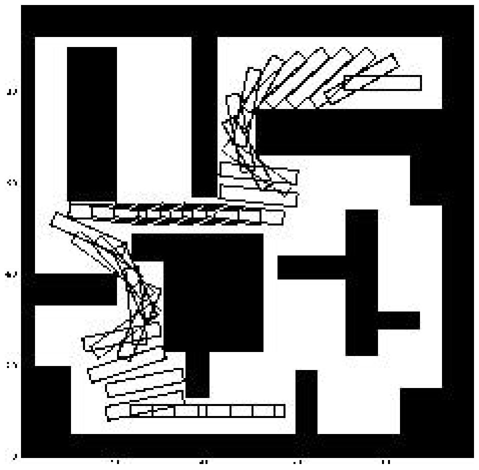
Robotic Navigation with Constraints



Overview

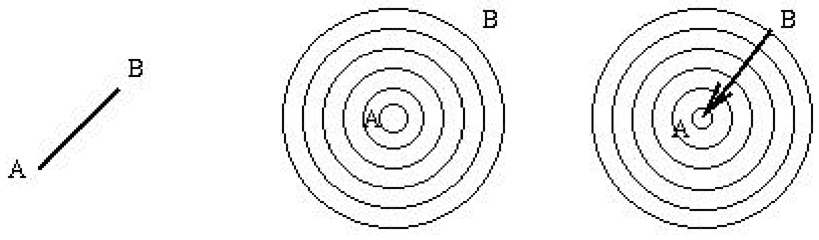
Suppose you live in an apartment with lots of corridors and long skinny halls. And, you've finally scraped up the money to buy a piano, which you need to get from the front door to the back room. The first question is, is it actually possible to twist and turn the piano in such a way as to get it back there? And second, if it is, what is the shortest path?

This is a problem in robotic navigation with constraints; the navigation part is to find the shortest path; the constraint part is the requirement that you don't accidentally add a few new holes in the walls as you move the piano. Fast Marching Methods, provide a very quick way to solve this problem.

Finding the Shortest Path

An easy version

As a warm-up, suppose you are standing in a giant parking lot. You are at point A, your car is at point B, and there are no other cars in the lot. If you want the shortest path to your car, just draw the straight line shown in the figure on the left.



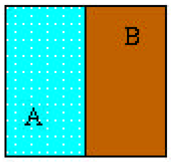
Straight line from A to B   Expanding front around A   Trace back to find path

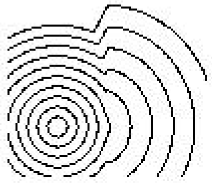
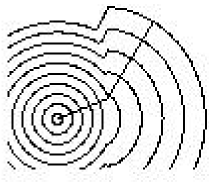
But there's a different way to find this path. Imagine a front expanding from point A in all directions. Since it doesn't "cost" you any more to walk in one direction over another, let the front expand with speed 1 in all directions. That means that the expanding front will be a growing circle around point A, which will eventually touch point B. Once the front touches point B, you can

find the shortest path by starting at point B and proceeding backwards along the path that is always perpendicular to the expanding front. If you do this, you'll get the straight line shown in the figure on the right.

A slightly harder version

Now, let's imagine that one half of the parking lot is full of snow, and it's slower to walk through snow. Furthermore, you're standing on the snowy side, and your car is on the dry side. In this case, the "shortest" path (that is, the one that takes the least time), as shown in the figure on the left, is not always a straight line. But we can still use our front propagation technique: we expand a front around point A, only this time the front expands faster when it is on dry pavement than it does over the snow. Once the front hits the car, we again trace backwards from B to A, always going perpendicular to the front, and construct the shortest path!

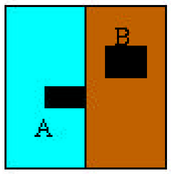


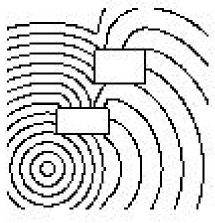
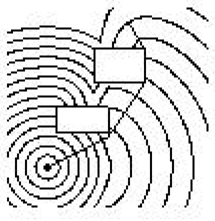



Left=snow, Right=dry Expand front around A Trace back to find path

An even harder version

Now, let's add other cars to the lot. We can represent those cars as places where the "cost" of traveling is infinity: it takes forever to walk through a car. Again, we solve our front propagation problem, trace back, and construct the optimal path.



Snow + Dry + Other Cars Expand front around A Trace back to find path

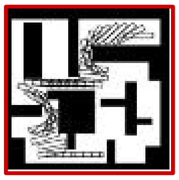
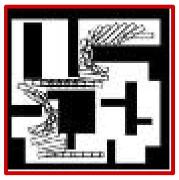
By the way, you were carrying a ladder

Yup, and the ladder can't be tilted on its side, and the cars are really big so that you can't pick the ladder over the cars. That means that you are no longer a point, but now have to negotiate the ladder through the narrow paths between cars. It's the same thing; we propagate a front outwards from A to B, adjusting the speed corresponding to our ability to walk or turn the ladder at any point, and then, when it reaches B, we can trace backwards to find the fastest path.

Finding the Shortest Path

The equation that describes the arrival time of this expanding front as it depends on the possible

speed at any point and direction is the Eikonal equation. The Fast Marching Method is used to solve the equation for the first arrival time, and then we can trace backwards from B to A to construct the actual path.

(51K) (71K)

Movies of Moving a Piano in a San Francisco Apartment

An Interactive Java Applet:

Design your own obstacle course and robot, and then let the Fast Marching Method compute the optimal path

Details

The calculations were made using a Fast Marching Method to solve the Eikonal equation given by | grad T | = f(x,y,theta) for first arrival times, coupled to back propagation using Heun's method for the O.D.E. u= grad T given by the gradient field. The Eikonal equation in configuration space is obtained by discretizing the domain in x and y, as well as the rotational angle theta. The constraints of the walls are manifested by modifying configuration space as a function of the angle of the stick relative to the domain geometry. Thus, the full problem is a three-dimensional configuration space, and the back propagation trajectory through configuration space is then shown in the movie at each time step. Additional movies

References

Kimmel, R., and Sethian, J.A., *Fast Marching Methods for Robotic Navigation with Constraints* Center for Pure and Applied Mathematics Report, Univ. of California, Berkeley, May 1996, submitted for publication, Int. Journal Robotics Research, 1998.

NEW BOOK ON FAST MARCHING METHODS AND LEVEL SET METHODS:

Sethian, J.A., Fast Marching Methods and Level Set Methods: Evolving Interfaces in

Computational Geometry, Fluid Mechanics, Computer Vision and Materials Sciences, Cambridge University Press, 1999.

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J.A. Sethian
sethian@math.berkeley.edu

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