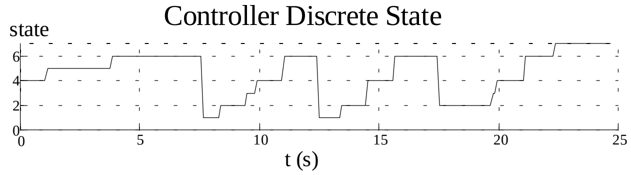
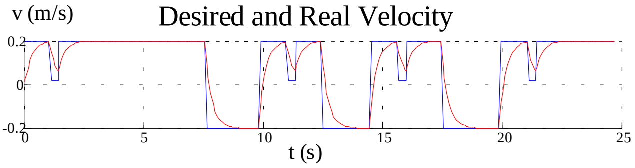
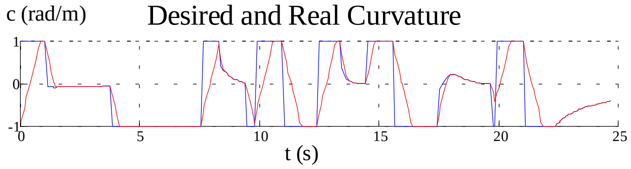
The last figure shows the robot trajectory, and marks with a circle all the switches of the control locations. The following graphs, represent the system variables along time.



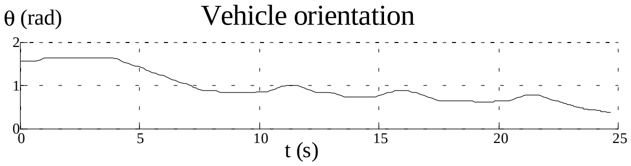
Controller Discrete State



Desired and Real Velocity



Desired and Real Curvature



Vehicle orientation

## 7. Conclusions

A new integrated approach to the global control problem of the locomotion of non-holonomic robots was developed, motivated by several observations of the functional and performance requirements. Its main features are:

- The organization of the global motion into elementary modalities together with a properly designed event-driven feedback enables to overcome the dimensionality problem associated with the continuous quest to reach for a global goal that encompasses some performance criterium
- Elementary modalities allows to overcome the difficulties associated with the vehicle non-holonomic constraints as well as the integration of obstacles data directly in the control loop.

A hybrid feedback controller was implemented for a case study within a class of problems that, to the best of our knowledge, has never been addressed and solved under our requirements in the literature before. Realistic simulation studies showed the robustness to perturbations ensuring a good performance.

## Acknowledgments

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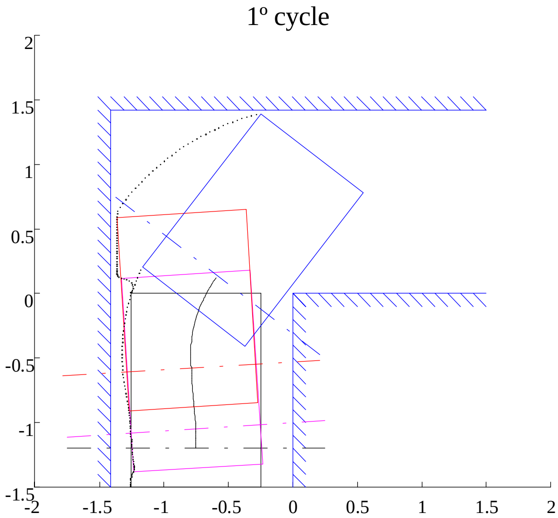
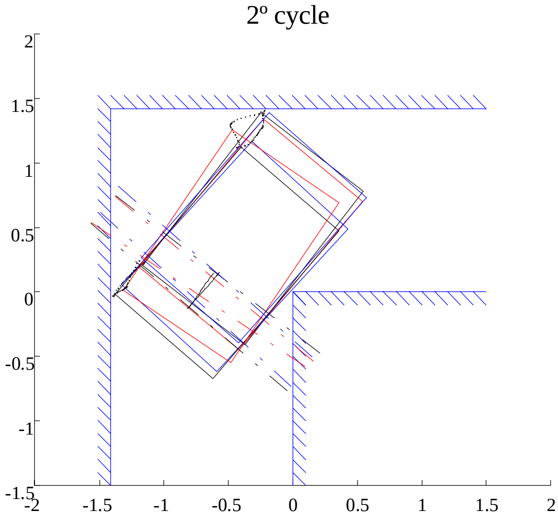
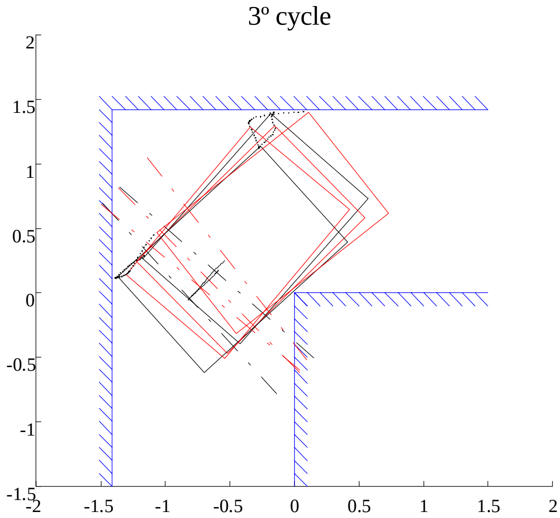
demonstrate the robustness of the controller with respect to model uncertainties.

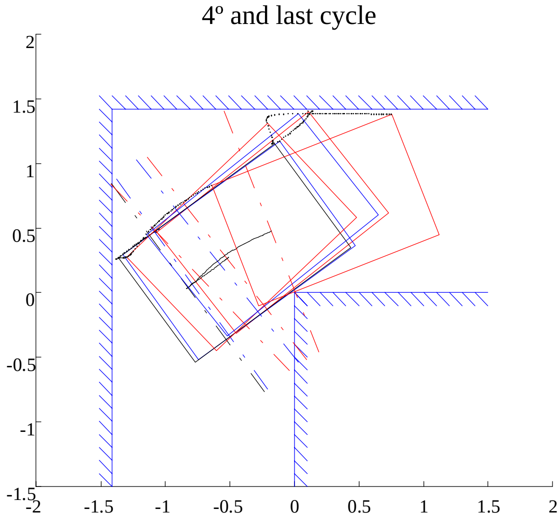
In this example, the width of the entrance and exit corridors of the ‘L’-shaped turn are both 1.4 m. The vehicle is initially in the entrance corridor with the configuration $(x_0,y_0,\theta_0)$ = (-0.75m, -1.2m, $\pi/2$ rad).

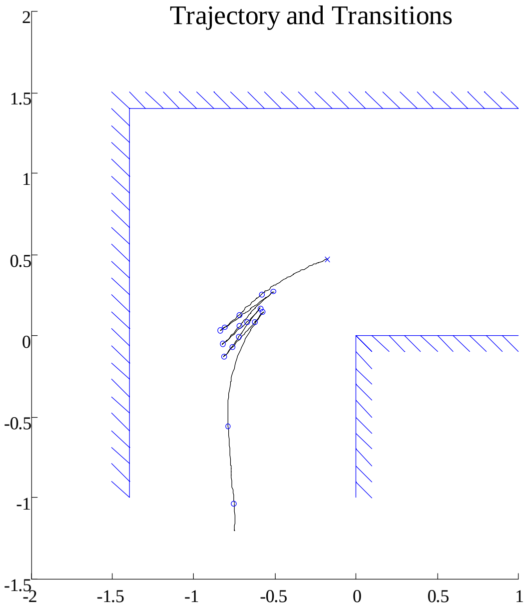
The following four figures represent the robot configuration at the discrete state transitions of the hybrid controller and the trajectory of the robot’s points P, $P_1$ and $P_4$.











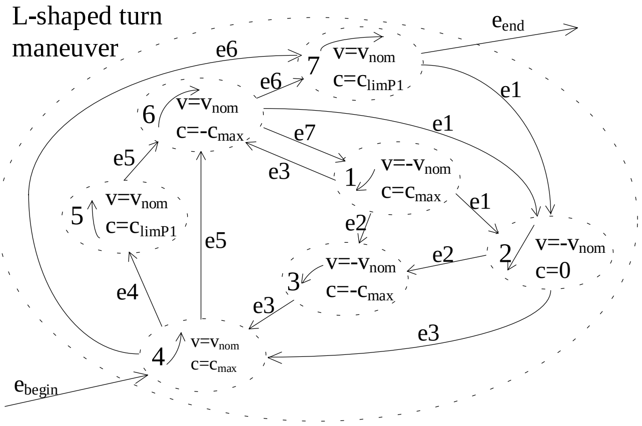


Figure 2. The controller organization.

This last modification does not represent a significant degradation in performance since it only affects a relatively small number of cycles in strongly constrained problems. Furthermore, this modification increases the robustness of the controller by considering viable non-extremal trajectories.

### 5.3.2 Elementary controllers

Given the structure of the kinematic model, control signals v(t) and c(t) can be generated independently.

In each control location the velocity is kept constant at some nominal value.

The curvature controllers can be organized into two classes.

The first class includes modalities 1, 3, 4, and 6. These modalities are not affected by external world constraints. In this case, the objective of the controller is to follow the extreme values in the control constraint set until some event occur.

The second class includes the modalities 2, 5 and 7. Here, geometric constraints may be 'almost active' at some points during the motion segments affecting the allowed control values. In this case, the objective of the controller is to keep the constraints 'almost active', i.e., to move the vehicle along the tangent to them. Therefore, the error to consider in the feedback loop is the 'distance' to those constraints,

| *Elementary controller* | *Distance Error Function* |
|---|---|
| 2 | distance( $\overline{P2Pd}$ , Resq - lag c2 ) |
| 5 | distance( P1, R2 + lag c5 ) |
| 7 | distance( P1, R1 - lag c7 ) |

Table 2. Error Functions

### 5.3.3 Guards synthesis

The relevant event detection is accomplished by mechanisms enabling the prediction of the cost impact on the overall performance of the hybrid controller switch of control location in the next cycle.

We can divide this mechanisms in two groups.

The first one (events e1, e3, e4, e6, e7 and $e_{end}$) is related with the assurance of geometric constraints. Although in this case, the condition is easily predicted because they involve a short temporal horizon, we must guarantee that the resulting transition occur in time to avoid collision. The second group (events e2 and e5) is related with the detection of maneuver optimization events. Since this type of predictions involve larger temporal horizons and needs to use both the controller organization and modalities, it may yield larger errors. Those errors can lead to inappropriate transitions. In those cases, it is more robust to have a delay than to anticipate the transitions. Finally, we can observe that while in the first type of mechanism we only need to know the transition condition and the future control law, in the second type it is necessary for the controller to use its own organization, control laws and transition conditions to predict the behavior of the controller.

| Event | Condition to be verified |
|---|---|
| e1 | Activation of Resq by $\overline{P2Pd}$ |
| e2 | Minimize θ during a future c2 |
| e3 | Activation of R2 by P4 |
| e4 | Activation of R2 by P1 |
| e5 | Minimize θ during a future c2 |
| e6 | Activation of R1 by P1 and θ≤θini |
| e7 | Activation of R1 by P1 |
| $e_{end}$ | θ≤θlim |

Table 3. Conditions

## 6. Simulation Results

In this section, we present Matlab/Simulink simulation results. The hybrid controller was implemented as an S file, containing both the elementary controllers and the relevant event detection mechanisms.

The performance of the controller was evaluated for 'L'-shaped turns with several dimensions and for vehicles at different initial conditions. Due to space limitations, we only show in this paper one example. The characteristics of the vehicle are:

Dimensions:

| | |
|---|---|
| Lf (front length) | 1.2 m. |
| Lb (back length) | 0.3 m. |
| w (width) | 1.0 m. |

Kinematics parameters:

| | |
|---|---|
| Vnom (Nominal velocity) | 0.2 m/s. |
| Cmax (Minimal turn radius) | 1 m. |

Dynamics:

We will consider a model where the velocity is given by a second order system which implies that there is an upper bound on the curvature rate corresponding to an upper bound (2 rads/s, in our example) on the rotation velocity of a fictitious wheel at the middle of the front axis. The consideration of a system with an order degree higher than the one considered in the underlying analysis, allows to course of the second stage. This first step yields a family of sets each one composed by elementary motion segments that will serve as the space in which a search will be performed in order to find the globally optimal trajectory.

The second stage involves a search procedure on a finite subset of the above family of elementary motion segments in such a way that the globally optimal one is selected to compose the feasible trajectory from the current configuration to the given endset. This search procedure will permit to define a set of conditions which will constitute the mechanism to detect the relevant events triggering the transition to a new elementary motion segment. In this way, a continous variable and discrete event based feedback control law is defined that ensures the optimization of a given optimality criterium.

In order to exclude motion trajectories composed by an infinite (possibly uncountable) number of elements, a reduction procedure based on a systems engineering process will permit to increase the computational efficiency of the search in the second stage at the price of sacrificing optimality.

The good news is twofold: on the one hand, the degree of suboptimality is a “controllable” engineering design parameter, and, on the other hand, the fact that we may choose strategies so that the state trajectory progresses within the reachable set thus keeping the flexibility required to react to perturbations during motion execution.

## 5.2 Elementary Motion Strategies

The motion is organized into a set of open loop optimal segment providing the specifications for the control locations, edges and the associated guards are defined ( see [2]). The obtained set of motion segments is:

c1(t)={v<0,c=Cmax}

c2(t)={v<0,c=0}

c3(t)={v<0,c=-Cmax}

c4(t)={v>0,c=Cmax}

$c5(t)=\{v>0,c(t)=Clim_{R2P1}(t)\}$

c6(t)={v>0,c=-Cmax}

$c7(t)=\{v>0,c(t)=Clim_{R1P1}(t)\}$.

Where $clim_{R1P1}(t)$ and $clim_{R2P1}(t)$ are defined as:

$$C\lim_{R1P1}(t) = -\frac{\sin(\theta(t))}{Lf\cos(\theta(t)) - \frac{w}{2}\sin(\theta(t))}$$

$$C\lim_{R2P1}(t) = \frac{\cos(\theta(t))}{Lf\sin(\theta(t)) + \frac{w}{2}\cos(\theta(t))}$$

The set of guards can be seen in Table 1.

| Transition | | Condition to be verified |
|---|---|---|
| Number | Meaning | Event |
| 1 | c1 → c2 | Activation of Resq by $\overline{P2Pd}$ |
| 2 | c1 → c3 | Minimize θ in a future c2 |
| 3 | c1 → c6 | Activation of R2 by P4 |
| 4 | c2 → c3 | Minimize θ in a future c2 |
| 5 | c2 → c4 | Activation of R2 by P4 |
| 6 | c3 → c4 | Activation of R2 by P4 |
| 7 | c4 → c5 | Activation of R2 by P1 |
| 8 | c4 → c6 | Minimize θ in a future c2 |
| 9 | c4 → c7 | Activation of R1 by P1 |
| 10 | c5 → c6 | Minimize θ in a future c2 |
| 11 | c6 → c7 | $\theta \le \theta ini$ |
| 12 | c6 → c1 | Activation of R1 by P1 |
| 13 | c7 → c2 | Activation of Resq by $\overline{P2Pd}$ |
| 14 | c7 → end | $\theta \le \theta lim$ |

Table 1. Set of Guards

We can summarize some additional notes about the above motion strategy structure presented:

- There is at most one segment c5 and it is used in the beginning of the maneuver.
- The existence of the segment c2 permits the decoupling of each cycle of the maneuver.
- In some cases, like those needing a large number of cycles or with small entrance width, it happens that a initial set of cycles of the optimal motion strategy do not have a c2-type of segment. In this case the decoupling is impossible and all the cycles in this set have to be optimized all together.

## 5.3 Synthesis of the Hybrid Controller

The previously defined optimal motion strategies provide the main guideline for the synthesis and organization of the control locations and guards.

The existence of execution errors and modeling uncertainty poses some issues to be addressed during the synthesis of the controller. For this reason, we have to introduce some flexibility at critical situations in order to either ensure the satisfaction of problem constraints or avoid highly performance sensitive configurations.

### 5.3.1 Controller Organization

The overall motion organization of the hybrid controller is described in Figure 2. In this figure, nodes represent the various control locations (or elementary modalities), and arcs indicate the edge. In each control location, the number represents a discrete state corresponding to a modality type, and the arrow shows the graphic shape of the trajectory generated by the indicated control. Close to the edge, there is an identifier of the event associated with that transition.

This organization does not necessarily represent the apriori global optimal motion strategy since:

- Transitions are added since new situations might occur due to execution errors or model uncertainty.
- The first cycles that interact together are decoupled whenever required in order to obtain a better computational efficiency.

feedback, previous work on execution control considers new types of feedback approaches. These approaches take the form of discontinuous time-invariant feedback (piecewise continuous [1,3] or sliding mode [5,6]), time-varying feedback ([12]) or hybrid feedback ([9]) control laws.

## 3. Our Approach to Motion Control

The main idea of our approach to the motion control of a mobile robot in semi-structured world is to increase the abstraction level of the planner. As a consequence, the plan is no longer a geometric sequence or a graph but takes the form of a structure supporting the coordination of a set of parametrized atomic hybrid controllers, that we also refer to by maneuvers.

This way, the approach involves two stages. The first one, hybrid controllers synthesis, involves the definition of a set of maneuvers solving specific classes of motion problems which are classified according to the patterns of the associated spatial constraints. This phase produces the resources to the planner. The second stage, planning, addresses the definition of a hybrid controller coordinating the real-time invocation of the most appropriate maneuver so that the desired motion is accomplished. Motion specification consists of two endpoints (or endsets) in the configuration space, a set of requirements and a performance criterium.

This paper focus on the first phase, i.e., the synthesis of the atomic hybrid controller, which is illustrated with the case study of a L-shaped turn maneuver. This exercise provides the main ingredients intervening in a general methodology for motion planning and control for nonholonomic vehicles.

## 4. The Problem

The problem addressed in this paper consists in the synthesis of a feedback controller enabling a car-like rectangular vehicle to move from the entrance to the exit configurations sets in the exit corridor, while satisfying all the constraints, and minimizing the performance criterium. Here, the performance criterium is the total number of maneuvers which correspond to the changes of sign of the longitudinal velocity. This changes are needed for the terminal endset to be reachable.

A sketch of the setting considered in this problem is depicted in Figure 1.

The considered car-like rectangular vehicle is steered by two front directional wheels and, as a consequence, it has a minimum turning radius. The kinematic model used for this vehicle is

$$\begin{cases} \dot{x} = v.\cos(\theta) \\ \dot{y} = v.\sin(\theta) \\ \dot{\theta} = v.c \end{cases}$$

with the control inputs $v$ and $c$ satisfying the constraints

$$v \in \left[-v_{max}, v_{max}\right] \text{ and } c \in \left[-c_{max}, c_{max}\right]$$

and where,

- $(x, y, \theta)$, is the system state, with $(x, y)$ representing the coordinates of the mid point of the rear wheels axis which is also the origin of the vehicle referential system, and $\theta$ the angular deviation between the global and the vehicle coordinate systems.
- $v$, is the linear velocity
- c, is the instantaneous curvature, which is the inverse of the instantaneous turning radius.

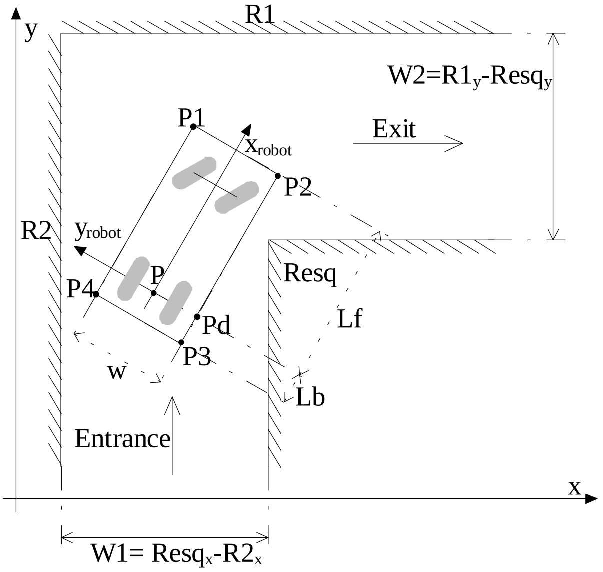


Figure 1. The robot, geometric constraints and objective

For simplicity and without any loss of generality, we will assume that the vehicle is entering the curve in a forward motion. The geometric constraints are simply parametrized by the widths of the entrance and exit corridors, respectively w1 and w2, which define two line segments, R1 and R2, and one corner, Resq.

## 5. The Solution

The description of the hybrid motion controller [11] involves the specification of a set of control locations, the transition structure and a set of guards. A control location corresponds to an elementary motion controller which is selected in real-time according to a dynamic set of edges, defining the transition structure of the hybrid controller. Associated with a set of edges, there is a set of guards representing conditions of occurrence relevant events. These are related to either the satisfaction of constraints or to opportunities to switch to another control location that better contributes to the improvement a global performance.

### 5.1 Design sketch of the organization

Our methodology to the organization design of the hybrid controller involves two stages:

In the first one, local variational optimality conditions in the form of a separated maximum principle are used in order to characterize elementary optimal motion strategies parametrized by their endpoint configurations as well as some coordinating parameter which will be defined in the

# ON THE DESIGN OF A HYBRID FEEDBACK CONTROL SYSTEM FOR A NONHOLONOMIC CAR-LIKE VEHICLE[*]

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**Keywords:** Hybrid Systems, Nonholonomic Systems, Mobile Robotics, Car-like Robot, Maneuvering.

## Abstract

This paper addresses the design of a hybrid feedback control system enabling a car-like, rectangular vehicle to perform a tight L-shaped turn maneuver. This case constitutes an effort towards the definition of a general purpose methodology for hybrid feedback control synthesis. The design effort encompasses the synthesis of a set of modalities and mechanisms to detect relevant events enabling the overall motion coordination so that a global goal is attempted. Simulations of the implemented controller have shown a strong robustness with respect to modeling uncertainty and execution errors.

## 1. Introduction

The problem addressed in this paper consists in deriving a feedback control law for a rectangular car-like vehicle to perform a right turn in a rectangular L-shaped corridor which might require a turning radius smaller than that of the vehicle. This problem may be considered as an instance of the design of a feedback controller for an autonomous dynamic system. An integrated approach to motion organization and execution control was adopted in order to solve this problem. The observation that solutions synthesized by human beings when addressing the same issue are much simpler and more intuitive, robust, efficient, integrated than those referred to in the literature for mobile robotics motivated the proposed approach. Although most of the previous research efforts in this area reveal important contributions to motion organization and execution, the methods applied so far usually fail to consider the required overall integration.

This approach is characterized by:

- considering nonholonomic constraints in the definition of both global and local components of the control strategies,
- taking into account data about obstacles in the closed loop control laws,
- using an event driven feedback in order to organize motion strategies so that some global goal is attempted, and
- enabling the topological-like representation of the global problem where the basic elements correspond to simple control subproblems.

As a consequence, the controller endows the system with a high degree of robustness with respect to execution errors and enables significant levels of performance.

This paper is organized as follows: in section 2, we review some work developed on the control of nonholonomic car-like vehicles. After describing the main ideas and the design stages, in section 3, we state the case study problem under consideration in this paper in section 4, and describe the solution in the section that follows by detailing the structure of the hybrid feedback controller and by stating the underlying assumptions. Finally, before drawing some conclusions, we present some simulation results in section 6.

## 2. Review on the Control of Non-holonomic Car-like Vehicles

A lot of work has been done in this field. An overview of recent developments in control of nonholonomic systems can be found in [8]. In this section, we will focus on the problem of motion planning and control for a car-like vehicle under state constraints.

In general, the motion control of nonholonomic vehicles consists of two phases: motion planning and execution control.

The first one has a global scope and involves the consideration of state and control constraints. This phase usually precedes the path execution.

The second one has a local scope and the objective is to track the planned reference while ensuring robustness to the execution errors.

There are two main approaches to the motion planning of non-holonomic vehicles.

In the first class of works (see [7,10]), the planning is decomposed in two phases: holonomic planning and synthesis of feasible trajectory that will serve as a reference for the vehicle. In the second class, e.g., [4], search and optimization techniques are used to find the best feasible trajectory in a discretized state space.

Due to the impossibility of stabilizing a driftless system that has fewer controls than states by smooth time-invariant

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