To Boldly Go: Bayesian Exploration for Mobile Robots

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Abstract

This work addresses the problem of *robot exploration*. That is, the task of automatically *learning* a map of the environment which is useful for mobile robot navigation and localization. The exploration mechanism is intended to be applicable to an arbitrary environment, and is independent of the particular representation of the world. We take an information-theoretic approach and avoid the use of arbitrary heuristics. Preliminary results are presented and we discuss future directions for investigation.

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1 Introduction

In order for an autonomous robot to operate in its environment, it requires knowledge of its task and how it can be accomplished. An essential aspect of this knowledge is a *map*. That is, information which enables the robot to estimate where it is, where it's going, and how to get there. To date, many researchers have devoted a significant amount of work to solving these three tasks, given a prior map [22, 38, 16, 33]. A few have tackled the problem of constructing a map semi-autonomously[47, 24], and some have attempted the map acquisition problem in a fully autonomous setting [20, 8, 31]. Of the latter works, the majority commit to a particular sensing modality (sonar) and assume that the world can be represented as a 2-D piecewise-linear map.

We will define exploration as the process of discovering those aspects of the world that allow us to reliably model its structure and/or behaviour. Like scientific investigation, exploration in the real world never terminates. However, for practical reasons, the robot can cease exploration when it has acquired sufficient information to perform its tasks with a certain measure of confidence.

Our work aims to develop a framework for autonomous exploration that is both domain- and sensor-independent. We accomplish this task by posing the problem as one of optimizing the robot's knowledge or *information* about the world. Most prior work in this domain is constrained to sonar sensing or, where vision sensors are used, a restricted class of models [33, 48, 10, 51, 2]. We seek to generalize the models employed, facilitating a wider domain of environments, and aim to address the open questions posed by other entropy-motivated approaches to exploration. The principle contribution of this work will be a *theory of exploration* which accommodates multiple hypotheses about the correct representation of the world and takes into account the uncertainty associated with the robot's actions.

The balance of this paper will divided as follows: First, we will define the problem, taking into account the question of sensor and odometric uncertainty. The problem statement will be followed by a review of prior work on the exploration problem. We will then present our approach to the problem, and provide some preliminary results. Finally, we will look at future directions for the work.

2 Problem Statement

Our goal is to develop a *robust* and *efficient* method for autonomous exploration of an arbitrary, unknown environment. The task is rife with uncertainty– odometry and sensor readings are unreliable and hence we must take these factors into account when planning where to move the agent. In this sense, our goal is to maximize our certainty about the world. However, the problem is further compounded by other factors, such as task-specific requirements (for example, *a prori* needs for higher accuracy in certain parts of the world), safety issues, the limitations imposed by any particular choice of model of the world, and questions of computational tractability. Finally, in light of our definition of exploration, we will also require *criteria* that allow us to determine when exploration can be terminated. It should be clear that any robust and efficient solution will require careful consideration of all of these issues.

3 Previous Work

In this section we consider previous work that is relevant to the exploration problem.

3.1 Approaches in Computational Geometry

Problems in computational geometry (CG) often provide insight about related real-world tasks. For example, sonar maps of the real world can sometimes be approximated in terms of polygons, and hence the body of research concerning polygons is immediately applicable. An important question that can be answered in the context of CG is that of complexity. If we know that a particular CG problem is difficult to solve, then we can conclude that the related real-world problem is at least as hard. Furthermore, if the world can be exactly expressed in terms of geometry, then the exact solution to a particular problem represents a lower bound on what must be accomplished in the presence of uncertainty, providing us with a measure by which to evaluate algorithms that tackle uncertainty. We first consider the *Watchman Route Problem*, whose solution represents the minimum distance a robot must travel in order to discover the world, as represented by a polygon.

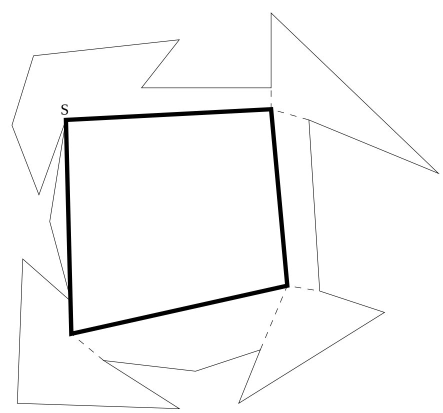

Figure 1: A polygon with inscribed shortest watchman route for starting point S. The route is marked by the bold line.

The *Watchman Route Problem* is related to the *Art Gallery Problem* [28, 35]. Chin and Ntafos define a *watchman route* for a polygon P as a closed walk in P such that every point of P is visible from some point in the walk [7].

Given a starting point S, and a polygon P, the watchman route problem is to find a shortest watchman route for P that starts at S (Figure 1). Chin and Ntafos demonstrate that the problem is NP-hard if the given polygon has holes. However, the problem is much simpler for polygons without holes, and Chin and Ntafos develop an $\mathcal{O}(n)$ algorithm for an orthogonal polygon [7]. In other work, they provide an $\mathcal{O}(n^4)$ algorithm for simple polygons [6]. Carlsson, *et al* have developed an $\mathcal{O}(n^6)$ algorithms for solving the problem when S is unspecified.

For the purposes of exploration with a single robot, one is faced with solving the watchman route problem in the face of only partial knowledge. There is a class of navigation algorithms, known as *Bug* algorithms, which deal with the task of finding a path from a start point to a goal in an environment populated with unknown obstacles [25, 29]. A *Bug*-style exploration strategy involves traveling through the world, searching for obstacles and circumnavigating obstacles as they are encountered. Taylor and Kriegman employ such an approach, building a set of local maps based on a set of visible *landmarks* [46]. Such *topological* representations of the world are considered in the next section.

A second class of CG problems which are relevant are *Geometric Probing* problems. Geometric probing considers problems of determining geometric structure from the results of a measuring device or *probe*. In the context of exploration, geometric probing is relevant because results from the problem domain specify lower-bounds on the number of discrete sites which must be sampled in order to completely reconstruct the environment. In many cases, the *probing models* reflect the idealized behaviour of real-world sensors, such as laser-range points, calipers, or the absorption of X-rays. Skiena provides a broad survey of the wide range of probing models and related results [41].

Of significant interest is the *finger probe* model, introduced by Cole and Yap [9]. A finger probe measures the first point of intersection between a directed line and an object. Such a model is equivalent to a perfect sonar measurement or laser-range sample. Cole and Yap show that $3n$ probes are sufficient to completely determine a convex polygon. However, their probing model is insufficient for determining arbitrary polygons. Alvizos, *et al* modify the probing model developed by Cole and Yap, allowing the probe to follow an arbitrary curve and returning not only the point of intersection with the polygon but also the surface normal at the point of intersection [1]. Under this probing model, the authors show that $3n - 3$ probes are sufficient to probe a polygon with n non-collinear edges. Of related interest is the problem of *geometric testing*, or object verification problem: given a set of objects and a target object, find the minimum set of probes which allow the target to be discriminated from all other objects in the set. Romanik provides a detailed survey of results in geometric testing [32].

Finally, Rekleitis and Dudek have developed a multi-robot collaborative algorithm for exploring a simple polygon [31, 30]. In this work, two robots commence exploration by positioning themselves at two adjacent vertices of the polygon and sweep the free space by traveling along edges while maintaining a line of sight. The traversal of the entire polygon results not only in a description of the polygon, but a topological representation which is based on a planar decomposition (for example, triangulation) of the polygon.

3.2 Topological Representations and Autonomous Map Construction

One of the earliest works that aims to address the autonomous exploration problem is that of Kuipers and Byun [21, 20]. That work models the world as a graph embedded in a 2-dimensional environment populated with point and line features. The goal is to automatically extract a topological representation where vertices are located at local maxima of a measure of *distinctiveness* on a subset of the sensory features, and vertices of the graph are recognized by a local *signature* of the environment. The vertices are connected to one another by arcs which define *local control strategies* for traveling between the vertices, and vertex recognition is verified via a *rehearsal* procedure. This approach to map-making has been duplicated somewhat by Choset's Generalized Voronoi Graph (GVG), which also deals with the particular issues of defining the local control mechanisms for moving from vertex to vertex [8].

Another topological approach that is of relevance to the work presented here is that of Tagare, *et al* [44]. In that work, the world is represented topologically as a set of places, each of which is characterized by a particular visual appearance. The problem of localizing in such a world is that of *recognizing* a known place based on the current input image.

The advantage of employing a topological representation is that it side-steps the difficult problem of maintaining the robot's pose in an absolute or global reference frame. Furthermore, it introduces a level of abstraction which can be employed for high-level inference (for example, understanding the command "Go to the *living room* via the *kitchen*.") One difficulty, however, with the approaches taken by Kuipers and Byun and Choset is that the robot is forced to operate in the context of the extracted topology, which may not be adequate for tasks that require specialized knowledge of places that are not well-represented by the graph. Simhon and Dudek resolve this issue by defining a set of metric maps, or *islands of reliability* in the neighbourhoods of distinctive places. The distinctive places themselves are selected on a measure that combines task-specific information with a quantitative measure of how well the robot is likely to be able to localize in the neighbourhood, selecting the best sensor for the task in the given neighbourhood.

Figure 2 summarizes the spectrum of representations that we have considered thus far, ranging from purely metric maps to purely topological maps. Clearly, any useful representation requires both metric and high-level information. Dudek has proposed that these requirements impose a hierarchy of representations that increase in generality as one ascends the hierarchy [14].

3.3 Inverse Problem Theory and Bayesian Analysis

Before we move on to the problems of map-building and exploration, we first examine the theory that motivates most current approaches to these tasks.

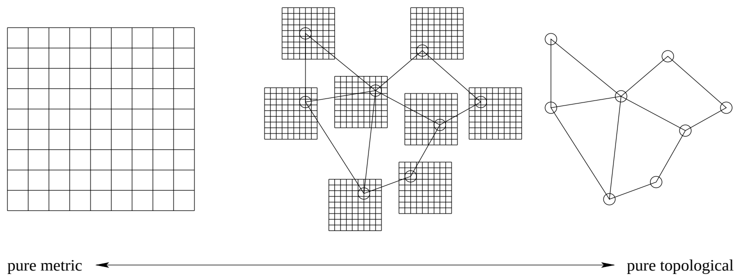

Figure 2: The spectrum of environment representations.

We commence by first defining some notation; the *pose* $\mathbf{q}$ of the robot represents the global parameters that capture its (zero-th order) state in the world. Typically, $\mathbf{q}$ is defined over a configuration space C which defines the set of admissible poses. Furthermore, an *observation* $\mathbf{z}$ is defined as the output of a sensor and belongs to the space Z of admissible sensor readings. For example, $\mathbf{z}$ might represent the rasterized set of gray-scale pixel intensity values in the image acquired by a camera. Finally, we will define a map m in the abstract as a method for *predicting* observations based on the pose of the robot.[1] A map will typically be coupled with a set of prior observations.

Tarantola's definitive work provides our foundation for *inverse problem theory* [45]. Inverse problem theory concerns the problem of inference based on observation. We proceed using the example of robot localization. The image $\mathbf{z}$ that the robot encounters from position $\mathbf{q}$ can be expressed by the relationship

$$\mathbf{z} = F(\mathbf{q}) \tag{1}$$

The localization problem is then that of inverting Equation 1:

$$\mathbf{q} = F^{-1}(\mathbf{z}) \tag{2}$$

and thus inferring $\mathbf{q}$ from $\mathbf{z}$. The problem is *ill-posed* in the sense of Hadamard, however, since there are no guarantees that $F(\cdot)$ is one-to-one. That is, more than one pose can observe the same sensor image.

Inverse problem theory comes to the rescue by taking the principle of least commitment. Where $F(\cdot)$ is not one-to-one, several different poses may be likely for a given image. Therefore, we represent the pose of the robot as a probability density function over C, given the input image $\mathbf{z}$ and a map m: $P(\mathbf{q}|\mathbf{z}, m)$.

The question of how to compute $P(\mathbf{q}|\mathbf{z}, m)$ analytically is not clear. However, if enough of the important parameters of the world are known (for example, lighting conditions, surface geometry and reflectance properties, etc), then we

[1]Consider the notion that a road map provides us with a method for predicting upcoming intersections, or a topographical map allows us to predict the slope of the trail ahead. The utility of a (human-readable) map as a localizer is rooted in our remarkable ability to rapidly search for places whose predictions match our observations.

can develop a *theory* that predicts our observation given the pose of the robot, $P(\mathbf{z}|\mathbf{q}, m)$. Clearly, the map m embodies those properties of the world which enable us to predict $\mathbf{z}$ given $\mathbf{q}$. For this reason, Bayesians usually parameterize m as $\mathbf{m}$ and refer to it as a *model*. In fact, the pose $\mathbf{q}$ represents a subset of these parameters. The reader should be aware, however, that parameterizing m implies that a particular computational framework has been selected. By referring to m in the abstract, we refer to a member of the universe of all computational methods for computing $\mathbf{z}$ from $\mathbf{q}$, without explicitly selecting, for example, explicit object models, regularization of observations in n-space or spline interpolation. The issue of choosing between computational models is addressed by MacKay [26]. Callari and Ferrie also deal with this issue in the context of object recognition, since the objects themselves represent a set of abstract classes[5].

How can we use $P(\mathbf{z}|\mathbf{q}, m)$ to infer pose? Simple application of Bayes' Law reveals that

$$P(\mathbf{q}|\mathbf{z}, m) = \frac{P(\mathbf{z}|\mathbf{q}, m)P(\mathbf{q}|m)}{P(\mathbf{z}|m)} \tag{3}$$

where $P(\mathbf{q}|m)$ refers to the *a priori* probability density function for $\mathbf{q}$ (i.e. where the robot thinks it is prior to taking the observation), and $P(\mathbf{z}|m)$ is referred to as the *evidence* for the map. The latter term is often assumed to be uniformly distributed and is treated as a normalizing constant. Note that if the prior and the evidence are uniformly distributed then the most likely pose of the robot, given the observation, is that which maximizes the likelihood of the observation $\mathbf{z}$ itself.

Computationally, the solution of Equation 3 for the maximum-likelihood pose of the robot may be difficult– the underlying probability density function may be multi-modal, complicating simple gradient-ascent methods. Furthermore, it is often necessary or desirable to compute a compact *representation* for the PDF. There are a variety of computational tools that can be employed to represent a probability density function. Two of the more common tools are mixture models and Monte Carlo simulation.

Mixture models represent a probability density function as a finite sum of elementary PDF's (for example, a Gaussian function)[19, 13]:

$$P(x) = \frac{1}{\sum w_i} \sum w_i P_i(x) \tag{4}$$

where each w_i represents a weighting term to be applied to elementary PDF $P_i(x)$. Such an approach is useful for representing multi-modal data. However, mixture models often require *a priori* knowledge of the number of modes of the data.

Monte Carlo simulation is a computational approach wherein the probability density function is represented implicitly by the set of outcomes of repeated stochastic simulation. For example, the PDF for the pose of a robot can be represented by a set of weighted particles, each of which is located at a particular point in the configuration space and weighted by the likelihood that the robot

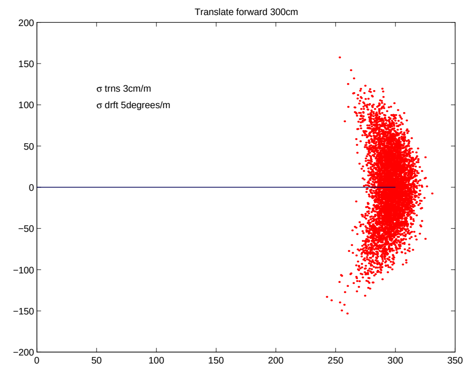

Figure 3: Monte Carlo simulation of a robot undergoing an uncertain motion. Uncertainty in the robot's heading yields a distinctly non-Gaussian distribution. Figure courtesy Ioannis Rekleitis

is located at that particular point. Figure 3.3 depicts a Monte Carlo simulation of the resulting pose of a robot undergoing an uncertain forward motion. Note that uncertainty in the robot's heading leads to a distinctly non-Gaussian pose distribution. In the limit, as the number of particles goes to infinity, the distribution of particles exactly represents the PDF of the robot's pose. The advantage of Monte Carlo simulation is that no *a priori* requirements are imposed on the structure of the PDF. The disadvantage is that the number of particles required to adequately represent the PDF increases exponentially with the dimensionality of the parameter space. Furthermore, faithfully simulating the stochastics of a physical system is a difficult problem in and of itself. Nevertheless, Monte Carlo simulation has been applied successfully to the problems of robot localization and visual tracking[10, 18].

Armed with Bayes Law, and the useful related tools, we now turn our attention to the task of constructing a map of the environment. Note that we are not yet discussing the problem of data acquisition or *exploration*, but considering what to do once the data has arrived.

3.4 Map-Construction in the Presence of Uncertainty

As a robot constructs a map of its environment, it must execute actions and take sensor readings. Each of these tasks involves a certain measure of uncertainty which can corrupt the resulting map, and since the uncertainty of the robot's actions compounds with each successive execution, the map eventually becomes useless (not to mention that the robot becomes hopelessly lost).

While it may seem like a classic chicken-and-egg problem, it is still possible to construct a useful map by posing the problem in a probabilistic framework. In this context, the problem of representing the world becomes one of selecting the representation which is *most probable*.

The most straightforward approach to probabilistic map construction is that of employing a Kalman Filter (KF) to update both the robot position and positions of landmarks (see, for example, [23] and [17]). While the concepts are straightforward, the difficulty posed by the Kalman Filter is that all uncertainties are embodied as Gaussian distributions, whereas the evolution of the actual uncertainty of the pose of the robot may be non-Gaussian. In addition to Gaussian assumptions, the Kalman Filter assumes that the problem is linear. The Extended Kalman Filter (EKF) accommodates non-linear formulations by first linearizing in the neighbourhood of the maximum-likelihood estimate via a Taylor series expansion. The difficulty with this approach is that linearization can cause the filter to diverge from the correct solution.

In order to avoid the problems encountered due to assumptions of linearity and Gaussian uncertainty, we turn to a more general framework for posing the map-building problem, as suggested by Thrun *et al* [47], (see also [48, 49]). Thrun's work takes advantage of a Markov assumption and Bayes' law [45] to show that, given a sequence of actions and observations as input data, the most probable map is that which maximizes the likelihood of the data. The method for producing the most probable map (and simultaneously the most probable set of robot motions) is based on the *Expectation Maximization* or *EM-algorithm*[12].

The *EM-algorithm* is an iterative hill-climbing strategy in likelihood space which operates in two phases, an *Expectation* phase, and a *Maximization* phase. Thrun's rendition of the *EM*-algorithm poses the *E*-step as that of computing the most likely set of robot actions assuming the current estimate of the map is correct. The *M*-step then computes the maximum likelihood map from the robot's observations, assuming the pose estimates from the *E* step are correct. The process continues iteratively until the map converges to a local maximum.

In more recent work, Thrun suggests an incremental method which adjusts the map as data arrives in real-time [50]. The method exploits a Monte Carlo representation of the pose of the robot and distributes the computed error in robot pose backwards in time over the prior observations. Such an approach maintains a set of good starting points for gradient ascent in likelihood space, which can be computed rapidly, foregoing the cost of searching the entire space of maps for the maximum likelihood estimate.

The *EM* approach has also been exploited by Dellaert in the context of computing Structure from Motion (SFM) using a camera [11]. This work is significant in that it can be exploited as an alternative to the map representation we will employ in Section 5.2. It should be noted that both the Kalman Filter and Markov-based approaches to map-building are motivated by previous successes in robot localization [22, 42, 16, 10].

The algorithm for map construction presented by Thrun is satisfying in that no specific representation of the underlying uncertainties is required. However, this approach assumes that the robot has already performed the task of *exploration*– the collection of observations from which to build the map. It makes no attempt to instruct the robot about where to next collect an observation. We will consider this task in the next section.

3.5 Exploration

Having considered the tasks of localization and map construction, we turn at last to the problem of exploration.

The majority of exploration approaches apply heuristics to incrementally discover unexplored space [4]. The difficulty with these methods is exactly that they are heuristics– no consideration is given to the quality of the results, and the results themselves are impossible to validate or evaluate.

This problem has been addressed by several authors in the context of Tarantola's inverse problem theory and Fedorov's theory of optimal experiments [45, 15]. MacKay's series of papers addresses the problem of Bayesian interpolation– interpolating a function from sample observations in the presence of observational uncertainty [26, 27]. MacKay exploits Shannon's entropy [36] to show that the optimal place from which to obtain the next sample is that where the prediction of an observation is least certain. In other words, one should take observations at the places where the "error-bars" on the interpolating function are largest. While the application of MacKay's work is only to 1-d functions, the fundamental theory behind his derivation will be preserved when we derive our own objective function for data selection.

Whaite and Ferrie employ MacKay's framework in their work on active exploration for the purposes of obtaining object models [51, 52]. They show that the best location from which to take an observation is that which maximizes the *prediction variance*, based on the covariance matrix of the set of model parameters describing the object. They formulate a decision process which is based only on a local measure of the model uncertainty, directing the agent to take sensor readings as it climbs (and simultaneously suppresses) the uncertainty gradient.

Arbel and Ferrie exploit this same idea in solving the object recognition problem, directing the sensor to places which will maximize the expected reduction in entropy of the probability distribution over possible object classes [3, 2]. In this case the authors benefit from the fact that the "entropy map" can be constructed in advance, given the known universe of objects.

Finally, the information theoretic approach has also been applied to the problem of robot navigation. Roy and Thrun define an objective function for navigating to a goal position in a known world that simultaneously aims to minimize the entropy of the probability density function defining the pose of the robot and to reach the goal along a reasonable path [34]. The resulting path planning method is referred to as "coastal navigation" for its tendency to direct the robot along the boundaries of obstacles, where the certainty of the robot's pose can be improved.

The approaches taken by MacKay and Whaite rely on several important assumptions and approximations. First, it is assumed that the pose of the robot is known exactly. No indication is given as to how to deal with the uncertain outcomes of the robot's actions. Second, it is assumed that the distribution of the model parameters is Gaussian, which facilitates an analytic solution for the best observation site. As Whaite only applies the Gaussian assumption in order to approximate the local gradient in the uncertainty space and make a local decision about where to move next, this assumption seems reasonable. However, the more critical assumption imposed by the theory is that the model, or representation, can exactly model the real-world phenomena that give rise to data. If this assumption is incorrect, then the agent can be directed to repeatedly collect samples from sites that the model cannot represent, leading to a sub-optimal improvement in the representation. Alternatively, over-confidence in the model can lead the robot to ignore regions which are poorly represented. MacKay refers to this issue as the "Achilles' Heel" of the approach. One of the goals of this work is to develop a framework for accommodating a representation's inability to completely capture the data.

4 Developing a Theory of Exploration

Having considered extensively the prior work on the topic of exploration and environment representation, we will proceed to develop the theory that motivates our proposed approach. We will employ an information-theoretic approach to the problem, also taking into account any task-specific requirements and accounting for uncertainty in the pose of the robot.

4.1 A Bayesian Approach

Given a map m, and a sensor reading $\mathbf{z}$, Equation 3 computed the probability of the pose $\mathbf{q}$ of the robot using Bayes' Law:

$$P(\mathbf{q}|\mathbf{z}, m) = \frac{P(\mathbf{z}|\mathbf{q}, m)P(\mathbf{q}|m)}{P(\mathbf{z}|m)} \tag{5}$$

where $P(\mathbf{z}|\mathbf{q}, m)$ represents our predictive *model* or *theory* that allows us to measure the likelihood of an observation based on what we know about the world.

In the context of exploration, a naive approach to data collection might be to move to places or select actions such that the uncertainty in the pose of the robot is maximized. This would amount to selecting the pose that maximizes the expected *posteriori* entropy H of $P(\mathbf{q}|\mathbf{z}, m)$:

$$H = -\int_{\mathbf{z}\in Z} P(\mathbf{q}|\mathbf{z}, m) \log P(\mathbf{q}|\mathbf{z}, m) d\mathbf{z} \tag{6}$$

It is interesting to contrast this idea with the navigational approach of Roy and Thrun, who move toward a goal in the known world by minimizing the entropy of the posteriori pose distribution. In fact, maximizing H is incorrect for exploration because it depends on fixed properties of the environment– different places in the world may look alike and no matter how many observations we take our uncertainty about being in one place versus another identical place should remain fixed.

Instead, we are more interested in the likelihood of our map given our current observation:

$$P(m|\mathbf{q},\mathbf{z}) = \frac{P(\mathbf{z}|\mathbf{q},m)P(m|\mathbf{q})}{P(\mathbf{z}|\mathbf{q})} \tag{7}$$

Equation 7 relates the likelihood of the map in the face of an observation taken from a particular pose to the likelihood of the observation and our *a priori* measure of the probability of the map. Note that $P(\mathbf{z}|\mathbf{q})$ is independent of $\mathbf{q}$ in the absence of a map.

The information theoretic approach to exploration is to find the pose $\hat{\mathbf{q}}$ which maximizes the expected reduction in entropy of the probability distribution $P(m|\mathbf{q},\mathbf{z})$. We state the expected change in entropy in terms of what MacKay defines as *cross-entropy* G:

$$G = -\int_{m\in M} P(m|\mathbf{q},\mathbf{z}) \log \frac{P(m)}{P(m|\mathbf{q},\mathbf{z})} dm \tag{8}$$

and

$$E[G] = \int_{\mathbf{d}\in Z} G d\mathbf{d} \tag{9}$$

where $P(m)$ represents the prior for m before the arrival of the datum $\mathbf{z}$; it may depend, however, on a set of previously collected data.

Evaluating Equation 8 requires an integration over the space of possible maps. This poses a difficulty. In contrast to methods which construct parametric models based on the data [51], we have not yet instantiated our maps in terms of an explicit set of parameters. In fact, we have consciously avoided any such instantiation in order to allow for generality– the parameters may be implicitly encoded by an inaccessible black-box or may be embodied solely in the set of prior observations.

At this juncture, we arrive at a crossroads for continued investigation. On the one hand, we can insist that our maps be parameterized. This leads to the formulations proposed by MacKay and Whaite *et al*, which are simplified by computing the first-order Taylor series expansion about the maximum-likelihood map $\hat{\mathbf{m}}$ and assuming that the likelihood space is normally distributed in the neighbourhood of $\hat{\mathbf{m}}$. One possible avenue for consideration is the explicit parameterization of m as the actual set of observations. This yields a parameterization $\mathbf{m} \in Z \times Z \times \ldots \times Z$ where the observation space is crossed with itself once for each datum. The author is currently investigating whether Equation 8 can be simplified in this context.

Alternatively, we can work with the text of MacKay's observation that we should collect data "at the point where the error bars on the interpolant are currently largest". The implication is that at these points the ability of the map to predict the data is weakest. Mathematically, however, the author has been unable to derive an explicit relationship between the change in entropy of the model and the entropy, or uncertainty, of the predicted observation. In fact, the nearest derivation suggests quite the opposite– it is more informative to take a low-noise observation than one that is noisy. This apparent contradiction stems from the fact that the model may be quite certain that the prediction arises from a noisy process, whereas it may be highly uncertain in predicting a datum for a process which is known to be relatively noiseless. The conclusion that we are forced to draw is that this second line of attack is theoretically unsound.

Finally, we must consider the uncertain pose of the robot. In this case, we cannot select the best pose to navigate to. We should instead select an action $\mathbf{a}$ to be executed by the robot so as to maximize the expectation of $E[G]$ given $\mathbf{a}$ over the configuration space C:

$$H(\mathbf{a}) = E[E[G]|\mathbf{a}] = \int_{\mathbf{q}\in C} P(\mathbf{q}|\mathbf{a})E[G]d\mathbf{q} \tag{10}$$

Let us assume for the moment that the configuration space of the robot is discretized onto a finite grid. If we restrict the possible actions to a small, discrete set (such as, move forward 10cm, or turn left 10^o), then it should be possible to formulate the task as a *reinforcement learning problem* [43]. In this context, the task is to compute a policy π that selects actions in order to maximize the expected reward, or certainty, accrued to the robot over the long-term. The principal difficulty with this approach is that tractability also requires a discretization of the robot's state of knowledge, which may not be straightforward. This is a question for future research.

4.2 Adding an Action-Cost Measure

The formulation, as presented, directs the robot to the place (or to execute an action) which is globally optimal for data collection. However, the globally optimal action may in fact require the robot to travel a significant distance, or put itself at risk due to environmental hazards. This fact has two effects– first, the physical cost of executing the optimum action may be too high, and second, the robot may become hopelessly lost in traveling to its goal. To accommodate this difficulty, we present a new objective function

$$F(\mathbf{a}) = \lambda H(\mathbf{a}) + (1-\lambda)c(\mathbf{a}) \tag{11}$$

where $c(\mathbf{a})$ represents the cost of executing action $\mathbf{a}$ and λ is a parameter determining the desired relative weight between the expected change in entropy H and c. It is not clear at this juncture whether $H(\mathbf{a})$ sufficiently penalizes highly uncertain actions. This is also a question for future investigation.

5 Preliminary Results

5.1 Simulation

In this section we consider the state of the work to date. Our first instantiation of the exploration method simulates a robot in a polygon-shaped environment (Figure 4(a)).

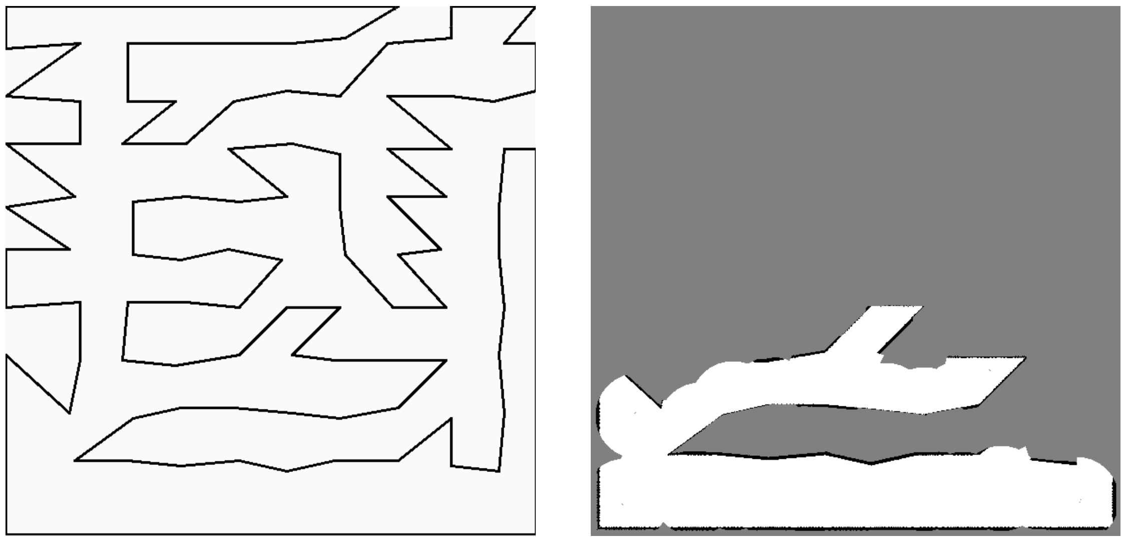

Figure 4: Simulation results. a) the robot's environment. b) the termination of exploration.

We model the environment as an occupancy grid, where each grid point (x, y) holds the value $p_{occ}(x, y)$, the probability that position (x, y) is intersected by an edge of the polygon. The occupancy grid is initialized to 0.5 everywhere, indicating the state of no information, and the robot is initialized to pose $(30, 30)$, in the lower left corner of the map.

Exploration is performed by first identifying a set of candidate poses which are eligible for exploration (those which are known to be reachable based on the current state of the occupancy grid), and using the occupancy grid to simulate the PDF of a simulated sonar scan at each position. The pose whose sonar PDF has highest entropy is selected as the optimal pose from which to take the next scan. Figure 4(b) shows the state of the occupancy grid after 30 iterations of the algorithm. Unfortunately, at this stage, the model repeatedly selects the same position from which to scan and the algorithm terminates. The reasons for termination are various– we concluded in Section 4.1 that the heuristic is incorrect. Furthermore, MacKay's argument that insufficient models lead to suboptimal behaviour also applies, since we have modeled the sonar scanner heuristically.

While the simulated exploration of Figures 4(a) and 4(b) demonstrate some of the important issues faced by information-theoretic exploration, each of which must be addressed in the course of this work, it is of significant interest to pursue a richer sensing model. We now turn our attention to the task of modeling the probability density function of a vision sensor in a real world setting.

Figure 5: Original scene and synthesized landmarks.

5.2 An instantiation of $P(\mathbf{z}|\mathbf{q}, m)$

Ultimately, our goal is to automate the acquisition of training inputs for a map-learning mechanism, such as the one discussed in Section 3.4. Fundamental to this mechanism is the development of the *theory*, that is, the ability to compute $P(\mathbf{z}|\mathbf{q}, m)$. We will discuss here our prior work in the domain of landmark learning and introduce an instantiation of $P(\mathbf{z}|\mathbf{q}, m)$ which can be applied in a Bayesian context to robot localization[38, 39, 40]. It should be noted that the prior work cited does not employ a Bayesian framework– the formulation presented here is, as yet, unpublished.

Our goal is to compute the probability density function of a camera image $\mathbf{z}$, given the pose $\mathbf{q}$ of the robot, and a map m. Our approach makes two simplifying assumptions– first, rather than generate the entire image $\mathbf{z}$, we concentrate only on generating the image in the neighbourhood of a set $L = \{\mathbf{l}_1, \mathbf{l}_2, \ldots, \mathbf{l}_n\}$ of visible salient points or *landmarks*[37]. Second, the probability density function of an observed landmark is normally distributed about a maximum likelihood observation $\hat{\mathbf{l}}_i$ which is computed explicitly from $\mathbf{q}$ and m.

The function that generates $\hat{\mathbf{l}}_i$ is based on an unsupervised learning mechanism which constructs the most likely image position and appearance of a landmark from a set of prior observations (obtained in the process of exploration). Furthermore, a *cross-validation* scheme is used to infer the covariance matrix of the distribution. Therefore, the covariance captures at once the stochastic nature of the sensor and the inadequacies of the model. Figure 5(a) depicts a scene whose landmarks were learned by uniform sampling of the pose space. Figure 5(b) depicts a synthesized image from a nearby pose in the pose space– only the image in the neighbourhood of each landmark has been reconstructed.

Given a method for computing $\hat{\mathbf{l}}_i$, we can formulate $P(\mathbf{z}|\mathbf{q}, m)$ as a mixture of Gaussian PDF's, each corresponding to a landmark in the image :

$$P(\mathbf{z}|\mathbf{q}, m) = \frac{1}{N} \sum_i P(\mathbf{l}_i|\mathbf{q}, m_i) \tag{12}$$

where $P(\mathbf{l}_i|\mathbf{q}, m_i)$ corresponds to the Gaussian probability density function of landmark l_i, given the pose and the learned predictor for that landmark m_i.

One further point of justification must be made for our choice of a mixture model in Equation 12. Whereas a correct formulation would be to compute a joint distribution multiplicatively, we have not considered the possibility of outliers in the landmark recognition phase, which can obliterate the results due to limited machine precision. While a summation model prevents such catastrophes at the cost of higher uncertainty in the PDF, it also imposes the incorrect assumption that the landmark observations are disjoint.

Figure 6(b) depicts $P(\mathbf{q}|\mathbf{z})$ over a 2m by 2m pose space in the laboratory setting depicted in Figure 6(a). The mode of the distribution predicts the pose of the robot to be at position $(131, 29)$ whereas the actual robot position was $(130, 22)$.

6 Future Directions

We have established a framework for autonomous information discovery in the context of robotic exploration. There are a wide variety of issues that require further discussion.

- The most significant issue is that of tractability. The integrals that must be solved in order to compute the entropy function require simplification. It is likely that we can exploit the mixture model formulation we have proposed in order to derive an analytic solution.

- The lack of explicit models is a handicap to the derivation and must be addressed. The question of model parameterization remains complicated, given the computational model that we have presented.

- The model we have proposed for modeling $P(\mathbf{z}|\mathbf{q}, m)$ in the image domain requires further investigation, particularly for more complicated configuration spaces.

- More work is required in the context of evaluating the expected entropy given the uncertainty in the robot's pose. This is necessary for evaluating a set of actions, as opposed to choosing a single vantage point from which to obtain a sensor reading. The question of whether the reinforcement learning paradigm can be applied will be considered.

- We have not yet addressed the inference of geometric constraints in the context of navigating through the world– presumably these can be added to the cost function. However, just as we are aiming to reconstruct the image function, it should be possible to reconstruct the sonar function.

- A separate, yet equally interesting issue for future work has to do with the natural topology of the environment which is expressed by the set of landmarks that are currently in view. That is, different regions of

Figure 6: Laboratory scene and probability density function of robot pose. The area depicted corresponds to the $2-D$ configuration space of the robot. Darker regions correspond to more likely poses, given the image.

space give rise to a different set of visible landmarks. Clearly, there are opportunities for path planning and inference based on navigating between the sets of visible landmarks. This approach lies very much in the domain of a hybrid representation between a set of topological places and local metric maps.

7 Conclusions

One of the most significant gaps in the domain of robotics research is that of autonomous exploration. Whereas the domain of machine learning has studied how to make sense of the data, very few researchers have considered the task of actually acquiring the data that optimally facilitates the task at hand. Our work aims to fill this gap in a way that is at once theoretically sound and practically feasible. We have proposed a theory of exploration which is derived from information theory. Specifically, the robot is directed to acquire sensor readings from places where its ability to model the world is weakest. We have established preliminary results which validate in part the feasibility of our goals and highlight some of the important difficulties that must be overcome. Our future work will continue to seek a marriage between a robust theory of exploration and the practical issues of implementation.

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