

David S. Touretzky
October 2023

Day 4: Analyzing Data

```
clear all
x = 0 : 0.1 : 5;
y = 3*x + rand(1, length(x));
scatter(x, y)
scatter(x, y, 20, 'r', 'filled')
lsline      % least squares line fit
```

1

4

Exploring Distributions

- Use **disttool** to display a distribution and manipulate its parameters.
- Switch between CDF and PDF.
- Click and drag a slider arrow to change a parameter.
- Try a Gamma distribution.
- Click and drag in the figure window to measure the value of the distribution (dashed red line will slide as you move the mouse).

2

Polynomial Fitting

```
clear all
x = -10 : 0.25 : 10;
y = x.^3/10 - x.^2 + 5*sin(x);
polytool(x,y)
    Try using a Degree of 2, then 3.
    Click the Export button.
beta
clf, plot(x, y, 'LineWidth', 2)
hold on, plot(x, polyval(beta, x), 'ro')
```

5

Random Distributions

- Use **randtool** to explore random distributions.
- Select Poisson distribution.
- Click the Resample button a few times.
- Increase the sample size to 1000 samples.
- Try resampling now.
- Click the Export button to export samples to the workspace.

```
hist(poissrv)
plot(sort(poissrv), 'o')
```

3

Anonymous Functions

```
f = @(x) 1 ./ (1+exp(-x.^2))
whos f
f(5)
f()
f(-1:4)
```

6

Plot Tools: Data Statistics

```
clf
x = randn(1000,1);
y = 1 - f(x)
plot(x, y, 'ro')
```

Undock the figure if it is docked.

*Select Tools > Data Statistics, then
check boxes for x mean and x std. dev.*

7

Neurophysiology Exercise (cont.)

- Notice the two variables in your workspace.
- Make a diameter-vs-velocity scatter plot.
- Fit a line to this data using the Basic Fitting tool.
- What is the predicted conduction velocity of an axon 22 microns in diameter?
- What diameter value would give a conduction velocity of 6 meters/second?

10

Plot Tools: Basic Fitting

Select Tools > Basic Fitting

Check "cubic"

Check "Show equations"

Click the → button

Click the next → button

In the "Find y=f(x)" panel, enter -2 : 0.5 : 1.5

*Click the Evaluate button, then
check "Plot evaluated results"*

8

Fitting A Gaussian (Live Script)

- Download
<https://www.cs.cmu.edu/~dst/Tutorials/Matlab/Likelihood.mlx>
- Run the script.

11

Neurophysiology Exercise

- How does axon diameter in microns relate to conduction velocity in meters/second?

`!wget www.cs.cmu.edu/~dst/Tutorials/Matlab/hursh.csv`

`type hursh.csv`

`clear all`

Select "Import Data" from the toolbar

Select the file hursh.csv

Select Import as: Column Vectors

Click the Import Selection button

9

Fitting A Gaussian (Manually)

- Load a dataset of gasoline prices:
`clear all, load gas.mat`
- Type the value of price2
- Let's look at the distribution of values:
`hist(price2)`
- Calculate some statistics:
`n = length(price2)`
`mu = mean(price2)`
`sigma = std(price2) * sqrt((n-1)/n)`

12

Plot the Gaussian

```
x = min(price2) : 0.25 : max(price2)
y = normpdf(x, mu, sigma);
scaled_y = y * 4/max(y);
hold on
plot(x, scaled_y, 'r')
```

Could also do: `histfit(price2, 10)`

13

Plot the Likelihood Surface

```
mus = 116 : 0.5 : 121;
sigmas = 2.5 : 0.1 : 5.5;
[x,y] = meshgrid(mus, sigmas);
z = gauslike(x, y, price2);
clf, surfc(x, y, z*10)
xlabel mu
ylabel sigma
zlabel Likelihood
rotate3d on
```

16

What Is the Likelihood?

- We estimated the mu and sigma parameters based on a small sample size (20 points).
- The true distribution may differ from our estimate.
- If we change mu and/or sigma slightly, how well does the new distribution fit our dataset?

14

Countour Plot of Likelihood

```
figure
[c,h] = contour(mus, sigmas, z*1e24);
clabel(c,h)
```

17

Calculating Likelihood

```
function z = gauslike(mu, sigma, points)
    n = length(points);
    z = ones(size(mu));
    for i = 1 : n
        z = z .* normpdf(points(i), mu, sigma);
    end
end
```

15

Interactive Contour Plot

- ```
fsurfht('gauslike', [116 122], [2.5 5.5], price2)
```
- Click and drag to move the crosshairs.
  - Type the mean 118.5 into the X Value box.
  - Type the sigma value 3.6401 into the Y Value box. Note the Z Value is 2.8386e-24
  - Compute `std(price2)` and type that value into the Y-value box: the Z Value decreases.
  - The peak is located at the sample mean, but not at the sample's standard deviation.
  - Moral: the sigma value giving the greatest likelihood may not be the sample's sigma.

18

## Nonlinear Regression

- Matlab can “tweak” parameters to fit an arbitrary model to a data set.
- First step: choose a model and determine its set of parameters. Example: a constant term plus an exponential function plus a noise term:

$$y_i = a_1 + a_2 \exp(-a_3 x_i) + \text{epsilon}_i$$

- Write a Matlab function to evaluate the model given a parameter vector a and data x:

```
mdl = @(a,x) a(1) + a(2)*exp(-a(3)*x)
```

19

## Examining the Fit

```
xrange = min(x) : 0.01 : max(x);
clf, hold on
scatter(x, y)
plot(xrange, mdl(a_hat, xrange), 'r')
```

22

## Generate Some Test Data

```
true_a = [1; 3; 2]
x = exprnd(2.5, 100, 1); % 100x1 exp. distrib.
noise = normrnd(0, 0.1, 100, 1);
y = mdl(true_a, x) + noise;
scatter(x,y)
```

20

## Box Plots

```
load carsmall

MPG

Origin

boxplot(MPG,Origin)

doc boxplot
```

23

## Fitting The Model to the Data

- Need a starting point for the parameter vector.
- Doesn't have to be accurate; just guess.

```
a_guess = [2; 2; 2]
```

- Now use nlinfit to estimate the parameters:

```
a_hat = nlinfit(x, y, mdl, a_guess)
```

- Pretty close to true\_a!

21

## Anova Example

```
doc anova1
```

*Scroll down to Example 2. Are steel beams as strong as special alloy beams?*

*Cut and paste the sample code into your Matlab command window. Then do:*

```
anova1(strength, alloy)
```

24

## To Learn More

- Browse the Statistics Toolbox documentation:  
doc stats
- Take Rob Kass' course on statistics for computational neuroscience.

25

## Live Scripts

- Function definitions must go at the end of the file.
- Figures can appear either to the right, or inline.
- Type “open <filename>” to open the file.

28

## Live Scripts

- Similar to Jupyter Notebooks (Python) or Mathematica notebooks
- Combine text, code, images, and hyperlinks
- File name ends in .mlx instead of .m

26

## Live Scripts

- New Live Script button
- Type Matlab code
- Press the Run button, or left click on the bar at the left edge of the editor, or Control-Return.
- To enter text: click on the Text button
- Text formatting: style, bold/italics, etc.
- To insert a figure: go to the Insert tab and click the Image button
- Save as a .mlx file

27