Blocked Literals are Universal

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Joint work with
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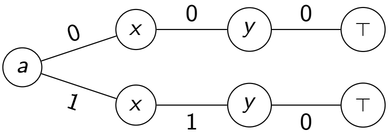
Introduction to QBF

A quantified Boolean formula (QBF) is a propositional formula where variables are existentially ($\exists$) or universally ($\forall$) quantified.

Consider $\forall a \exists x \forall b \exists y . (a \vee \neg x \vee y) \wedge (\neg a \vee x \vee y) \wedge (b \vee \neg y)$

A model is:



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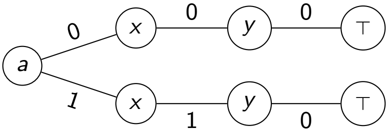
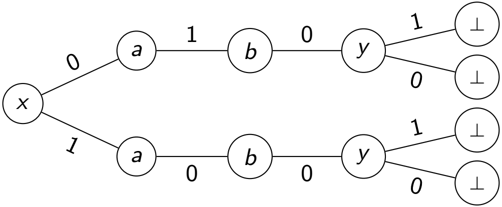


Consider $\exists x \forall a, b \exists y.(\neg x \vee a \vee y) \wedge (x \vee \neg a \vee y) \wedge (b \vee \neg y)$

A counter-model is:



Promises of QBF

- QSAT is the prototypical problem for PSPACE.
- QBFs are suitable as host language for the encoding of many application problems like
  - verification
  - synthesis
  - artificial intelligence
  - knowledge representation
  - game solving
- In general, QBF allow more succinct encodings then SAT

Introduction to QBF Preprocessing

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Existing QBF preprocessing techniques can eliminated all clauses in the above formula making it trivially satisfiable.

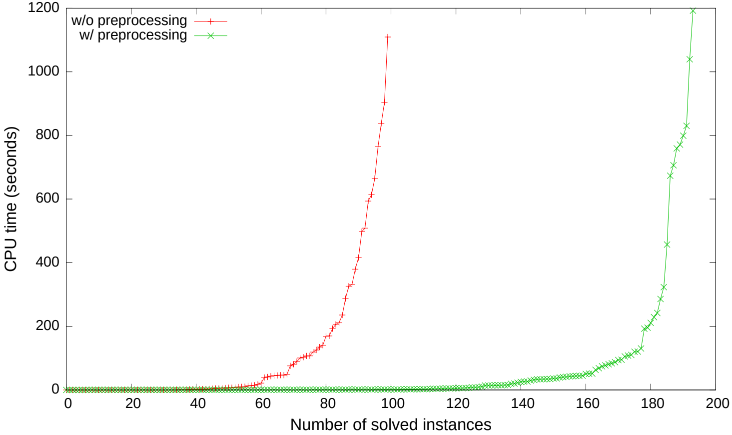
Consider $\exists x \forall a, b \exists y.(\neg x \vee a \vee y) \wedge (x \vee \neg a \vee y) \wedge (b \vee \neg y)$

Our new QBF preprocessing technique can eliminated all universal literals, thereby reducing the problem to SAT.

QBF Preprocessing

Preprocessing is crucial to solve most QBF instances efficiently.

Results of DepQBF w/ and w/o bloqqer on QBF Eval 2012



Challenges for Quantified Boolean Formulas (QBF)

Preprocessing is crucial to solve most QBF instances efficiently.

Proofs are useful for applications and to validate solver output.

Main challenges regarding QBF and preprocessing [Janota'13]:

1. produce proofs that can be validated in polynomial time;
2. develop methods to validate all QBF preprocessing; and
3. narrow the performance gap between solving with and without proof generation.

In our IJCAR'14 paper [1], we meet all three challenges!

[1] Marijn J. H. Heule, Matina Seidl and Armin Biere:
A Unified Proof System for QBF Preprocessing.
IJCAR 2014, LNCS 8562, pp 91-106 (2014)

Here we show present a new preprocessing technique called Blocked Literal Elimination that follows from [1].

Blocked Literal Elimination

Quantified Blocked Clauses [BiereLonsingSeidl 2011]

Definition (Quantified Blocking Literal)

An existential literal l in a clause C of a QBF $\pi.\varphi$ blocks C w.r.t. $\pi.\varphi$ if for every clause $D \in \varphi$ with $\neg l \in D$ holds that there exists k s.t. $k \in C$, $\neg k \in D$, $l \neq k$ and $k \leq_\pi l$.

With respect to a fixed QBF $\pi.\varphi$ and its clauses we have:

Definition (Quantified Blocked Clause)

A clause is blocked if it contains a literal that blocks it.

Quantified Blocked Clauses [BiereLonsingSeidl 2011]

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Example

$$\forall a \exists x \forall b \exists y.(a \vee \neg x \vee y) \wedge (\neg a \vee x \vee y) \wedge (b \vee \neg y)$$

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$$\forall a \exists x \forall b \exists y.(a \vee \neg x \vee y) \wedge (\neg a \vee x \vee y) \wedge (b \vee \neg y)$$

Proposition 1

Removal of quantified blocked clauses preserves unsatisfiability.

Blocked Literals [This paper]

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Example

$$\exists x \forall a, b \exists y.(\neg x \vee a \vee y) \wedge (x \vee \neg a \vee y) \wedge (b \vee \neg y)$$

Proposition 2

Removal of a quantified blocked literal preserves satisfiability.

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Proposition 2

Removal of a quantified blocked literal preserves satisfiability.

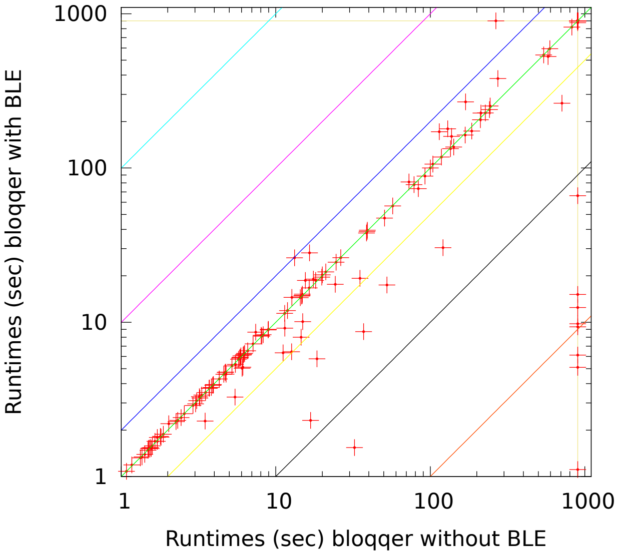
Proposition 3

Elimination of blocked literals is not confluent in contrast to quantified blocked clause elimination:

$$\forall a, b \exists x.(a \vee b \vee x) \wedge (\neg a \vee \neg b \vee \neg x)$$

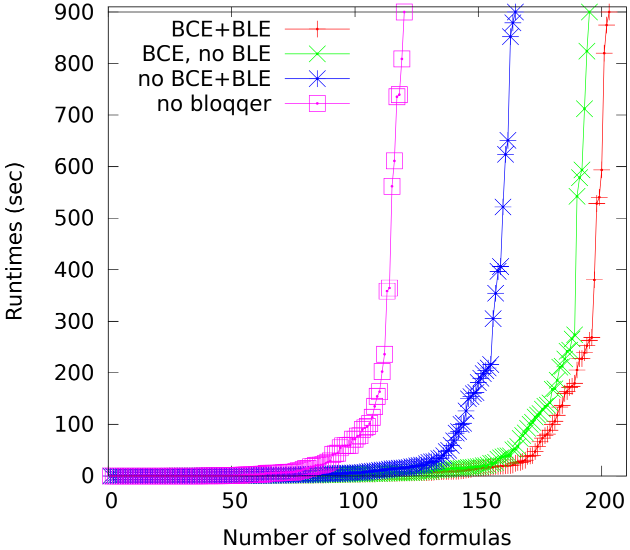
Experimental Results

Experimental Results: Runtime Scatter Plot



Above the diagonal: faster due to blocked literal elimination (BLE)

Experimental Results: Runtime Cactus Plot



Experimental Results: Only solvable with BLE

	formula statistics			preprocessing		solving	
formula	#vars	#cl	#Q	#bl	time	time	val
adder-6-sat	1727	1259	4	1278	0.74	0.36	T
C88020_0_0_inp	1046	2644	21	3	0.2	874.32	F
cache-coh-2-fixp-5	9604	28198	2	3599	9.32	–	F*
ethernet-fixpoint-3	12514	33884	2	3879	9.76	–	F*
k_branch_n-14	7068	33865	33	389	5.09	–	T*
k_branch_n-20	13821	78949	44	1397	12.45	–	T*
k_branch_p-15	8035	39595	34	239	6.12	–	F*
k_branch_p-21	15161	88627	46	1532	15.12	–	F*
s820_d7_s	24757	26960	3	5365	54.7	11.44	T

* solved directly by bloqqer

#vars : number of variables
#cl : number of clauses
#Q : number of quantifier alternations
#bl : number of eliminated blocked literals

Conclusions

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We presented a new preprocessing technique:

- Blocked Literal Elimination (BLE) is useful in practice
- BLE is the dual of Blocked Clause Elimination (BCE)
- BLE is not confluent, in contrast to BCE

Directions for future work:

- Can the addition of blocked literals be helpful?
  - For example in combination with universal expansion
- Which other QRAT simplifications are useful in practice?
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Thanks!