Kinematics

15-494 Cognitive Robotics
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Outline

Kinematics is the study of how things move.

- Homogeneous coordinates
- Kinematic chains
  - Robots are described as collections of kinematic chains
- Reference frames
- Kinematics and PostureEngine classes
- Forward kinematics: calculating limb positions from joint angles. (Straightforward matrix multiply.)
- Inverse kinematics: calculating joint angles to achieve desired limb positions. (Hard.)

Homogeneous Coordinates

- Represent a point in N-space by an (N+1)-dimensional vector. Extra component is an inverse scale factor.
  - In “normal” form, last component is 1.

$$\vec{v} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  - Points at infinite distance: last component is 0.
- Allows us to perform a variety of transformations using matrix multiplication:

  Rotation, Translation, Scaling

- Tekkotsu uses 3D coordinates (so 4-dimensional vectors) for everything.

Transformation Matrices

- Let θ be rotation angle in the x-y plane.
  Let dx, dy, dz be translation amounts.
  Let 1/s be a scale factor.

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & dx \\ -\sin\theta & \cos\theta & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & s \end{bmatrix}$$

$$T\ \vec{\mathrm{v}} = \begin{bmatrix} x\cos\theta + y\sin\theta + dx \\ -x\sin\theta + y\cos\theta + dy \\ z + dz \\ s \end{bmatrix} = \begin{bmatrix} (x\cos\theta + y\sin\theta + dx)/s \\ (-x\sin\theta + y\cos\theta + dy)/s \\ (z + dz)/s \\ 1 \end{bmatrix}$$

Transformations Are Composable

- To rotate about point p, translate p to the origin, rotate, then translate back.

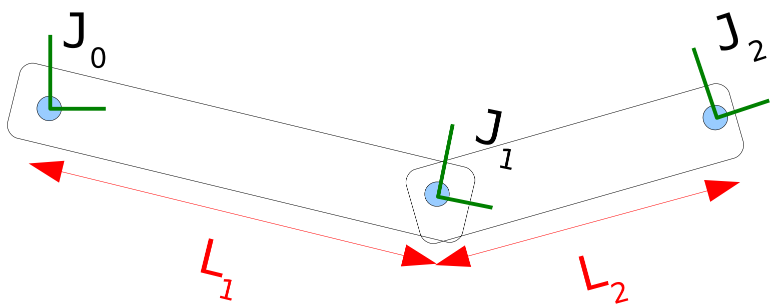
$$Translate(p) = \begin{bmatrix} 1 & 0 & 0 & p.x \\ 0 & 1 & 0 & p.y \\ 0 & 0 & 1 & p.z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rotate(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RotateAbout(p,\theta) = Translate(p) \cdot Rotate(\theta) \cdot Translate(-p)$$

Kinematic Chains

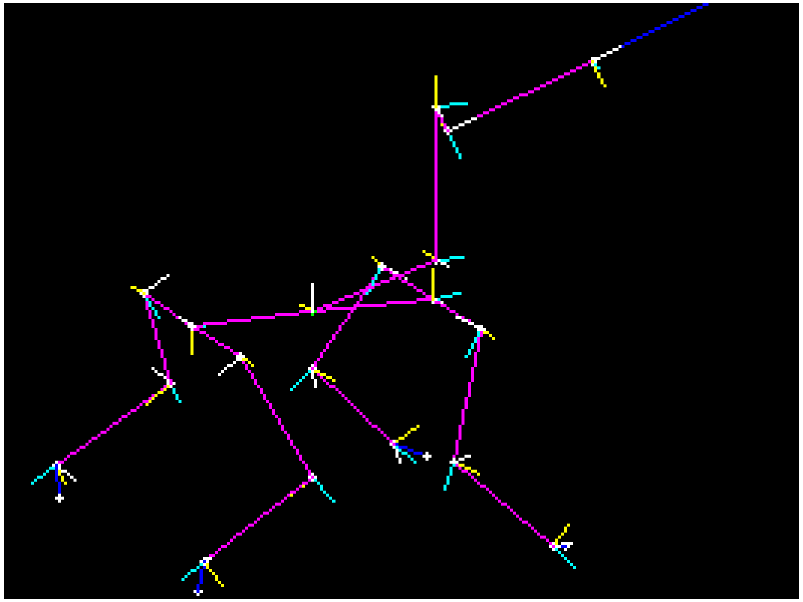
- Sequence of joints separated by links.

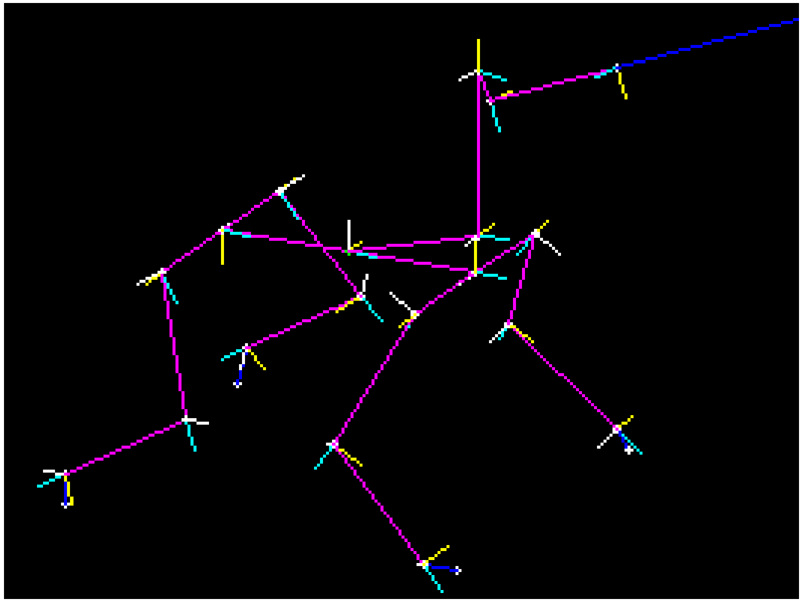


- We can use transformation matrices to calculate the position of the tip of the chain (joint J_2) from the joint angles θ_0, θ_1 and the link lengths L_1, L_2.
- Each joint has a rotation transform; each link has a translation transform.

AIBO Kinematic Chains

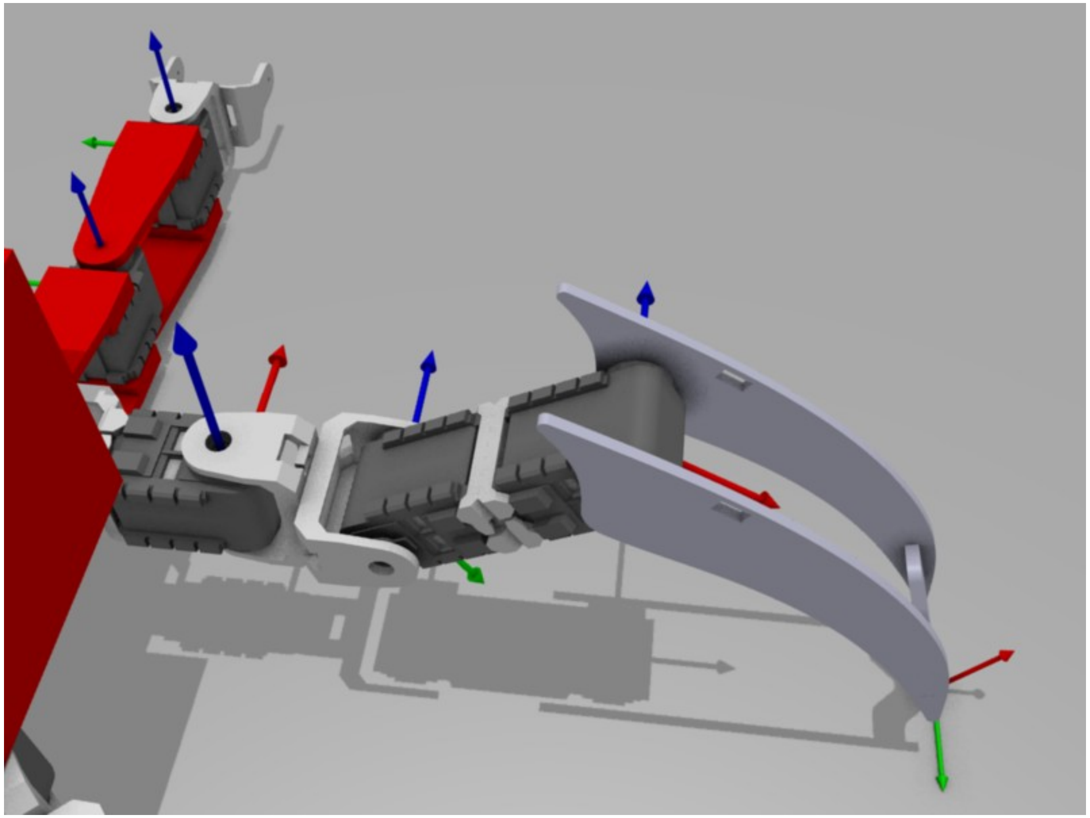
- The AIBO has 9 kinematic chains instead of 6 because branched chains were formerly not supported:
  - 4 for the legs
  - 1 for the head (ending in the camera), 1 for the mouth
  - 3 for the IR range sensors
- All chains begin at the center of the body (base frame).





Chiara Kinematic Chains

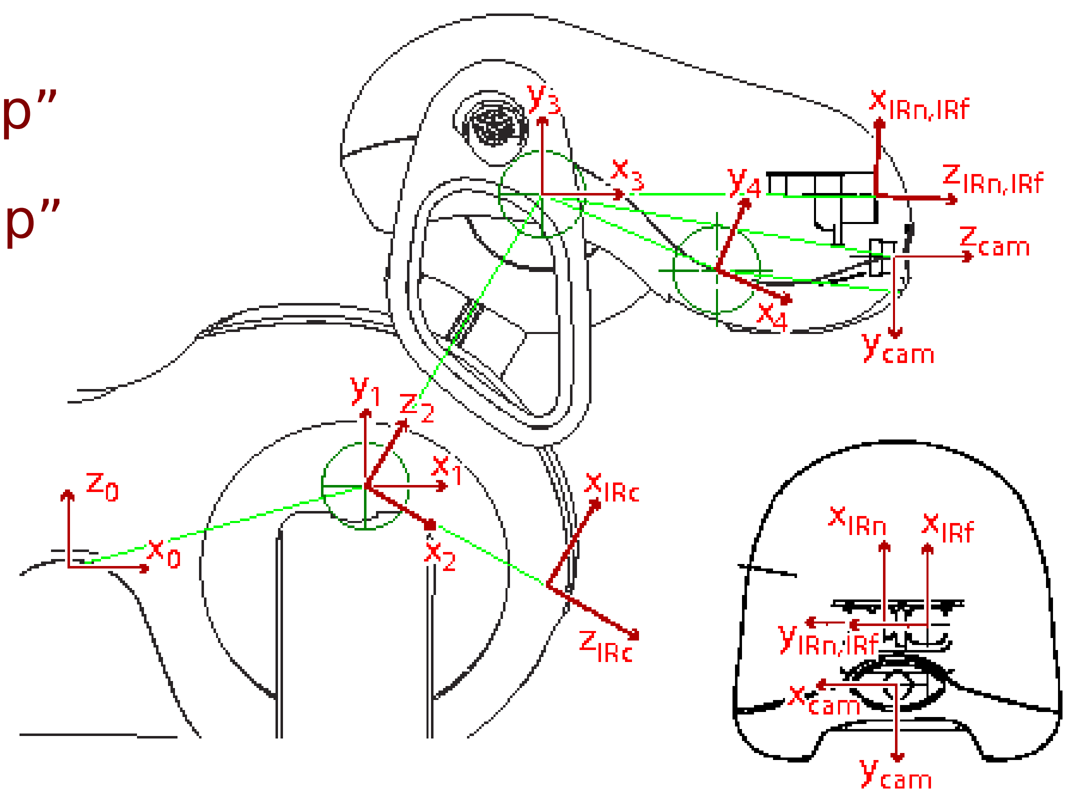
- The Chiara has 8 major kinematic chains:
  - Head / camera / IR
  - Arm
  - Left front leg
  - Right front leg (4-dof)
  - Left middle leg
  - Right middle leg
  - Left back leg
  - Right back leg



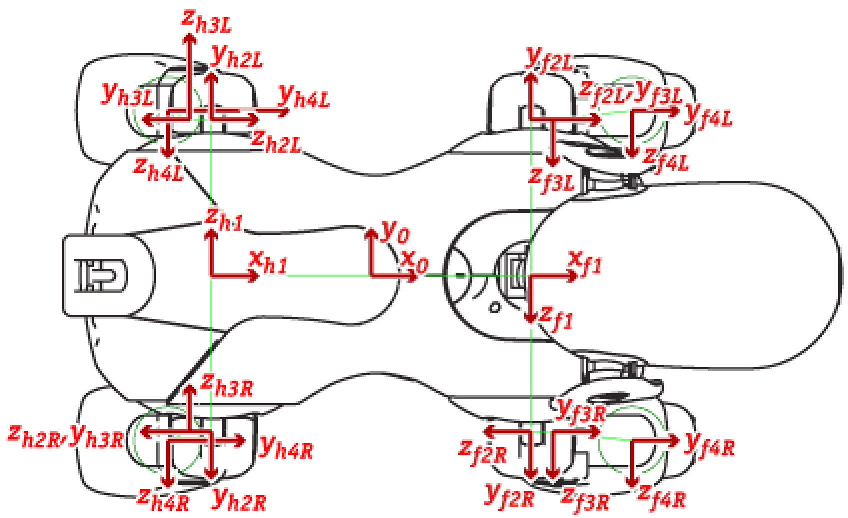
- Chains are defined in project/ms/config/chiara.kin

Reference Frames

- Every link has an associated reference frame.
- Denavit-Hartenberg conventions: all links move about their reference frame's z-axis.
- The head chain:
  - Base frame 0 z_0 = “up”
  - Tilt joint 1 y_1 = “up”
  - Pan joint 2
  - Nod joint 3
  - Camera 4



Leg Reference Frames

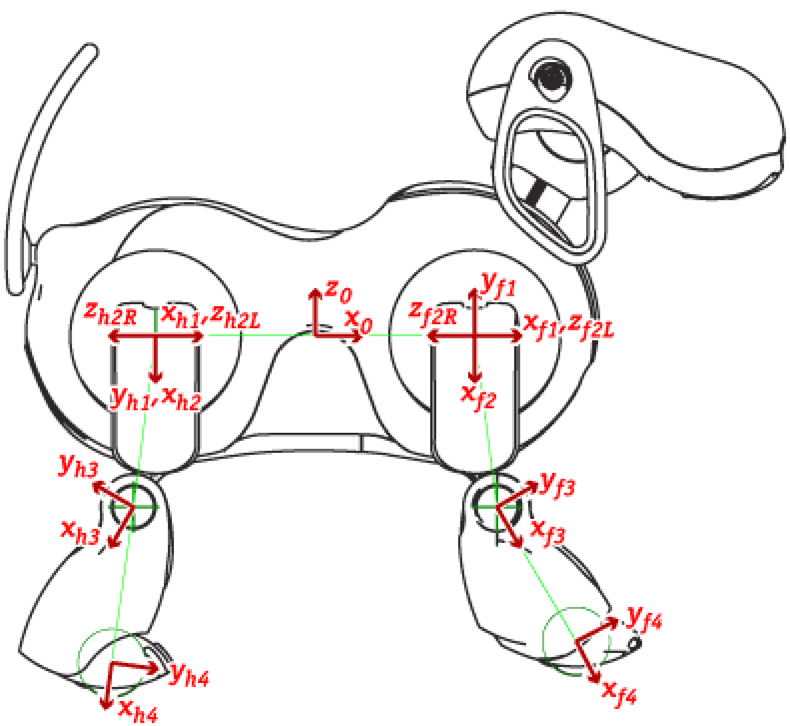


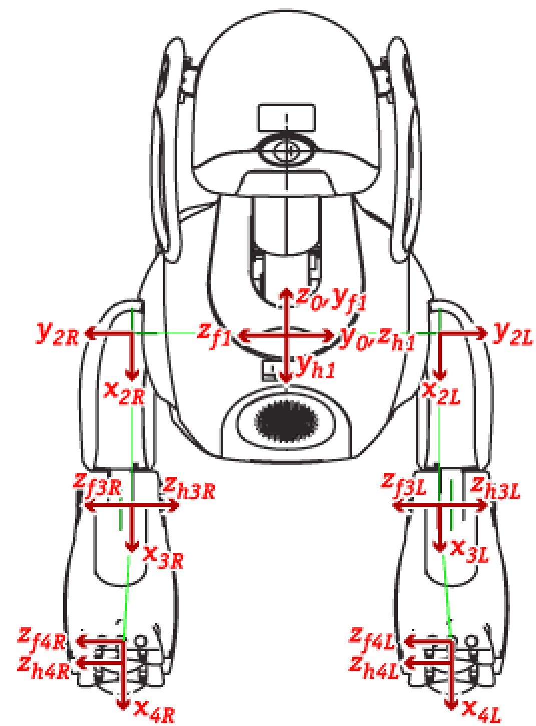
ERS-7 Legs

	Δx	Δy	Δz
1. - shoulder	65	0	0
2. - elevator	0	0	62.5
3. - knee	69.5	0	9
f4. - ball	*69.987*	*-4.993*	4.7
h4. - ball	*67.681*	*-18.503*	4.7

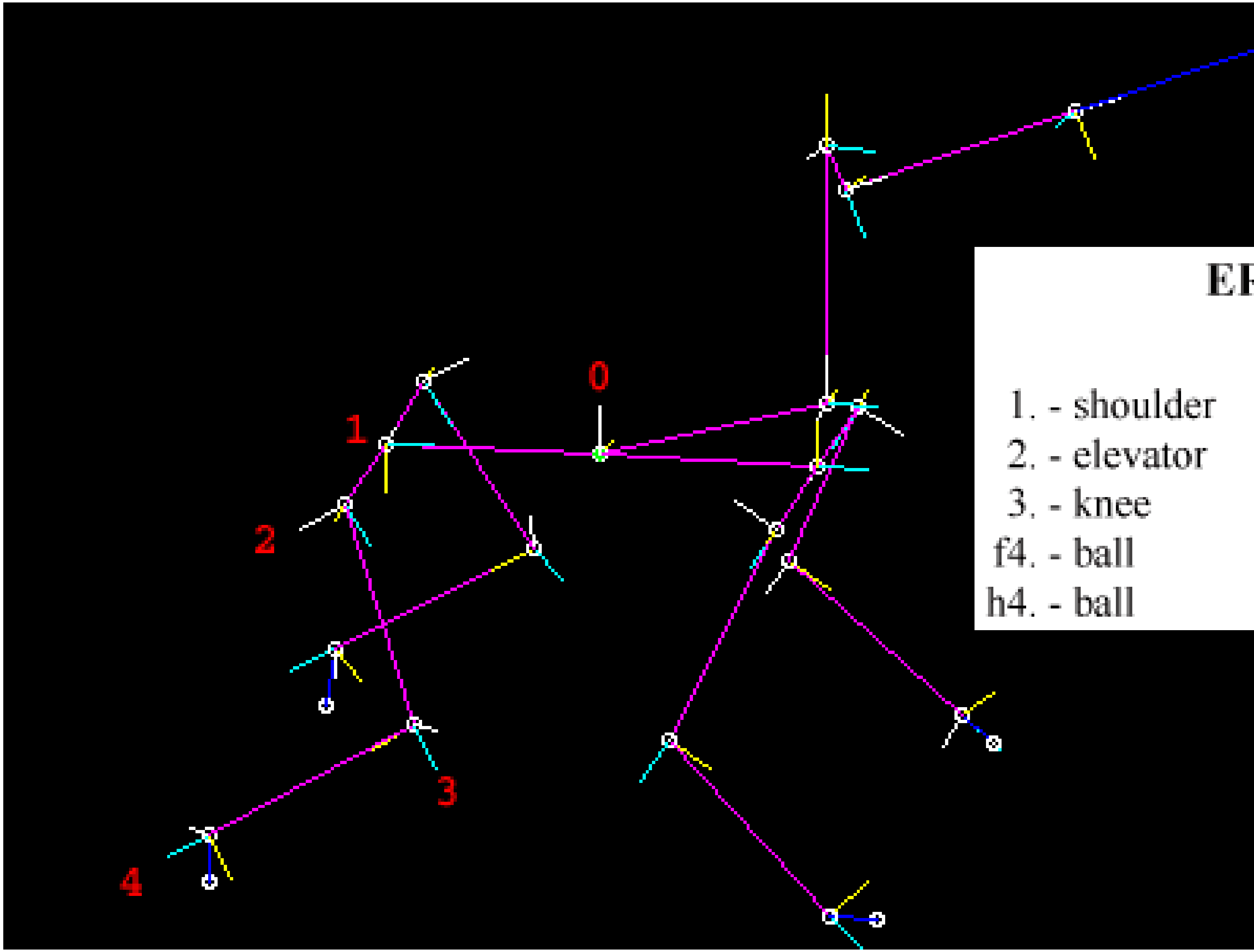
Diameter of ball of foot is *23.433*mm
Each link offset is relative to previous link

The shins shown in this diagram appear to be slightly distorted compared to a real robot. Corresponding measurements have been taken from actual models.





Leg Reference Frames



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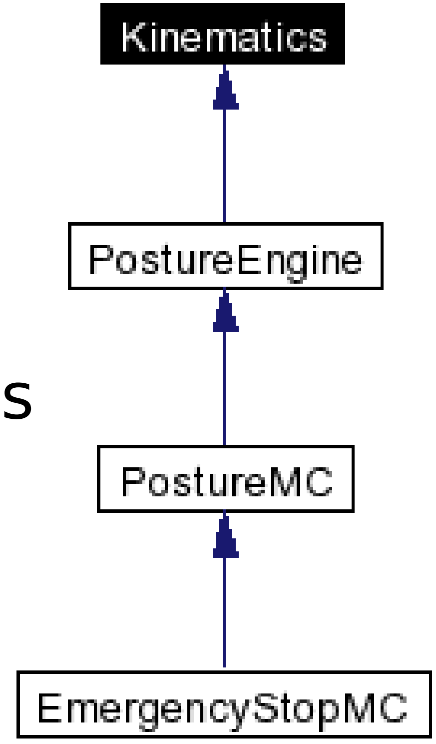
Reference Frame Naming Conventions

- Use a similar offset-based indexing scheme as for joint names in motion commands and world state vectors:
  - BaseFrameOffset
  - HeadOffset + TiltOffset
  - CameraFrameOffset
  - LFrLegOffset + ElevatorOffset

- Denavit-Hartenberg conventions specify how to express the relationship between one reference frame and the next: d, θ, r, α.
  - See DH video.

Kinematics Class

- Tekkotsu contains its own kinematics engine for kinematics calculations, modeled after ROBOOP.
- The Kinematics class provides access to basic functionality for forward kinematics.
- Global variable **kine** holds a special Kinematics instance:
  - Joint values reference WorldState.
- PostureEngine is a child of Kinematics so it can do kinematics calculations too. It adds inverse kinematics.
  - Joint angle results are stored in the PostureEngine instance.



fmat

- Tekkotsu uses the fmat package to represent coordinates and transformation matrices.
- fmat is optimized for efficient representation of small, fixed-size matrices and vectors.

```
fmat::Column<4> v, w;
v = fmat::pack(5.75, 30.0, 115, 1);
w = fmat::pack(17, -4.2f, 100, 1);

fmat::Matrix<4,4> T;
T = v * w.transpose();
```

fmat::Transform

- Transformation matrices using homogenous coordinates are 4×4.
- But the last row is always [0 0 0 1].
- So fmat eliminates the last row and overloads the arithmetic operators to make the math work correctly.
- fmat::Transform is really a Matrix<3,4>

Converting Between Reference Frames

- Most common conversion is between the base frame (body coordinates) and a limb frame, or vice versa.

- Conversion requires computing a transformation matrix:

```
fmat::Transform linkToBase(unsigned int link) {...}

fmat::Transform baseToLink(unsigned int link) {...}

fmat::Transform linkTolink(unsigned int ilink,
                           unsigned int olink) {}
```

Reference Frame Conversion 1

- Transform Base to Base:

```
fmat::Transform T = kine->linkToBase(BaseFrameOffset);
cout << T.fmt("%8.3f") << endl;
```

- Result:

1.000	0.000	0.000	0.000
0.000	1.000	0.000	0.000
0.000	0.000	1.000	0.000
0.000	0.000	0.000	1.000

Reference Frame Conversion 2

Translate AIBO head tilt frame to base frame:

```
const float headtilt = state->outputs[HeadOffset+TiltOffset];
cout << "Head tilt is " << headtilt * 180/M_PI
     << " degrees." << endl;

fmat::Transform TtiltL(kine->linkToBase (HeadOffset+TiltOffset));

cout << "tilt linkToBase=\n" << TtiltL.fmt(“%8.3g”) << endl;
```

At ~Zero Degree Tilt Angle

```
Head tilt is 1.25 degrees.

tilt linkToBase=
  1.000  -0.022   0.000  67.500
  0.000   0.000  -1.000   0.000
  0.022   1.000   0.000  19.500
```

ERS-7 Head

	Δx	Δy	Δz
1. - tilt_0	67.5	0	19.5
2. - pan_1	0	0	0
3. - nod_2	0	0	80
4. - jaw_3	40	-17.5	0
cam. - camera_3	81.06	-14.6	0
IRn. - NearIR_3	76.9	*1.917*	*2.795*
IRf. - FarIR_3	76.9	*1.052*	*-8.047*
IRc. - $\mathrm{ChestIR}_0$	109.136	-3.384	0

$x_3 \angle x_4 = -23.6294°$

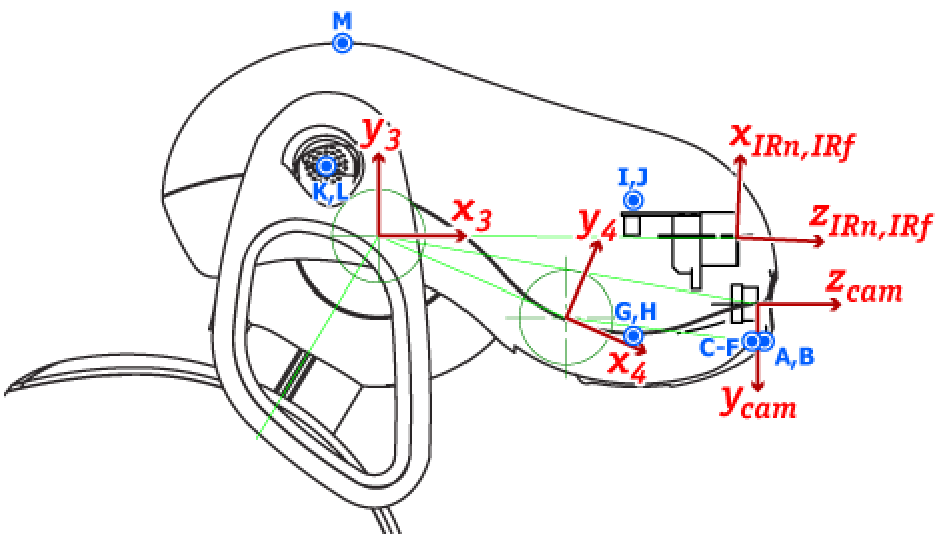
At ~ -30 Degree Tilt Angle

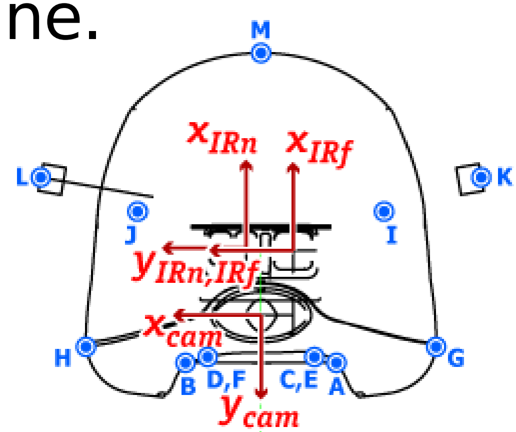
```
Head tilt is -29.5 degrees.


tilt linkToBase=
   0.871   0.492   0.000  67.500
   0.000   0.000  -1.000   0.000
  -0.492   0.871   0.000  19.500
```

$\cos(-30^{o}) = 0.866$
$\sin(-30^{o}) = 0.500$

Interest Points

- Interest points on the head, legs, and body can be predefined for use in kinematics calculations.
- Not yet supported in new kinematics engine.





Interest Points:

A - LowerLeftLowerLip$_4$
B - LowerRightLowerLip$_4$
C - UpperLeftLowerLip$_4$
D - UpperRightLowerLip$_4$
E - LowerLeftUpperLip$_3$
F - LowerRightUpperLip$_3$
G - LowerLeftSnout$_3$
H - LowerRightSnout$_3$
I - UpperLeftSnout$_3$
J - UpperRightSnout$_3$
K - LeftMicrophone$_3$
L - RightMicrophone$_3$
M - HeadButton$_3$

Leg Interest Points

Interest Points:

A - Toe{L,R}{Fr,Bk}Paw_4
B - Lower{Inner,Outer}Front{L,R}{Fr,Bk}$Shin_3$
C - Lower{Inner,Outer}Middle{L,R}{Fr,Bk}$Shin_3$
D - Lower{Inner,Outer}Back{L,R}{Fr,Bk}$Shin_3$
E - Middle{Inner,Outer}Middle{L,R}{Fr,Bk}$Shin_3$
F - Upper{Inner,Outer}Front{L,R}{Fr,Bk}$Shin_3$
G - Upper{Inner,Outer}Back{L,R}{Fr,Bk}$Shin_3$
H - Lower{Inner,Outer}Front{L,R}{Fr,Bk}$Thigh_2$
I - Lower{Inner,Outer}Back{L,R}{Fr,Bk}$Thigh_2$
J - Upper{Inner,Outer}Front{L,R}{Fr,Bk}$Thigh_2$
K - Upper{Inner,Outer}Back{L,R}{Fr,Bk}$Thigh_2$
L - Upper{L,R}$Chest_0$
M - Lower{L,R}$Chest_0$
N - {L,R}{Fr,Bk}$Belly_0$
O - Lower{L,R}$Rump_0$
P - Upper{L,R}$Rump_0$

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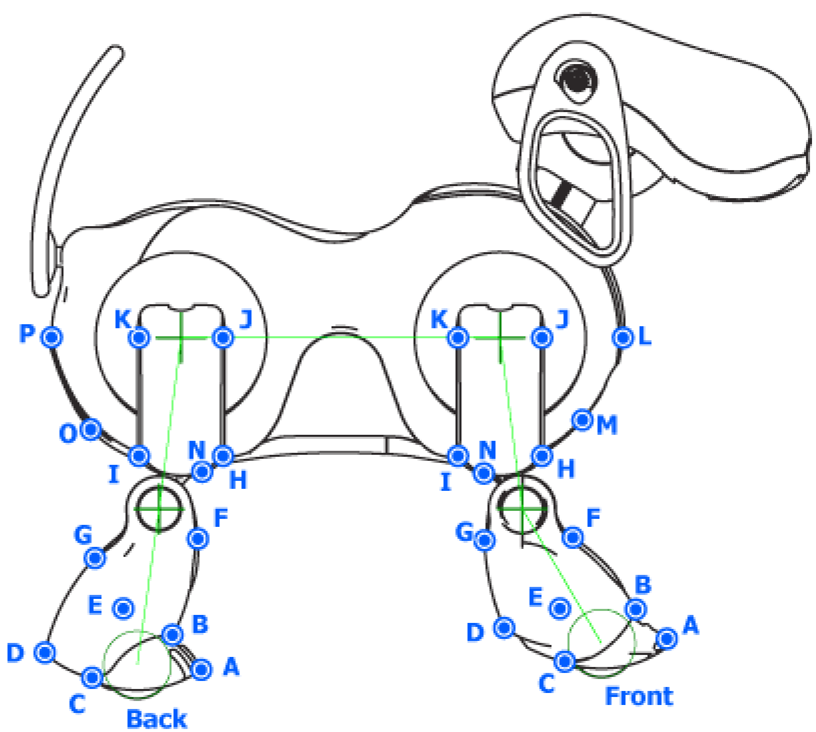
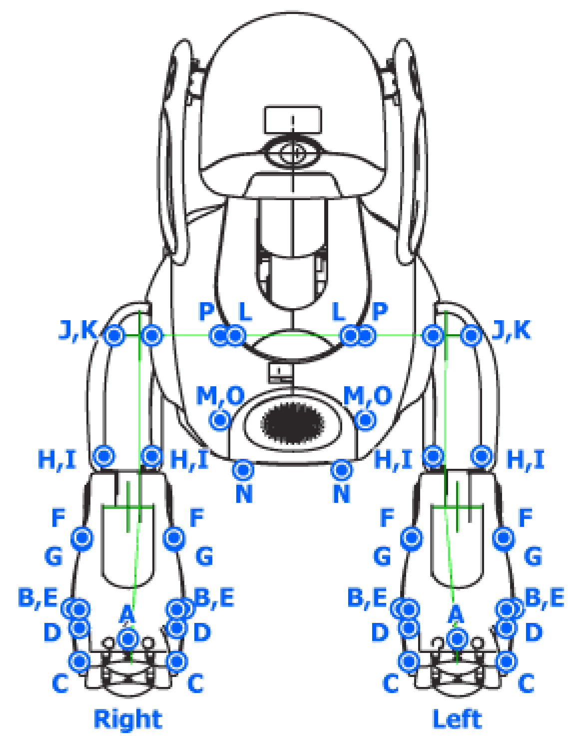




Retrieving Interest Points

- Each interest point is attached to a link:

```
void getInterestPoint(const std::string &name,
                      unsigned int &link,
                      fmat::Column<4> &coords)
```

  – Returns the link associated with the named interest point, and its coordinates in the link's reference frame.

- Interest points can be expressed in any reference frame:

```
fmat::Column<4>
    getInterestPoint(unsigned int link,
                     const std::string &name)
```

Forward Kinematics: Measure Distance From RFr Leg to Gripper

```
#nodemethod processEvent

  fmat::Transform rfrFoot =
    kine->linkToBase(FootFrameOffset+RFrLegOrder);
  fmat::Column<3> rfrFootPos = rfroot.translation();

  fmat::Transform gripper =
    kine->linkToBase(GripperFrameOffset);
  fmat::Column<3> gripperPos = gripper.translation();

  float dist = (rfrFootPos-gripperPos).norm();

  cout << “Distance is “ << setw(5) < dist << “ mm.” << endl;
```

Inverse Kinematics: lookAtPoint

- Inverse kinematics finds the joint angles to put an effector at a particular point in space.
- Hard problem:
  - solution space can be discontinuous
  - can be highly nonlinear
  - multiple solutions may be possible
  - maybe no solution (so find closest approximation)
- Example: lookAtPoint(x,y,z)
  - point described in base frame coordinates
  - calculates head joint angles

CameraTrackGripper Demo

Root Control > Framework Demos > Kinematics Demos > CameraTrackGripper

```
#nodeclass CameraTrackGripper : StateNode : armRelaxer(), headMover()

  MotionPtr<PIDMC> armRelaxer;
  MotionPtr<HeadPointerMC> headMover;

  #nodemethod DoStart
    addMotion(armRelaxer);
    addMotion(headMover);
    erouter->addListener(this,EventBase::sensorEGID);
```

TrackGripper Behavior 2

```
#nodemethod processEvent

  fmat::Column<3> Pgripper =
     kine->linkToBase(GripperFrameOffset).translation();

  cout << “Transform:“ << Tgripper.fmt(“%8.3f”) << endl;

  headMover->lookAtPoint(Pgripper[0],
                         Pgripper[1],
                         Pgripper[2]);
```

General Inverse Kinematics

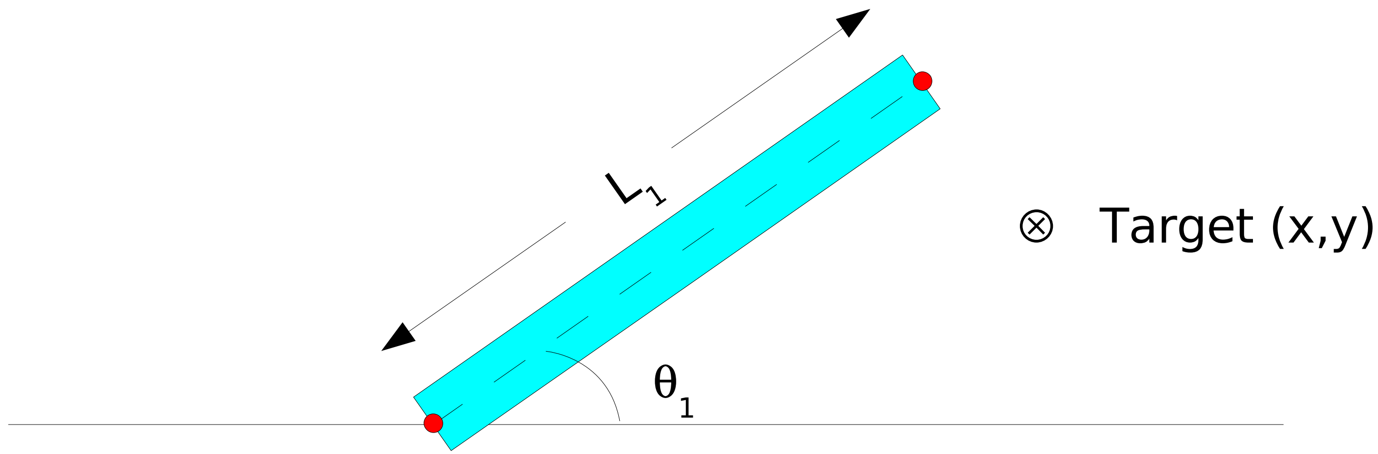
- Inverse kinematics solver included in PostureEngine:

  ```
  solveLinkPosition(const fmat::Column<3> &Ptgt,
                    unsigned int link,
                    const fmat::Column<3> &Peff)
  ```

  - Ptgt is the target point to move to (in base frame coordinates)
  - link is the index of some effector on the body, e.g., ArmOffset+GripperOffset
  - Peff is a point on the effector that is to be moved to Ptgt, in the reference fame of that effector.

- Returns true if a solution was found. False if no solution exists (e.g., joint limits exceeded, distance too far, etc.)
- Solution is stored in the PostureEngine as joint values.

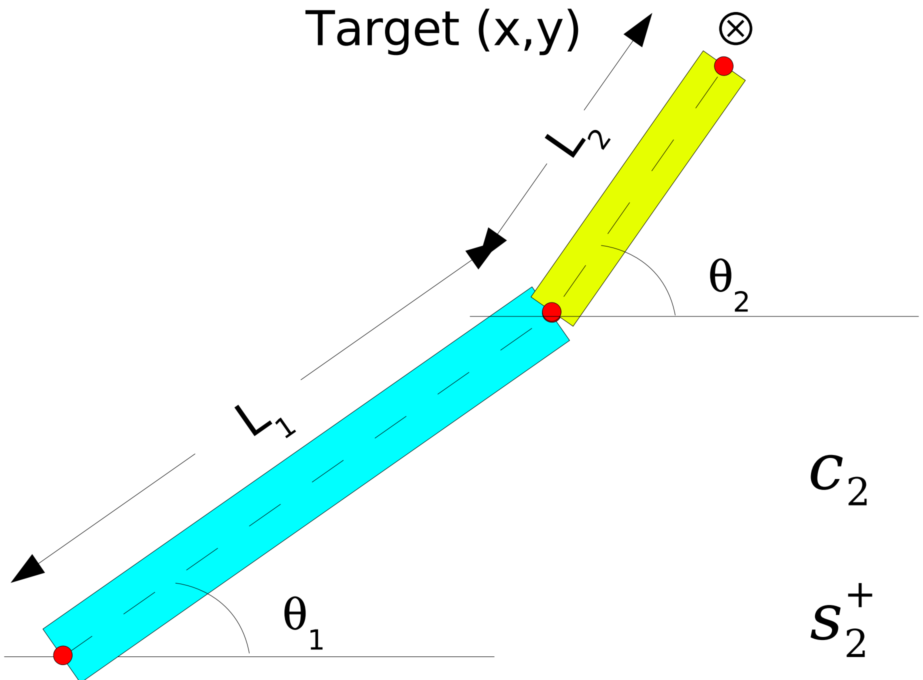
Solving the 1-Link Arm



Reachable if: $L_1 = \sqrt{x^2 + y^2}$

Solution: $\theta_1 = \text{atan2}(y, x)$

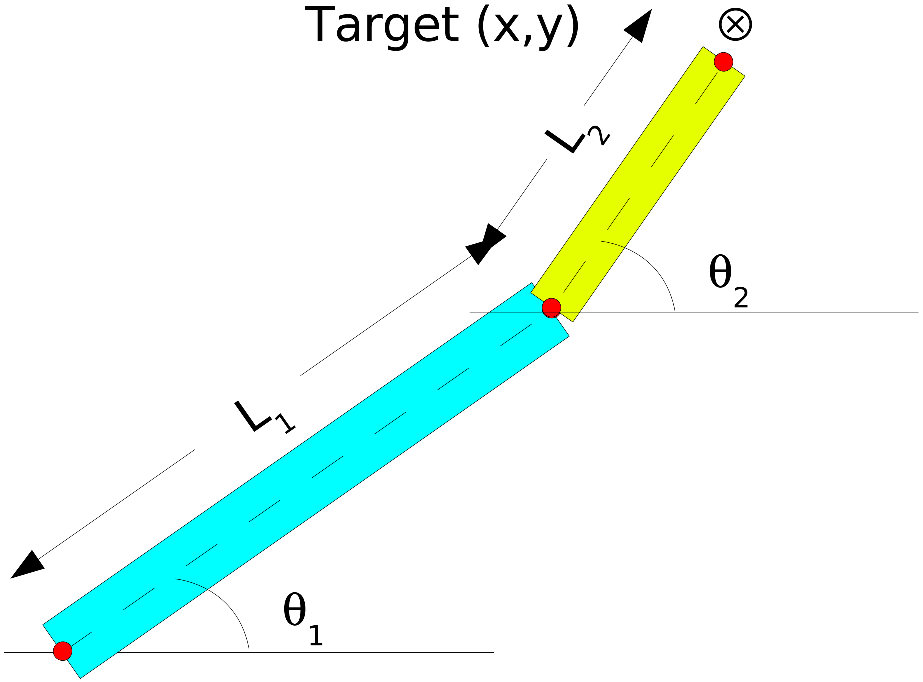
Solving the 2-Link Planar Arm

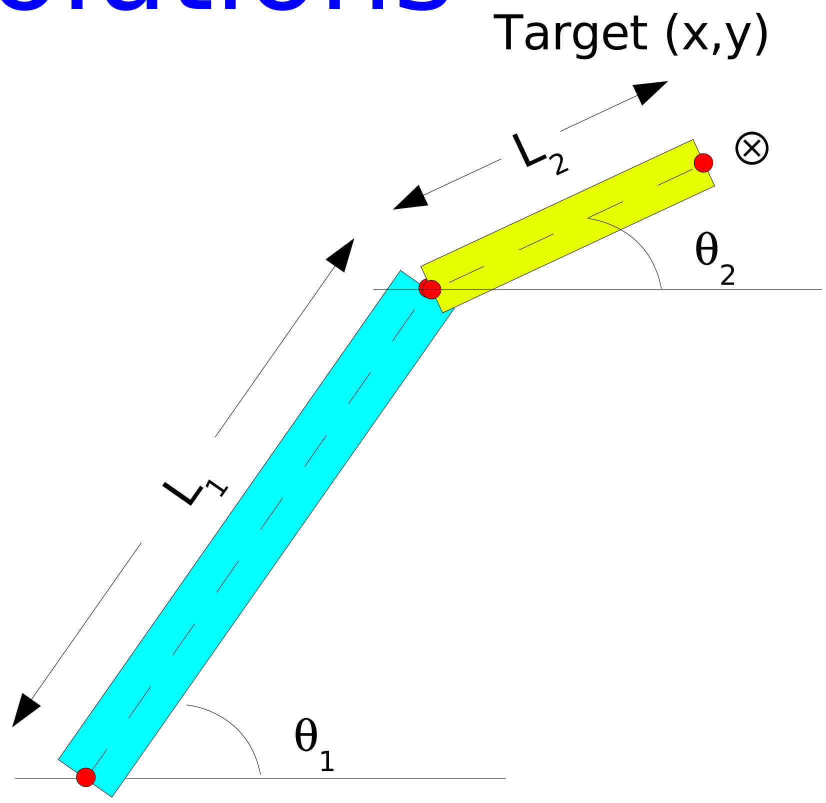


$$
\begin{aligned}
c_2 &= \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2} \\
s_2^+ &= \sqrt{1-c_2^2} \\
\theta_2^+ &= \operatorname{atan2}(s_2^+, c_2) \\
K_1 &= L_1 + c_2 L_2 \\
K_2 &= s_2^+ L_2 \\
\theta_1 &= \operatorname{atan2}(y, x) - \operatorname{atan2}(K2, K1)
\end{aligned}
$$

Reachable if: $c_2^2 \le 1$

Two Possible Solutions

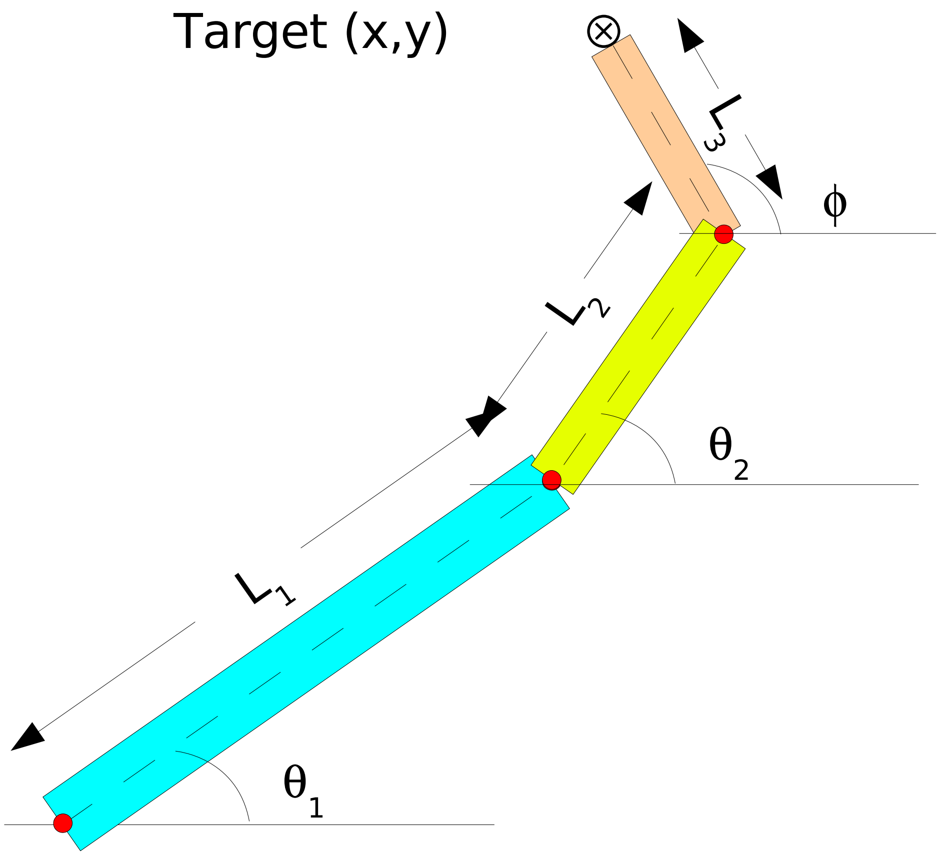




$$s_2^+ = \sqrt{1-c_2^2}$$
$$\theta_2^+ = \mathrm{atan2}(s_2^+, c_2)$$

$$s_2^- = -\sqrt{1-c_2^2}$$
$$\theta_2^- = \mathrm{atan2}(s_2^-, c_2)$$

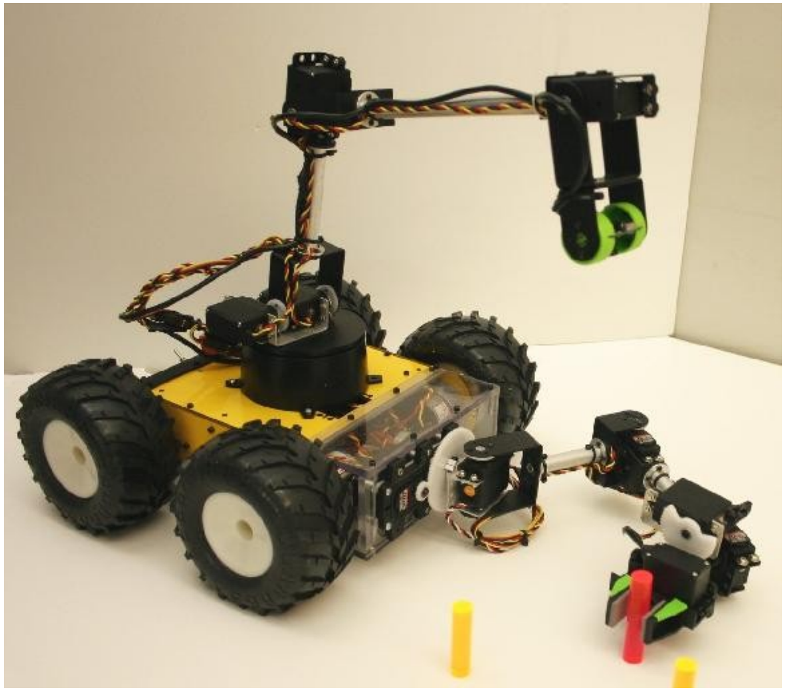
Solving the 3-Link Planar Arm



- Choose tool angle ϕ
- Given target position x_t, y_t, calculate wrist position: x_w and y_w
- Solve 2-link problem to put wrist at x_w, y_w.

Customized Kinematics Solvers

- For some simple kinematic chains, such as a pan/tilt, we can write analytical solutions to the IK problem.
- For the general case, must use gradient descent search.



See IK videos.