Rau, M. A., Aleven, V., & Rummel, N. (2009). Intelligent Tutoring Systems with Multiple Representations and Self-Explanation Prompts Support Learning of Fractions. In V. Dimitrova, R. Mizoguchi, & B. du Boulay (Eds.), *Proceedings of the 14th International Conference on Artificial Intelligence in Education* (pp. 441-448). Amsterdam, the Netherlands: IOS Press.

Intelligent Tutoring Systems with Multiple Representations and Self-Explanation Prompts Support Learning of Fractions

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Abstract. Although a solid understanding of fractions is foundational in mathematics, the concept of fractions remains a challenging one. Previous research suggests that multiple graphical representations (MGRs) may promote learning of fractions. Specifically, we hypothesized that providing students with MGRs of fractions, in addition to the conventional symbolic notation, leads to better learning outcomes as compared to instruction incorporating only one graphical representation. We anticipated, however, that MGRs would make the students' task more challenging, since they must link the representations and distill from them a common concept or principle. Therefore, we hypothesized further that self-explanation prompts would help students benefit from working with MGRs. To investigate these hypotheses, we conducted a classroom study in which 112 6th-grade students used intelligent tutors for fraction conversion and fraction addition. The results of the study show that students learned more with MGRs of fractions than with a single representation, but only when prompted to self-explain how the graphics relate to the symbolic fraction representations.

Keywords. Multiple representations, fractions, intelligent tutoring systems

Introduction

In the educational psychology literature, there is a substantial amount of evidence demonstrating that the use of multiple representations of learning content (MRs) can significantly enhance student learning in complex domains, compared to learning with only a single representation [1,2]. However, simply providing a learner with multiple representations (e.g., textual description plus graphic, or multiple graphical representations [MGRs]) does not necessarily result in flexible knowledge acquisition [3]. It has been argued that learners must perform a number of cognitive tasks in order to benefit from MRs [4]. In particular, learners acquire a deep understanding only if they are able to link MRs of the same concept and to coordinate between them [5-7]. If students fail to integrate the information from the different representations, their learning may be jeopardized [3]. Unfortunately, it has been demonstrated that most students do not spontaneously attempt engage in integration activities [8].

MGRs are used extensively in middle-school math curricula to help students learn about fractions. A wide variety of graphical representations of fractions are used,

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including area models (e.g., pies, rectangles), linear models (e.g., number lines, or liquid container models), or discrete models depicting sets of objects. Fractions pose a very significant challenge for students in the elementary and middle grades, and are difficult to understand even for college students and pre-service teachers [9]. Yet, fractions are an important foundation for later learning in mathematics, such as algebra [6]. There are many different conceptual interpretations of fractions (or "subconstructs"), namely, part-whole, ratio, operator, quotient, and measurement [10]. Much of the difficulty that students have when learning fractions may be due to the fact that it is hard to see connections between these subconstructs, and the way they are expressed in graphical representations.

Several observational studies demonstrate the promise of providing instruction that makes use of graphical representations of fractions. For example, Mack [11] showed that graphical representations such as pies can help students connect formal knowledge of fractions with their existing informal knowledge of sharing and dividing. A curriculum created by Moss and Case [12] emphasizes various linear representations of fractions, and was shown to be more effective than a standard curriculum for the fractions topics that were covered. Caldwell [13] found that area models of fractions promote learning for 5th- and 6th-graders. Finally, Yang and Reys [14] pointed out the importance of using graphical representations of fractions in order to help students understand rational number concepts. Although this research suggests the promise of learning with MGRs in the domain of fractions, it is mostly based on observational studies, and does not systematically compare learning with single graphical representations (SGRs) to learning with MGRs. That is, in this challenging area of mathematics learning, the advantage of MGRs over an SGR is still unproven.

There is evidence that the positive effect of learning with MRs is mediated by an increased engagement in self-explanation (SE) activities, the process of generating explanations to oneself with the goal to make sense of what one is learning [15]. Several studies have demonstrated that the quality of SEs generated by learners when studying multi-representational learning material differs between successful and less successful learners. Ainsworth and Loizou [1] conclude that learning with MRs may be beneficial because it can promote the SE effect. Berthold, Eysink, and Renkl [16] prompted students to self-explain while studying multi-representational worked examples. They found that prompts that assisted the learner in relating the different representations resulted in the best learning outcomes regarding both conceptual and procedural knowledge. The existing evidence suggests that prompting SEs could be effective in helping students' deep understanding of different graphical representations. However, the differential effects of prompting students to self-explain when learning with SGRs or with MGRs remain to be investigated systematically.

We investigated the value of MGRs for fractions learning in the context of a proven intelligent tutoring system (ITS) technology, namely, Cognitive Tutors [19-21]. Specifically, we developed a set of example-tracing tutors for fractions learning, a type of tutors that are behaviorally similar to Cognitive Tutors, but that rely on examples of correct and incorrect solution paths rather than on a cognitive model underlying student behavior. We created these tutors with the Cognitive Tutor Authoring Tools (CTAT [22]). The use of an ITS as research platform to investigate the value of MGRs is attractive for several reasons. First, Cognitive Tutors have a proven track record in improving students' mathematics achievement [19-21]. Furthermore, SE can be effectively supported by such simple means as menu-based selection [18]. Finally,

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some studies show that computer-based interactive representations promotes students' learning more than physical representations [17].

We conducted an *in vivo* experiment (i.e., a rigorously controlled experiment in a real educational setting) to investigate the two open research questions identified above. We compared learning results of students working with multiple versions of the CTAT-built fractions tutors. Students worked either with an SGR of fractions (namely, a number line), or with MGRs, which were presented consecutively. (In both conditions, students also worked with the standard symbolic notation of fractions.) This factor was crossed with a second experimental factor, namely, whether or not students were prompted to self-explain how the graphical representations correspond to the symbolic notation. In light of the literature discussed above, we predicted an advantage of learning with MGRs over learning with an SGR, but only when students are prompted to self-explain. We further expected prompting students to self-explain to promote learning. Finally, we anticipated that the effects would be apparent in both reproduction and transfer of procedural and conceptual knowledge.

1. Methods

1.1. Experimental Design and Procedure

A total of 132 6^{th}-graders in a US middle school participated in the study during regular mathematics instruction. All students worked with a set of ITS for fractions (example-tracing tutors, as mentioned) designed and created specifically for this study. Students were randomly assigned to one of four conditions: they either learned with one SGR or with MGRs, and half of the students in each of these groups were prompted to self-explain how the graphical representation(s) correspond to the symbolic representation.

Students' knowledge about fractions was assessed before, immediately after, and six days after the experimental sessions. On the first day of the study, students completed a 20-minute prior knowledge test. On the two following days, students worked individually for a total of 2.5 hours on fraction conversion and fraction addition problems with the computer-based fractions tutors. At the end of the third study day, students completed a 30-minute post-test. Six days after the immediate post-test, students were given a 30-minute delayed post-test.

1.2. Material

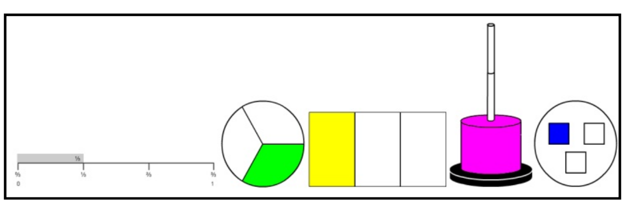

Figure 1. Number line, pie chart, rectangle, stack, set (from left to right) as used in the study.

The tutors used in the study included five different graphical representations of fractions, shown in Figure 1. Each graphical representation emphasizes certain aspects of the different interpretations of fractions [9]. Measurement representations emphasize that fractions can be compared in terms of their magnitude, and that they fall in

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between whole numbers. Part-whole representations depict fractions as parts of an area that is partitioned into equally-sized parts (pie chart, rectangle). Ratio representations present fractions in the context of discrete objects that can have several features (set).

1.3. Fractions tutors

All students worked through a set of fraction conversion and fraction addition problems, supported by the ITS. In fraction conversion problems, students had to convert given fractions to multiple denominators greater than that of the given fraction. In fraction addition problems, students had to add fractions with like and unlike denominators. In some of the problems, the sum fraction was reducible. Students solved each problem by manipulating both the symbolic representation of fractions, and a graphical representation. In the SGR condition, all problems involved an interactive number line representation (see Figure 2). In the MGR condition, on the other hand, each problem was presented five times, once for each of the five graphical representations shown in figure 1, with the interactive number line always coming first. (Each individual problem involved only a single graphical representation.) As shown in figure 2, in number line problems, the student could specify the number of divisions on the number line, and could set the length of the fraction bar to show the sum fraction. The students then performed the same steps symbolically. The four other graphical representations were not as fully interactive as the number line, although the tutor problems involving these representations *were* interactive. They gave students an opportunity to revisit a problem that they had solved with the number line from a somewhat different perspective. In order to convert a fraction using the graphics, students entered the number of divisions (e.g., the number of pieces in the pie chart) into a text field. They then were shown a graphical representation of the sum fraction, and were asked whether the new graphic showed the sum correctly.

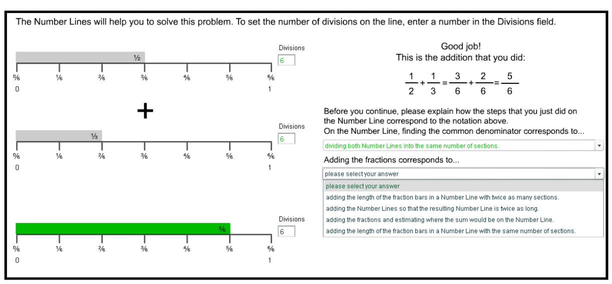

Figure 2. Fraction addition with the number line representation.

Students in the prompted SE conditions were asked to self-explain how the given graphics correspond to the numerator and the denominator of the fraction, or how the procedure they performed symbolically corresponds to the manipulation of the graphics. Students selected their answer from a drop-down menu, as shown in figure 2.

The tutors provided error feedback and hints on all problem-solving steps. Error feedback messages were designed to make the student reconsider their answer by either reminding them of a previously-introduced principle, or by providing them with an explanation of their error. Hint messages usually had three levels. First, students received a clarification of the goal. They were then given conceptually oriented help, by reminding them of a specific concept. Finally, students received the correct answer.

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1.4. Test instruments

Students' understanding of fractions was assessed with respect to reproduction and transfer of both conceptual and procedural knowledge. Test items were adapted from standardized national tests as well as from examples from the fractions literature. The prior knowledge test was a shorter version of the post-tests. Students were given different versions of the same test as immediate and delayed post-test. The theoretical structure of the knowledge types was validated by a confirmatory factor analysis.

2. Results

Data from 112 students were included in the data analysis. Students were excluded from the analysis if they had been absent during at least two of the study days ($n = 7$), if they performed more than two standard deviations worse than their classmates on both the immediate and the delayed post-test ($n = 6$), or if they had spent more than two standard deviations more or less time on the tutors than their classmates ($n = 6$). The number of excluded students did not differ between conditions, χ^2 (3, N = 21) < 1.

Students' scores on the prior knowledge test were included in the analysis as a covariate. Table 1 shows the adjusted means and standard deviations. Repeated measures ANCOVAs with immediate and delayed post-test scores as dependent variables and number of representations and SE prompts as independent variables were used for data analysis. In addition, *a priori* contrasts on the effect of number of representations within both the prompted conditions and the no-prompt conditions were computed in order to clarify the predicted interaction effect. Finally, we used post-hoc comparisons to clarify the results from the ANCOVAs.

Table 1. Estimated marginal means and standard deviations (in brackets) for all knowledge types. The maximum score was 3 for all knowledge types.

		Means (SD)			
		SGR	**SGR + SE**	**MGR**	**MGR + SE**
immediate post-test	reproduction conceptual	2.45 (0.67)	2.32 (0.96)	1.92 (1.12)	2.75 (0.64)
	reproduction procedural	2.95 (0.62)	2.79 (0.62)	2.68 (0.84)	2.95 (0.58)
	transfer conceptual	1.60 (0.64)	1.60 (0.92)	1.65 (1.01)	1.71 (1.14)
	transfer procedural	2.23 (1.26)	1.65 (1.27)	1.27 (1.21)	2.36 (0.84)
delayed post-test	reproduction conceptual	1.98 (1.10)	1.97 (1.09)	1.42 (1.03)	2.40 (0.70)
	reproduction procedural	2.55 (0.73)	2.57 (0.66)	2.31 (0.88)	2.73 (0.48)
	transfer conceptual	2.30 (0.87)	2.21 (0.99)	2.06 (1.03)	2.66 (0.58)
	transfer procedural	2.39 (0.96)	2.11 (1.02)	1.94 (1.14)	2.35 (0.77)

We found a significant main effect in favor of prompted SE with regard to reproduction of conceptual knowledge, $F(1, 108) = 9.13$, $p < .01$, $\eta^2 = .08$, but not with respect to other knowledge types. We found no main effect for the number of graphical representations for any knowledge type (*F*s < 1). As shown in Figure 3, we found significant interaction effects between the number of graphical representations and SE prompts for reproduction of conceptual knowledge, $F(1, 108) = 13.02$, $p < .01$, and transfer of procedural knowledge, $F(1, 108) = 11.35$, $p < .01$, on the immediate post-test. The *a priori* contrasts showed that students in the prompted conditions performed better when learning with MGRs, whereas students within the no-prompt conditions performed worse when provided with MGRs as compared to learning with an SGR. Within the no-prompt groups, these differences were significant only for transfer of

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procedural knowledge on the immediate post-test ($p < .05$). By contrast, in the prompted SE conditions, *a priori* contrasts showed significant advantages for learning with MGRs for transfer of procedural knowledge on the immediate post-test ($p < .05$), and marginally significant effects for reproduction of conceptual knowledge on the immediate post-test ($p < .10$), and the delayed post-test ($p < .10$), and on transfer of conceptual knowledge on the delayed post-test ($p < .10$).

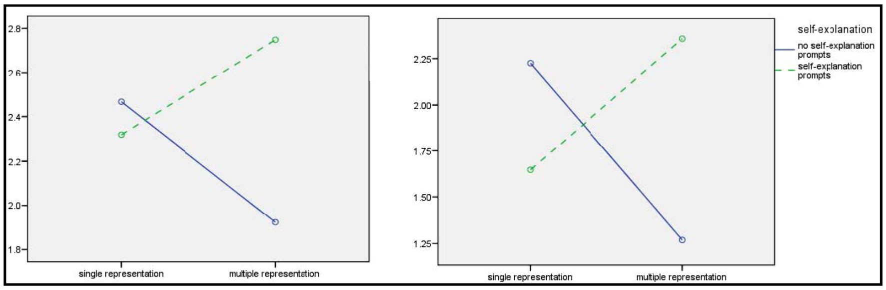

Figure 3. Interaction effects for reproduction of conceptual knowledge in the immediate post-test (left) and transfer of procedural knowledge in the immediate post-test (right).

To find out why there was no overall advantage of SE prompts for knowledge types other than reproduction of conceptual knowledge, we compared the groups within the SGR and MGR conditions post-hoc. Within the MGR conditions, significant differences supporting the SE effect were found on reproduction of conceptual knowledge for the immediate and the delayed post-test (*p*s < .01), transfer of conceptual knowledge for the delayed post-test ($p < .05$), and transfer of procedural knowledge for the immediate post-test ($p < .01$). A marginally significant effect was identified for reproduction of procedural knowledge on the delayed post-test ($p < .10$). No significant differences were found within the SGR conditions.

Finally, we compared the two most successful conditions; namely, the non-prompted SGR group, and the prompted MGR group. Marginally significant differences were found for reproduction and transfer of conceptual knowledge on the delayed post-test (*p*s < .10) in favor of the prompted MGR condition.

3. Discussion and Conclusion

The results largely support our hypothesis that students learn better with MGRs of fractions (presented consecutively) than they do with an SGR, but only when prompted to self-explain. As expected, we found an interaction effect of SE prompts and MGRs, but no main effect for MGRs. The contrast comparisons showed an advantage for learning with MGRs within the prompted groups, and the post-hoc comparisons showed an effect for SE prompts within the MGR conditions. The marginal advantage of the prompted MGR condition over the SGR group without SE prompts regarding conceptual knowledge as identified by post-hoc comparisons further underlines the benefit from learning with MGRs over learning with only an SGR. One comparison revealed a disadvantage for learning with MGRs within the no-prompt conditions, suggesting that without SE prompts, MGRs may be confusing rather than helpful.

The experiment provided partial support for the hypothesis that the beneficial effect of MGRs combined with SE prompts would be seen across all knowledge types:

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reproduction and transfer of both conceptual and procedural knowledge. We found interaction effects (showing an advantage for MGRs when combined with SE prompts) for reproduction of conceptual knowledge and transfer of procedural knowledge. The lack of an effect on transfer of conceptual knowledge may be explained by the fact that the conceptual problems in the fraction tests were especially hard. Performance was lowest on these items. With a more appropriate test for conceptual transfer, we might still find the hypothesized interaction effect. With respect to reproduction of procedural knowledge, the lack of an effect may indicate that MGRs do not help much in learning to perform a procedure per se. The benefit of MGRS may be that they help students to learn to flexibly apply a procedure to multiple situations, as assessed by test items on transfer of procedural knowledge.

The hypothesis that SE prompts improve student learning with graphical representations of fractions was largely confirmed. We found a main effect for SE prompts for reproduction of conceptual knowledge, but not for the other knowledge types. For the other knowledge types, no evidence for an SE effect within the SGR groups could be identified by post-hoc comparisons. However, differences *were* found within the MGR groups. The lack of a main effect was thus due to the lack of an effect for students in the SGR conditions. It is possible that the simple support for SE by means of menu-based selection allowed for superficial elaboration strategies, such as answering the prompts by trial and error. This "gaming" behavior [23] might have occurred more in the SGR conditions than in the MGR conditions. Also, the problems in the tutors with MGRs were more diverse, which may have positively affected students' motivation to engage in active sense-making processes, including more effective (and less superficial) use of the tutor's hints. Finally, it is possible that use of an SGR, even when combined with SE prompts, undermined students' ability to generalize their understanding of fractions to fractional subconstructs not emphasized or expressed in that particular representation. We plan to analyze the tutor log data to find out how students' learning behavior differed between conditions.

In conclusion, the current work extends the ITS and educational psychology literature on learning with multiple representations in three ways. First, to the best of our knowledge, the work presented here is the first rigorous experimental investigation of MGRs v SGR in the challenging domain of fractions learning, in a classroom or in the lab. Second, it is the first such investigation in the context of an ITS. Finally, the work reproduces, within the context of an ITS for fractions, the often-found result that MGRs are typically effective only when accompanied by some form of support that helps students make sense of the representations, and connect them to a deep conceptualization of the domain. We are not aware of any prior work that illustrates this finding in the context of an ITS for fractions learning.

Future work might address a limitation in our study, namely, that students were not asked to make direct connections between the different graphical representations of fractions. Rather, SE prompts asked students to elaborate, separately for each graphical representation, how it relates to the standard symbolic notation of fractions. The benefits from learning with MGRs may be even stronger if students are supported more directly in making connections between the different subconstructs of fractions as emphasized differentially by different MGRs. It remain an interesting open question how such cross-representational sense-making can be supported most effectively in an ITS for fractions learning, and what the effect on students' learning outcomes will be.

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