Collective Localization: A distributed Kalman filter approach to localization of groups of mobile robots*

Stergios I. Roumeliotis[1]†and George A. Bekey[1,2]

stergios|bekey@robotics.usc.edu

[1]Department of Electrical Engineering
[2]Department of Computer Science
Institute for Robotics and Intelligent Systems
University of Southern California
Los Angeles, CA 90089-0781

Abstract

This paper presents a new approach to the cooperative localization problem, namely collective localization. A group of M robots is viewed as a single system composed of robots that carry, in general, different sensors and have different positioning capabilities. A single Kalman filter is formulated to estimate the position and orientation of all the members of the group. This centralized schema is capable of fusing information provided by the sensors distributed on the individual robots while accommodating independencies and interdependencies among the collected data. In order to allow for distributed processing, the equations of the centralized Kalman filter are treated so that this filter can be decomposed in M modified Kalman filters each running on a separate robot. The collective localization algorithm is applied to a group of 3 robots and the improvement in localization accuracy is presented.

1 Introduction

In order for a mobile robot to autonomously navigate, it must be able to localize itself [7]; i.e. to know its position and orientation (pose). Localization has always been a problem for both indoor and outdoor mobile robots. Different types of sensors [8] and techniques have been employed to attack this problem (e.g. [6], [5], [17], [20]). The basic idea behind most of the current localization systems is to combine measurements from proprioceptive sensors that monitor the motion of the vehicle with information collected by exteroceptive sensors that provide a representation of the environment and its signals. The first category of sensing devices includes wheel encoders, accelerometers, gyroscopes, etc. By applying appropriate integration of the measured quantities, the displacement of a robot in the space of motion can be estimated. However, the integration of noise contaminating these signals, causes the position estimate to drift away from its real value [10], [1]. The sensors in the second category focus on extracting directly the position and/or orientation of the robot by measuring the unique characteristics of an area. For example, in an outdoor environment a compass records the heading of the robot while a GPS receiver provides the longitude and latitude coordinates of the current position. In indoor environments, sensors such as sonars, laser scanners and cameras can be used for differentiating between locations in a building [16], [14], [23]. Uncertainty is the limiting factor in this case. Areas that appear similar prohibit the exteroceptive sensing module to single out a location among a set of possible ones. By using sensors from both categories and combining both approaches in an expectation-measurement cycle [21], [22], the exteroceptive sensor uncertainty can be reduced while the proprioceptive sensor noise is filtered out.

Many robotic applications require that robots work in collaboration in order to perform a certain task [9], [15]. Most existing localization approaches refer to the case of a single robot. Even when a group of, say M, robots is considered, the group localization problem is usually resolved by independently solving M pose estimation problems. Each robot estimates its position based on its individual experience (proprioceptive and exteroceptive sensor measurements). Knowledge from the different entities of the team is not combined and each member must rely on its own resources (sensing and processing capabilities). This is a relatively simple approach since it avoids dealing with the complicated problem of fusing information from a large number of independent and interdependent sources. On the other hand, a more coordinated schema for localization has a number of advantages that can compensate for the added complexity.

First let us consider the case of a homogeneous group of robots. As we mentioned earlier, robotic sensing modalities suffer from uncertainty and noise. When a number of robots equipped with the same sensors detect a particular feature of the environment, such as a door, or measure a characteristic property of the area, such as the local vec-

*This work is partially supported by NASA-JPL (contract 959816) and DARPA (contracts F04701-97-C-0021 and DAAE07-98-C-L028).

†Contact author for correspondence.

tor of the earth's magnetic field, a number of *independent* measurements originating from the different members of the group is collected. Combining all this information will result in a single estimate of increased accuracy and reduced uncertainty. A better estimate of the position and orientation of a landmark can drastically improve the outcome of the localization process and thus this group of robots can benefit from this collaboration schema.

The advantages stemming from the exchange of information among the members of a group are more crucial in the case of heterogeneous robotic colonies. When a team of robots is composed of different platforms carrying different proprioceptive and exteroceptive sensors and thus having different capabilities for self-localization, the quality of the localization estimates will vary significantly across the individual members. For example, a robot equipped with a laser scanner and expensive INS/GPS modules will outperform another member that must rely on wheel encoders and cheap sonars for its localization needs. Communication and flow of information among the members of the group constitutes a form of sensor sharing and can improve the overall positioning accuracy. In fact, as will be evident later, if each robot could sense and communicate with its colleagues at all times then every member of the group would have less uncertainty about its position than the robot with the best localization results (if it were to localize itself without communicating with the rest of the group).

In the following section we will refer to previous approaches to the collaboration of robots in order to perform localization and we will state the main differences between these approaches and the proposed *collective localization schema*.

2 Previous Approaches

An example of a system that is designed for cooperative localization is presented in [13]. The authors acknowledge that dead-reckoning is not reliable for long traverses due to the error accumulation and introduce the concept of "portable landmarks". A group of robots is divided into two teams in order to perform cooperative positioning. At each time instant, one team is in motion while the other remains stationary and acts as a landmark. In the next phase the roles of the teams are reversed and this process continues until both teams reach the target. This method can work in unknown environments and the conducted experiments suggest accuracy of 0.4% for the position estimate and 1 degree for the orientation [12]. Improvements over this system and optimum motion strategies are discussed in [11].

A similar realization is presented in [18], [19]. The authors deal with the problem of exploration of an unknown environment using two mobile robots. In order to reduce the odometric error, one robot is equipped with a camera tracking system that allows it to determine its relative position and orientation with respect to a second robot carrying a helix target pattern and acting as a portable landmark.

Both previous approaches have the following limitations: (a) Only one robot (or team) is allowed to move at a certain time instant, and (b) The two robots (or teams) must maintain visual contact at all times. Although practices like those previously mentioned can be supported within the proposed *collective localization* framework, the key difference is that it provides a solution to the most general case where all the robots in the group can move simultaneously while visual contact is not required.

In order to treat the group localization problem, we begin from the reasonable assumptions that the robots within the group can communicate with each other (at least 1-to-1 communication) and carry two types of sensors: 1. Proprioceptive sensors that record the self motion of each robot and allow for position tracking, 2. Exteroceptive sensors that monitor the environment for (a) (static) features and identities of the surroundings of the robot to be used in the localization process, and (b) other robots (treated as dynamic features). The goal is to integrate measurements collected by different robots and achieve localization across all the robotic platforms constituting the group.

The key idea for performing *collective localization* is that the group of robots must be viewed as one entity - the "group organism" - with multiple "limbs" (the individual robots in the group) and multiple virtual "joints" visualized as connecting each robot with every other member of the team. The virtual "joints" provide 3 degrees of freedom $(\Delta x, \Delta y, \Delta\phi)$[1] and thus allow the "limbs" to move in every direction within a plane without any limitations. Considering this perspective, the "group organism" has access to a large number of sensors such as encoders, gyroscopes, cameras etc. In addition, it "spreads" itself across a large area and thus it can collect far more rich and diverse exteroceptive information. When one robot detects another member of the team and measures its relative pose, it is equivalent to the "group organism's" joints measuring the relative displacement of these two "limbs". When two robots communicate for information exchange, this can be seen as the "group organism" allowing information to travel back and forth from its "limbs". This information can be fused by a centralized processing unit and provide improved localization results for all the robots in the group. At this point it can be said that a realization of a two-member "group organism" would resemble the multiple degree of freedom robot with compliant linkage shown to improve localization implemented by J. Borenstein [2], [3], [4].

The main drawback of addressing the cooperative localization problem as an information combination problem within a single entity ("group organism") is that it requires centralized processing and communication. The solution would be to attempt to decentralize the sensor fu-

[1] 6 dof in a 3 dimensional space of motion.

sion within the group. The *collective localization* approach uses the previous analogy as its starting point and treats the processing and communication needs of the group in a distributed fashion. This is intuitively desired; since the sensing modalities of the group are distributed, so should be the processing modules.

In order to deal with the cross-correlation terms (localization interdependencies) that can alter the localization result (Section 3), the data processed during each *collective localization* session must be propagated among all the robots in the group. While this can happen instantly in groups of 2 robots, in Section 4 we will show how this problem can be treated by reformulating the *collective localization* approach so it can be applied in groups of 3 or more robots.

3 Group Localization Interdependencies

In order to examine the effect of the interdependencies in the group localization process we study the simple case of two robots constrained to move in a single dimension capable of exchanging positioning data. Every time the two robots meet they have available two independent estimates of their position. One is the estimate based on their own sensors $x_A(-)$ (or $x_B(-)$) and the other is derived from the measurement of their relative position with respect to the other robot $x_A(meas) = \Delta x_{A,B} + x_B(-)$ (or $x_B(meas) = \Delta x_{B,A} + x_A(-)$). The uncertainty related to this additional estimate is given by:

$$P_{x_A(meas)} = P_{\Delta x_{A,B}} + P_{x_B(-)} = R + P_{x_B(-)} \tag{1}$$

where $R = P_{\Delta x_{A,B}}$ is the uncertainty corresponding to the relative position measurement. The quality of the additional estimate $x_A(meas)$ depends on the quality of the relative position measurement and the position estimate of robot B. Small values of $P_{x_A(meas)}$ require precise localization of robot B and an accurate relative position measurement. The two estimates $x_A(-)$ and $x_A(meas)$, assuming that they are independent, can be combined as:

$$P^{-1}_{x_A(+)} = P^{-1}_{x_A(-)} + P^{-1}_{x_A(meas)} \tag{2}$$

$$P^{-1}_{x_A(+)}\, x_A(+) = P^{-1}_{x_A(-)}\, x_A(-) + P^{-1}_{x_A(meas)}\, x_A(meas) \tag{3}$$

The previous equation expresses that the location of robot A is the weighted average of where robot A itself believes it is and where robot B believes that robot A is.

This combination of positioning information between two robots is valid only when the robots meet for the first time. If they meet again later, the independence assumption is not valid anymore. The new propagated estimates contain information exchanged during their previous rendezvous that is now shared by both robots.[2] The effect of this is explained in the following numerical example:

Example: In this one-dimensional case, let us assume that robots A and B started from the same point and moved (probably with different speeds) along the same direction for some distance and then meet for the first time. The uncertainty for A is $P_A(k) = 4$ and for B, $P_B(k) = 4$. In order to simplify the calculations, let us also assume that the relative position measurement is almost perfect (zero uncertainty associated with it). Applying Equation (2), the result of combining these two independent sources of information would be $P_A^+(k) = P_B^+(k) = 2$.

The robots move again and we assume that their uncertainty has increased by $\Delta P_A(k, k+1) = \Delta P_B(k, k+1) = 8$. Then each of them will have total uncertainty $P_A(k+1) = P_A^+(k) + \Delta P_A(k, k+1) = 10$ and $P_B(k+1) = P_B^+(k) + \Delta P_B(k, k+1) = 10$. If the two robots meet again and we assume (falsely) that their current estimates are independent, then their updated covariance estimates would be: $P_A^+(k+1) = P_B^+(k+1) = 5$, which is over-optimistic. The reason is that the independent part of information now is the one due to the motion after the first rendezvous and the associated uncertainty is $\Delta P_A(k, k+1)$ and $\Delta P_B(k, k+1)$. Therefore, it is only legitimate to combine these last two quantities and infer that due to the exchange of information between the two robots the uncertainty associated with the last part of their motion is updated to $\Delta P_A^+(k, k+1) = \Delta P_B^+(k, k+1) = 4$. The overall position uncertainty would now be: $P_A^+(k+1) = P_A^+(k) + \Delta P_A^+(k, k+1) = P_B^+(k+1) = P_B^+(k) + \Delta P_B^+(k, k+1) = 2 + 4 = 6 > 5$.

It is obvious that if the cross-correlation terms are not considered properly in the formulation of the localization information fusion, the resulting positioning estimates will not be realistic. In the extreme case, if every time the two robots have moved infinitesimal distance they take another snapshot of each other, their position uncertainty will decrease exponentially (becoming almost half of its previous value during each update).

4 Problem Statement

We state the following assumptions:

1. A group of M independent robots move in an $N-$ *dimensional* space. The motion of each robot is described by its own linear or non-linear equations of motion,

2. Each robot carries proprioceptive and exteroceptive sensing devices in order to propagate and update its own position estimate. The measurement equations can differ from robot to robot depending on the sensors used,

3. Each robot carries exteroceptive sensors that allow it to detect and identify other robots moving in its vicinity and measure their respective displacement (relative position and orientation),

[2] Another pathological situation would be if one or both of the initially met robots A and B subsequently meet other robots C and D which have no knowledge of A and B having met in the past. If C and D meet subsequently, their position estimates are not independent and any exchange of information will produce levels of uncertainty that are over-optimistic.

4. All the robots are equipped with communication devices that allow exchange of information within the group.

The problem is to determine a principal way to exploit the information exchanged during the interactions among members of a group taking under consideration possible independencies and interdependencies. It is also within our focus to formulate the problem in such a way that it will allow for distributed processing with minimal communication requirements.

As we mentioned before, our starting point is to consider this group of robots as a single centralized system composed of each and every individual robot moving in the area and capable of sensing and communicating with the rest of the group. In this centralized approach, the motion of the group is described in an $N \times 3$-dimensional space and it can be estimated by applying Kalman filtering techniques. The goal now is to treat the Kalman filter equations of the centralized system so as to distribute the estimation process among M Kalman filters, each of them operating on a different robot.

Here we will derive the equations for a group of $M = 3$ robots. The same steps describe the derivation for larger groups.

4.1 Cross-correlation terms

At this point we show how the cross-correlation terms are introduced in the system and how their calculation can be distributed. Starting from the covariance propagation equation for a single system:

$$P(t_{k+1}^-) = \Phi(t_{k+1}, t_k) P(t_k^+) \Phi^T(t_{k+1}, t_k) \; + \; Q_d(t_{k+1}) \qquad (4)$$

in our case initially we would have:

$$P(t_k^+) = \begin{bmatrix} P_{11} & 0 & 0 \\ 0 & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix}_{(t_k^+)} \qquad (5)$$

The assumption is that at the beginning each robot knows only its own position in global coordinates and the uncertainty related to it. Since there is no *a priori* shared knowledge amongst the robots, the covariance matrix for the centralized system is diagonal and each of the diagonal elements is the covariance of each of the participating robots. Under the assumption that the motion of each robot does not affect, at least directly, the motion of the other robots, the centralized system matrix will also be diagonal:

$$\Phi(t_{k+1}, t_k) = \begin{bmatrix} \Phi_1 & 0 & 0 \\ 0 & \Phi_2 & 0 \\ 0 & 0 & \Phi_3 \end{bmatrix}_{(t_{k+1}, t_k)} \qquad (6)$$

Each of the Φ_i matrices describes the motion of robot i. Similarly, the system noise matrix Q_d for the centralized system would be:

$$Q_d(t_{k+1}) = \begin{bmatrix} Q_{d1} & 0 & 0 \\ 0 & Q_{d2} & 0 \\ 0 & 0 & Q_{d3} \end{bmatrix}_{(t_{k+1})} \qquad (7)$$

Where Q_{di} corresponds to the system noise matrix associated with robot i. While no shared dual update has occurred, i.e. no relative position information has been exchanged, Equation (4) describes the update for the centralized system position uncertainty and by substituting from Equations (5), (6), and (7) we have:

$$P(t_{k+1}^-) =$$

$$\begin{bmatrix} \Phi_1 P_{11} \Phi_1^T + Q_{d1} & 0 & 0 \\ 0 & \Phi_2 P_{22} \Phi_2^T + Q_{d2} & 0 \\ 0 & 0 & \Phi_3 P_{33} \Phi_3^T + Q_{d3} \end{bmatrix}_{(t_k^+)} \qquad (8)$$

It is obvious from Equation (8) that the propagated covariance of the centralized system is also a diagonal matrix as was the initial covariance matrix (Equation (5)). Therefore the state covariance propagation can easily be decentralized and distributed amongst the individual robots. Each robot can propagate its own part of the centralized system covariance matrix. This is the corresponding diagonal matrix element P_{ii} that describes the uncertainty associated with the position of $robot_i$:

$$P_{ii}(t_{k+1}^-) = \Phi_i \; P_{ii}(t_k^+) \; \Phi_i^T + Q_{di}(t_{k+1}), \; i = 1..3 \qquad (9)$$

If no relative position information is exchanged between any of the robots of the group then there is no global update and thus the covariance remains the same:

$$P(t_{k+1}^+) = P(t_{k+1}^-) \qquad (10)$$

Applying Equation (4) repetitively, to propagate to the next step we will again have a diagonal covariance matrix for the centralized system and its computation can be distributed amongst the 3 robots. All the quantities in Equation (9) are local to robot i and thus the centralized system covariance propagation can be distributed with all the necessary computations being local.

When for example, robot 1 meets robot 2, they use the exteroceptive sensing to measure their relative position and orientation:

$$\mathbf{z}_{k+1} = \begin{bmatrix} \mathbf{\Delta}\vec{\mathbf{x}}_{12} \end{bmatrix} = \begin{bmatrix} \Delta x_{12} \\ \Delta y_{12} \\ \Delta \phi_{12} \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \\ \phi_1 - \phi_2 \end{bmatrix} =$$

$$\begin{bmatrix} I & -I & 0 \end{bmatrix} \begin{bmatrix} \vec{\mathbf{x}}_1 \\ \vec{\mathbf{x}}_2 \\ \vec{\mathbf{x}}_3 \end{bmatrix} = H(t_{k+1}) \; \vec{\mathbf{x}}(t_{k+1}) \qquad (11)$$

Where $\vec{\mathbf{x}}_i$ is the pose estimate for robot i. This measurement is used to update the overall (centralized system) pose estimate and the covariance of this estimate.[3] This update is described by the following equations:

$$S(t_{k+1}) = H(t_{k+1}) P(t_{k+1}^-) H^T(t_{k+1}) + R_{12}(t_{k+1}) \qquad (12)$$

[3] One of the reasons for formulating the group localization problem in a centralized way is that when one robot in the group senses another robot, a relative position and orientation measurement is recorded. There is no function relation between the state of one of the two robots with the relative measurement. An observation model for this type of data would consist of a functional containing the position and orientation states of both robots. Motivated by this observation we formulate the group localization problem as a centralized one and then describe how it can be distributed among the group of the robots.

Where $R_{12}(t_{k+1})$ represents the noise associated with the relative position and orientation measurement between robots 1 and 2.

$$K(t_{k+1}) = P(t_{k+1}^-)H^T(t_{k+1})S^{-1}(t_{k+1}) \tag{13}$$

$$\widehat{\mathbf{x}}(t_{k+1}^+) = \widehat{\mathbf{x}}(t_{k+1}^-) + K(t_{k+1})\left[\mathbf{z}_{k+1} - H(t_{k+1})\widehat{\mathbf{x}}(t_{k+1}^-)\right] \tag{14}$$

$$P(t_{k+1}^+) = P(t_{k+1}^-) - P(t_{k+1}^-)H^T(t_{k+1})S^{-1}(t_{k+1})H(t_{k+1})P(t_{k+1}^-) \tag{15}$$

First we calculate the residual covariance. Substituting $H(t_{k+1})$ from Equation (11) in Equation (12) we have:

$$S(t_{k+1}) = \begin{bmatrix} I & -I & 0 \end{bmatrix} \begin{bmatrix} P_{11} & 0 & 0 \\ 0 & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix}_{(t_{k+1}^-)} \begin{bmatrix} I \\ -I \\ 0 \end{bmatrix} + R_{12}(t_{k+1})$$

$$= P_{11}(t_{k+1}^-) + P_{22}(t_{k+1}^-) + R_{12}(t_{k+1}) \tag{16}$$

Then we calculate the covariance update for the pose estimate. By applying Equation (15) we have:

$$P(t_{k+1}^+) =$$

$$\begin{bmatrix} P_{11} - P_{11}S^{-1}P_{11} & P_{22}S^{-1}P_{11} & 0 \\ P_{11}S^{-1}P_{22} & P_{22} - P_{22}S^{-1}P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix}_{(t_{k+1}^-)} \tag{17}$$

By inspection of Equation (17) we can derive the following conclusions:

1. The covariances of robots 1 and 2 are the only ones that change:

$$P_{11}(t_{k+1}^+) = P_{11}(t_{k+1}^-) - P_{11}(t_{k+1}^-)S^{-1}(t_{k+1})P_{11}(t_{k+1}^-) \tag{18}$$

$$P_{22}(t_{k+1}^+) = P_{22}(t_{k+1}^-) - P_{22}(t_{k+1}^-)S^{-1}(t_{k+1})P_{22}(t_{k+1}^-) \tag{19}$$

The matrix $S(t_{k+1})$, as calculated in Equation (16), depends only on quantities local to robots 1 and 2. Thus this update can be performed locally at robots 1 and 2 which actively participate in the update. It is not necessary for the other robots to know about this update. Therefore the communication is limited to robots 1 and 2 only. They must exchange matrices $P_{11}(t_{k+1}^-)$ and $P_{22}(t_{k+1}^-)$ in order for each of them to calculate the covariance of the residual, $S(t_{k+1})$, required for the update.

2. The covariances of the rest of the robots (in this case robot 3) remain the same:

$$P_{33}(t_{k+1}^+) = P_{33}(t_{k+1}^-) \tag{20}$$

Thus, no computations need to take place at the rest of the robots and no information from the exchange amongst robots 1 and 2 needs to be communicated to any of the rest of the group.

3. Cross coupling terms appear, changing the form of the overall (centralized system) covariance matrix. The new elements are:

$$P_{12}(t_{k+1}^+) = P_{11}(t_{k+1}^-)S^{-1}(t_{k+1})P_{22}(t_{k+1}^-) \tag{21}$$

$$P_{21}(t_{k+1}^+) = P_{22}(t_{k+1}^-)S^{-1}(t_{k+1})P_{11}(t_{k+1}^-) \tag{22}$$

$$P_{12}(t_{k+1}^+) = P_{21}^T(t_{k+1}^+) \tag{23}$$

These new elements represent the shared knowledge in the robotic colony and need to be included in the calculations during the next propagation and a later update.

4.2 Propagation

In the previous section, we derived the equations for the propagation of the initial, fully decoupled system. In this section we will examine how Equations (4) and (12) are modified in order to include the cross-correlation terms introduced after a few updates of the system. Starting again from Equation (4) we have;

$$P(t_{k+1}^-) = \Phi P(t_k^+)\Phi^T + Q_d =$$

$$\begin{bmatrix} \Phi_1 P_{11}(t_k^+)\Phi_1^T + Q_{d1} & \Phi_1 P_{12}(t_k^+)\Phi_2^T & \Phi_1 P_{13}(t_k^+)\Phi_3^T \\ \Phi_2 P_{21}(t_k^+)\Phi_1^T & \Phi_2 P_{22}(t_k^+)\Phi_2^T + Q_{d2} & \Phi_2 P_{23}(t_k^+)\Phi_3^T \\ \Phi_3 P_{31}(t_k^+)\Phi_1^T & \Phi_3 P_{32}(t_k^+)\Phi_2^T & \Phi_3 P_{33}(t_k^+)\Phi_3^T + Q_{d3} \end{bmatrix} \tag{24}$$

where $P(t_{k+1}^-)$ now contains cross-correlation terms

$$P(t_{k+1}^-) = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}_{(t_{k+1}^-)} \tag{25}$$

Equation (24) is repeated at each step of the propagation and it can be distributed among the robots after appropriately splitting the cross-correlation terms:

Robot 1:

$$P_{11}(t_{k+1}^-) = \Phi_1 P_{11}(t_k^+)\Phi_1^T + Q_{d1} \tag{26}$$

$$\sqrt{P_{12}(t_{k+1}^-)} = \Phi_1\sqrt{P_{12}(t_k^+)} \tag{27}$$

$$\sqrt{P_{13}(t_{k+1}^-)} = \Phi_1\sqrt{P_{13}(t_k^+)} \tag{28}$$

Robot 2

$$\sqrt{P_{21}(t_{k+1}^-)} = \Phi_2\sqrt{P_{21}(t_k^+)} \tag{29}$$

$$P_{22}(t_{k+1}^-) = \Phi_2 P_{22}(t_k^+)\Phi_2^T + Q_{d2} \tag{30}$$

$$\sqrt{P_{23}(t_{k+1}^-)} = \Phi_2\sqrt{P_{23}(t_k^+)} \tag{31}$$

Robot 3

$$\sqrt{P_{31}(t_{k+1}^-)} = \Phi_3\sqrt{P_{31}(t_k^+)} \tag{32}$$

$$\sqrt{P_{32}(t_{k+1}^-)} = \Phi_3\sqrt{P_{32}(t_k^+)} \tag{33}$$

$$P_{33}(t_{k+1}^-) = \Phi_3 P_{33}(t_k^+)\Phi_3^T + Q_{d3} \tag{34}$$

After a few steps, if we want to calculate the (full) cross-correlation terms of the centralized system, we will have to multiply their respective components. For example:

$$P_{32}(t_{k+1}^-) = \sqrt{P_{32}(t_{k+1}^-)}\,\sqrt{P_{23}(t_{k+1}^-)}^T \tag{35}$$

4.3 Update

If now we assume that robots 2 and 3 are exchanging relative position and orientation information, the residual covariance matrix is updated as:

$$S(t_{k+1}) = \begin{bmatrix} 0 & I & -I \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}_{(t_{k+1}^-)} \begin{bmatrix} 0 \\ I \\ -I \end{bmatrix} + R_{23}(t_{k+1})$$

$$= P_{22}(t_{k+1}^-) - P_{32}(t_{k+1}^-) - P_{23}(t_{k+1}^-) + P_{33}(t_{k+1}^-) + R_{23}(t_{k+1}) \quad (36)$$

In order to calculate matrix S only the covariances of the two meeting robots are needed along with their cross-correlation terms. All these terms can be exchanged when the two robots detect each other, and then used to calculate the S matrix ($N \times N$ instead of $3N \times 3N$). For the covariance update we apply Equation (15) again, yielding the final formula:

$$P(t_k^+) = \left[\begin{matrix} P_{11} - (P_{12} - P_{13})S^{-1}(P_{21} - P_{31}) \\ P_{21} - (P_{22} - P_{23})S^{-1}(P_{21} - P_{31}) \\ P_{31} - (P_{32} - P_{33})S^{-1}(P_{21} - P_{31}) \end{matrix} \right|$$

$$\left| \begin{matrix} P_{12} - (P_{12} - P_{13})S^{-1}(P_{22} - P_{32}) & P_{13} - (P_{12} - P_{13})S^{-1}(P_{23} - P_{33}) \\ P_{22} - (P_{22} - P_{23})S^{-1}(P_{22} - P_{32}) & P_{23} - (P_{22} - P_{23})S^{-1}(P_{23} - P_{33}) \\ P_{32} - (P_{32} - P_{33})S^{-1}(P_{22} - P_{32}) & P_{33} - (P_{32} - P_{33})S^{-1}(P_{23} - P_{33}) \end{matrix} \right]_{(t_k^-)} \quad (37)$$

This centralized system covariance matrix calculation can be divided into $3(3+1)/2 = 6$, $N \times N$ matrix calculations and distributed among the robots of the group.[4] Finally, the calculation of the Kalman gain (Equation 13) can be split to the following 3 equations:

$$K_1 = (P_{12} - P_{13})S^{-1} \quad (38)$$

$$K_2 = (P_{22} - P_{23})S^{-1} \quad (39)$$

$$K_3 = (P_{32} - P_{33})S^{-1} \quad (40)$$

5 Experimental Results

The proposed *collective localization* method was implemented and tested in simulation for the case of 3 mobile robots. The most significant result is the reduction of the uncertainty regarding the position and orientation estimates of each individual member of the group.

The 3 robots start from 3 different locations and they move within the same area. Every time a meeting occurs, the two robots involved measure their relative position and orientation. Information about the cross-correlation terms is exchanged among the members of the group and the distributed modified Kalman filters update the pose estimates for each of the robots. In order to focus on the effect of the *collective localization* algorithm, no absolute localization information was available to any of the robots. Therefore the covariance of the position estimate for each of them is bound to increase while the position estimates will drift away from their real values.

[4]In general $M(M+1)/2$ matrix equations distributed among M robots, thus $(M+1)/2$ matrix calculations per robot.

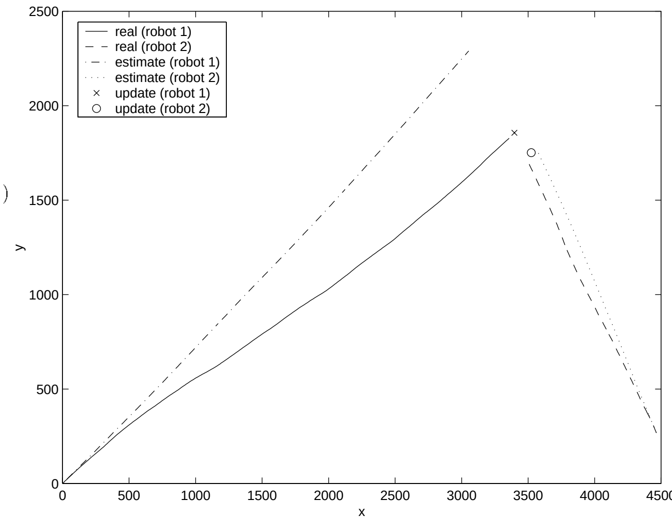

Figure 1: Collective localization position updates for robots 1 ($\times$) and 2 (o).

At time t=320 robot 1 meets robot 2 and they exchange relative localization information. At time t=720 robot 1 meets robot 3 and they also perform *collective localization*. As it can be seen in Figure 2, after each exchange of information, the covariance, representing the uncertainty of the position and orientation estimates, of robots 1 and 2 (t=320) and 1 and 3 (t=720) is significantly reduced. Robot 1 that met with other robots of the group twice, has significantly lower covariance values at the end of the test (t=1000).

Finally in Figure 1 parts of the real and estimated trajectories of robots 1 and 2 before and after they meet are shown. When they detect each other, the localization estimates are updated and the difference between the next position estimate and the real trajectory is significantly reduced.

6 Conclusions

In this paper, a new method for cooperative mobile robot localization was presented. In order to improve the overall localization accuracy, a team of mobile robots was initially treated as a centralized system. A single Kalman filter was used to optimally combine the information gathered by all the sensors distributed among the robots of this group. The transition to a fully distributed system of M modified Kalman filters (one for each robot) was then described.

The application of the *collective localization* approach results in groups homogeneous as far as the type and amount of knowledge shared by its members while there is no homogeneity requirement for the members themselves. Within the proposed framework, each robot propagates the uncertainty related to its motion independent of the other robots. The individual uncertainty propagation is decentralized and depends on quantities local to each robot. This way the overall system has increased flexibility and can support collective localization of homogeneous as well as heterogeneous groups of robots. The motion of different robots can be described by different models depending on their particular capabilities, mission, local area morphology, etc. Similarly, each robot can carry different sets of sensors described by different models. The individual, local updates are also carried out without any communication with other members of the group.

Exteroceptive sensing allows for relative positioning and this introduces localization coupling within the group. When two robots sense each other, we have a shared, dual, coupled update that has global characteristics compared to a local update that takes place when one robot detects a landmark, for example. Every time two robots meet they exchange information not only for themselves but also for the rest of the group (cross-correlation terms). These sessions cause diffusion of the overall certainty amongst all the robots and push toward a more homogeneous (in terms of positional knowledge) robotic colony.

References

[1] B. Barshan and H. F. Durrant-Whyte. Inertial navigation systems for mobile robots. *IEEE Transactions on Robotics and Automation*, 11(3):328–342, June 1995.

[2] J. Borenstein. Control and kinematic design of multi-degree-of freedom mobile robots with compliant linkage. *IEEE Transactions on Robotics and Automation*, 11(1):21–35, Feb. 1995.

[3] J. Borenstein. Internal correction of dead-reckoning errors with a dual-drive compliant linkage mobile robot. *Journal of Robotic Systems*, 12(4):257–273, April 1995.

[4] J. Borenstein. Experimental results from internal odometry error correction with the omnimate mobile robot. *IEEE Transactions on Robotics and Automation*, 14(6):963–969, Dec. 1998.

[5] J. Borenstein and L. Feng. Gyrodometry: A new method for combining data from gyros and odometry in mobile robots. In *Proceedings of the 1996 IEEE International Conference on Robotics and Automation*, pages 423–428, 1996.

[6] J. Borenstein and L. Feng. Measurement and correction of systematic odometry errors in mobile robots. *IEEE Transactions on Robotics and Automation*, 12(6):869–880, Dec. 1996.

[7] I. J. Cox. Blanche-an experiment in guidance and navigation of an autonomous robot vehicle. *IEEE Transactions on Robotics and Automation*, 7(2):193–204, April 1991.

[8] H. R. Everett. *Sensors for Mobile Robots*. AK Peters, 1995.

[9] M.S. Fontan and M.J. Mataric. Territorial multi-robot task division. *IEEE Transactions on Robotics and Automation*, 14(5):815–822, Oct. 1998.

[10] Y. Fuke and E. Krotkov. Dead reckoning for a lunar rover on uneven terrain. In *Proceedings of the 1996 IEEE International Conference on Robotics and Automation*, pages 411–416, 1996.

[11] R. Kurazume and S. Hirose. Study on cooperative positioning system: optimum moving strategies for cps-iii. In *Proceedings of the 1998 IEEE International Conference in Robotics and Automation*, volume 4, pages 2896–2903, Leuven, Belgium, 16-20 May 1998.

[12] R. Kurazume, S. Hirose, S. Nagata, and N. Sashida. Study on cooperative positioning system (basic principle and measurement experiment). In *Proceedings of the 1996 IEEE International Conference in Robotics and Automation*, volume 2, pages 1421–1426, Minneapolis, MN, April 22-28 1996.

[13] R. Kurazume, S. Nagata, and S. Hirose. Cooperative positioning with multiple robots. In *Proceedings of the 1994 IEEE International Conference in Robotics and Automation*, volume 2, pages 1250–1257, Los Alamitos, CA, 8-13 May 1994.

[14] M. J. Mataric. Integration of representation into goal-driven behavior-based robots. *IEEE Transactions on Robotics and Automation*, 8(3):304–312, 1992.

[15] M.J. Mataric. Coordination and learning in multi-robot systems. *IEEE Intelligent Systems*, 13(2):6–8, March-April 1998.

[16] H.P. Moravec and A. Elfes. High resolution maps from wide angle sonar. In *Proceedings of the 1985 IEEE International Conference on Robotics and Automation*, pages 116–121, St. Louis, MO, 1985.

[17] C.F. Olson and L.H. Matthies. Maximum likelihood rover localization by matching range maps. In *Proceedings of the 1998 IEEE International Conference on Robotics and Automation*, pages 272–277, Leuven, Belgium, 16-20 May 1998.

[18] I.M. Rekleitis, G. Dudek, and E.E. Milios. Multi-robot exploration of an unknown environment, efficiently reducing the odometry error. In M.E. Pollack, editor, *Proceedings of the Fifteenth International Joint Conference on Artificial Intelligence (IJCAI-97)*, volume 2, pages 1340–1345, Nagoya, Japan, 23-29 Aug. 1997.

[19] I.M. Rekleitis, G. Dudek, and E.E. Milios. On multiagent exploration. In *Visual Interface*, pages 455–461, Vancouver, Canada, June 1998.

[20] S. I. Roumeliotis, G. S. Sukhatme, and G. A. Bekey. Smoother-based 3-d attitude estimation for mobile robot localization. In *Proceedings of the 1999 IEEE International Conference on Robotics and Automation*, Detroit, MI, May 10-15 1999.

[21] S.I. Roumeliotis. Reliable mobile robot localization. Technical Report IRIS-99-374, University of Southern California, April 1999. (http://iris.usc.edu/ irislib/).

[22] S.I. Roumeliotis and G.A. Bekey. Bayesian estimation and kalman filtering: A unified framework for mobile robot localization. In *Proceedings of the 2000 IEEE International Conference on Robotics and Automation*, San Fransisco, CA, April 24-28 2000.

[23] S. Thrun. Bayesian landmark learning for mobile robot localization. *Machine Learning*, 33(1):41–76, Oct. 1998.

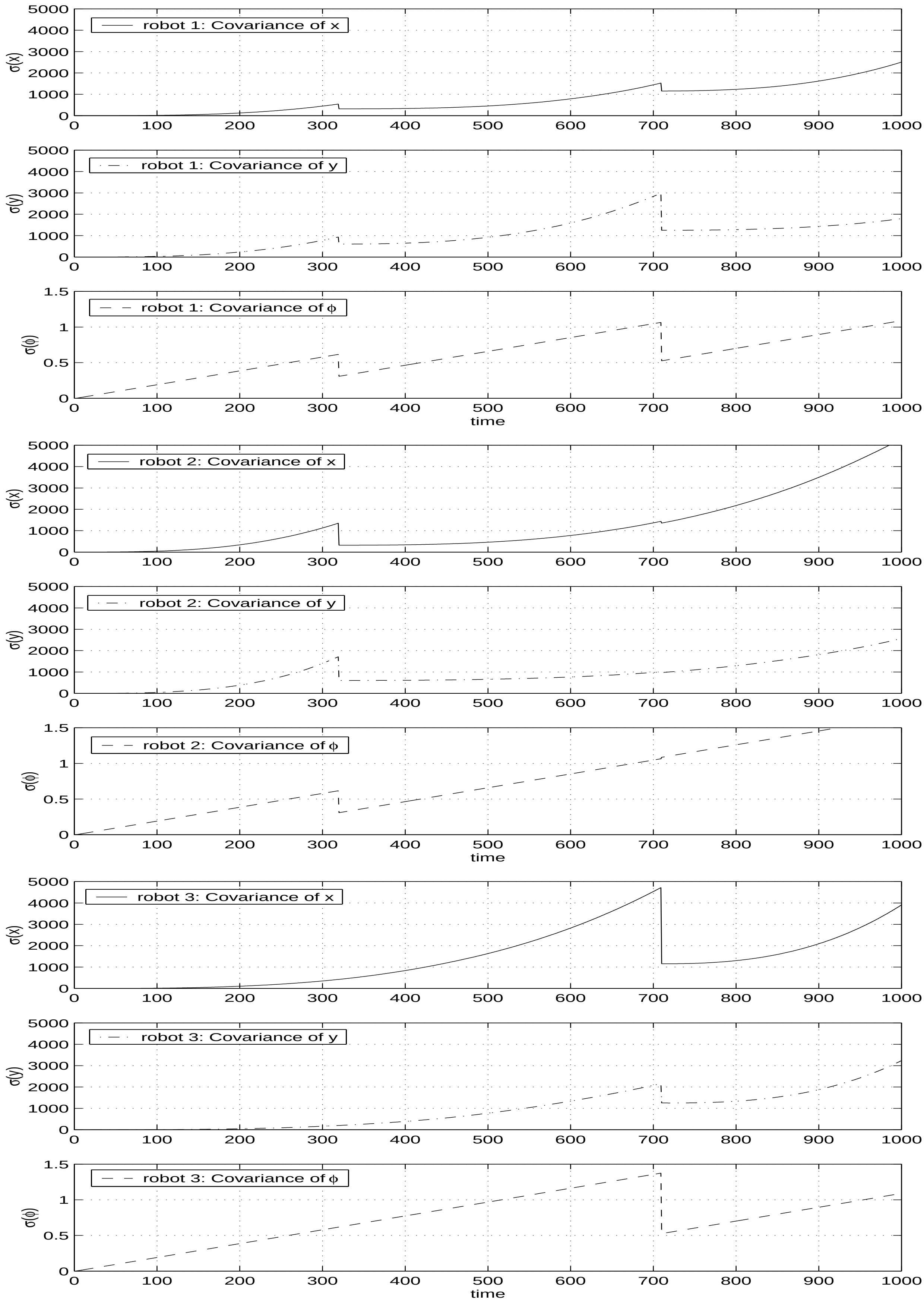

Figure 2: Collective localization results: The covariance of the x (plots 1, 4, 7), y (plots 2, 5, 8), and ϕ (plots 3, 6, 9) estimates for each of the three robots in the group.