An Optimal Competitive Strategy for Walking in Streets

Christian Icking* Rolf Klein* Elmar Langetepe*

Abstract

We present an optimal strategy for searching for a goal in a street which achieves the competitive factor of $\sqrt{2}$, thus matching the best lower bound known before. This finally settles an interesting open problem in the area of competitive path planning many authors have been working on.

Key words: computational geometry, autonomous robot, competitive strategy, LR-visibility, on-line navigation, path planning, polygon, street.

1 Introduction

In the last decade, the path planning problem of autonomous mobile systems has received a lot of attention in the communities of robotics, computational geometry, and on-line algorithms; see e. g. Rao et al. [21], Blum et al. [4], and the upcoming surveys by Mitchell [20] and Berman [3].

Among the basic problems is searching for a goal in an unknown environment. One is interested in strategies that are correct, in that the goal will always be reached whenever this is possible, and in performance guarantees that allow us to relate the length of the robot's path to the length of the shortest path from start to goal, or to other measures of the complexity of the scene.

It is well known that there are some differences between the outdoor setting, where the robot has to circumnavigate a set of compact obstacles in order to get to the target, and the indoor setting where the obstacles are situated in a—not necessarily rectangular—room whose walls may further impede the robot; see e. g. Angluin et al. [1]. Therefore, it is reasonable to study the indoor problem in its most simple form, that is, where the walls of the room are the only obstacles the robot has to cope with.

Suppose a point-shaped mobile robot equipped with a 360° vision system is placed inside a room whose walls are modeled by a simple polygon. Neither the floorplan nor the position of the target point are known to the robot. As the robot moves around it can build a partial map of those parts that have so far been visible. Also, it will recognize the target point on sight.

It is quite easy to see that in arbitrary simple polygons no strategy can guarantee a search path at most a constant times as long as the shortest path from start to goal. In fact, imagine a hall from which m straight corridors of equal length give on to smaller chambers. The robot cannot but inspect these chambers one by one, moving forth and back through the corridors. In the worst case, the target point is contained in the chamber inspected last; then the robot's path is $2m - 1$ times as long as the shortest path from the hall to the target.

*FernUniversität Hagen, Praktische Informatik VI, D-58084 Hagen, Germany.

This research was supported by the Deutsche Forschungsgemeinschaft, grant no. Kl 655/8-3.

The question arose if there are subclasses of polygons for which a constant performance ratio can be achieved. At FOCS '91, Klein [13] introduced the concept of *streets*. A polygon P with two distinguished vertices s and t is called a street if the two boundary chains leading from s to t are mutually weakly visible, i. e. if each point on one of the chains can see at least one point of the other; see Figure 1 for an example. Equivalently, from each s-to-t path inside P each point of the polygon is at least once visible.

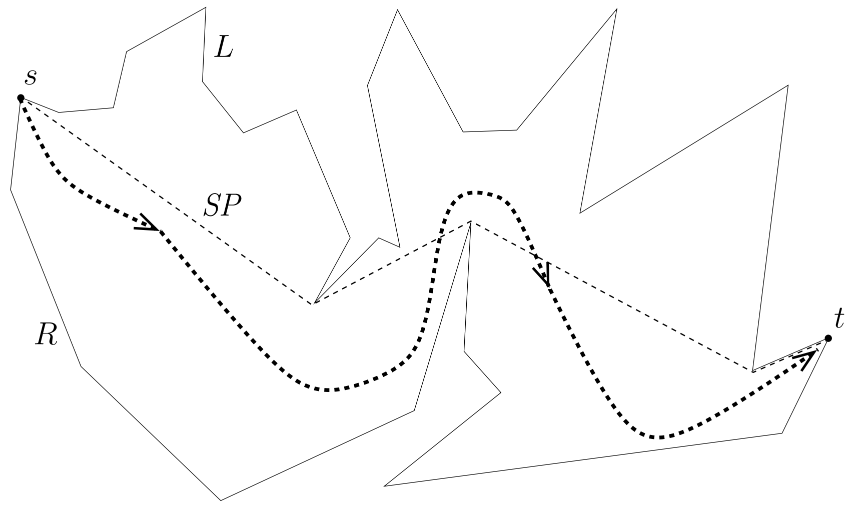

Figure 1: A street.

In [13, 14] Klein provided the first competitive strategy for searching for the target point, t, of a street, starting from s. He proved an upper bound of 5.72 for the ratio of the length of the robot's path over the length of the shortest path from s to t in P. Also, it was shown that no strategy can achieve a competitive ratio of less than $\sqrt{2} \approx 1.41$. This lower bound applies to randomized strategies, too.

Since then, the street problem has attracted considerable attention. Some research was devoted to structural properties. Tseng et al. [24] have shown how to report all pairs of vertices (s, t) of a given polygon for which it is a street; for star-shaped polygons many of such vertex pairs exist. Das et al. [6] have improved on this result by giving an optimal linear time algorithm. Datta and Icking [9] introduced generalized streets, a concept further generalized by Datta et al. [8] and by López-Ortiz and Schuierer [18]. Ghosh and Saluja [10] have described how to walk an unknown street incurring a minimum number of turns.

Other research addressed the gap between the $\sqrt{2}$ lower bound and the first upper bound of 5.72 known for the class of street polygons. The upper bound was lowered to 4.44 in Icking [11], then to 2.61 in Kleinberg [15], to 2.05 in López-Ortiz and Schuierer [17], to 1.73 in López-Ortiz and Schuierer [19], to 1.57 in Semrau [23], and to 1.51 in Icking et al. [12]. Further attempts were made by Dasgupta et al. [7] and by Kranakis and Spatharis [16].

But it has remained open, until now, if $\sqrt{2}$ is really the largest lower bound, and how to design an optimal strategy for searching the target in a street; compare the open problems mentioned in Mitchell [20].

In this paper both questions are finally answered. We introduce a new strategy and prove that the search path it generates, in any particular street, is at most $\sqrt{2}$ times as long as the shortest path from s to t. This result makes the street problem one of the few problems in on-line navigation whose competitive complexity is precisely known (the only other example we are aware of is the result by Baeza-Yates et al. [2] on multiway search).

One might wonder if this paper is but another small step in a chain of technical improvements. We do not think so, for the following reason. Unlike many approaches discussed in previous work, the optimal strategy we are presenting here is not an artifact. Rather, its definition is well motivated by backward reasoning.

The crucial subproblem can be parametrized by a single angle, ϕ. For each possible value of ϕ a lower bound can be established, see Section 3.1. For the maximum value $\phi = \pi$ the existence of a strategy matching this bound is obvious. We state a requirement in Section 3.2 that would allow us to extend an optimal strategy from a given value of ϕ to smaller values. From this requirement we can infer how the strategy should proceed; see Section 3.3. The main difficulty is in proving that the strategy we arrive at does in fact fulfil our requirement. This is done in Section 3.4, using well-known facts from planar geometry and analysis.

After this work was finished, and made publicly available via Internet, we learned that Schuierer and Semrau [22] have simultaneaously and independently studied the same strategy. However, their analytic approach is quite different from our proof.

2 Definitions and known properties

We briefly repeat necessary definitions and known facts, mostly from [14].

A simple polygon P is considered as a room, the edges are opaque walls. Two points are mutually *visible*, i. e. see each other, if the connecting line segment is contained within P. As usual, two sets of points are said to be *mutually weakly visible* if each point of one set can see at least one point of the other set.

Definition 1 A simple polygon P in the plane with two distinguished vertices s and t is called a *street* if the two boundary chains from s to t are weakly mutually visible, for an example see Figure 1. Streets are sometimes also denoted as *LR-visible polygons* [6, 24], where L denotes the left and R the right boundary chain from s to t.

A strategy for searching a goal in an unknown street is an on-line algorithm for a mobile system (robot), modeled by a point, that starts at vertex s, moves around inside the polygon and eventually arrives at the goal t. The robot is equipped with a vision system that provides the visibility polygon, $\text{vis}(x)$, for the actual position, x, at each time, and everything which has been visible is memorized. When the goal becomes visible the robot goes there and its task is accomplished.

Compared to the shortest path, *SP*, from s to t inside P, it seems clear that most of the time a detour is unavoidable. Our aim is to bound that detour.

Definition 2 A strategy for searching a goal in a street is *competitive with factor c* (or *c-competitive*, for short) if its path is never longer than c times the length of the shortest path from s to t.

The shortest path from the startpoint s to the goal t inside a simple polygon P, which only turns at reflex[1] vertices of P, is a useful guide for any strategy. At each time, either the next vertex on the shortest path to t is known and there is no question where to go. Or there is some uncertainty, but we will see that only two candidates remain for the next vertex on the shortest path to t. Each part of the polygon which has never been visible is called a *cave*, and each cave is hidden behind a reflex vertex. Such a reflex vertex v that causes a cave is called *left* reflex vertex if its adjacent segments on P lie to the left of the ray from the actual position of the robot through v, and analogously for *right* vertices.

[1] A *reflex vertex* is one whose internal angle exceeds 180°.

First, we consider the situation at the beginning. From the startpoint s we order clockwise around s the set of the left and right reflex vertices, obviously they appear in the same clockwise order on the boundary of P. As seen from s, let v_l be the clockwise most advanced left reflex vertex and v_r the counterclockwise most advanced right reflex vertex, see Figure 2.

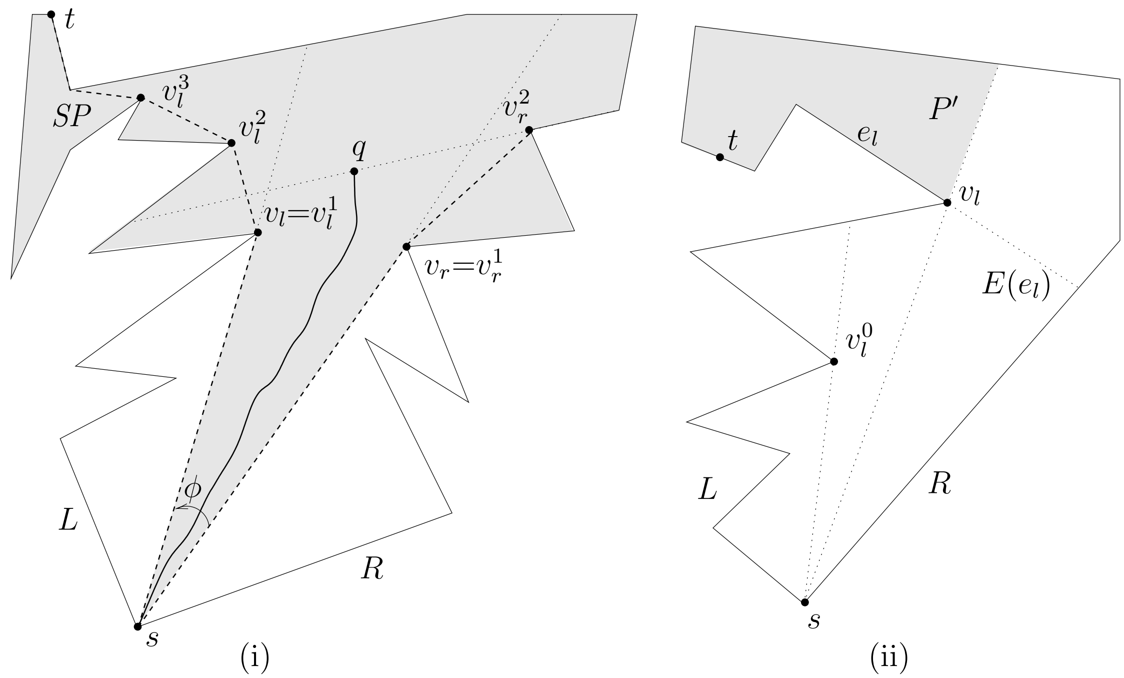

Figure 2: Typical situations in streets.

If they exist, vertex v_l belongs to the left chain L, and v_r belongs to the right chain R. Only the following situations occur. If both, v_l and v_r, exist, see Figure 2 (i), then the goal has to be in one of the caves behind v_r and v_l, thus SP passes over either v_r or v_l. If there is no vertex v_r, see Figure 2 (ii), then the goal has to be inside the cave behind v_l, and the robot moves straight towards v_l along SP. We proceed correspondingly if only v_r exists.

To prove these properties assume that w. l. o. g. the goal is inside the cave behind a left reflex vertex v_l^0, see Figure 2 (ii), and v_l^0 appears before v_l clockwise from s. Then the boundary of P inside the cave P' behind vertex v_l would belong to the right chain R. The extension, $E(e_l)$, of the invisible clockwise adjacent edge, e_l, of v_l cannot hit the left chain L. Therefore no point inside the cave P' can see a part of L, a contradiction to the street property.

By the same arguments we can prove that the counterclockwise angle $\phi \geq 0$ between sv_r and sv_l is always smaller than π, see Figure 2 (i). In other words, the situations in Figure 3 cannot occur. Therefore in the vicinity of s the robot should always walk into the triangle $v_l\, s\, v_r$ to avoid unnecessary detours to v_r and v_l.

Now we look at the general situation. We assume that a strategy has led the robot to an actual position somewhere in the polygon. We will see that the properties discussed for the start essentially remain valid. Vertices v_l and v_r are defined as before, i. e. $v_l \in L$ is the clockwise most advanced left reflex vertex and $v_r \in R$ the counterclockwise most advanced right reflex vertex.

There is no reason for a strategy to loose the current v_l or v_r out of sight, so we assume that v_l and v_r are always visible, as long as they exist. As already discussed above, the only non-trivial case is if both, v_l and v_r, actually exist. We call this a *funnel situation.*

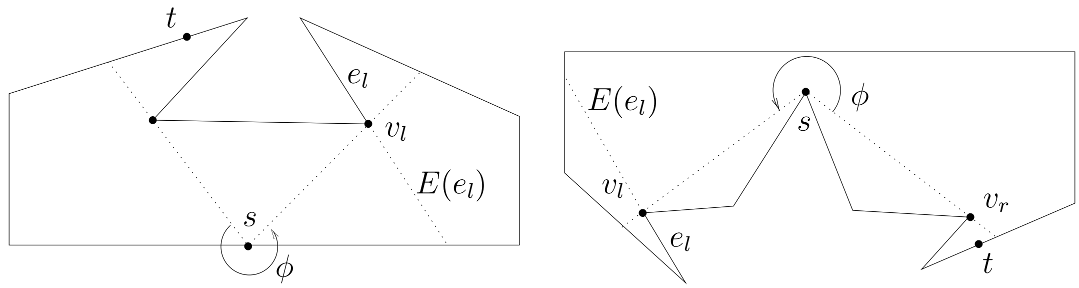

Figure 3: If the counterclockwise angle ϕ between $v_r s$ and $s v_l$ at s is greater than or equal to π then the polygon is not a street.

The angle, ϕ, between the directions from the actual position to v_l and to v_r is called the *opening angle*, it is always smaller than π.

While exploring P in a funnel situation sequences of reflex vertices $v_l \in \{v_l^1, v_l^2, \ldots, v_l^m\}$ and $v_r \in \{v_r^1, v_r^2, \ldots, v_r^n\}$ occur until the funnel situation ends, see e. g. point q in Figure 2 (i). If at this time only $v_l = v_l^m$ exists (analogously for v_r) then we know that the goal t is contained in the cave of v_l, we walk to v_l, and the left convex chain $v_l^1\, v_l^2 \ldots v_l^m$ belongs to SP.

So any reasonable strategy will proceed in the following way. If the goal is visible or only one of v_l and v_r exists, then walk into that direction. Otherwise we have a funnel situation, we choose a walking direction within the opening angle, i. e. between v_l and v_r, and repeat this continuously until the first case applies again.

It is important to note that at the robot's current position is a vertex of the shortest path SP whenever a funnel situation newly appears *and* when the next vertex has been reached after the funnel situation was solved. For example, at point q in Figure 2 (i) it is clear that we have to go to vertex $v_l^2 \in SP$ where the next funnel situation starts. Therefore, if a strategy achieves a competitive factor c in each funnel situation (i. e. compared to the shortest path between the two visited vertices of SP) then it achieves the same factor in arbitrary streets.

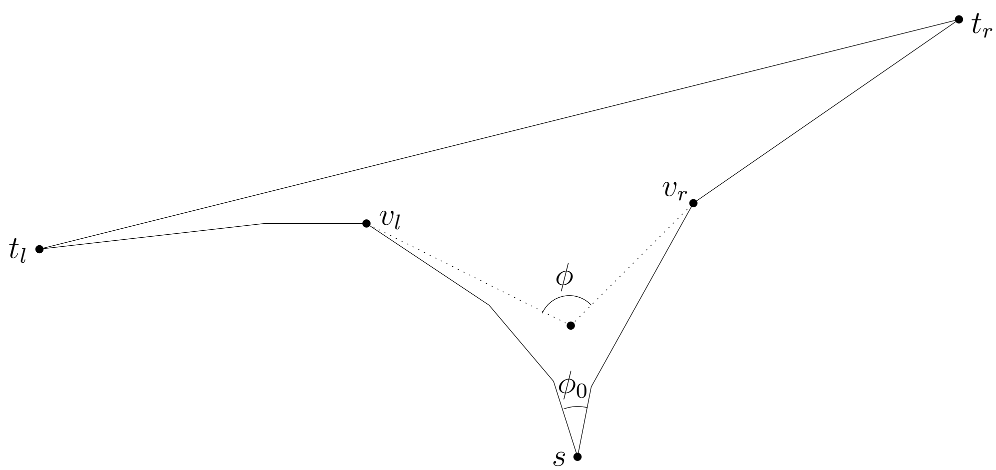

Figure 4: A funnel.

As a consequence, we can restrict our attention to very special polygons, the so-called *funnels*. A funnel consists of two chains of reflex vertices with a common start point s, see Figure 4 for an example. The two reflex chains end in vertices t_l and t_r, respectively, and the line segment $t_l\, t_r$ closes the polygon. A funnel polygon represents a funnel situation in which the goal t lies arbitrarily close behind either t_l or t_r, and the strategy will know which case applies only when the line segment $t_l\, t_r$ is reached. For analyzing a strategy, both cases have to be considered and the worse of them determines the competitive factor. Other funnel situations which end with a smaller opening angle or where the goal is further away from t_l or t_r will produce a smaller detour.

Since the walking direction is always within the opening angle, ϕ is always *strictly increasing*. It starts at the angle, ϕ_0, between the two edges adjacent to s and reaches, but never exceeds, 180° when finally the goal becomes visible. By this property, it is quite natural to take the opening angle ϕ for *parameterizing a strategy*.

We can further restrict ourselves to consider only funnels with initial opening angle $\phi_0 \geq 90°$. As was shown in [12, 23], any strategy which achieves a factor $\geq \sqrt{2}$ for all funnels with $\phi_0 \geq 90°$ can be adapted to the general case without changing its factor in the following way. First, we start with a simple walk along the 'static' angular bisector of the first pair v_l and v_r until an opening angel of $\pi/2$ is reached. Then we proceed with the given strategy.

3 A strategy which always takes the worst case into account

3.1 A generalized lower bound

We start with a generalized lower bound for initial opening angles $\geq 90°$. For an arbitrary angle ϕ, let

$$K_\phi := \sqrt{1 + \sin\phi}\,.$$

Lemma 3 *Assume an initial opening angle $\phi_0 \geq \frac{\pi}{2}$. Then no strategy can guarantee a smaller competitive factor than K_{ϕ_0}.*

Proof. We take an isosceles triangle with an angle ϕ_0 at vertex s, the other vertices are t_l and t_r; see Figure 5.

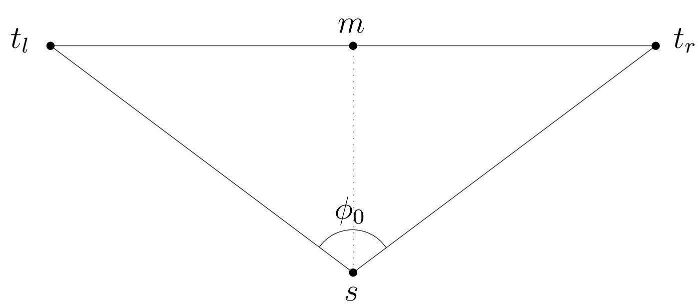

Figure 5: Establishing a generalized lower bound.

The goal becomes visible only when the line $t_l\, t_r$ is reached. If this happens to the left of the midpoint m then the goal may be to the right, and vice versa. In any case the path length is at least the distance from s to m plus the distance from m to t_l. For the ratio, c, of the path length to the shortest path we obtain by simple trigonometry

$$c \geq \cos\frac{\phi_0}{2} + \sin\frac{\phi_0}{2} = \sqrt{1 + \sin\phi_0} = K_{\phi_0}\,. \qquad \square$$

For $\phi_0 = \frac{\pi}{2}$, we have the well-known lower bound of $\sqrt{2}$ stemming from a rectangular isoceles triangle [14].

Remark also that the bound K_{ϕ_0} also applies for any non-symmetric situation, since at the start the funnel is unknown except for the two edges adjacent to s and it may turn into a nearly symmetric case immediately after the start. This means in other words that for an initial opening angle ϕ_0 a competitive factor of K_{ϕ_0} is always the best we can hope for.

In the following we will develop a strategy which achieves exactly this factor.

3.2 Sufficient requirements for an optimal strategy

In a funnel with opening angle π the goal is visible and there is a trivial strategy that achieves the optimal competitive factor $K_\pi = 1$. So we look backwards to decreasing angles.

Let us assume for the moment that the funnel is a triangle, and that we have a strategy with a competitive factor of K_{ϕ_2} for all triangular funnels with initial opening angle ϕ_2. How can we extend this to initial opening angles ϕ_1 with $\pi \geq \phi_2 > \phi_1 \geq \frac{\pi}{2}$?

Starting with an angle ϕ_1 at point p_1 we walk a certain path of length w until we reach an angle of ϕ_2 at point p_2 from where we can continue with the known strategy; see Figure 6. The left and right reflex vertices, v_l and v_r as defined in Section 2, do not change.

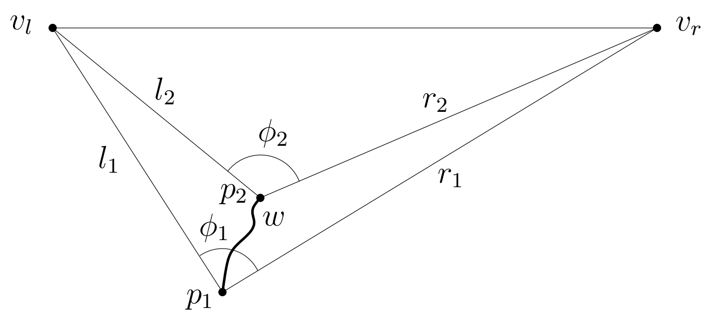

Figure 6: Getting from angle ϕ_1 to ϕ_2.

Let l_1 and l_2 denote the distances from p_1 resp. p_2 to v_l at the left side and r_1 and r_2 the corresponding distances at the right. If $t = v_l$ then the path length from p_1 to t is not greater than $w + K_{\phi_2} l_2$. If now $K_{\phi_1} l_1 \geq w + K_{\phi_2} l_2$ holds and the analogous inequality $K_{\phi_1} r_1 \geq w + K_{\phi_2} r_2$ for the right side, which can also be expressed as

$$w \leq \min(K_{\phi_1} l_1 - K_{\phi_2} l_2, K_{\phi_1} r_1 - K_{\phi_2} r_2)\,, \tag{1}$$

then we have a competitive factor not bigger than K_{ϕ_1} for triangles with initial opening angle ϕ_1.

Note that condition (1) is additive in the following sense. If it holds for a path w_{12} from ϕ_1 to ϕ_2 and for a continuing path w_{23} from ϕ_2 to ϕ_3 then it is also true for the combined path $w_{12} + w_{23}$ from ϕ_1 to ϕ_3. This will turn out to be very useful: if (1) holds for arbitrarily small, successive steps w then it is also true for all bigger ones.

Now let us go further backwards and observe what happens if the current v_l or v_r change. We assume that condition (1) holds for path w from p_1 to p_2 and that v_l changes at p_2; see Figure 7. The visible left chain is extended by l_2'. Nothing changes on the right side of the funnel, and for the left side of the funnel we have

$$w \leq K_{\phi_1} l_1 - K_{\phi_2} l_2 = K_{\phi_1} l_1 - K_{\phi_2} l_2 + K_{\phi_2} l_2' - K_{\phi_2} l_2' \leq K_{\phi_1}(l_1 + l_2') - K_{\phi_2}(l_2 + l_2')\,. \tag{2}$$

The last inequality holds because $K_\phi = \sqrt{1 + \sin\phi}$ is decreasing with increasing ϕ. Here, $l_1 + l_2'$ and $l_2 + l_2'$ are the lengths of the shortest paths from p_1 and p_2 to v_l', respectively. But (2) in fact means that (1) remains valid even if changes of v_l or v_r occur.

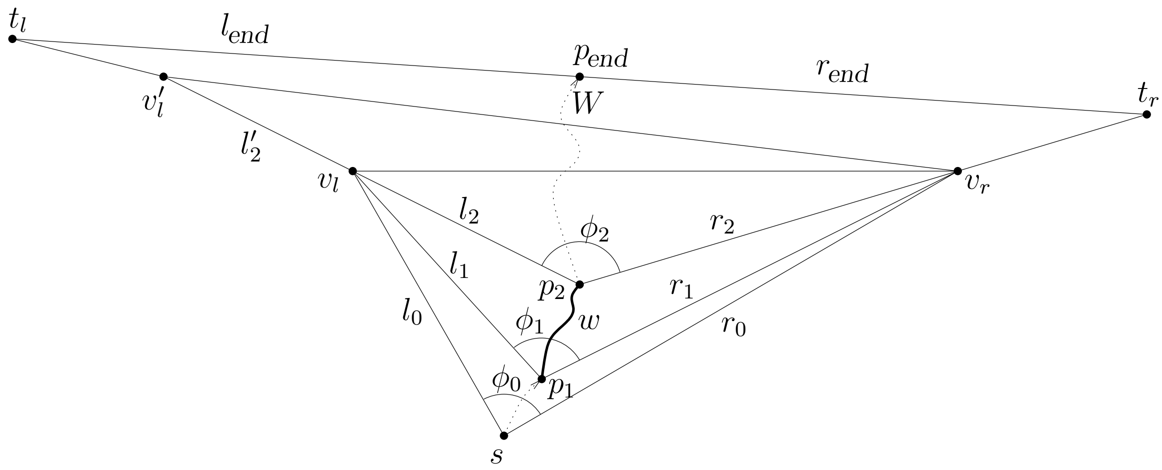

Figure 7: When p_2 is reached, the most advanced visible point to the left jumps from v_l to v_l'.

Under the assumption that (1) holds for all small steps where v_l and v_r do not change we can make use of the additivity of (1) and obtain the following for the path length, W, from an initial opening angle ϕ_0 to the point p_{end} where the line segment $t_l\, t_r$ is reached; see Figure 7.

$$W \leq \min\Big(K_{\phi_0}(\text{length of left chain}) - K_\pi l_{end}, \quad K_{\phi_0}(\text{length of right chain}) - K_\pi r_{end} \Big)$$

But, since $K_\pi = 1$, this inequality exactly means that we have a competitive factor not bigger than K_{ϕ_0}. It only remains to find a curve that fulfills (1) for small steps.

3.3 Developing the curve

One could try to fulfill condition (1) by analyzing, for fixed p_1, ϕ_1, and ϕ_2, which points p_2 meet that requirement. To avoid this tedious task, we argue as follows. For fixed ϕ_2, the point p_2 lies on a circular arc through v_l and v_r. While p_2 moves along this arc, the length l_2 is strictly increasing while r_2 is strictly decreasing. Therefore, we maximize our chances to fulfill (1) if we require

$$K_{\phi_2} l_2 - K_{\phi_1} l_1 = K_{\phi_2} r_2 - K_{\phi_1} r_1$$

or the equivalent

$$K_{\phi_2}(l_2 - r_2) = K_{\phi_1}(l_1 - r_1)\,. \tag{3}$$

In other words: if we start with initial values ϕ_0, l_0, r_0, we have a fixed constant $A := K_{\phi_0}(l_0 - r_0)$ and for any $\phi_0 \leq \phi \leq \pi$ with corresponding lengths l_ϕ and r_ϕ we want that

$$K_\phi(l_\phi - r_\phi) = A\,. \tag{4}$$

In the symmetric case $l_0 = r_0$ this condition means that we walk along the bisector of v_l and v_r.

Otherwise condition (4) defines a nice curve which can be determined in the following way. We choose a coordinate system with horizontal axis $v_l\, v_r$, the midpoint being the origin. We scale such that the distance from v_l to v_r equals 1.

W. l. o. g. let $l_0 > r_0$. For any $\phi_0 \le \phi < \pi$ the corresponding point of the curve is the intersection of the hyperbola

$$\frac{X^2}{\left(\frac{A}{2K_\phi}\right)^2} - \frac{Y^2}{\left(\frac{1}{2}\right)^2 - \left(\frac{A}{2K_\phi}\right)^2} = 1$$

and the circle

$$X^2 + \left(Y + \frac{\cot\phi}{2}\right)^2 = \frac{1}{4\sin^2\phi}.$$

Solving the equations gives, after some transformations, the following solutions.

$$X(\phi) = \frac{A}{2} \cdot \frac{\cot\frac{\phi}{2}}{1+\sin\phi}\sqrt{\left(1+\tan\frac{\phi}{2}\right)^2 - A^2} \quad (5)$$

$$Y(\phi) = \frac{1}{2}\cot\frac{\phi}{2}\left(\frac{A^2}{1+\sin\phi} - 1\right) \quad (6)$$

Since $A < \sqrt{1+\sin\phi}$ holds, the functions $X(\phi)$ and $Y(\phi)$ are well defined and continuous and the curve is contained in the triangle defined by ϕ_0, l_0, r_0.

Figure 8 shows how these curves look like for all possible values of ϕ and A and also for $l_0 \le r_0$. All points with an initial opening angle of $\frac{\pi}{2}$ lie on the lower half circle.

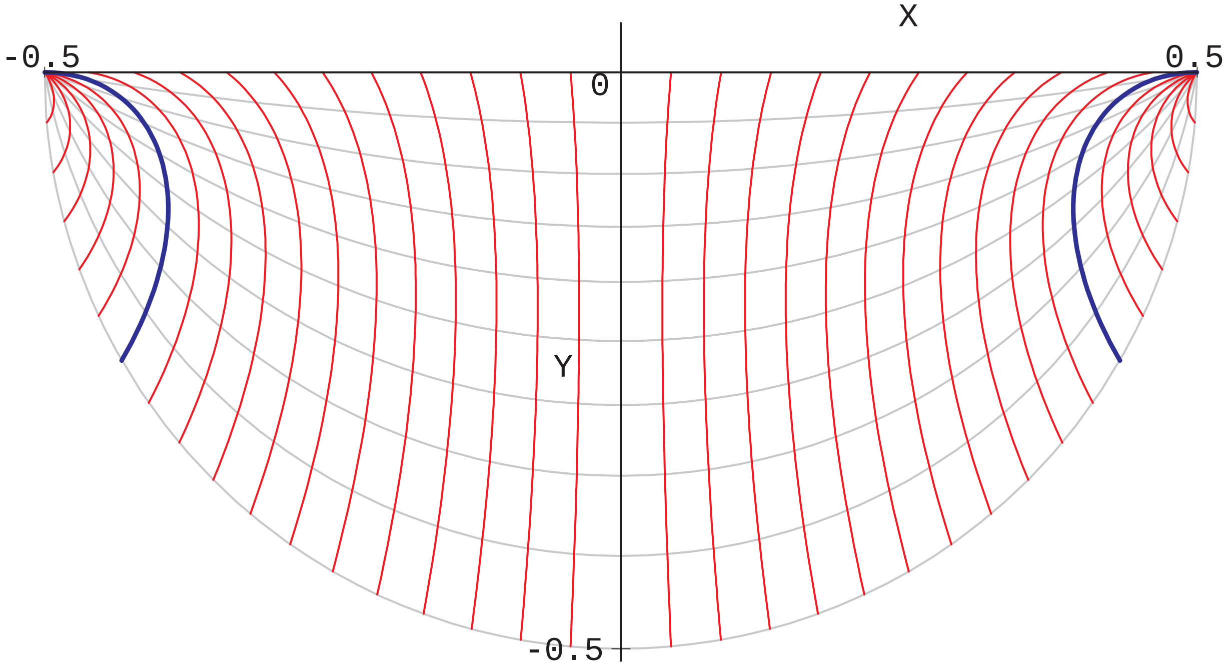

Figure 8: The curves fulfilling condition (4) for all values of ϕ and A. This figure was created by using the computer algebra system Maple [5] which was also very helpful in checking the transformations of the formulae in Sections 3.3 and 3.4.

Two cases can be distinguished. For $A \le 1$ the curves can be continuously completed to an endpoint on the line $v_l\, v_r$ with $X(\pi) = \pm\frac{A}{2}$ and $Y(\pi) = 0$ where also (4) is fulfilled. For $A > 1$ the curves end up in v_l and v_r, resp., with parameter $\phi = \arcsin\sqrt{A^2-1} < \pi$. The curves for the limiting case $A = 1$ are emphasized with a thick line in Figure 8.

3.4 Checking the requirements

We want to check that the given curve fulfills condition (1). Because of the additive property of (1) it is sufficient to verify this for very small intervals. The arc length of the curve from angle ϕ to $\phi + \epsilon$ has to be compared to the right side of (1). Because of (3) the min can be dropped.

For $l_0 = r_0$ we trivially have equality in (1). Otherwise we check if

$$\sqrt{X'(\phi)^2 + Y'(\phi)^2} < -(K_\phi l_\phi)' \,. \tag{7}$$

Here, $X'(\phi)$ and $Y'(\phi)$ denote the derivatives of $X(\phi)$ and $Y(\phi)$ from (5) and (6). Then by integration we have

$$\forall \epsilon > 0 \quad \int\limits_{\phi}^{\phi+\epsilon} \sqrt{X'(\phi)^2 + Y'(\phi)^2}\, d\phi \le K_\phi l_\phi - K_{\phi+\epsilon} l_{\phi+\epsilon} \,.$$

This is what we need. For the hyperbola, we have

$$l_\phi = \frac{\frac{1}{2}}{\frac{A}{2K_\phi}} X(\phi) + \frac{A}{2K_\phi} \quad \text{and therefore} \quad K_\phi l_\phi = \frac{K_\phi^2}{A} X(\phi) + \frac{A}{2} \,.$$

This can be used in (7), and after squaring the following remains to show.

$$\begin{aligned} F(\phi, A) &< 0 \quad \text{for all} \quad \frac{\pi}{2} < \phi < \pi \quad \text{and} \quad 0 < A < \sqrt{1 + \sin\phi} \quad \text{where} \\ F(\phi, A) &:= \frac{X'(\phi)^2}{A^2}(A^2 - (1 + \sin\phi)^2) - \frac{\cos^2\phi}{A^2} X(\phi)^2 \\ &\quad -2\frac{\cos\phi(1 + \sin\phi)}{A^2} X(\phi) X'(\phi) + Y'(\phi) \end{aligned}$$

We insert the values of the derivatives into $F(\phi, A)$. Using trigonometric identities we simplify $F(\phi, A)$ in such a way that a factor of $\frac{1}{4}\cot^2\frac{\phi}{2}$ can be extracted and only even powers of A remain in the rest.

Substituting A^2 by B, we define

$$G(\phi, B) := F(\phi, A) \frac{(1 + \tan\frac{\phi}{2})^2 - A^2}{\frac{1}{4}\cot^2\frac{\phi}{2}} \,.$$

This does not change the sign since $A^2 < 1 + \sin\phi < (1 + \tan\frac{\phi}{2})^2$. As a polynom in B, $G(\phi, B)$ has degree 2 and can be written in the form

$$G(\phi, B) = (V(\phi)B + H(\phi))B \,.$$

Since $B > 0$, $G(\phi, B)$ can only become 0 if $B = -H(\phi)/V(\phi)$, but $B < 1 + \sin\phi$ and $-H(\phi)/V(\phi) - (1 + \sin\phi)$ simplifies to $-\dfrac{\sin\phi\cos\phi}{\cos\phi + 2} > 0$, which proves that $G(\phi, B) \neq 0$ for all $\frac{\pi}{2} < \phi < \pi$ and $0 < B < 1 + \sin\phi$. So the sign of $G(\phi, B)$ is constant, i. e. constantly negative, as one can verify. This proves (7) and therefore (1) for the curves of Section 3.3.

3.5 The main result

To summarize, our strategy for searching a goal in an unknown street works as follows.

> Strategy *WCA* (worst case aware):
>
> If the initial opening angle is less than 90° walk along the angular bisector of v_l and v_r until a right angle is reached; see the end of Section 2.
>
> Depending on the actual parameters ϕ_0, l_0, and r_0, walk along the corresponding curve from Section 3.3 until one of v_l and v_r changes. Switch over to the curve corresponding to the new parameters ϕ_1, l_1, and r_1. Continue until the line $t_l\, t_r$ is reached.

Theorem 4 *By using strategy WCA we can search a goal in an unknown street with a competitive factor of at most* $\sqrt{2}$. *This is optimal.*

The proof is contained in Sections 3.1 through 3.4.

4 Conclusions

We have developped a competitive strategy for walking in streets which guarantees an optimal factor of at most $\sqrt{2}$ in the worst case, thereby settling an old open problem. Furthermore, the strategy behaves even better for an initial opening angle $\phi_0 > 90°$ in which case an optimal factor $K_{\phi_0} = \sqrt{1 + \sin\phi_0}$ between 1 and $\sqrt{2}$ is achieved.

The idea for this strategy comes from the generalized lower bound in Lemma 3 and from the two conditions (1) and (3), which are not strictly necessary for the optimal competitive factor but turn out to be very useful. Therefore, we do not claim that this is the only optimal strategy. It would be interesting if there are substantially different but also optimal strategies.

References

[1] D. Angluin, J. Westbrook, and W. Zhu. Robot navigation with range queries. In *Proc. 28th Annu. ACM Sympos. Theory Comput.*, pages 469–478, May 1996.

[2] R. Baeza-Yates, J. Culberson, and G. Rawlins. Searching in the plane. *Inform. Comput.*, 106:234–252, 1993.

[3] P. Berman. On-line searching and navigation. In A. Fiat and G. Woeginger, editors, *Competitive Analysis of Algorithms.* Springer-Verlag, 1998.

[4] A. Blum, P. Raghavan, and B. Schieber. Navigating in unfamiliar geometric terrain. *SIAM J. Comput.*, 26(1):110–137, Feb. 1997.

[5] B. W. Char, K. O. Geddes, G. H. Gonnet, B. L. Leong, M. B. Monagan, and S. M. Watt. *Maple V Language Reference Manual.* Springer-Verlag, New York, 1991.

[6] G. Das, P. Heffernan, and G. Narasimhan. LR-visibility in polygons. *Comput. Geom. Theory Appl.*, 7:37–57, 1997.

[7] P. Dasgupta, P. P. Chakrabarti, and S. C. DeSarkar. A new competitive algorithm for agent searching in unknown streets. In *Proc. 16th Conf. Found. Softw. Tech. Theoret. Comput. Sci.*, volume 1180 of *Lecture Notes Comput. Sci.*, pages 147–155. Springer-Verlag, 1996.

[8] A. Datta, C. A. Hipke, and S. Schuierer. Competitive searching in polygons: Beyond generalised streets. In *Proc. 6th Annu. Internat. Sympos. Algorithms Comput.*, volume 1004 of *Lecture Notes Comput. Sci.*, pages 32–41. Springer-Verlag, 1995.

[9] A. Datta and C. Icking. Competitive searching in a generalized street. In *Proc. 10th Annu. ACM Sympos. Comput. Geom.*, pages 175–182, 1994.

[10] S. K. Ghosh and S. Saluja. Optimal on-line algorithms for walking with minimum number of turns in unknown streets. *Comput. Geom. Theory Appl.*, 8:241–266, 1997.

[11] C. Icking. *Motion and Visibility in Simple Polygons*. PhD thesis, FernUniversität Hagen, 1994.

[12] C. Icking, A. López-Ortiz, S. Schuierer, and I. Semrau. Going home through an unknown street. Technical Report 228, Dep. of Comp. Science, FernUniversität Hagen, 1998. http://wwwpi6.fernuni-hagen.de/Publikationen/tr228.pdf, subm. f. publication.

[13] R. Klein. Walking an unknown street with bounded detour. In *Proc. 32nd Annu. IEEE Sympos. Found. Comput. Sci.*, pages 304–313, 1991.

[14] R. Klein. Walking an unknown street with bounded detour. *Comput. Geom. Theory Appl.*, 1:325–351, 1992.

[15] J. M. Kleinberg. On-line search in a simple polygon. In *Proc. 5th ACM-SIAM Sympos. Discrete Algorithms*, pages 8–15, 1994.

[16] E. Kranakis and A. Spatharis. Almost optimal on-line search in unknown streets. In *Proc. 9th Canad. Conf. Comput. Geom.*, pages 93–99, 1997.

[17] A. López-Ortiz and S. Schuierer. Going home through an unknown street. In *Proc. 4th Workshop Algorithms Data Struct.*, volume 955 of *Lecture Notes Comput. Sci.*, pages 135–146. Springer-Verlag, 1995.

[18] A. López-Ortiz and S. Schuierer. Generalized streets revisited. In *Proc. 4th Annu. European Sympos. Algorithms*, volume 1136 of *Lecture Notes Comput. Sci.*, pages 546–558. Springer-Verlag, 1996.

[19] A. López-Ortiz and S. Schuierer. Walking streets faster. In *Proc. 5th Scand. Workshop Algorithm Theory*, volume 1097 of *Lecture Notes Comput. Sci.*, pages 345–356. Springer-Verlag, 1996.

[20] J. S. B. Mitchell. Geometric shortest paths and network optimization. In J.-R. Sack and J. Urrutia, editors, *Handbook of Computational Geometry*. Elsevier Science Publishers B.V. North-Holland, Amsterdam, 1998.

[21] N. S. V. Rao, S. Kareti, W. Shi, and S. S. Iyengar. Robot navigation in unknown terrains: introductory survey of non-heuristic algorithms. Technical Report ORNL/TM-12410, Oak Ridge National Laboratory, 1993.

[22] S. Schuierer and I. Semrau. Eine optimale Suchstrategie für Straßen. Technical Report 97, Institut für Informatik, Universität Freiburg, Germany, 1998.

[23] I. Semrau. Analyse und experimentelle Untersuchung von Strategien zum Finden eines Ziels in Straßenpolygonen. Diploma thesis, FernUniversität Hagen, 1996.

[24] L. H. Tseng, P. Heffernan, and D. T. Lee. Two-guard walkability of simple polygons. *Internat. J. Comput. Geom. Appl.*, 8(1):85–116, 1998.