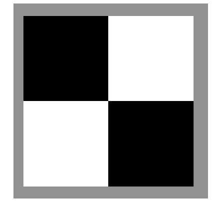
Aliasing

OUTLINE:

- What is aliasing?
- What causes it?
  - frequency domain explanation using Fourier Transforms

What is Aliasing?



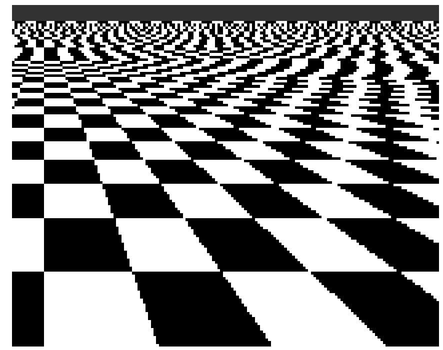
checkerboard

Aliasing comes in several forms:

SPATIAL ALIASING, IN PICTURES

moire patterns arise in
image warping & texture mapping

jaggies arise in rendering



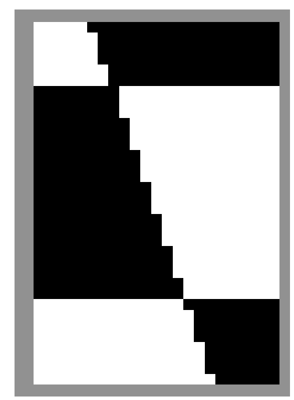
Warped checkerboard. Note moire near horizon, jaggies in foreground.

TEMPORAL ALIASING, IN AUDIO

when resampling an audio signal at a lower sampling frequency, e.g. 50KHz (50,000 samples per second) to 10KHz

TEMPORAL ALIASING, IN FILM/VIDEO

strobing and the “wagon wheel effect”



jaggies

When Does Spatial Aliasing Occur?

During image synthesis:

when sampling a continuous (geometric) model to create a raster image, e.g. scan converting a line or polygon.

Sampling: converting a continuous signal to a discrete signal.

During image processing and image synthesis:

when resampling a picture, as in image warping or texture mapping.

Resampling: sampling a discrete signal at a different sampling rate.

Example: “zooming” a picture from n_x by n_y pixels to sn_x by sn_y pixels

$s>1$: called **upsampling** or **interpolation**

can lead to blocky appearance if point sampling is used

$s<1$: called **downsampling** or **decimation**

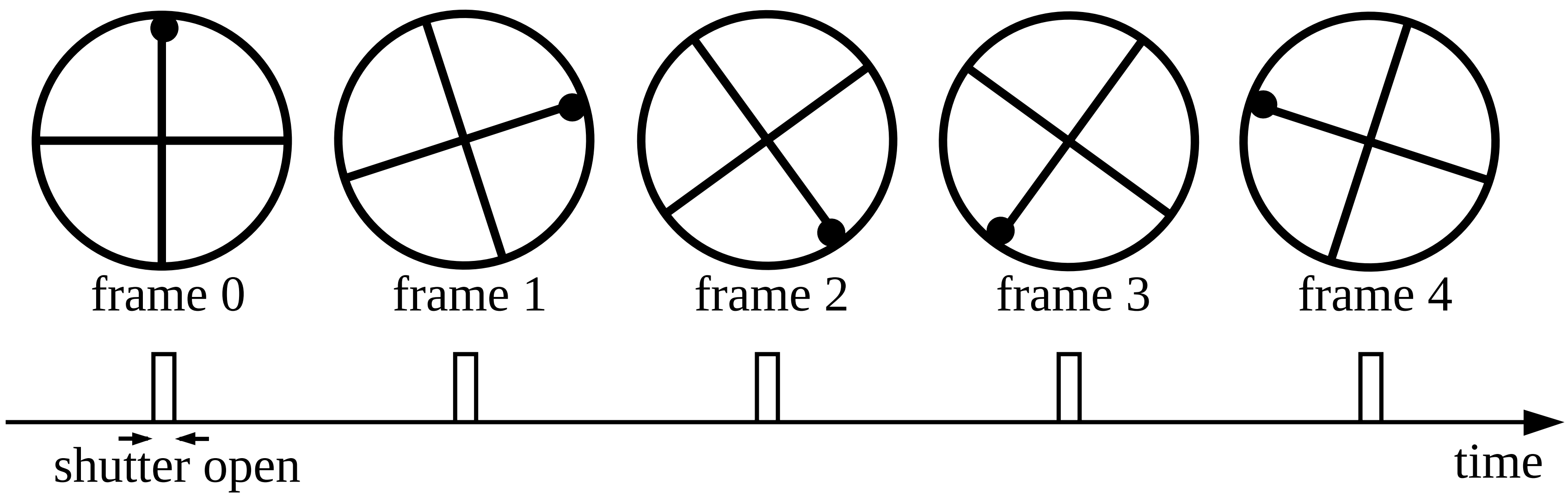
can lead to moire patterns and jaggies

Wagon Wheel Effect

an example of temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Aliasing is Bad!

Jaggies, moire patterns, temporal aliasing, and other symptoms of aliasing are **undesirable artifacts**.

- In a still picture, these artifacts look poor, unrealistic.
- In audio, they sound bizarre.
- In animation, they are very distracting, particularly in training simulations, such as flight simulators.

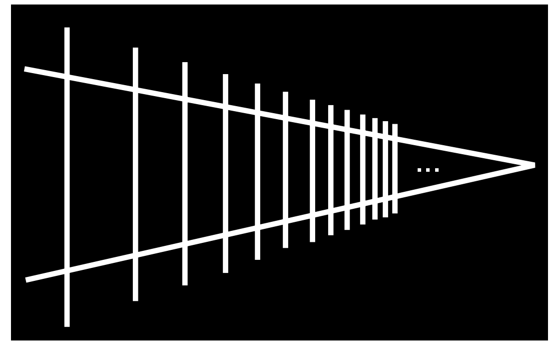
So **we want to eliminate aliasing. But how?**

First, let's figure out what causes aliasing...

Aliasing – *Related to High Frequencies?*

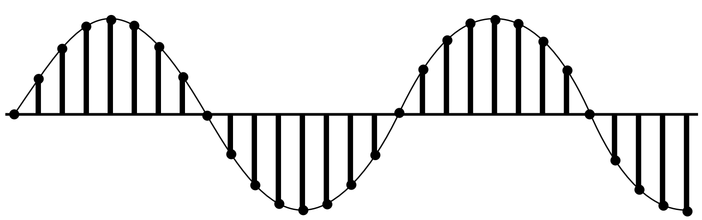
Suppose we wanted to make a picture of a long, white picket fence against a dark background, receding into the distance.

It will alias in the distance.

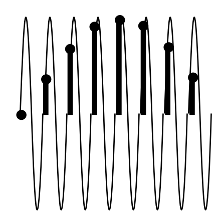


Along a horizontal scanline of this picture, the intensity rises and falls.

We can approximate the rising & falling with a sinusoid:



low frequency sinusoid:
no aliasing



high frequency sinusoid:
aliasing occurs
(high freq. looks like low freq.)

Frequency Domain

We can visualize & analyze a signal or a filter in either the spatial domain or the frequency domain.

Spatial domain: x, distance (usually in pixels).

Frequency domain: can be measured with either:

ω, **angular frequency** in radians per unit distance, or

f, **rotational frequency** in cycles per unit distance. $\omega=2\pi f$.

We'll use ω mostly.

The **period** of a signal, $T = 1/f = 2\pi/\omega$.

Examples:

The signal [0 1 0 1 0 1 ...] has frequency $f=.5$ (.5 cycles per sample).

The signal [0 0 1 1 0 0 1 1 ...] has frequency $f=.25$.

Fourier Transform

The **Fourier transform** is used to transform between the spatial domain and the frequency domain. A **transform pair** is symbolized with "$\leftrightarrow$", e.g. $f \leftrightarrow F$.

SPATIAL DOMAIN $\leftrightarrow$ FREQUENCY DOMAIN

signal $f(x)$ $\leftrightarrow$ **spectrum** $F(\omega)$

$$\text{Fourier Transform}: \quad F(\omega) = \int_{-\infty}^{+\infty} f(x) e^{-i\omega x} dx$$

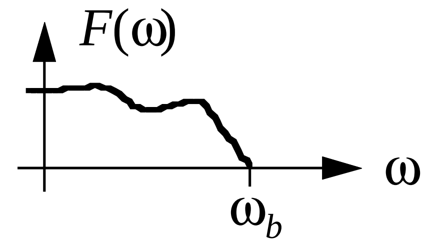
$$\text{Inverse Fourier Transform}: \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega x} d\omega$$

where $i = \sqrt{-1}$. Note that F will be complex, in general.

Some Fourier Transform Pairs

SPATIAL DOMAIN		FREQUENCY DOMAIN
impulse train, period T	$\leftrightarrow$	impulse train, period $2\pi/T$
discrete with sample spacing T	$\leftrightarrow$	periodic with period $2\pi/T$
convolution of signals: $f(x)\otimes g(x)$	$\leftrightarrow$	multiplication of spectra: $F(\omega)G(\omega)$
multiplication of signals: $f(x)g(x)$	$\leftrightarrow$	convolution of spectra: $F(\omega)\otimes G(\omega)/2\pi$
$(\omega_c/\pi)\,\mathrm{sinc}\big((\omega_c/\pi)x\big)$ where $\mathrm{sinc}(x)=\dfrac{\sin\pi x}{\pi x}$	$\leftrightarrow$	$\mathrm{box}(\omega/2\omega_c)$ where $\mathrm{box}(x) = \{1 \text{ if } \lvert x\rvert<1/2,\ 0 \text{ otherwise}$ and ω_c is cutoff frequency

Aliasing is Caused by Poor Sampling



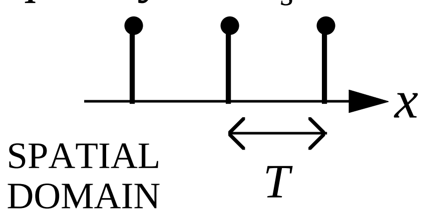
A **bandlimited** signal is one with a highest frequency.

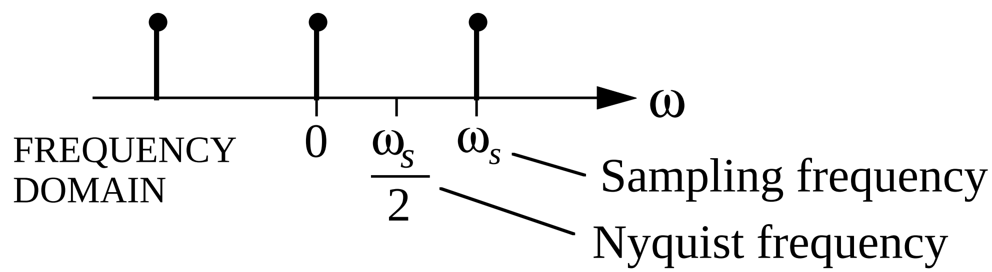
The highest frequency is called the **bandwidth** ω_b.

If sample spacing is T, then sampling frequency is $\omega_s = 2\pi/T$.

(If samples are one pixel apart, then T=1).

The highest frequency that can be represented by a discrete signal with this sampling frequency is the **Nyquist frequency**, which is half the sampling frequency: $\omega_s/2 = \pi/T$.





Sampling Theorem: **A bandlimited signal can be reconstructed exactly from its samples if the bandwidth is less than Nyquist frequency**: $\omega_b < \omega_s/2$.

Otherwise, aliasing occurs: high frequencies **alias**, appearing to be a lower frequency. *(Q: what frequency does frequency ω appear to be?)*

Further Reading

see figures 14.27 & 14.28 in

Foley-van Dam-Feiner-Hughes,

Computer Graphics - Principles & Practice, 2nd ed.

for illustrations of aliasing in the frequency domain

see also

Blinn, "Return of the Jaggy"

IEEE Computer Graphics & Applications, March 1989

for a nice explanation of aliasing & antialiasing