The SPmap: A Probabilistic Framework for Simultaneous Localization and Map Building

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***Abstract*—This article describes a rigorous and complete framework for the simultaneous localization and map building problem for mobile robots: the symmetries and perturbations map (SPmap), which is based on a general probabilistic representation of uncertain geometric information. We present a complete experiment with a LabMate™ mobile robot navigating in a human-made indoor environment and equipped with a rotating two-dimensional (2-D) laser rangefinder. Experiments validate the appropriateness of our approach and provide a real measurement of the precision of the algorithms.**

***Index Terms*—Correlations, probabilistic model, simultaneous localization and map building.**

I. Introduction

Successful path planning and navigation of a mobile robot in a human-made indoor environment requires the availability of both a sufficiently reliable estimation of the current vehicle location, and a sufficiently precise map of the navigation area. *A priori* model maps are rarely available, costly to obtain, and when they are available, they usually introduce inaccuracies in the planning tasks. An automatic construction of the map of the environment in which the robot navigates would be desirable, and it has become an important research direction in today's robotics community.

The precision of the constructed map is highly influenced by the accuracy of the dead-reckoning system of the mobile robot, whose location estimations drift with time. An improved solution would require the relocation of the mobile robot along its trajectory to avoid biases introduced by odometry, hence, an approach based on the simultaneous localization and map building would be necessary (Fig. 1). Exact mathematical approaches to the simultaneous localization and map building problem were originally addressed by Smith *et al.* [1], [2] who introduced the concept of *stochastic map*, a representation of spatial relationships, their uncertainties, and their interdependencies with respect to a base reference. Later, this concept was used in the works of Moutarlier *et al.* [3], Leonard *et al.* [4]–[6], and Hébert *et al.* [7]. Recently, Uhlmann *et al.* [8] have reported interesting work related to the problem of correlations in the simultaneous localization and map building problem.

This article proposes a new probabilistic framework adapted to the problem of simultaneous localization and map building: the symmetries and perturbations map (SPmap) [9] which is based on a general representation of uncertain geometric information. Our main contributions are as follows.

1) SPmap represents a rigorous and complete solution to the simultaneous localization and map building problem for mobile robots.
2) SPmap consistently formulates the representation and integration of sensorial information gathered by different sensors.

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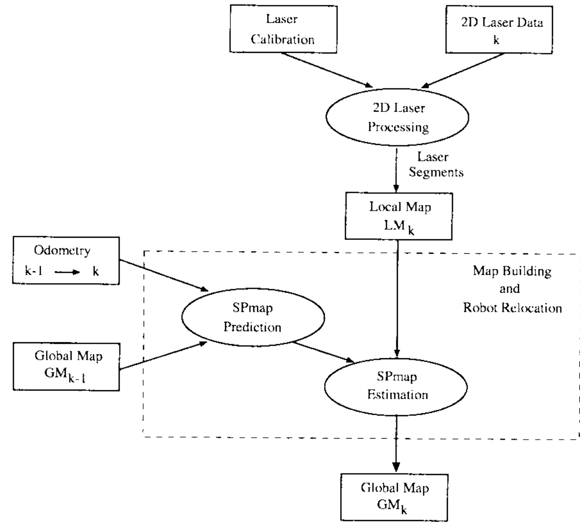


Fig. 1. Simultaneous localization and map building.

3) SPmap overcomes the difficulties reported in previous works dealing with singularities in the representation of geometric features.
4) SPmap has been experimentally validated by a complete experiment which profited from ground-truth to accurately validate the precision and the appropriateness of the approach.

This article also extends our preliminary results reported in [10] to demonstrate the importance of maintaining the correlations between the estimation of the entities involved in the simultaneous localization and map building problem, to avoid optimistic estimations of uncertainty associated to the precision of the locations of features.

The rest of the article is structured as follows. Section II describes the probabilistic representation of uncertain geometric information. Sections III and IV present our probabilistic framework for the simultaneous localization and map building problem. Experimental results are described in Section V while some conclusions and further work are shown in Section VI.

II. Symmetries and Perturbations Model

In our feature-based approach, uncertain geometric information is represented using a probabilistic model: the symmetries and perturbation model (SPmodel) [11], [12] which combines the use of probability theory to represent the imprecision in the location of a geometric element, and the theory of symmetries to represent the partiality due to characteristics of each type of geometric element.

In the SPmodel, the location of a geometric element E with respect to a base reference W is given by a *location vector* $\mathbf{x}_{WE} = (x,\, y,\, \phi)^T$. The estimation of the location of an element is denoted by $\hat{\mathbf{x}}_{WE}$, and the estimation error is represented locally by a *differential location vector* $\mathbf{d}_E$ relative to the reference attached to the element. Thus, the true location of the element is

$$\mathbf{x}_{WE} = \hat{\mathbf{x}}_{WE} \oplus \mathbf{d}_E \tag{1}$$

where $\oplus$ represents the composition of location vectors. To account for the symmetries of the geometric element, we assign in $\mathbf{d}_E$ a null value to the degrees of freedom corresponding to them, because they do not represent an effective location error. We call *perturbation*

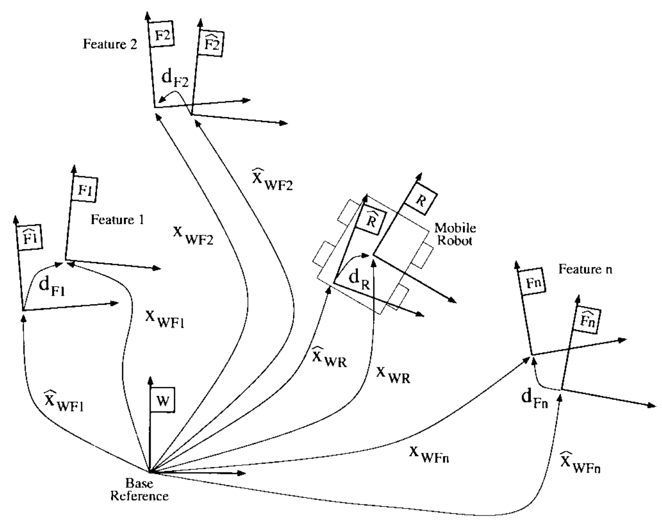


Fig. 2. Representation of the SPmap with generic features and the mobile robot expressed with respect to a base reference W. The location vector of each entity is obtained by the composition of its estimated location vector and its differential location vector.

vector the vector $\mathbf{p}_E$ formed by the non null elements of $\mathbf{d}_E$. Both vectors can be related by a row selection matrix $\mathbf{B}_E$ that we call *self-binding matrix* of the geometric element

$$\mathbf{d}_E = \mathbf{B}_E^T \mathbf{p}_E; \qquad \mathbf{p}_E = \mathbf{B}_E \mathbf{d}_E. \tag{2}$$

Then, the *uncertain location* of every geometric entity is represented in the SPmodel by a quadruple $\mathbf{L}_{WE} = (\hat{\mathbf{x}}_{WE}, \hat{\mathbf{p}}_E, \mathbf{C}_E, \mathbf{B}_E)$, where the transformation $\hat{\mathbf{x}}_{WE}$ is an estimation taken as base for perturbations, $\hat{\mathbf{p}}_E$ is the estimated value of the perturbation vector, and $\mathbf{C}_E$ its covariance.

III. Symmetries and Perturbations Map

The symmetries and perturbations map (SPmap) is a complete representation of the environment of the robot which includes the uncertain location of the mobile robot $\mathbf{L}_{WR}$, the uncertain locations of the features obtained from sensor observations $\mathbf{L}_{WF_i}$, $i \in \{1 \cdots N_F\}$ and their interdependencies (Fig. 2).

The SPmap can be defined as a quadruple

$$\mathbf{SPmap} = \left(\hat{\mathbf{x}}^W, \hat{\mathbf{p}}^W, \mathbf{C}^W, \ \mathbf{B}^W\right) \tag{3}$$

where $\hat{\mathbf{x}}^W$ is the *estimated location vector* of the SPmap and $\mathbf{p}^W$ is the *perturbation vector* of the SPmap

$$\hat{\mathbf{x}}^W = \begin{bmatrix} \hat{\mathbf{x}}_{WR} \\ \hat{\mathbf{x}}_{WF_1} \\ \vdots \\ \hat{\mathbf{x}}_{WF_{N_F}} \end{bmatrix}; \qquad \mathbf{p}^W = \begin{bmatrix} \mathbf{d}_R \\ \mathbf{p}_{F_1} \\ \vdots \\ \mathbf{p}_{F_{N_F}} \end{bmatrix}. \tag{4}$$

The true location of the robot and the map features is

$$\mathbf{x}^W = \hat{\mathbf{x}}^W \oplus (\mathbf{B}^W)^T \mathbf{p}^W \tag{5}$$

where the composition operator $\oplus$ applies in this case to each of the components of the vectors, and $\mathbf{B}^W$ is the *binding matrix* of the SPmap, a diagonal matrix formed by the self-binding matrix of the robot and the self-binding matrices of the map features

$$\mathbf{B}^W = \text{diag}\left(\mathbf{B}_R, \mathbf{B}_{F_1}, \cdots, \mathbf{B}_{F_{N_F}}\right). \tag{6}$$

The covariance matrix of the SPmap represents the covariance of the estimation of the robot and the map feature locations, the cross-covariances between the robot and the map features, and finally, the cross-covariances between the map features themselves

$$\mathbf{C}^W = \begin{pmatrix} \mathbf{C}_R & \mathbf{C}_{RF_1} & \cdots & \mathbf{C}_{RF_{N_F}} \\ \mathbf{C}_{RF_1}^T & \mathbf{C}_{F_1} & \cdots & \mathbf{C}_{F_1F_{N_F}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{RF_{N_F}}^T & \mathbf{C}_{F_1F_{N_F}}^T & \cdots & \mathbf{C}_{F_{N_F}} \end{pmatrix}. \tag{7}$$

Note that we may represent any type of geometric entity within this general framework. The proposed approach can include different types of features obtained by different types of sensors, and thus it is suitable to deal with a multisensor system.

IV. Incremental Construction of the SPmap

This section describes the incremental construction of the SPmap using suboptimal estimation techniques based on the extended Kalman filter [13].

A. Uncertain Displacement of the Mobile Robot

An estimation of the displacement of the mobile robot between two intermediate points along its trajectory can be obtained by dead-reckoning

$$\mathbf{x}_{R_{k-1}R_k} = \hat{\mathbf{x}}_{R_{k-1}R_k} \oplus \mathbf{d}_{R_{k-1}R_k} \tag{8}$$

where $\hat{\mathbf{x}}_{R_{k-1}R_k}$ represents the estimated displacement of the robot and $\mathbf{d}_{R_{k-1}R_k} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{R_{k-1}R_k})$ represents the imprecision in its estimation (i.e., dead-reckoning errors). Other nonrandom sistematic errors are not considered because they can be corrected by an appropriate calibration procedure.

Thus, the predicted location $\mathbf{x}_{WR_k}$ of the mobile robot at time k can be calculated by the composition

$$\begin{aligned} \mathbf{x}_{WR_k} &= \mathbf{x}_{WR_{k-1}} \oplus \mathbf{x}_{R_{k-1}R_k} \\ &= \hat{\mathbf{x}}_{WR_{k-1}} \oplus \mathbf{d}_{R_{k-1}} \oplus \hat{\mathbf{x}}_{R_{k-1}R_k} \oplus \mathbf{d}_{R_{k-1}R_k} \\ &= \hat{\mathbf{x}}_{WR_k} \oplus \mathbf{J}_{R_kR_{k-1}} \mathbf{d}_{R_{k-1}} \oplus \mathbf{d}_{R_{k-1}R_k} \end{aligned} \tag{9}$$

where $\mathbf{J}_{R_kR_{k-1}}$ is the Jacobian of the transformation $\hat{\mathbf{x}}_{R_kR_{k-1}}$ between the location vectors of the robot at time k and that at time $k-1$.

After the displacement of the vehicle, only the location of the mobile robot changes as estimated by dead-reckoning, while the location of map features, being static entities, remain the same as the estimated in the previous time instant $k-1$. Nevertheless, the displacement of the mobile robot produces changes in the dependencies existing between the location of the robot and those of the map features. The complexity of this phase grows linearly with the number of features, $\mathcal{O}(N_F)$.

B. Matching Local and Global Maps

The predicted mobile robot location is improved by matching local observations expressed with respect to the robot reference R, with map features represented with respect to the base reference W. At each point of the robot's trajectory it is desirable to obtain as much pairings as possible because they represent the links between new observations and previous stored knowledge of the navigation area.

Fig. 3 exemplifies the pairing between a local observation E obtained at time k and represented with respect to the robot by $\mathbf{L}_{RE} = (\hat{\mathbf{x}}_{RE}, \hat{\mathbf{p}}_E, \mathbf{C}_E, \mathbf{B}_E)$, and a global map feature F available at time $k-1$ and represented with respect to the base reference W

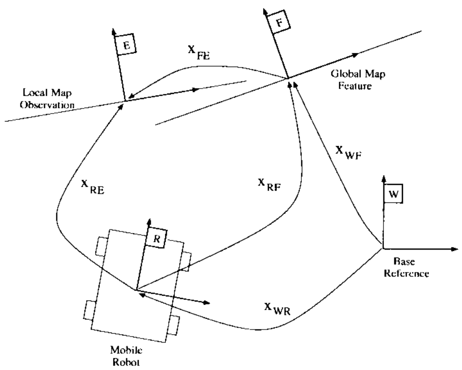


Fig. 3. Matching local observation E with global feature F.

by $\mathbf{L}_{WF} = (\hat{\mathbf{x}}_{WF}, \hat{\mathbf{p}}_F, \mathbf{C}_F, \mathbf{B}_F)$. From their relative location we formulate a nonlinear measurement equation [11]

$$\begin{aligned}
&\mathbf{f}_{k,m}(\mathbf{d}_R, \mathbf{p}_F, \mathbf{p}_E)\\
&\quad= \mathbf{B}_{FE}\mathbf{x}_{FE}\\
&\quad= \mathbf{B}_{FE}(\ominus \mathbf{x}_{WF} \oplus \mathbf{x}_{WR} \oplus \mathbf{x}_{RE})\\
&\quad= \mathbf{B}_{FE}(\ominus \mathbf{B}_F^T\mathbf{p}_F \oplus \hat{\mathbf{x}}_{FE} \oplus \mathbf{J}_{ER}\mathbf{d}_R \oplus \mathbf{B}_E^T\mathbf{p}_E)\\
&\quad= \mathbf{0}
\end{aligned} \tag{10}$$

where $\mathbf{B}_{FE}$ is *the binding matrix of the pairing* [11], a row selection matrix which selects the components of $\mathbf{x}_{FE}$ which must be zero. Due to uncertainty, a hypothesis test based on the squared Mahalanobis distance D^2, validates the compatibility between E and F

$$D^2 = (\mathbf{B}_{FE}\hat{\mathbf{x}}_{FE})^T[\mathbf{B}_{FE}\mathbf{C}(\mathbf{x}_{FE})\mathbf{B}_{FE}^T]^{-1}(\mathbf{B}_{FE}\hat{\mathbf{x}}_{FE}) \tag{11}$$

where matrix $\mathbf{C}(\mathbf{x}_{FE})$ is computed from the linearization of (10) taking into account the correlations between the map feature F and the[1] robot:

$$\mathbf{C}(\mathbf{x}_{FE}) = \mathbf{J}_{2\oplus}\{\hat{\mathbf{x}}_{FE}, \mathbf{0}\}\mathbf{C}(\mathbf{d}_E^F)\mathbf{J}_{2\oplus}^T\{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \tag{12}$$

where

$$\begin{aligned}
\mathbf{C}(\mathbf{d}_E^F) = {}&\mathbf{J}_{ER}\mathbf{C}_R\mathbf{J}_{ER}^T + \mathbf{J}_{EF}\mathbf{B}_F^T\mathbf{C}_F\mathbf{B}_F\mathbf{J}_{EF}^T\\
&- \mathbf{J}_{EF}\mathbf{B}_F^T\mathbf{C}_{RF}^T\mathbf{J}_{ER}^T - \mathbf{J}_{ER}\mathbf{C}_{RF}\mathbf{B}_F\mathbf{J}_{EF}^T\\
&+ \mathbf{B}_E^T\mathbf{C}_E\mathbf{B}_E.
\end{aligned} \tag{13}$$

Under the Gaussianity hypothesis, D^2 follows a χ^2 distribution. For a given significance level α, the local feature E is compatible with the global feature F if $D^2 \le \chi^2_{r,\alpha}$, with $r = \text{rank}(\mathbf{B}_{FE})$ degrees of freedom, otherwise the matching between the E and F is discarded. In general, when there exist multiple possible pairings for a particular local feature, the pairing with the smallest Mahalanobis distance is chosen.

C. Estimating the SPmap at Time k

The matching between E and F is used both to relocalize the robot at the current trajectory point and, simultaneously, to update the estimated location of the map features. The perturbation vector of the local observation $\mathbf{p}_E \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_E)$ constitutes the measurement used to improve the estimation of $\mathbf{p}_k^W$ through the relation established by (10). Linearization of (10) is done by considering a first order Taylor expansion

$$\begin{aligned}
\mathbf{h}_m &= \mathbf{f}_{k,m}(\hat{\mathbf{p}}_k^W, \hat{\mathbf{P}}_E) = \mathbf{B}_{FE}\hat{\mathbf{x}}_{FE}\\
\mathbf{H}_m &= \left.\frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_k^W}\right|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{P}}_E)} = \left(\mathbf{H}_m^R\, \mathbf{0}\cdots\mathbf{0}\, \mathbf{H}_m^F\, \mathbf{0}\cdots\mathbf{0}\right)\\
\mathbf{H}_m^R &= \left.\frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{d}_R}\right|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{P}}_E)} = \mathbf{B}_{FE}\mathbf{J}_{2\oplus}\{\hat{\mathbf{x}}_{FE}, \mathbf{0}\}\mathbf{J}_{ER}\\
\mathbf{H}_m^F &= \left.\frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_F}\right|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{P}}_E)} = -\mathbf{B}_{FE}\mathbf{J}_{1\oplus}\{\mathbf{0}, \hat{\mathbf{x}}_{FE}\}\mathbf{B}_F^T\\
\mathbf{G}_m &= \left.\frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_E}\right|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{P}}_E)} = \mathbf{B}_{FE}\mathbf{J}_{2\oplus}\{\hat{\mathbf{x}}_{FE}, \mathbf{0}\}\mathbf{B}_E^T.
\end{aligned} \tag{14}$$

Integration of local observations LM_k (i.e., new knowledge of the navigation area) into the SPmap known up to the previous time instant GM_{k-1} by using the EKF equations, produces a reestimation of its whole perturbation vector $\mathbf{p}_k^W$ and the uncertainties represented by its covariance matrix $\mathbf{C}_k^W$. Location estimations of the complete set of entities included in the SPmap are reestimated after the integration of new information. Furthermore, correlations between their estimations are also updated. The complexity of this phase grows polynomially with the number of features $\mathcal{O}(N_F^2 M)$, where M is the number of matched observations.

D. Adding Nonmatched Features to the Global Map

Local observations obtained at time k which cannot be paired with any of the global features of GM_{k-1} are interpreted as knowledge about the environment which has not yet been learned. These local observations are added to the SPmap by considering the composition

$$\begin{aligned}
\mathbf{x}_{WE} &= \mathbf{x}_{WR} \oplus \mathbf{x}_{RE}\\
&= \hat{\mathbf{x}}_{WE} \oplus \mathbf{J}_{ER}\mathbf{d}_R \oplus \mathbf{B}_E^T\mathbf{p}_E.
\end{aligned} \tag{15}$$

The covariance matrix of the SPmap is extended to represent the correlation between the mobile robot and the nonpaired feature E, the cross-correlation between the previous map features and the nonpaired feature E, and the covariance of the nonpaired feature. When the whole local map is composed only of nonpaired observations, the robot's location estimation obtained by odometry cannot be improved.

V. Experimenting with the SPmap

This section presents the experimental verification of the previous ideas by considering a LabMate™ mobile robot navigating indoors, and equipped with a two-dimensional (2-D) rotating laser scanner. We also analyze the effects of neglecting correlations between the location estimation of the features. The vehicle was programmed to follow a trajectory (53 m approximately), stopping at regular intervals to take measurements. Complementary information was taken, by hand, with a pair of theodolites which provided real locations of the robot with respect to a base reference.

From the laser readings gathered at each point along the trajectory, a straight-line segment-based local map LM_k was constructed. A segmentation algorithm [14] was used. The resultant 2-D segments (Fig. 4) were expressed with respect to the mobile robot reference frame R by using the SPmodel formulation; also, estimations of their lengths were computed from their endpoints. Typically the number of straight-line segments was kept small for each local map (below ten segments per local map) by considering only meaningful (i.e., long) segments.

Pairings between local observations and previously stored knowledge of the navigation area were obtained by using a 5% significance level. In the following figures, precision of the results has been

[1] Matrices $\mathbf{J}_{1\oplus}$ and $\mathbf{J}_{2\oplus}$ are the Jacobians of the composition of location vectors [1]

$$\mathbf{J}_{1\oplus}\{\mathbf{x}_1, \mathbf{x}_2\} = \left.\frac{\partial(\mathbf{y}\oplus\mathbf{z})}{\partial \mathbf{y}}\right|_{\mathbf{y}=\mathbf{x}_1,\, \mathbf{z}=\mathbf{x}_2}$$

$$\mathbf{J}_{2\oplus}\{\mathbf{x}_1, \mathbf{x}_2\} = \left.\frac{\partial(\mathbf{y}\oplus\mathbf{z})}{\partial \mathbf{z}}\right|_{\mathbf{y}=\mathbf{x}_1,\, \mathbf{z}=\mathbf{x}_2}.$$

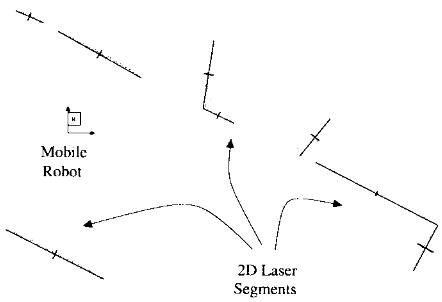


Fig. 4. Straight-line segment representation of a local environment of the vehicle R at the first point of its trajectory. Lateral uncertainty for each detected 2-D segment has been represented (magnified three times).

represented by 95% error ellipses (i.e., $\pm 2\sigma$ bounds). This probabilistic matching technique has proven to be simple and effective for laser segments. However, other sensor systems, such as monocular or stereo vision, require more robust matching strategies to properly solve the data association problem.

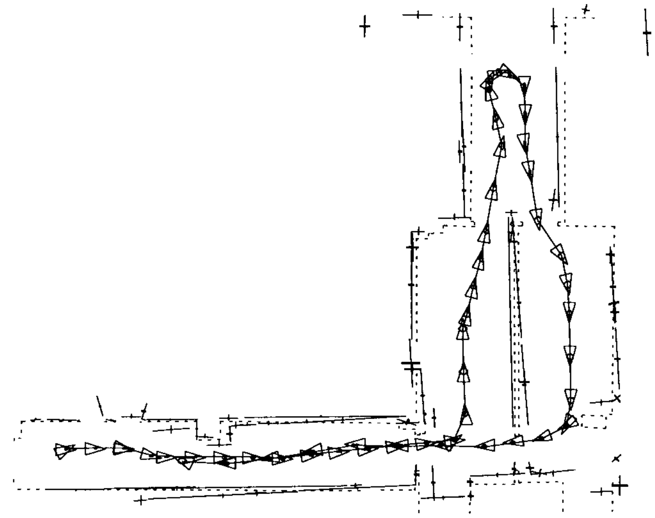
A. Neglecting Correlations Between Entities

When neglecting correlations, the EKF estimation phase is decomposed into the estimation of the mobile robot location and subsequently the estimation of the map features [10] from a common set of observations. Fig. 5(a) shows the result of this process. It can be seen that this two-step process produces optimistic estimations of uncertainty because the same observations are used twice in the estimation algorithms.

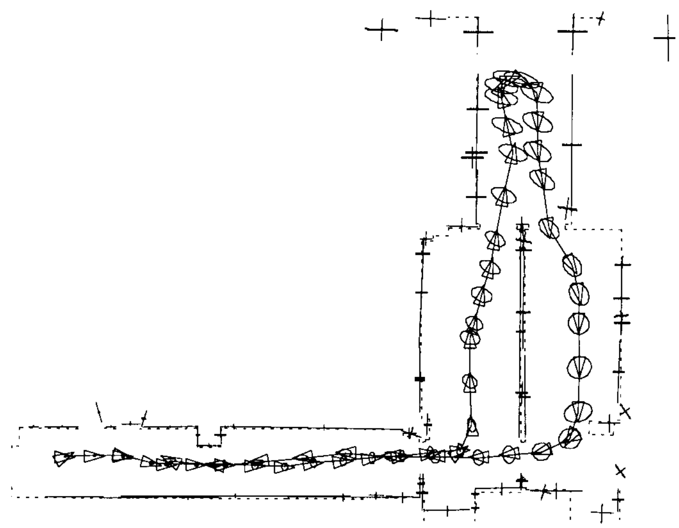
Table I presents the maximum errors and the error growing-rate along the vehicle trajectory when neglecting correlations between features. An increasing discrepancy of the estimated robot location with respect to ground truth was observed (only 5.6% of the estimations were compatible with the real solution). This discrepancy became larger as the vehicle moved to previously learned places in the navigation area, and was due to optimistic estimations of the uncertainty associated to the location of environment features, producing a high rejection rate in the matching process. Thus, when the robot revisited places already learned, very few pairings were obtained and neither the mobile robot was effectively relocalized nor the global map was accurately built. Most of the local map features were directly added to the representation, creating multiple location hypotheses for the same map feature. Later, matchings involving those false hypotheses (i.e., data association errors) induce the final solution to diverge from ground truth.

B. Maintaining Correlations within the SPmap

Fig. 5(b) shows the result of the robot localization and map building process considering the SPmap approach. Even though we simultaneously were building the map and relocating the mobile robot, whenever the vehicle navigated in previously unknown areas, uncertainty continually increased, that is, integration of new observations only reduced the uncertainty growing rate downtown the measurement error of the sensor used, but not further. On the contrary, whenever the vehicle revisited places in the environment already learned, uncertainty decreased, converging to the values of the location uncertainty of the reobserved global map features. Also, the "indirect estimation" effect appeared, that is, location uncertainty decreased for all the features of the SPmap, even for those not visible from the current robot location but statistically correlated to current observations through the off-diagonal elements of the covariance matrix of the SPmap.



(a)



(b)

Fig. 5. Estimated robot trajectory and built map (a) neglecting correlations between entities, where the estimated solution diverges from ground-truth, and (b) using the SPmap approach. An *a priori* model map is drawn for reference purposes.

TABLE I
SUMMARY OF THE SOLUTIONS OBTAINED BY THE DIFFERENT APPROACHES: MAXIMUM ERRORS ($x_{\max}$, $y_{\max}$, AND $t_{\max}$), DISTANCE d_{rel} AND ANGULAR t_{rel} ERROR GROWING-RATES, AND χ^2 TEST RESULTS

	Odometry	Neg. Corr.	SPmap
x_{max} (mm)	1518.22	416.24	272.09
y_{max} (mm)	2066.69	352.36	208.99
ϕ_{max} (deg)	15.50	2.89	2.05
d_{rel} (mm/m)	46.3	9.5	5.9
ϕ_{rel} (deg/m)	0.293	0.055	0.039
χ^2_{test} (%)	100	5.6	98.1

Table I compares the largest errors obtained by the SPmap approach with the approach neglecting correlations. Clearly, the SPmap approach obtained estimations for the mobile robot localization with an upper bound of around 5.9 mm/m of the total trajectory length for the distance error and 0.04°/m for the orientation error, which represented an order of magnitude below dead-reckoning

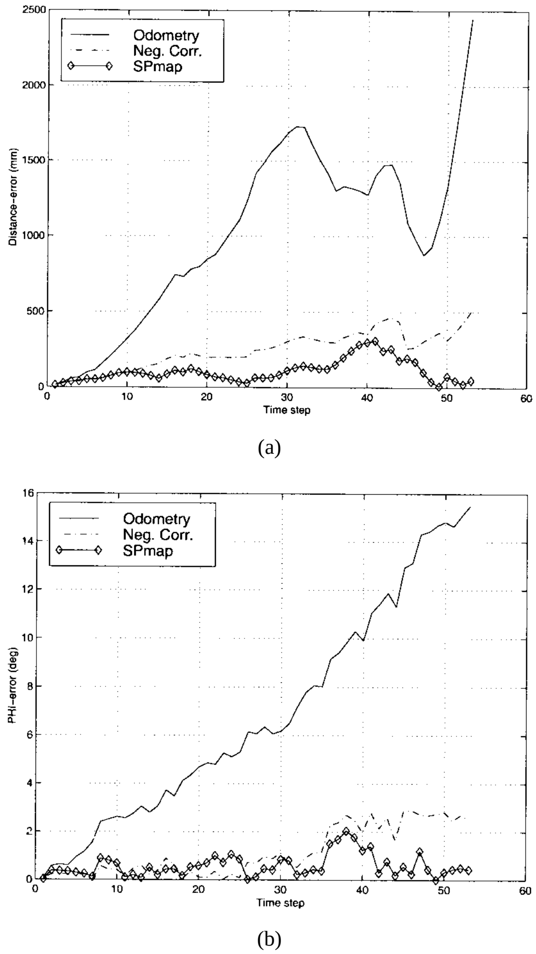


Fig. 6. Comparison of errors along the robot trajectory: (a) distance error and (b) angular error.

errors. Also, the location uncertainty of the map features was not underestimated, with a compatibility of $\chi^2_{\text{test}} \simeq 98\%$, therefore, pairings between local observations at time k and previously stored knowledge up to time $k-1$ were found at each point along the robot trajectory even when the vehicle returned to previously learned places of the navigation area. An average of 74% of the number of available observations were matched with previous known features. Fig. 6 compares the real errors along the robot path, obtained by each approach. The highest errors correspond to odometry-based navigation, while the smallest correspond to the SPmap approach. From the Fig. 6 note how the location uncertainty of the robot decreases when the vehicle revisits previously learned places (i.e., from trajectory point 40 onwards).

VI. Conclusion

This article has presented the symmetries and perturbations map (SPmap), a probabilistic framework for the simultaneous localization and map building problem for mobile robots. We have presented a complete experiment, where a LabMate mobile robot equipped with a rotating 2-D laser rangefinder navigated indoors. Experimentation showed the importance of maintaining correlations between entities. Satisfactory results have been obtained concerning the problem of revisiting previously learned places of the navigation area.

Our recent work has motivated us with further extensions of the concept of SPmap, both to extent its applicability to real-life environments and to increase its robustness:

1) sensor cooperation to obtain more robust and reliable observations from the navigation area [15];
2) increase in the structuration and the semantical contents of the representation toward a topological description where human-language-like instructions could be commanded to the vehicle;
3) search for optimal representations of the navigation area to reduce the complexity $\mathcal{O}(N^2)$ of the current approach, when larger environments are considered;
4) design of strategies to maintain the constructed map, such as those required to remove features not visible for a long time.

Also, we believe that when the vehicle revisited places of the navigation area already learned after travelling for a long time in unknown areas, the Mahalanobis distance would be insufficient to match local observations with previously stored features in the SPmap. In those cases, new data association mechanisms would be required.

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