b \succ c \succ a$. The Condorcet winner is a. Let s be the strategy profile where $s_a = s_b = s_c = 1$ and s_v is "truthful voting" for all voters v. It is easily verified that s is a strong C-equilibrium and $o_{Borda}(s) = b$. In fact, it can be checked that the Condorcet winner is not chosen in any strong C-equilibrium with truthful voting. (The only other strong C-equilibrium under truthful voting has candidates b and c running and also yields outcome b.)*

Duke UNIVERSITY

strong C-eq., truthful voting (2)

Example 7. *Consider the following preference profile with candidates a, b, c and 14 voters.*

4	4	6
a	b	c
b	a	b
c	c	a

The preferences of the candidates are such that a prefers c over b and b prefers c over a. Whereas the preferences of the voters are single-peaked with respect to the ordering $a \lhd b \lhd c$, this is not true for the preferences of the candidates. (Therefore, this profile is not single-peaked according to the definition in Section 3.1.) The Condorcet winner is b and the Condorcet loser is c. Let s be the strategy profile where all candidates run and all voters vote truthfully. It is easily verified that s is a strong C-equilibrium and $o_{plurality}(s) = c$. In fact, "everybody running" is the only strong C-equilibrium under truthful voting.