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Extracting Topology-Based Maps from Gridmaps

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Abstract

We propose topology-based maps as a new representation of the workspace of a mobile robot. These maps capture the structure of the free space in the environment in terms of the basic topological notions of connectivity and adjacency. Topology-based maps can be automatically extracted from an occupancy grid built from sensor data using techniques borrowed from the image processing field. Since these techniques can be soundly defined on fuzzy values, our approach is well suited to deal with the uncertainty inherent in the sensor data. Topology-based maps are fairly robust with respect to sensor noise and to small environmental changes, and have nice computational properties.

1 Introduction

Building a representation of the environment is an important task for a mobile robot that aims at moving autonomously in the surrounding space. In robotics, the most common descriptions of the space are *metric maps* and *topological maps*. A metric map represents the environment according to the absolute geometric position of the objects. A topological map is a more abstract representation that describes relationships among features of the environment, without any absolute reference system. Topological maps are usually represented in graph form [8, 14, 6, 2].

Being more abstract, a topological maps has the advantage of being more compact and more stable with respect to sensor noise and to small changes in the environment. Unfortunately, the semantics associated to topological maps are still somehow ambiguous. For example, in the maps defined by Kuipers and Byun [8] nodes represent *places*, characterized by sensor data, and arcs represent *paths* between places, characterized by control strategies. By contrast, the maps defined by Thrun [14] are obtained by partitioning a probabilistic occupancy grid into *regions* (nodes) separated by *narrow passages* (arcs) according to some measure of clearance. Perhaps more puzzling, the topological maps found in the literature seem to be rather detached from a description of the environment in terms of the usual notions of (mathematical) topology.

In this paper, we propose to build a representation of the working space of the robot using concepts from general topology. We call a representation of this type a *topology-based map*. The semantics of these maps is uniquely defined in terms of the topological structure of the information. We assume that this information is represented in a discrete form by a two-dimensional array of cells, or gridmap. Standard topology, however, cannot be directly applied to gridmaps because many concepts need a continuous space, so we resort to *digital topology*. This discipline, used to study the topological properties of images, reformulates the general topology for finite sets in a consistent way. The use of digital topology makes our approach well founded, since we define topological properties directly in a discretized space, the only one available to the robot.

In this work, we focus on topology-based maps built from information about the free space in the environment, represented by an occupancy grid. Studying directly the topology of free space, however, would not be very useful since typically all the free space mapped by the robot is connected. We then study the topology induced by the *shape* of the free space. In particular, we focus on large open spaces connected by narrow passages, since these features are useful for behavior-based navigation and self-localization. To extract shape information, we use another tool from the field of image processing: mathematical morphology [1]. By the combined use of mathematical morphology and digital topology we can build maps that represent the topological structure of the space according to some specific morphological criterion.

We use fuzzy logic to improve the robustness of our maps in two ways. First, we account for the inherent uncertainty of the sensor data using a fuzzy technique to build the initial gridmap [10]. Second, we account for the vagueness of the morphological concepts that interest us, like "large open space," using fuzzy mathematical morphology. Note that both mathematical morphology and digital topology have been consistently extended to the case of fuzzy values. Experimental evidence suggests that the resulting topology-based maps are fairly robust with respect to sensor noise and to small modifications to the environment.

The rest of this paper is organized as follows. The next section briefly recalls the main ingredients needed for our construction: fuzzy gridmaps, (fuzzy) digital topology, and (fuzzy) mathematical morphology. The next two sections detail our extraction technique in conceptual and in algorithmic terms, respectively. Sec. 5 presents an experiment performed on real data.

2 Background

We define a *gridmap* I to be any two-dimensional array of integer-coordinate points, whose elements are called *cells*. In a *binary gridmap* the cells have values in $\{0, 1\}$, while in a *fuzzy gridmap* the cells have values in the real interval $[0, 1]$. The values of the cells define a subset M of I: in a binary gridmap, M is the set cells that have value 1; in a fuzzy gridmap, M is the fuzzy subset of cells identified by the membership function $\mu : I \rightarrow [0, 1]$. In this paper, we only consider fuzzy gridmaps whose elements take a finite number of values in $[0, 1]$. A useful way to think of a fuzzy gridmap is as a *stack* of binary gridmaps, each one obtained by cutting the fuzzy gridmap at some level $\alpha \in [0, 1]$. This stacking property is guaranteed by the fact that the α-cuts are nested when α increases.

2.1 Digital topology

Digital topology is the study of topological properties of images. First introduced by Rosenfeld in the seventies for binary images [5], the approach has been reformulated from Kovalesky [7] in the contest of general topology. In 1979, Rosenfeld extended the digital topology to gray-level images, considered as fuzzy sets [11]. All the definitions we use in this section are the one proposed by Rosenfeld [11], except the notion of influence zone, defined by [15].

Let I be a fuzzy gridmap, and μ the corresponding membership function. Two cells in the array are said to be *4-adjacent* (resp., *8-adjacent*) if they are distinct and differ in at most one (resp., two) of their coordinates. For a bidimensional gridmap with square cells, 4-adjacent cells share a common side, while 8-adjacent cells have a side or a corner in common. In the following, we use "adjacent" to indicate both 4-adjacent and 8-adjacent cells. The adjacency relation is fundamental to define the *neighborhood* of a cell, a basic concept in topology. Given a cell p, the *neighborhood* of p is the union of p with the set of its adjacent cells.

Another fundamental concept in topology is connectedness. In binary digital topology, this concept is defined as follows. Given two cells p and q of I, a *path* from p to q is a sequence of points $\varrho = \langle p_1, \ldots, p_n \rangle$ such that $p_0 = p, p_n = q$, and p_i is adjacent to p_{i+1} for $i = 1, \ldots, n-1$. We then say that p and q are *connected* in M if there exists a path from p to q entirely in M. "Connected" is an equivalence relation, and hence it partitions M in classes that we call *connected components*. To extend this definition to fuzzy sets, we need to introduce some more concepts.

Given a path ϱ, the *strength* of ϱ with respect to μ is defined as $s_\mu(\varrho) = \min_{0 \leq i \leq n} \mu(p_i)$. The *degree of connectedness* c_μ of two points p and q is then defined by $c_\mu(p, q) = \max_\varrho s_\mu(\varrho)$, where ϱ ranges over all the paths from p to q. Two points p and q are then said to be *connected* with respect to μ if $c_\mu(p, q) = \min(\mu(p), \mu(q))$, that is, if there is a path $\varrho = \langle p_1, \ldots, p_n \rangle$ from p to q such that $\mu(p_i) \geq \min(\mu(p), \mu(q))$ for all i.

Unlike the binary case, "connected" is not an equivalence relation and it does not give us a partition. However, it is still possible to define connected components with partition-like properties. Let us define a *top* (resp. *bottom*) of μ as a maximal connected subset Π of I on which μ has constant and locally maximum (resp. minimum) value. Two tops can be expanded until they meet at some common border using the notion of geodesic distance: a distance computed taking into account the profile of μ on the gridmap. We define the *influence zone* Z_Π of a top Π as the set of points $p \in A_\Pi$ such that the geodesic distance of p from Π is less than the geodesic distance of p from any other top of μ. For a a given μ, the influence zones Z_{Π_i} are uniquely determined, and they form a partition of I. (See [15] for more formal definitions.)

In the rest of this paper, we take the sets Z_{Π_i} as our connected components. Two components Z_Π and $Z_{\Pi'}$ are said to be *adjacent* if there is a point $p \in I$ which is adjacent to a point $p_1 \in Z_\Pi$ and a point $p_2 \in Z_{\Pi'}$.

2.2 Mathematical morphology

Mathematical morphology is a branch of image processing concerned with the extraction of shape features from a digital image. Initially defined for binary images, mathematical morphology has been later extended to grey-scale images [12] and to fuzzy images [1]. A nice characteristic of mathematical morphol-

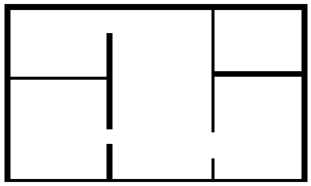

Fig. 1: An ideal gridmap.

ogy is that most operations are defined starting from a small structuring element, used as a probe, and two elementary operations, called *dilation* and *erosion*.

Let I and K be two binary gridmaps, where I contains the original image and K is the *structural element*. The dilation of I by K is a new gridmap, denoted by $I \oplus K$, that has value 1 at point p if and only if $I(q) = 1$ for *some* cell q in the K-neighborhood of p, i.e., the cells for which $K(q) = 1$ once K is centered on p. The erosion of I by K, denoted by $I \ominus K$, is defined in a similar way by replacing *some* by *all*. Dilation and erosion have a nice duality property: $\neg(I \oplus K) = (\neg I) \ominus \check{K}$, where $\check{K}(x) = K(-x)$.

Dilation and erosion with the same structuring element K can be combined to obtain the *closure* ($\bullet$) and *opening* ($\circ$) operators by:

$$\begin{aligned} I \bullet K &= (I \oplus K) \ominus K \\ I \circ K &= (I \ominus K) \oplus K \end{aligned} \tag{1}$$

Duality also holds for closure and opening. Closure and opening are mainly used to single out parts of the gridmap that do not match a given shape. For example, consider a binary gridmap with black holes of different diameters on a white surface: closing with a binary disc erases the holes smaller than the disc.

The morphological operators have been extended to fuzzy gridmaps in several ways. In this paper, we follow [1]. Let I be a fuzzy gridmap and K a fuzzy structuring element, with membership functions μ and ν, respectively. Then we let

$$\begin{aligned} I \oplus K(x) &= \sup_{y \in I} \min[\mu(y), \nu(y-x)], \\ I \ominus K(x) &= \inf_{y \in I} \max[\mu(y), 1-\nu(y-x)]. \end{aligned} \tag{2}$$

Fuzzy closure and fuzzy opening are still defined by (1) using definitions (2) for $\oplus$ and $\ominus$.

3 Topology-based maps

We now describe how we can use the above ingredients to define and extract our topology-based maps. The ability to define a consistent topology on a (fuzzy) digital grid allows us to give a topological representation

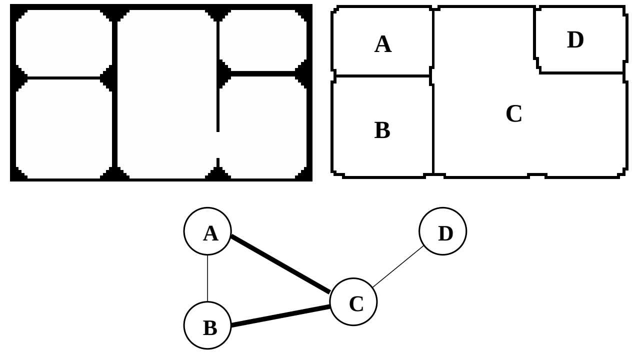

Fig. 2: Left: morphological opening. Right: connected components. Bottom: topological graph.

of any characteristic of the robot's workspace that can be represented on a gridmap. In particular, we can summarize the topological structure of this gridmap in a graph where nodes represent connected components, and arcs represent adjacency relationships between these components. Interestingly, this does not depend on the specific meaning of the information represented in the gridmap: we can extract a topological description of any useful characteristics of the environment that can be mapped on a gridmap.

For example, suppose that the robot has a map of the free space in the environment in the form of the idealized (i.e., with no uncertainty) occupancy grid shown in Fig. 1, where the white cells represent free areas. A topological description of this gridmap gives us the connectivity of the free-space. As we have noted above, this description is not very interesting (we just have two nodes) and we prefer to focus on topological descriptions induced by the *shape* of the free space, like the connectivity between large open spaces.

In order to extract the information about the "large open spaces," we apply a morphological opening to the grid using as structuring element a disc of a given diameter, say 17 cells. The choice of a disc is natural one given that we do not assume any privileged direction in the environment. The result is shown in Fig. 2 (left): the white cells in the transformed grid are those that belong to a large open space, that is, a free space in which we can fit our disc; all the "narrow passages" that cannot accommodate our disc have been closed. The connected components of the transformed grid are shown in Fig. 2 (right). The associated graph, shown in Fig. 2 (bottom), consists of nodes representing connected components, and arcs representing the adjacency relation. Since this graph must be used for navigation, it may be useful to label the arcs with additional information. For instance, we can check if an

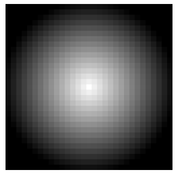

Fig. 3: A fuzzy structural element to extract “large open spaces.”

arc corresponds to a traversable passage by verifying that there is a cell which is adjacent to the components connected by this arc, and which is free (white) in the original gridmap. Traversable arcs are marked by thick lines in the picture.

The above procedure critically depends on a sharp definition of what constitutes a “large open space” in our environment, i.e., on the structuring element used in the morphological opening. In our example, the 17-disc was too narrow to close the door on the right side, hence the middle and the bottom-right rooms have been considered as one open space (node C) in the resulting topological graph. Using a larger disc would result in a graph where these two rooms are represented by distinct nodes. This behavior is clearly undesirable, since it makes the result of the procedure extremely sensitive to noise in the data. Fuzzy mathematical morphology allows us to extract shapes that are less sharply defined by using a fuzzy structural element. For example, we can perform a fuzzy morphological opening using the structural element depicted in Fig. 3, which incorporates a fuzzy notion of large. A simple way to interpret this operation is to think of this element as a discrete stacking of several discs. Each disc is used for a crisp opening of the original grid, and the results are stacked accordingly.

In the transformed grid, shown in Fig. 4 (left) the value (grey level) in each cell is proportional to the size of the largest disc that can be fit in the free space and that covers that cell. Hence, this value precisely captures the information of how much that cell belongs to a large open space.

The result of a fuzzy opening is a fuzzy gridmap, hence we need to use fuzzy digital topology in order to extract its topological structure. Fig. 4 (right) shows the connected components of this gridmap according to definition given in the previous section. Interestingly, the borderlines, and hence the components themselves, remain stable when the diameter of the base of the conic structural element is varied from 25 cells to the full size of the gridmap. The corresponding topological graph is shown in Fig. 4 (bottom).

As before, we can enrich this representation to add

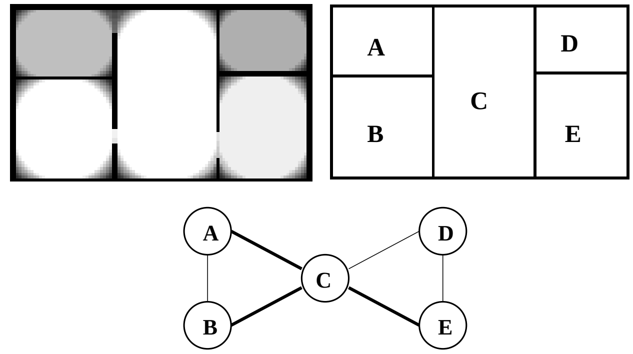

Fig. 4: Left: fuzzy morphological opening. Right: influence zones. Bottom: topological graph.

extra topological information which is useful for navigation. For example, we can label our graph using the information contained in the gridmap obtained by fuzzy erosion of the original one using our conic element. This transformed gridmap, shown in Fig. 5 (left), encodes information about the clearance at each cell [13]: the maximum value of the cells in a connected component gives the width of a space; the maximum value of the cells in a borderline gives the clearance of a passage. These values can be used to label the nodes and arcs of the topological graph (Fig. 5, right).

The simple gridmap used in this section is ideal, in that it does not contain any uncertainty about the occupancy state of each cell. In practice, gridmaps built from sensor data will inevitably be affected by uncertainty and imprecision. A nice feature of fuzzy morphological operators is that they can be applied to fuzzy gridmaps as well. The uncertainty in the initial data is propagated to the transformed gridmap, and the topology is extracted from it. This makes the resulting topology more robust than if we performed a thresholding on the initial map.

4 Algorithm description

The procedure described in the previous section is summarized by the following algorithm.

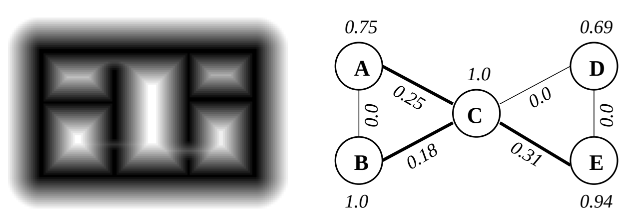

Fig. 5: Left: computing clearance by morphological erosion. Right: labelled topological graph.

```
procedure build_topology (gridmap, n, δ)
1  K ← MakeConicElement(n)
2  G1 ← FuzzyErode(gridmap, K)
3  G2 ← CFuzzyDilate(G1, K)
4  W ← Watersheds(G2)
5  CC ← FuseNodes(W, δ)
6  graph ← MakeGraph(CC)
7  return graph
end.
```

This algorithm takes as input a gridmap (either crisp or fuzzy), the size n of the conic structuring element, and a filtering parameter δ.

Steps 2 and 3 perform the stacked opening defined above. Strictly speaking, this transformation requires to perform one opening for each grey level in K, and then to stack all the results. Unfortunately, this is not equivalent in general to performing just one fuzzy opening using the entire K. For a conic K, however, we can define a specialized CFuzzyDilate operator such that the composition of FuzzyErode and CFuzzyDilate computes the desired transformation in time $O(nm)$ where m is the size of the gridmap.

Step 4 computes the connected components of $G2$ using an algorithm based on the segmentation technique proposed by [15]. The intuition is to see the gridmap as a landscape with (white) valleys and (black) peaks, and to extract the watershed that separates the valleys. This watershed partitions the gridmap $G2$ into the Z_{Π_i} components defined in Sec. 2: W is a labeling of the cells in $G2$ where all the cells in the same component are assigned the same label. Since noise in the original gridmap produces small undulations in the landscape which may result in spurious watersheds, in step 5 we fuse labels that are only separated by a valley of depth less than δ (see [9] for a similar type of filtering).

Finally, in step 6 we build the topological graph: we create a node for each label in CC; and an arc whenever a cell in the watershed is adjacent to two cells with different labels in CC. The graph can be annotated with additional information as explained above.

5 Experimental results

The proposed approach has been tested on gridmaps built from real data collected in indoor environments. The robot used is a Nomad 200: it has cylindrical shape and is equipped with a ring of 16 Polaroid ultrasonic sensors. In the experiment reported here, the robot explored an environment of 18×9 meters consisting of two parallel corridors merging into a large hall, a small area connected to the hall, and two rooms. Fig. 6 shows the fuzzy gridmap representing the free space that was built using the technique described in [10]. Each cell represents a square of side 10 cm. White cells have received evidence of being empty; darker cells have not–they are either occupied or unexplored. (A dual gridmap, not used here, represents the occupied space.) The test environment was difficult to reconstruct from sonar data, since the corridors had windowed walls and the rooms were cluttered with furniture; therefore the gridmap appears rather noisy.

Fig. 6: Fuzzy occupancy grid used in our experiment.

The original gridmap was morphologically opened with a fuzzy cone having diameter 23 cells—that is, we regard every space larger than 2.3 m as a fully open space.[1] This resulted in the map shown in Fig. 7 (top). Large open spaces are characterized by lighter gray areas, while narrow spaces are darker. Note that the nature of the information represented in the original map is completely different from the information contained in the transformed map: in the former, each cell only represents its own state; in the latter, it summarizes some characteristic of its neighborhood.

The transformed map can be seen as a landscape where tops are lighter areas, while valleys are darker. This is best seen in the contour representation shown in Fig. 7 (mid). A connected component, as defined above, is composed of the top and the surrounding regions down to the valleys. Fig. 7 (bottom) shows the connected components found by the watershed algorithm, using a filtering parameter $\delta = 50$. The free-space has been correctly partitioned into a set of connected open spaces. Note that this partition corresponds intuitively to the "natural" topology of the given environment for a human observer.

This topology is represented as an annotated graph in Fig. 8 (top). Nodes and arcs have been labeled by their clearance, obtained from the eroded map shown

[1] This parameter is chosen according to the scale of the environment, but it is not critical. In our experiment, all values between 19 and 49 result in the same topology.

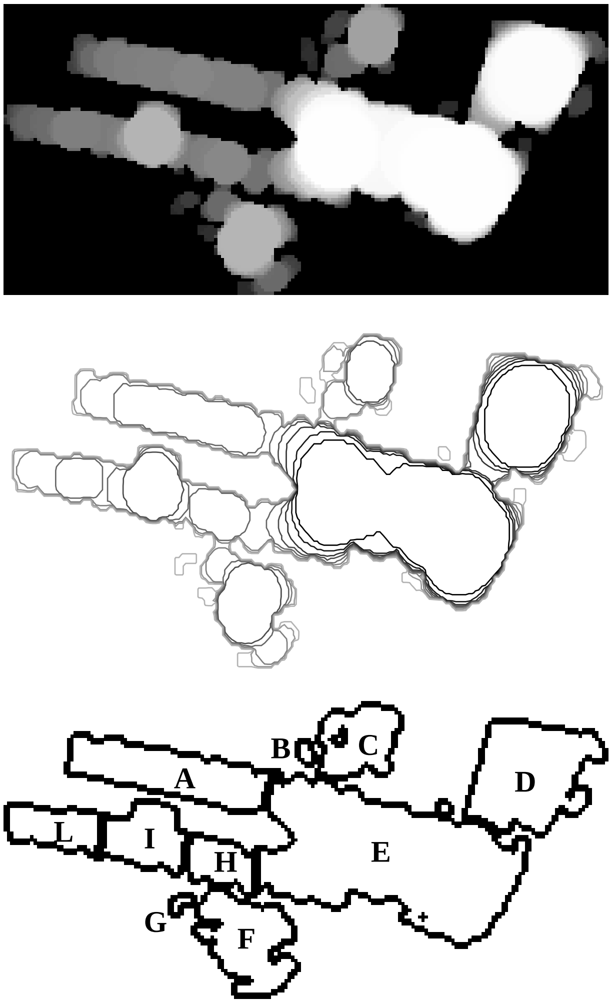

Fig. 7: Extracting a topology-based map from the grid. Top: morphological opening. Middle: contour representation. Bottom: connected components.

in Fig. 8 (bottom) as explained above. The B and G nodes are due to false reflections that created false open spaces in the original map; these spurious nodes can be easily detected because they are not connected (B) or they are too small (G).

It is interesting to note that we could extract the topology from the eroded map instead of the opened map. This would result in a graph where nodes are spaces separated by local minima of clearance, in the spirit of [14]. A comparison between Fig. 7 (mid) and Fig. 8 (bottom), however, reveals that this topology would be extremely sensitive to small irregularities in the environment and to noise in the data. For example, this topology would have two distinct nodes for E and several nodes for A, while it would not be able to separate node I from L. Different data from the same

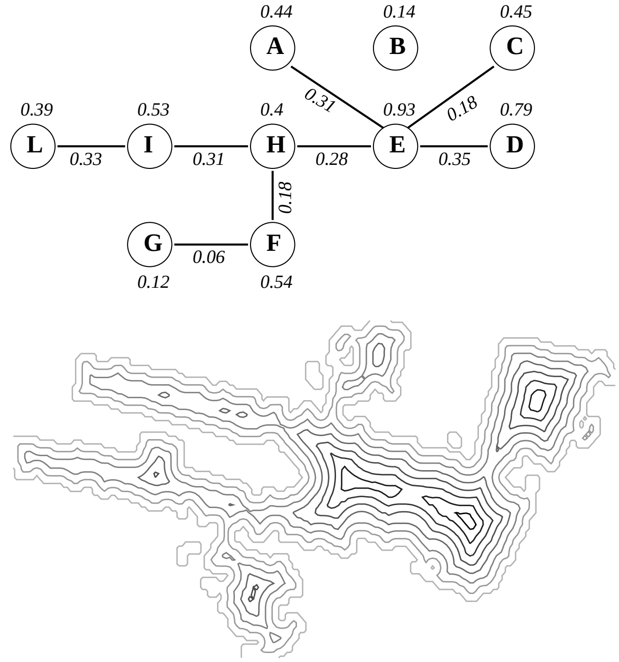

Fig. 8: Top: the annotated topology graph. Bottom: the eroded map used to compute clearance.

environment would result in a different topology.

Our algorithm took 580 msec on a Pentium-II processor at 400 MHz to process a gridmap of 128×256 cells. Extraction of topology-based maps is quite fast, and can be performed in real time during navigation.

6 Conclusions

Topology-based maps provide a new notion of topological maps. The distinctive feature of these maps is the fact that they have a well-founded semantics based on concepts of general topology. Topology-based maps also offer a number of practical advantages. First, they can be used to describe the structure of any feature that can be mapped on a grid, not just free space. Second, they accommodate the uncertainty inherent in sensor data in a natural way since the topology is directly defined on fuzzy digital grids. Third, these maps are robust with respect to sensor noise and to small environmental changes. Finally, the extraction procedure of these maps is almost parameter-free, and it can be performed in real-time.

A peculiar aspect of our approach is the use of image processing techniques to extract features from a sonar-based representation of the space. Although it was earlier suggested by Elfes [3], this idea has not been extensively exploited until now (but see [16]).

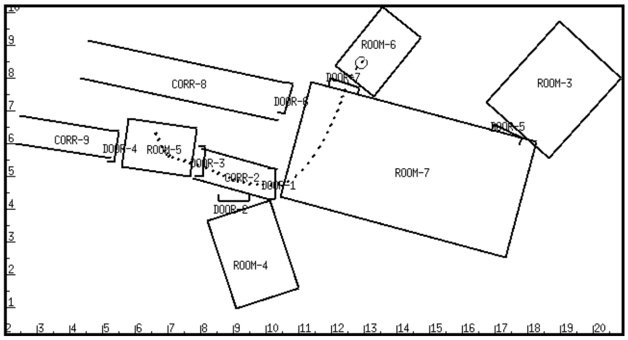

Fig. 9: Using the topology-based map for navigation.

We have used topology-based maps to plan and execute behavior-based navigation strategies on our mobile robots. Fig. 9 shows the result of inputting the map in Fig. 8 (top) into our navigation system. The map has been enriched by information about the shape and orientation of each region, again computed by using image processing techniques. Spaces have been classified into rooms and corridors according to their eccentricity. The robot uses different behaviors to traverse a room, a corridor, or a passage between regions. (See [4] for details.)

In the near future, we will perform experiments where we extract topology-based maps from features other than free space; in particular, we plan to use an artificial nose to produce a topological olfactive map of the environment. We will also explore the integration between topology-based maps and maps at other levels of abstraction; and investigate the important issue of using our maps for performing self-localization.

Acknowledgments

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