

# Sonar Mapping for Mobile Robots

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## Abstract

We address the problem of building environment maps from ultrasonic range data obtained from multiple viewpoints. We present a novel environment modelling technique called the ‘response grid’ that allows us to build occupancy maps in highly specular environments. We present three different approaches that utilise this technique: a Bayesian probabilistic approach, a Dempster-Shafer evidential reasoning approach and a winner-take-all heuristic approach. All three approaches can be implemented in real-time with modest computational resources, and as such are suitable for use in mobile robot navigation tasks. We present and compare the experimental results obtained by these three methods in a highly specular indoor environment.

## 1 Introduction

Building environment maps from sensory data is an important aspect of mobile robot navigation, particularly for those applications in which robots must function in unstructured environments. Ultrasonic range sensors are, superficially, an attractive sensor modality to use in building such maps, due mainly to their low cost, high speed and simple output. Unfortunately, these sensors have a number of properties that make map building a non-trivial process. In particular, standard sensors have very poor angular resolution and can generate misleading range values in specular environments. The first of these problems can be largely overcome by combining range measurements from multiple viewpoints. Elfes [1] and Moravec [2] describe an approach in which range measurements from multiple viewpoints are combined in a two-dimensional ‘occupancy grid’. Each cell in

the grid is assigned a value indicating the probability that the cell is occupied. Unfortunately, the occupancy grid approach does not work well in specular environments. Specular reflection may occur whenever an ultrasonic pulse encounters a smooth extended surface [3]. In such cases the pulse may not be reflected back to the ultrasonic sensor; in effect, the surface may appear to be invisible. In ordinary office environments which contain smooth walls and glass doors specular reflection is common. In this paper, we improve on earlier grid-based approaches by introducing the concept of a ‘response grid’. The intent of the response grid framework is to produce an approach which has the advantages of the occupancy grid framework, but also performs well in specular environments.

The response grid framework attempts to model the behaviour of ultrasonic range sensors in a more physically realistic fashion. A number of other authors have considered the physical behaviour of such sensors in some detail [4, 5, 6]; in this paper, however, we are only concerned with physical behaviour in-so-far as it allows us to generate two dimensional occupancy maps in specular environments. The basic notion that the response grid encapsulates is that a cell may generate a response (ie appears to be occupied) when viewed in one direction, but will not generate a response when viewed from another. For example, a smooth planar surface will only generate a response when the angle of incidence between the surface normal and the beam emitted by the sensor is close to zero. At larger angles of incidence the surface will generate no response. In the original occupancy map framework, this would present a contradiction, since this approach assumes that an occupied cell should generate responses in *every* direction.

In this paper, we present three different methods for generating occupancy maps from ultrasonic range data within the reflection grid framework. The difference between the three methods lies in the different techniques they use to combine data obtained from multiple viewpoints. We present and compare a Bayesian probabilistic reasoning approach, a Dempster-Shafer evidential reasoning approach and a heuristic winner-take-all approach. The Bayesian approach can be viewed, to some extent, as a generalisation of the approach described by Elfes [1] and by Moravec [2]. This earlier work appears as a special case in our approach. The second approach utilises the Dempster-Shafer theory of evidential reasoning to combine evidence. We will see that these first two methods produce very similar results. To our knowledge, Dempster-Shafer techniques have not previously been applied to ultrasonic map building; they have, however, been applied to the somewhat similar task of building occupancy maps from stereo image pairs [7]. The final method we present is based on a simple winner-take-all heuristic for combining evidence. In contrast with the other two methods, this method makes no assumptions about the statistical independence (or otherwise) of repeated measurements. On the other hand, it is not as robust in the presence of errors. In Section 5 we compare the experimental results obtained by all three methods and consider their relative advantages. Note, however, that all three methods can be implemented in real-time with modest computational

resources, which makes them well suited to mobile robot navigation tasks.

In the following sections, we describe some of the properties of ultrasonic range sensors and develop the response grid framework in detail. The theory and implementation of the three mapping methods is described, together with some experimental results obtained in a highly specular environment.

## 2 Framework

### 2.1 Properties of ultrasonic range finders

In order to motivate the discussion to follow, it is important to understand some of the physical properties of ultrasonic range sensors (detailed discussions of the physical properties of such devices can be found in [4]). Standard ultrasonic range sensors, such as the widely used Polaroid device, emit a series of short ultrasonic pulses in a beam which is about  $20^\circ$  wide. The pulses propagate through the air, reflect off solid objects and arrive back at the sensor. By timing the interval between the emitted pulse and the returned echo, these devices can measure ranges up to about 10 meters with an accuracy of the order of a few centimeters [3]. The difficulty with using these devices is two-fold. Firstly, the broad beam results in very poor angular resolution; the location of the object generating the reflection is known accurately in only one dimension (that along the beam axis). This limitation can be overcome by taking measurements from multiple viewpoints and combining the data. Secondly, when interacting with an extended surface the ultrasonic pulse will tend to undergo specular reflection. This means that for certain surface orientations there will be no return pulse received by the sensor: the surface is effectively invisible. Alternatively, the pulse reflected from a surface may then reflect off another surface before returning to the sensor. This ‘multiple reflection’ effect produces totally misleading range measurements. Although the environment model described in the next section copes well with ‘invisible’ surfaces, it has no effective way of dealing with multiple reflections.

### 2.2 The Response Grid

We model the environment as a set of cells arranged in a regular two-dimensional grid. A given cell in the grid may either be entirely empty, or else it may contain one or more surfaces which reflect ultrasonic pulses. A pulse entering a cell will do one of three things:

- If the cell is entirely empty, the pulse will pass through the cell unaffected.
- If the cell contains one or more reflective surfaces, the pulse may be reflected back to the ultrasonic detector.

- If the cell contains one or more reflective surfaces, the pulse may be reflected away from the ultrasonic detector.

The behaviour of a pulse reflecting off a surface contained in the cell will depend upon the orientation of the surface and the direction of the incoming pulse. If a pulse which is propagating in some given direction is reflected back to the detector, the cell is said to have a *response* in that direction. In this paper, we determine the *occupancy* of each cell by assuming that any cell that generates a response in one or more directions must contain at least one surface and is therefore *occupied*. Note that this leaves open the possibility of a cell being occupied, but not having a response in *any* direction. This situation can arise either when the cell contains a material that entirely absorbs incident pulses, or else contains a surface that reflects the pulses in such a way that they can *never* return to the detector. The former of these cases arises rarely in real world situations and can be ignored, but the latter case requires more careful consideration. The model that is proposed here is two dimensional, being intended for use with mobile robots whose ultrasonic sensors are generally constrained to motion in a plane. To an ultrasonic sensor so constrained, certain three-dimensional shapes are effectively invisible. Thus any algorithm using ultrasonic sensor data to determine occupancy must fail under certain environmental conditions. Fortunately, such conditions are rare in many environments, particularly human-made environments such as offices and factories.

A second problem arises when a surface reflects pulses in such a way that they travel to another surface and are then reflected back to the detector. Without some way of knowing that this ‘multiple echo’ effect has occurred, the detected response will be attributed to the wrong cell. Consequently, a cell which is in fact empty may appear to be occupied. This effect is quite common; it is clearly visible in the experimental results presented in Section 5.

The model described above can be expressed mathematically as follows. The occupancy of a cell at location  $(x, y)$  is measured by the state variable *Occ* which can have one of two values:

$$\text{Occ}(x, y) = [\textit{occupied}, \textit{unoccupied}]. \quad (1)$$

The response of a cell in some direction  $\phi$  is measured by the state variable *Res*:

$$\text{Res}(x, y, \phi) = [\textit{response}, \textit{no response}]. \quad (2)$$

The direction  $\phi$  is allowed to take discrete values between 1 and  $n$ . The two state variables are bound together by a logical implication. In order to express this clearly, we define a set of propositions:

$$\begin{aligned} O & : \text{Occ}(x, y) = \textit{occupied} \\ R_\phi & : \text{Res}(x, y, \phi) = \textit{response}. \end{aligned} \quad (3)$$

That is, the proposition  $O$  states that the cell at  $(x, y)$  is occupied and the proposition  $R_\phi$  states that the cell at  $(x, y)$  generates a response in direction  $\phi$ . We can therefore write the implication:

$$O \Leftarrow R_1 \vee R_2 \cdots \vee R_{n-1} \vee R_n. \quad (4)$$

In this paper we present three methods for using ultrasonic range data to determine the response properties of each cell, and thence to determine occupancy.

## 3 Mathematical Background

### 3.1 Method 1: Bayesian

The objective of the Bayesian method is to determine, for each cell, the *probability* that the cell is occupied. That is, for a cell at  $(x, y)$  we wish to determine the probability that the proposition  $O$  is true. Rather than attempting to calculate this probability directly from range data, we compute it indirectly. Since  $O$  is related to the cell responses by the logical implication expressed in Equation 4, the probability that the cell is occupied must be given by:

$$p(O) = p(R_1 \vee R_2 \cdots \vee R_{n-1} \vee R_n). \quad (5)$$

To expand the right hand side of this equation, we make use of the observation that for two independent propositions  $A$  and  $B$ , the probability of either  $A$  or  $B$  being true is given by [8]

$$p(A \vee B) = p(A) + p(B) - p(A)p(B). \quad (6)$$

Applying this to the above equation, one obtains after a little algebra:

$$p(O) = 1 - \prod_\phi (1 - p(R_\phi)). \quad (7)$$

This expression can be used to compute the probability that a cell is occupied once we have determined the cell response probabilities. Note that this involves the determination of  $n$  *separate* probabilities, since  $\{R_1 \cdots R_n\}$  is a set of *independent* propositions.

Consider now just one proposition  $R_\phi$  corresponding to the response in direction  $\phi$ . We can apply Bayes' rule to determine the probability that  $R_\phi$  is true, given a range measurement  $r$ :

$$p(R_\phi | r) = \frac{p(r | R_\phi)p(R_\phi)}{p(r)}. \quad (8)$$

In this expression  $p(R_\phi)$  is the *prior probability* of obtaining a response. In Bayesian approaches it is very common to set the prior probability to 0.5 to

indicate no opinion. However, in our case this leads to undesirable results. Consider a single cell for which we have no measurements; applying Equation 7 to determine the probability that the cell is occupied would yield  $p(O) = 1 - 0.5^n$ . So for  $n = 8$  we would have  $p(O) = 0.996$ . That is, on the basis of no measurements whatsoever we would conclude with near certainty that the cell is occupied. A better way to set the prior probabilities in this case is to demand that  $p(O) = 0.5$  in the absence of any measurements. This implies that we should set the prior probability to:

$$p(R_\phi) = 1 - (0.5)^{\frac{1}{n}}. \quad (9)$$

The other important term in Equation 8 is the *sensor model*  $p(r | R_\phi)$ . The sensor model indicates the probability of obtaining the measurement  $r$ , given that the proposition  $R_\phi$  is true. Let  $s$  be the distance between the cell and the sensor; the sensor model we use is:

$$p(r | R_\phi) = \begin{cases} 0.05 & \text{if } s < r \\ 0.5 & \text{if } s > r \\ \alpha/r & \text{if } s = r \end{cases}. \quad (10)$$

Qualitatively this can be expressed as follows:

- If the cell is closer to the sensor than the recorded range, then cell probably does not generate responses in this direction:  $p(r | R_\phi) = 0.05$ .
- If the cell is farther from the sensor than the recorded range, then we know nothing about this cell:  $p(r | R_\phi) = 0.5$ .
- If the cell is the same distance from the sensor as the recorded range, then the cell *may* generate a response in this direction:  $p(r | R_\phi) = \alpha/r$ .

The first two parts of this model are self-explanatory, but the third requires some explanation. When the sensor records a range  $r$  we know that *something* at this range has generated a response. Due to the finite angular resolution of the detector, however, there may be a number of cells that may have generated this response. In general, the number of such cells will be proportional to the measured range, so the probability that any individual cell generated the response is *inversely proportional* to the measured range. In effect, we give more weight to short range measurements than to long range measurements. This rule is both physically plausible and intuitively appealing. The  $\alpha$  that appears in the above rule is a normalisation constant: summing over the probabilities assigned to each cell at range  $r$  should yield a total probability of 1. The value of  $\alpha$  is determined by the spatial dimensions of the cells making up the map and by the angular resolution of the ultrasonic sensor.

The model we use ignores statistical errors associated with the range value itself; that is, our model is *ideal*. We justify this simplification by noting that

for common types of ultrasonic sensors such errors are of the order of one or two centimetres. In practical applications, such as mobile robotics, these errors are insignificant compared to errors arising from other sources, such as robot (mis)localisation.

Equations 7, 8 and 10 provide us with the mathematical tools required to construct a Bayesian occupancy map. We will postpone discussion of practical implementations of such a map until Section 4, where the implementation of all three methods presented in this paper is considered.

### 3.2 Method 2: Dempster-Shafer

The Dempster-Shafer Theory of Evidence [9] can be applied to our problem in a straight-forward fashion. The objective is to determine the *support* for the proposition  $O$ . From the logical implication expressed in Equation 4, we can write the following:

$$\text{Sup}(O) = \text{Sup}(R_1 \vee R_2 \cdots \vee R_{n-1} \vee R_n). \quad (11)$$

To expand the right hand side of this equation, we use the Dempster-Shafer analogue of Equation 7. Given two independent propositions  $A$  and  $B$ , the support for the combined proposition  $A \vee B$  is given by [10]:

$$\text{Sup}(A \vee B) = \text{Sup}(A) + \text{Sup}(B) - \text{Sup}(A)\text{Sup}(B). \quad (12)$$

Applying this to the expression above, one obtains:

$$\text{Sup}(O) = 1 - \Pi_\phi(1 - \text{Sup}(R_\phi)), \quad (13)$$

which allows us to calculate the support for the proposition that a cell is occupied, once we have determined the support for each of the propositions  $\{R_1 \cdots R_n\}$ .

Consider now just one possible response direction. In order to determine the support for the proposition that the cell responds in this direction, the Dempster-Shafer approach requires that we construct a relevant *frame of discernment*, which is a set which contains all the propositions of interest. In our case, the frame of discernment contains just two propositions: that the cell responds, or that it does not:

$$\Theta = \{R_\phi, \neg R_\phi\}. \quad (14)$$

In general, support for these propositions cannot be determined directly from the available evidence. Instead, support is computed indirectly via the *mass distribution*. The mass distribution allows us to allocate a ‘weight’ to any element in the frame of discernment, or to any proper subset of the frame of discernment. Thus in our application, the mass distribution can assign weight to any element in the set:

$$2^\Theta = \{R_\phi, \neg R_\phi, R_\phi \vee \neg R_\phi\}. \quad (15)$$

The weight assigned to an element in this set indicates the amount of evidence that supports that element directly. Support for propositions in the frame of discernment can be calculated according to the rule:

$$\text{Sup}(A) = \sum_{A \subseteq B} m(B). \quad (16)$$

Thus the support for the proposition  $R_\phi$  is trivially equal to  $m(R_\phi)$ . The mass distribution is somewhat similar to a probability distribution in that it must be non-negative and must sum to one. Consider the following mass distribution:

$$\begin{aligned} m(R_\phi) &= 0 \\ m(\neg R_\phi) &= 0 \\ m(R_\phi \vee \neg R_\phi) &= 1. \end{aligned} \quad (17)$$

For brevity we write this as  $(0, 0, 1)$ . This distribution corresponds to complete ignorance, since it will yield no support for either  $R_\phi$  or its negation. Contrast this with the distribution  $(1, 0, 0)$ , which indicates complete support for  $R_\phi$  and no support for its negation; and with  $(0.5, 0.5, 0)$ , which indicates a contradiction - there is equal support for  $R_\phi$  and its negation. One of the attractive features of the Dempster-Shafer approach is the way in which the ignorance and contradiction are clearly distinguished; this is not true of the Bayesian approach.

In our problem, every new measurement arriving from the ultrasonic sensor is treated as a new piece of evidence for which we must generate a mass distribution. These separate distributions must then be combined to generate a collective opinion. If  $m(\cdot | r_1)$  and  $m(\cdot | r_2)$  represent mass distributions arising from independent measurements, the combined mass distribution  $m(\cdot | r_1, r_2)$  is given by Dempster's rule [9]:

$$m(C | r_1, r_2) = \frac{1}{1 - \kappa} \sum_{A \cap B = C} m(A | r_1) m(B | r_2), \quad (18)$$

where  $\kappa$  is defined as

$$\kappa = \sum_{A \cap B = \emptyset} m(A | r_1) m(B | r_2), \quad (19)$$

and  $A$ ,  $B$  and  $C$  are elements of the set  $\{R_\phi, \neg R_\phi, R_\phi \vee \neg R_\phi\}$ . Inspecting the above equation, one can see that when a contradiction exists (for example when there is support for both a proposition and its negation),  $\kappa$  will be non-zero. As a result  $\kappa$  is usually interpreted as indicating the degree of contradiction between the two mass distributions. Dempster's rule can be thought of as Dempster-Shafer equivalent of Bayes' rule.

The rule we use for generating mass distributions from range measurements is analogous to the sensor model used in the Bayesian case. Let  $m(\cdot | r)$  be the mass



distribution resulting from the measurement  $r$ ; let  $s$  be the distance between the cell and the sensor; the sensor model we use is:

$$m(\cdot | r) = \begin{cases} (0, 0.95, 0.05) & \text{if } s < r \\ (0, 0, 1) & \text{if } s > r \\ (\alpha/r, 0, 1 - \alpha/r) & \text{if } s = r \end{cases} . \quad (20)$$

Qualitatively, this can be expressed as follows:

- If the cell is closer to the sensor than the recorded range, then the cell has not generated a response in this direction:  $m(\cdot | r) = (0, 0.95, 0.05)$ .
- If the cell is farther from the sensor than the recorded range, then we know nothing about the response of this cell:  $m(\cdot | r) = (0, 0, 1)$ .
- If the cell is the same distance from the sensor as the recorded range, then the cell *may* respond in this direction:  $m(\cdot | r) = (\alpha/r, 0, 1 - \alpha/r)$ .

The reasoning used to arrive at this model is identical to that used in the Bayesian case. We note only that the initial, or prior, mass distribution we use corresponds to complete ignorance –  $(0, 0, 1)$ . The implementation of this method is considered in Section 4.

### 3.3 Method 3: Winner-take-all

In this section we present a simple alternative method for building occupancy maps. This method arises from the observation that both the Bayesian and Dempster-Shafer methods assume that range measurements are *independent*. It is not clear that this is the case when building maps from ultrasonic range data. The *winner-take-all* method described here makes no such assumptions and thereby allows the effect of that assumption to be determined.

Consider first the determination of cell occupancy. Let  $\text{Bel}(O)$  represent the strength of belief that the cell is occupied and let  $\text{Bel}(R_\phi)$  be the belief that the cell responds in direction  $\phi$  (this should not be confused with the Dempster-Shafer concept of support, which is sometimes also referred to as belief). The belief that the cell is occupied is computed according to the following rule:

$$\text{Bel}(O) = \max_\phi \text{Bel}(R_\phi), \quad (21)$$

that is, we take the belief to be the maximum of all the cell response beliefs. Using this rule, a strong belief that the cell responds in one direction is both necessary and sufficient to obtain a strong belief that the cell is occupied; it is not sufficient for there to be a small amount of belief in many directions.

Looking at the sensor models used in the Bayesian and Dempster-Shafer methods, we can see that evidence really comes in two forms – evidence suggesting that

the cell responds and evidence suggesting that it does not. Let  $\text{Ev}(R_\phi)$  represent the strength of the evidence suggesting that a cell does respond, and  $\text{Ev}(\neg R_\phi)$  the evidence that it does not. We use the following rule to convert evidence to belief:

$$\text{Bel}(R_\phi) = \begin{cases} \text{Ev}(R_\phi) & \text{if } \text{Ev}(R_\phi) > \text{Ev}(\neg R_\phi) \\ 0 & \text{otherwise} \end{cases} . \quad (22)$$

This is a winner-take-all rule: we discard all the evidence supporting  $R_\phi$  if there is more evidence supporting its negation. We also require some rule for combining evidence. The rule we use is as follows. Given two pieces of evidence  $\text{Ev}(\cdot | r_1)$  and  $\text{Ev}(\cdot | r_2)$ , the combined evidence  $\text{Ev}(\cdot | r_1, r_2)$  is given by:

$$\text{Ev}(\cdot | r_1, r_2) = \max[\text{Ev}(\cdot | r_1), \text{Ev}(\cdot | r_2)] . \quad (23)$$

In other words, when presented with two pieces of evidence, we simply take the best one and discard the other. All that remains now is to determine how to compute  $\text{Ev}(R_\phi)$  and  $\text{Ev}(\neg R_\phi)$  from a range reading. As we have argued in previous sections, the best evidence to the measurement in which the distance between the cell and the sensor is *least*. In fact, the strength of evidence is inversely proportional to the distance between the cell and the sensor. This leads to the following result: for a cell at a distance  $s$  from the sensor, a range reading  $r$  generates evidence for  $R_\phi$ :

$$\text{Ev}(R_\phi | r) = \begin{cases} 1/s & \text{if } s = r \\ 0 & \text{otherwise} \end{cases} , \quad (24)$$

and for  $\neg R_\phi$ :

$$\text{Ev}(\neg R_\phi) = \begin{cases} 0 & \text{if } s = r \\ 1/s & \text{otherwise} \end{cases} . \quad (25)$$

Although this method appears superficially complicated, it is very simple to implement. Nor is the method entirely ad-hoc: it resembles, in many ways, a Fuzzy-Logic approach to the problem.

## 4 Implementation

The implementation of all three methods described in the preceding sections is remarkably similar. The map is represented by a two-dimensional array, with each cell corresponding to a region of the environment. Each cell has associated with it a value which represents the cell's occupancy, and an array of values representing the cell's responses. The meaning of these values will depend upon the method being used. When a measurement is taken, the first step is to determine which of the cells in the array should be updated to reflect this measurement. If we imagine that the pulse emitted by the ultrasonic sensor propagates outward through a conical region of space, then only the cells corresponding to this region of space

should be updated. For the Polaroid sensors used in our experiments, this will be all cells within about  $\pm 10^\circ$  of the center line of the sensor. Furthermore, for each of the candidate cells, only one of the responses needs to be updated – that corresponding to the direction of the emitted pulse. Once this determination has been made, the appropriate cell response can be updated and the overall cell occupancy recomputed. The details of the update process depend upon the method used.

## 4.1 Bayesian

Consider first the Bayesian method. The cell occupancy is represented by the probability  $p(O)$  and the cell responses an array of probabilities  $p(R_\phi)$ . These probabilities are initialised according to the rules discussed in Section 3.1:

$$\begin{aligned} p(R_\phi) &\leftarrow 1 - 0.5^{\frac{1}{n}} \\ p(O) &\leftarrow 0.5. \end{aligned} \tag{26}$$

Let  $r$  be a particular range measurement produced by a pulse emitted in direction  $\theta$ . A single response probability is updated as follows:

$$p(R_\theta) \leftarrow \frac{p(r | R_\theta)p(R_\theta)}{p(r | R_\theta)p(R_\theta) + (1 - p(r | R_\theta))(1 - p(R_\theta))} \tag{27}$$

This is simply Bayes' law (equation 8) recast in an incremental form, with the current response probability used as the prior probability. The sensor model, equation 10, is used to determine  $p(r | R_\theta)$ . The new cell occupancy  $p(O)$  can be recomputed using equation 7.

## 4.2 Dempster-Shafer

In the Dempster-Shafer implementation, cell occupancy is represented by the support  $\text{Sup}(O)$  and cell responses by an array of mass distributions  $m(\cdot)$ . These are initialised to 'don't know' values:

$$\begin{aligned} m(R_\phi) &\leftarrow 0 \\ m(\neg R_\phi) &\leftarrow 0 \\ m(R_\phi \vee \neg R_\phi) &\leftarrow 1 \\ \text{Sup}(O) &\leftarrow 0. \end{aligned} \tag{28}$$

Let  $r$  be a particular range measurement produced by a pulse emitted in direction  $\theta$ . The following update scheme is used:

$$m(R_\theta) \leftarrow \frac{m(R_\theta | r)m(R_\theta) + m(R_\theta \vee \neg R_\theta | r)m(R_\theta) + m(R_\theta | r)m(R_\theta \vee \neg R_\theta)}{m(R_\theta | r)m(\neg R_\theta) + m(\neg R_\theta | r)m(R_\theta)}$$

$$\begin{aligned}
m(\neg R_\theta) &\leftarrow \frac{m(\neg R_\theta | r)m(\neg R_\theta) + m(R_\theta \vee \neg R_\theta | r)m(\neg R_\theta) + m(\neg R_\theta | r)m(R_\theta \vee \neg R_\theta)}{m(R_\theta | r)m(\neg R_\theta) + m(\neg R_\theta | r)m(R_\theta)} \\
m(R_\theta \vee \neg R_\theta) &\leftarrow \frac{m(R_\theta \vee \neg R_\theta | r)m(R_\theta \vee \neg R_\theta)}{m(R_\theta | r)m(\neg R_\theta) + m(\neg R_\theta | r)m(R_\theta)}
\end{aligned} \tag{29}$$

These three expressions represent an expanded version of Dempster’s rule, equation 18. The sensor model, equation 20, is used to determine  $m(\cdot | r)$ . The new cell occupancy  $\text{Sup}(O)$  can be recomputed using equation 12.

### 4.3 Winner-take-all

In the winner-take-all implementation, cell occupancy is represented by the belief  $\text{Bel}(O)$  and cell responses by an array of evidence values  $\text{Ev}(\cdot)$ . These are initialised as follows:

$$\begin{aligned}
\text{Ev}(R_\phi) &\leftarrow 0 \\
\text{Ev}(\neg R_\phi) &\leftarrow 0 \\
\text{Bel}(O) &\leftarrow 0.
\end{aligned} \tag{30}$$

Let  $r$  be a particular range measurement produced by a pulse emitted in direction  $\theta$ . The appropriate response evidence value is updated as follows:

$$\begin{aligned}
\text{Ev}(R_\theta) &\leftarrow \max[\text{Ev}(R_\theta | r), \text{Ev}(R_\theta)] \\
\text{Ev}(\neg R_\theta) &\leftarrow \max[\text{Ev}(\neg R_\theta | r), \text{Ev}(\neg R_\theta)].
\end{aligned} \tag{31}$$

The sensor model, equations 24 and 25, is used to determine  $\text{Ev}(\cdot | r)$ . The new occupancy belief value  $\text{Bel}(O)$  is computed by applying, in order, equations 22 and 21.

## 5 Experimental results

The results presented in this section were obtained using a small mobile robot with a single Texas Instruments/Polaroid ultrasonic range-finder attached to a pivoting head. The sensor has an unobstructed view and can rotate through  $360^\circ$  in  $7.5^\circ$  increments. Consequently, 48 range readings are generated by each ‘sweep’ of the pivoting sensor head. Figure 1 shows the environment in which the experiments were conducted; this is a relatively complex environment containing a number of boxes, a hatstand and a chair. The robot is allowed to travel in a more-or-less straight line between the obstacles, taking range readings as it goes. The results shown in this section include readings from about 30 complete sweeps of the sensor head, i.e. about 1440 individual readings. The robot’s location is determined by simple odometry. The robot has an on-board 386 processor which



Figure 1: Test environment

is fast enough to generate maps in real time (i.e. it can incorporate new range measurements at the rate at which they are acquired).

Figure 2 shows the maps produced for varying values of  $n$  (i.e. varying numbers of response directions). Each pixel in these maps represents a region 4 centimetres square. Cells which are probably occupied (or for which we have strong support or belief) appear darker than cells which are probably unoccupied. The dotted line shows the path of the robot.

Consider first the  $n = 8$  result for the three methods. The Bayesian and Dempster-Shafer maps are quite similar (notwithstanding the fact that in the Bayesian map, ‘unknown’ cells appear as gray, whereas in the Dempster-Shafer map they appear white). These maps clearly show the various boxes, the hat-stand and the chair in the test environment. The winner-take-all map also shows these features, but in addition it has a number of ghostly shapes at the top and bottom of the map. These shapes are the result of multiple reflections – some of the surfaces in the environment act like mirrors under certain circumstances. The winner-take-all map is also generally ‘noisier’ than the other maps. The Bayesian and Dempster-Shafer methods do appear to produce better results than the winner-take-all method, presumably because they combine evidence in such a way that all measurements can make a contribution. The independence assumption made in both these methods does not seem to adversely affect the results. We note, however, that the winner-take-all method does have one advantage over the other two methods: it is significantly faster. Whereas the other two methods use relatively slow multiplication to combine evidence, the winner-take-all method uses faster max operations throughout.

Comparing the Bayesian result to the Dempster-Shafer result for  $n = 8$  we

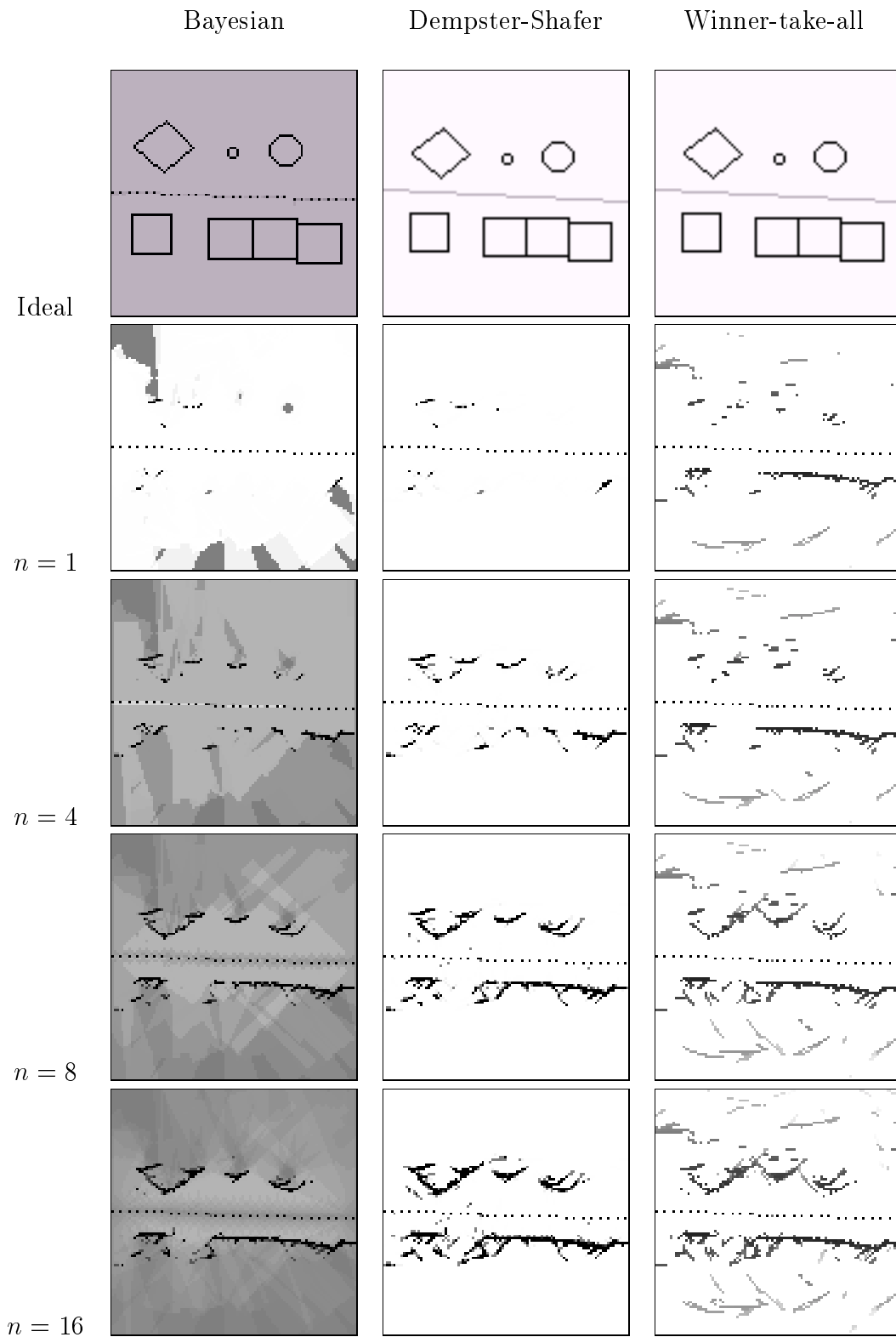


Figure 2: Occupancy maps  
14

can see that there is not a great deal of difference. As far as the implementation is concerned, the Bayesian method is slightly faster (it has fewer multiplications) and is more efficient in its use of memory (it requires that we store  $n + 1$  values per cell, as opposed to  $2n + 1$  for the Dempster-Shafer method). Note that in the Bayesian map, cell values are always in the range  $[0.5, 1]$  (the lower bound comes from our choice of prior probabilities – it could be set to an arbitrarily low, non-zero value). The Bayesian method we have developed cannot tell us which cells are probably *empty*, it can only tell us the cells that are probably *occupied*. This holds true of the other two methods also; it is a feature not of any particular method, but rather of the environment model we have chosen and the nature of ultrasonic range sensors.

Comparing the  $n = 16$  result with the  $n = 1$  case, it can be seen that there is a sharp decrease in the quality of the maps as  $n$  becomes small. Note particularly the  $n = 1$  case, where we are effectively ignoring the response behaviour and attempting to compute cell occupancy directly (the Bayesian  $n = 1$  case corresponds to the method described by Elfes [1]). The reason for this fall off is simple – the evidence obtained from multiple measurements is contradictory when  $n$  is small. Consider a smooth surface for example. A pulse whose angle of incidence on this surface is greater than about  $40^\circ$  will not be reflected back to the sensor. Consequently, this measurement will suggest that there is no response from the cell corresponding to the surface. Imagine that a second pulse is emitted, this time at an angle of incidence less than  $40^\circ$ . This time a response is recorded. If  $n$  is large, these two measurements are consistent – the cell responds in one direction, but not in another. On the other hand, if  $n$  is small, then a contradiction presents itself – two pulses were emitted in more-or-less the same direction (they correspond to the same direction ‘bin’), but one generated a response and the other did not. In the extreme case,  $n = 1$ , the entire approach collapses. This is a feature not of any particular method – it is consistent across all three methods – but of the environment model we have proposed.

On the other hand, close comparison of the  $n = 16$  and  $n = 8$  maps reveals that the  $n = 8$  map is somewhat less noisy. Once again, this is a feature of the environment model and not of any particular method. For large values of  $n$ , the chances that two measurements will fall into the same response direction ‘bin’ becomes very small. Consequently, there is no combination of evidence occurring at this level. For our particular experimental configuration, the optimal value of  $n$  seems to be about 8.

## 6 Conclusion

The remarkable thing about the three methods presented in this paper is the similarity of the results they produce. The best results are produced by the Bayesian and Dempster-Shafer methods, with the Bayesian method appearing somewhat

faster and slightly more efficient in its use of memory. The experimental results clearly demonstrate the advantage of the response grid framework in highly specular environments. Note, however, that there are some limitations to the work presented here. Most importantly, the occupancy maps we generate allow us to assert, with a high degree of confidence, that a cell is occupied. They do not allow us to assert that a cell is *unoccupied*. This is a property of the response grid framework, which in turn derives from the properties of ultrasonic range sensors. When such a sensor detects some object we can safely assume that there is in fact an object present (ignoring the case of multiple reflections). On the other hand, when a sensor fails to detect an object, we *cannot* assume that there is no object present. Unfortunately, in many robot navigation tasks, detecting ‘free space’ is more important than detecting obstacles. This leads us to suggest a natural extension to the response grid framework presented. Each cell could have associated with it a value indicating how ‘well measured’ that cell is. This value might be one if the cell has been measured from every possible direction and zero if it has not been measured at all. We would then be justified in concluding that a cell that does not appear to be occupied and is well measured is, in fact, unoccupied. We are currently exploring this extension, together with some dynamic extensions which will make the response grid approach suitable for use in changing environments.

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