Kinematics

15-494 Cognitive Robotics
David S. Touretzky &
Ethan Tira-Thompson

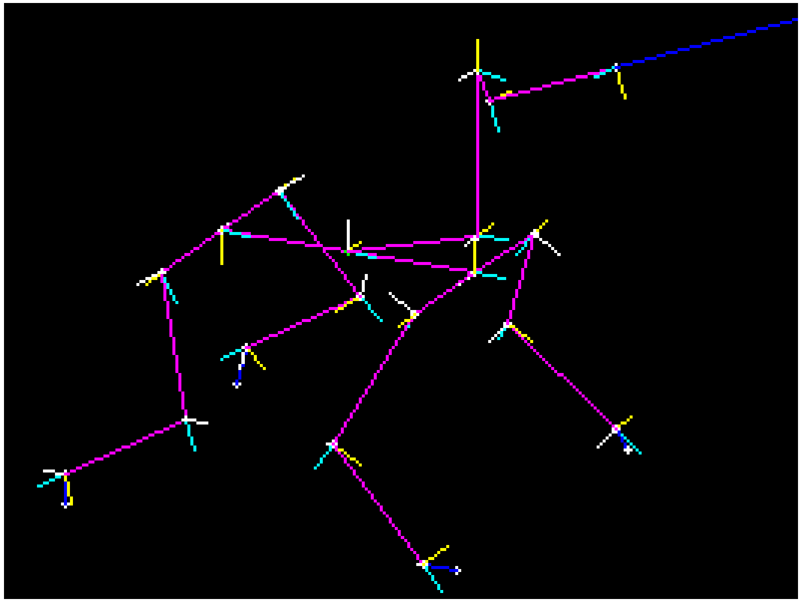
Carnegie Mellon
Spring 2012

Outline

Kinematics is the study of how things move.

- Kinematic chains
  - Robots are described as collections of kinematic chains
- Reference frames
- Homogeneous coordinates
- Kinematics and PostureEngine classes
- Forward kinematics: calculating limb positions from joint angles. (Straightforward matrix multiply.)
- Inverse kinematics: calculating joint angles to achieve desired limb positions. (Hard.)

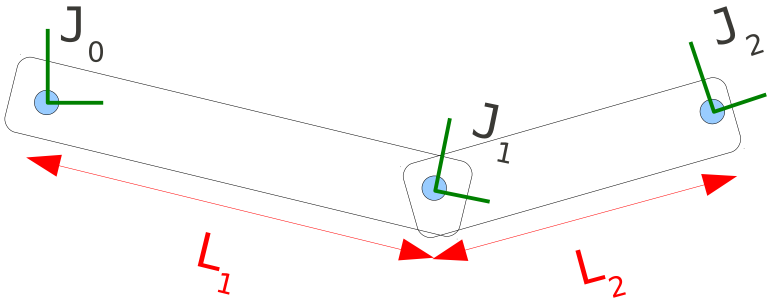
Robots As Kinematic Chains



- Tekkotsu allows branching chains, so robots are trees.
- The root of the tree is called the *BaseFrame* in Tekkotsu.
- It is typically at the center of the robot's body.

Chains = Joints + Links

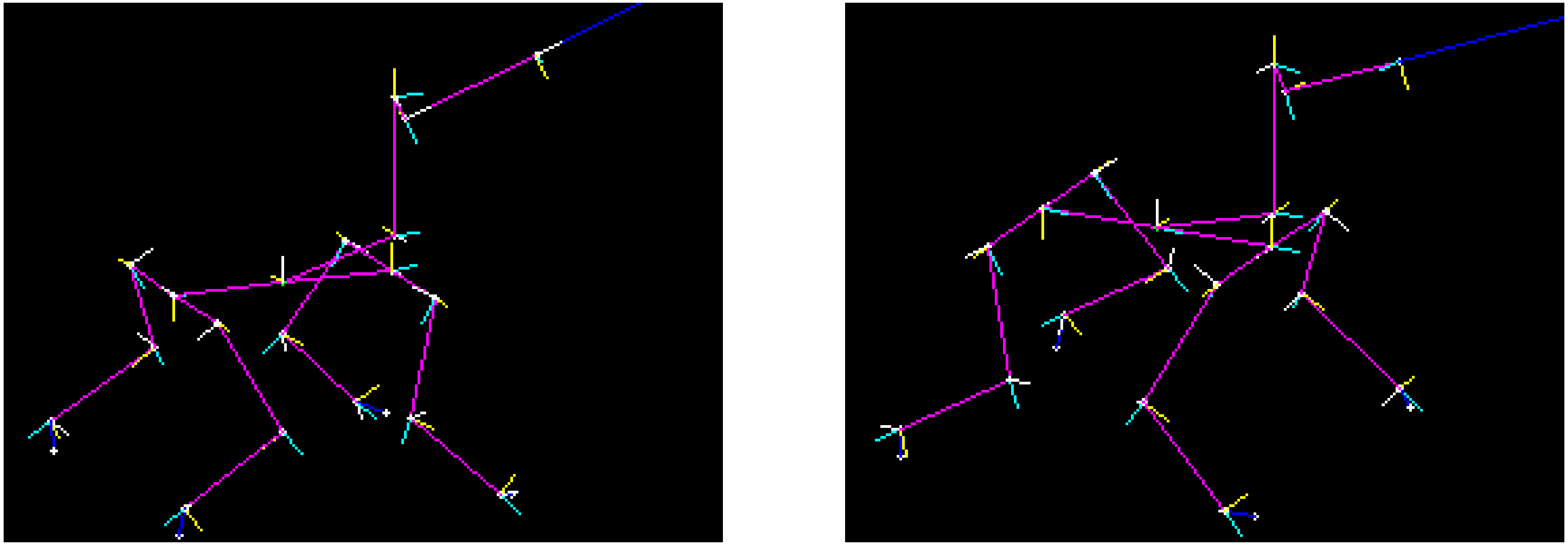
- A chain is a sequence of joints separated by links.



- We can use transformation matrices to calculate the position of the tip of the chain (joint J_2) from the joint angles θ_0, θ_1 and the link lengths L_1, L_2.
- Each rotational joint has a rotation transform; each link has a translation transform.
- The math for this will be shown later in this lecture.

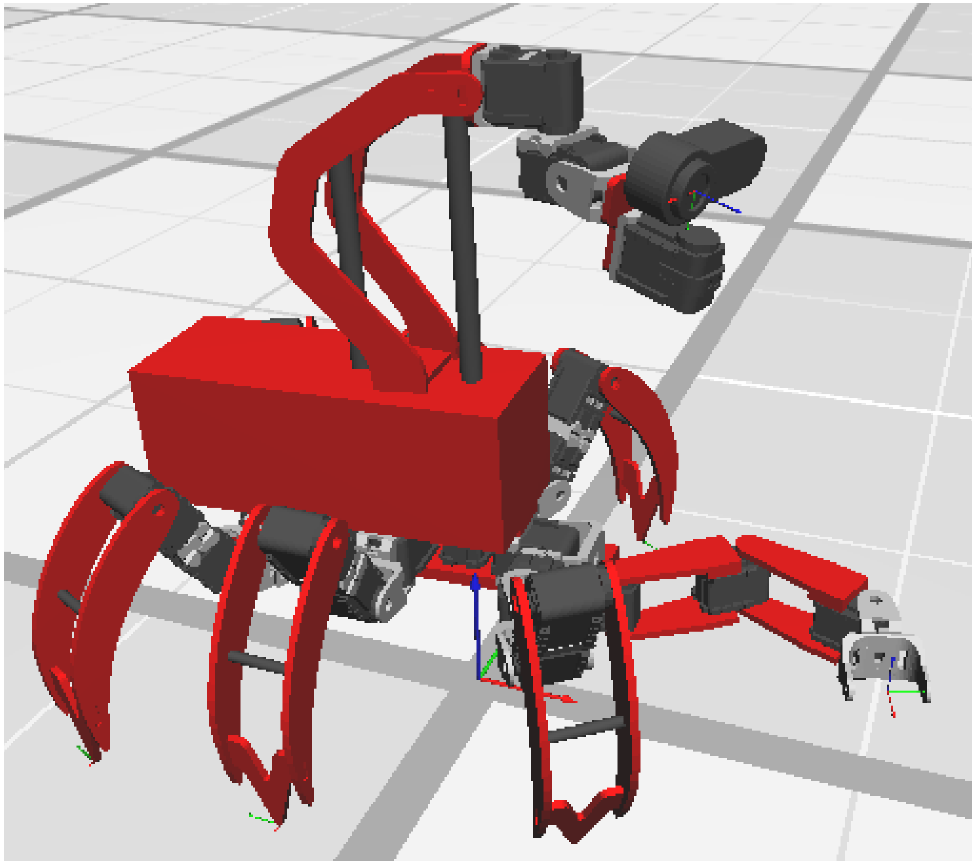
AIBO Kinematic Chains

- The AIBO has 9 kinematic chains instead of 6 because branched chains were formerly not supported:
  - 4 for the legs
  - 1 for the head (ending in the camera), 1 for the mouth
  - 3 for the IR range sensors
- All chains begin at the center of the body (base frame).



Chiara Kinematic Chains

- The Chiara has 8 major kinematic chains:
  - Head / camera / IR
  - Arm
  - Left front leg
  - Right front leg (4-dof)
  - Left middle leg
  - Right middle leg
  - Left back leg
  - Right back leg



Calliope Kinematic Chains

BaseFrame

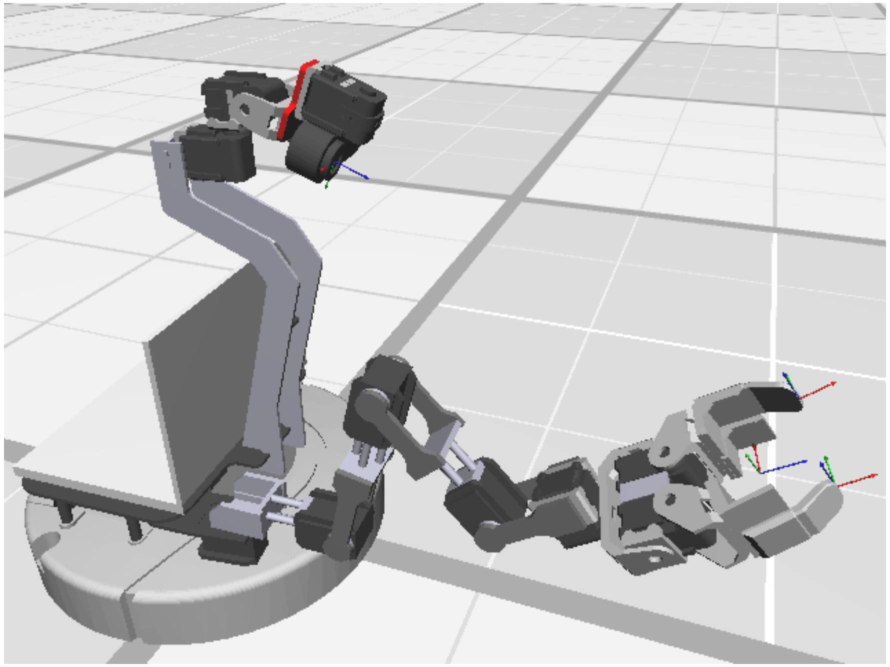
- center of axle
  - WHEEL:L, WHEEL:R

- NECK:PAN
  - NECK:TILT
    - **CameraFrame**

- ARM:base
  - ARM:shoulder
    - ARM:elbow
      - ARM:wrist
        - ARM:wristrot
          - **GripperFrame**
          - ARM:gripperleft
            - **LeftFingerFrame**
          - ARM:gripperright
            - **RightFingerFrame**

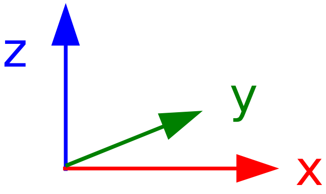
Use the DisplayKinTree demo to show the kinematic tree of the robot.

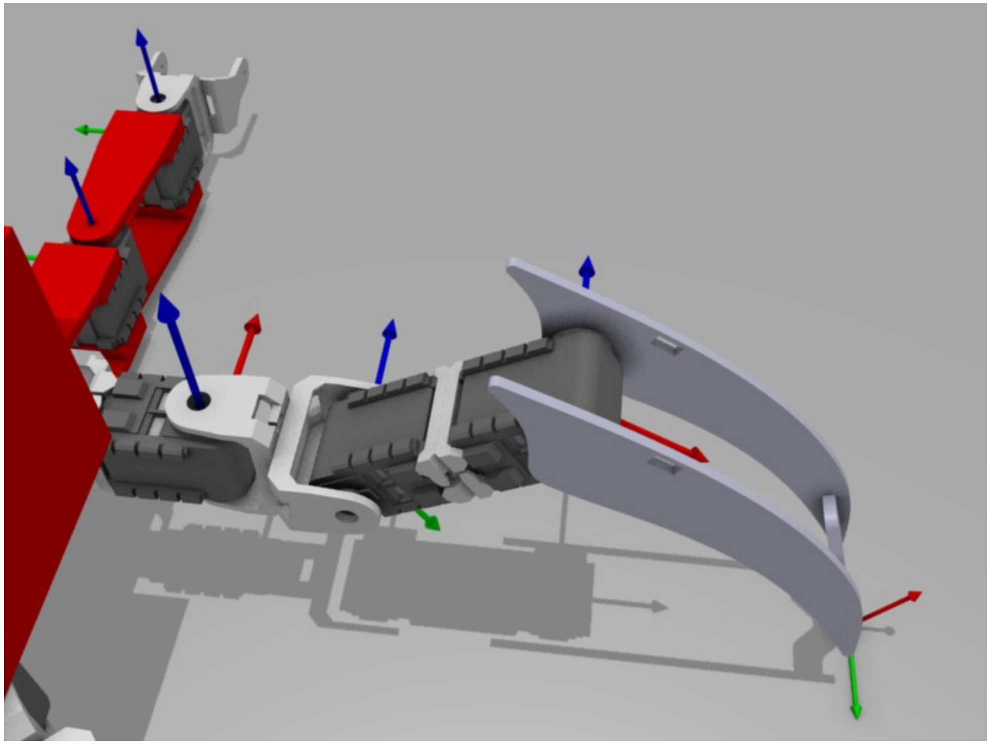
Root Control
  > Framework Demos
    > Kinematics Demos
      > DisplayKinTree



Reference Frames

- Every joint has an associated reference frame.
- Additional reference frames for camera, toes, etc.





- Denavit-Hartenberg conventions: joints rotate about their z-axes.
- The x and y axes follow the *right hand rule*.

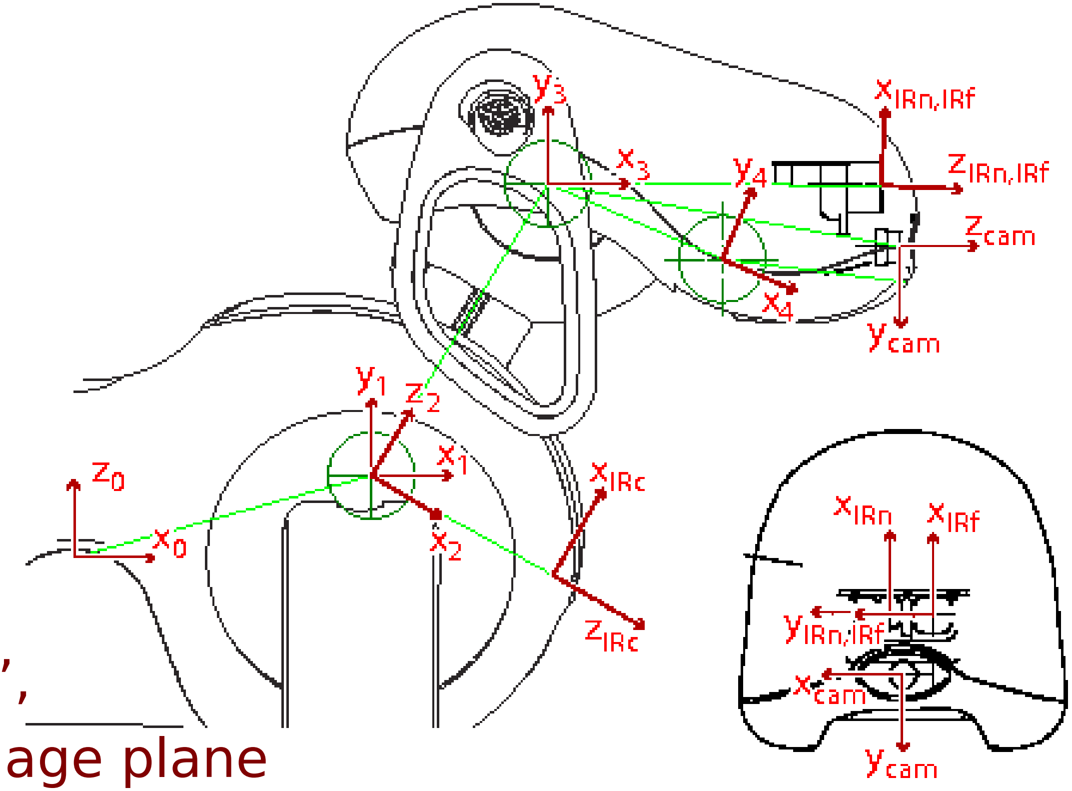
Chain of Reference Frames

- BaseFrame: z is up, x is forward, y is left.
  - This convention is also used for localShS and worldShS.

- Axis of rotation determines z for a joint.

- The head chain:

– Base frame	0	z_0 = "up"
– Tilt joint	1	y_1 = "up"
– Pan joint	2	
– Nod joint	3	
– Camera	4	z_4 = "out", x_4,y_4 = image plane



Reference Frame Naming Conventions

- Use the same offset-based indexing scheme as for joint names in motion commands and world state vectors:
  - **BaseFrameOffset**
  - HeadOffset+TiltOffset, HeadOffset+PanOffset
  - **CameraFrameOffset**
  - ArmShoulderOffset, ArmElbowOffset, ArmWristOffset, etc.
  - **GripperFrameOffset**

- Denavit-Hartenberg conventions specify how to express the relationship between one reference frame and the next: d, θ, r, α.

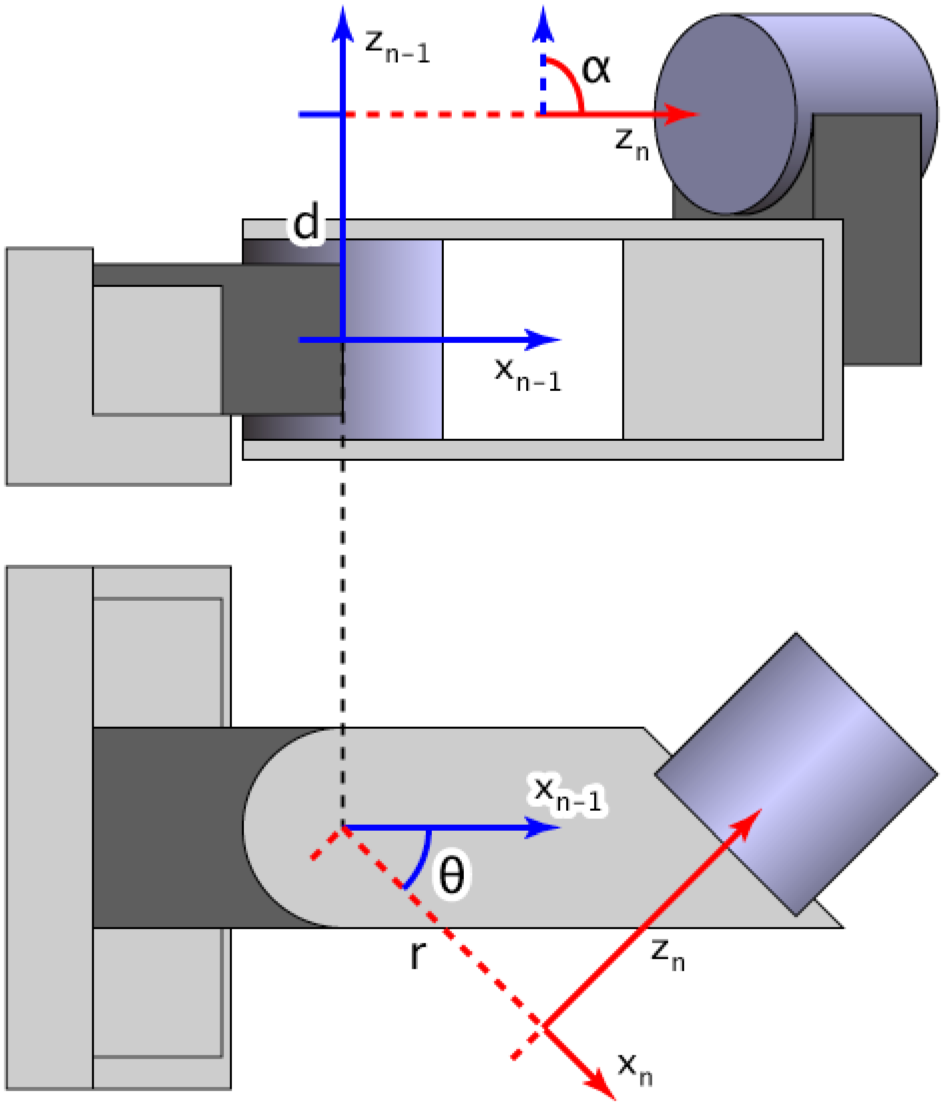
Denavit-Hartenberg Video

Denavit–Hartenberg Reference Frame Layout

Produced by Ethan Tira–Thompson

http://www.youtube.com/watch?v=rA9tm0gTln8

Summary of D-H Conventions



1) Move by d along z_{n-1}

2) Rotate by θ around z_{n-1}

3) Move by r along x_n, which is the common normal of z_{n-1} and z_n

4) Rotate by α along x_n

When z_{n-1} and z_n are parallel:

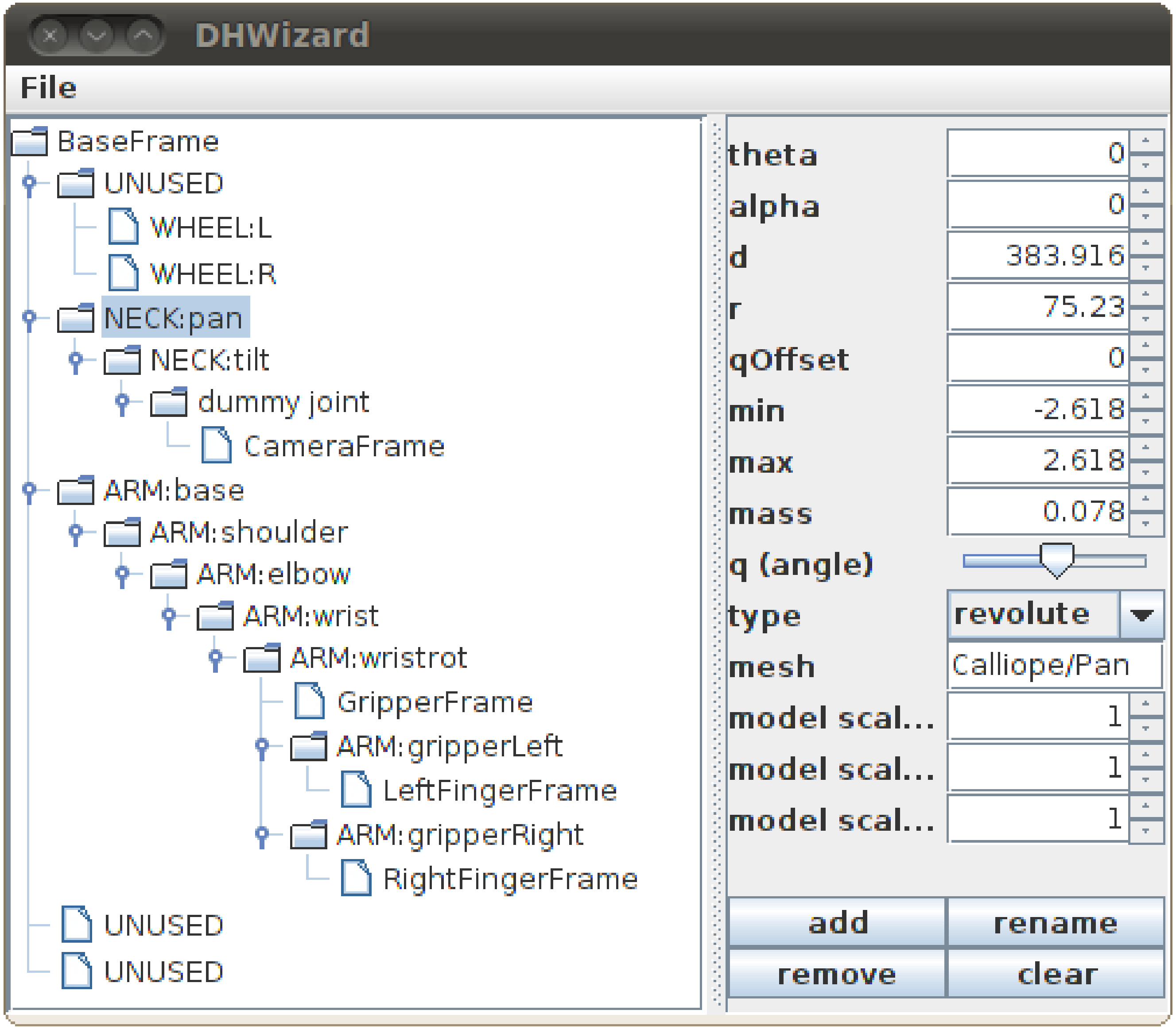
- d is arbitrary
- α is 0

The Tekkotsu .kin File

- See project/ms/config/Calliope5KP.kin
- Contains four types of information:
  - Kinematic description of the robot following D-H conventions, used by Tekkotsu's kinematics solvers.
  - Additional joint and link information, such as min, max, and offset values, mass, center of mass, etc.
  - Paths to mesh files (models) for selected joints, used by Mirage to render the robot.
  - Collision models for selected components, used by Mirage to determine how the robot interacts with the world.

DH Wizard

- Tool for editing kinematic descriptions. Outputs a kin file.

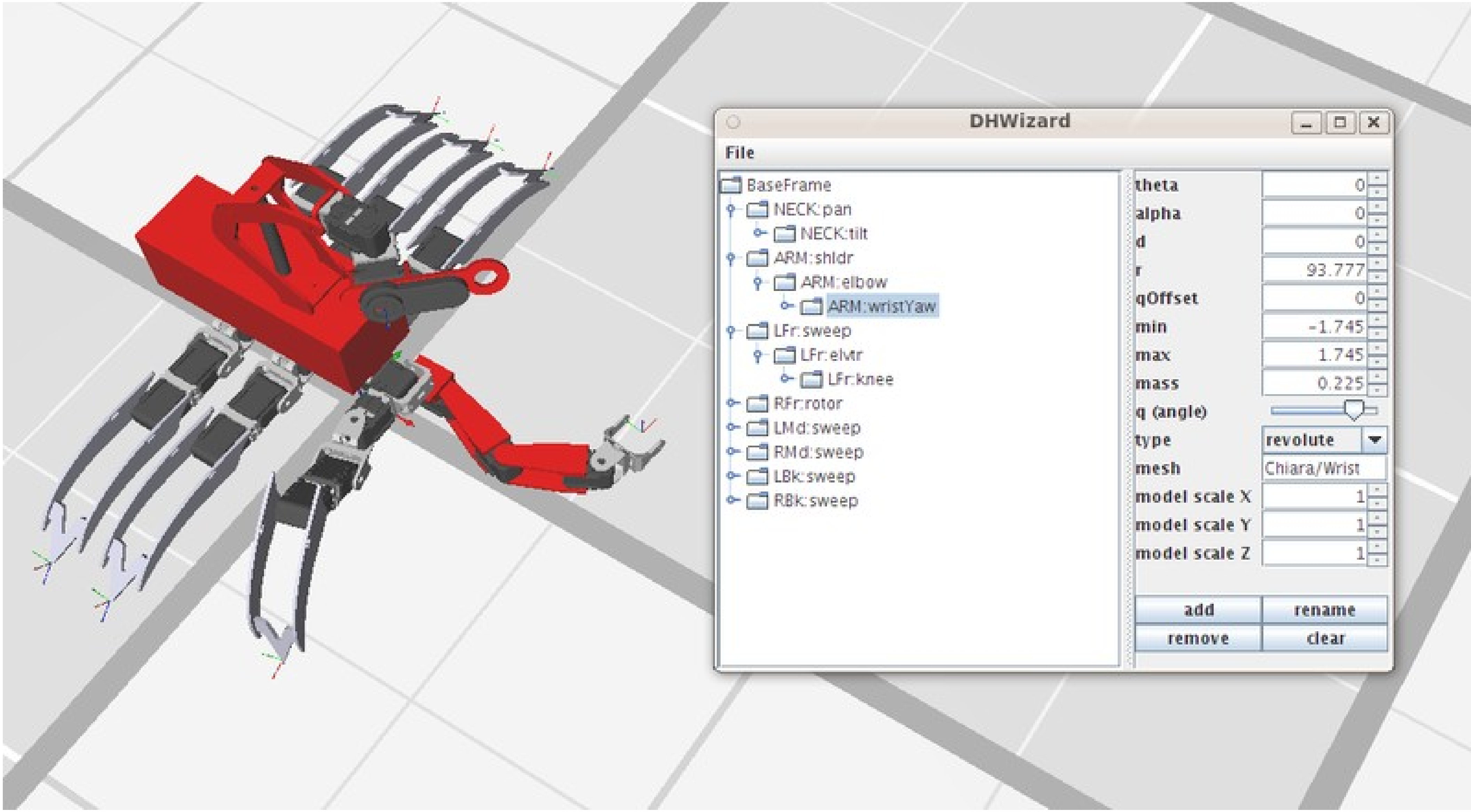


- BaseFrame
  - UNUSED
    - WHEEL:L
    - WHEEL:R
  - NECK:pan
    - NECK:tilt
      - dummy joint
        - CameraFrame
  - ARM:base
    - ARM:shoulder
      - ARM:elbow
        - ARM:wrist
          - ARM:wristrot
            - GripperFrame
            - ARM:gripperLeft
              - LeftFingerFrame
            - ARM:gripperRight
              - RightFingerFrame
  - UNUSED
  - UNUSED

theta	0
alpha	0
d	383.916
r	75.23
qOffset	0
min	-2.618
max	2.618
mass	0.078
q (angle)	
type	revolute
mesh	Calliope/Pan
model scal...	1
model scal...	1
model scal...	1

D-H params

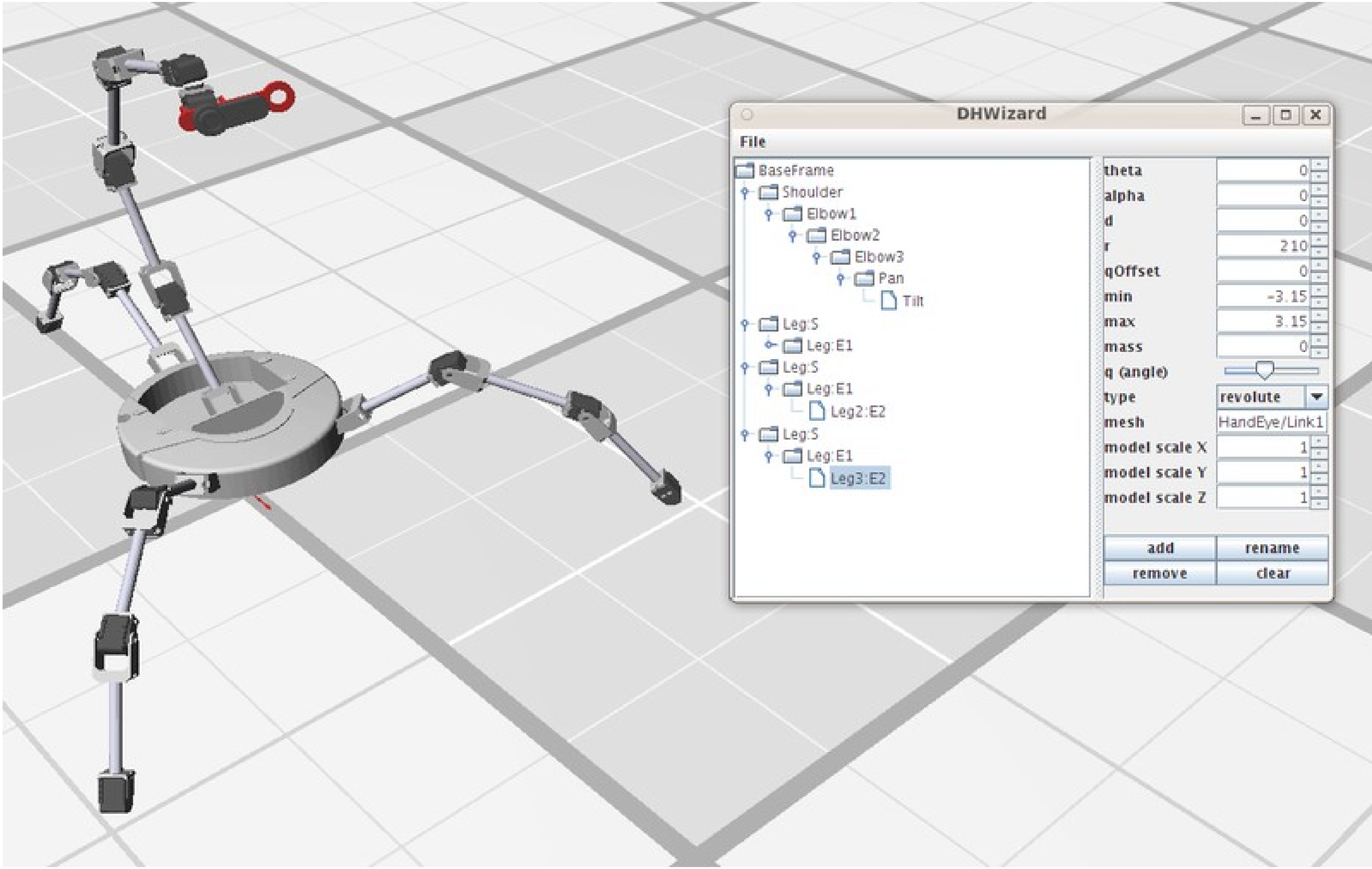
DH Wizard



- BaseFrame
  - NECK:pan
    - NECK:tilt
  - ARM:shldr
    - ARM:elbow
      - ARM:wristYaw
  - LFr:sweep
    - LFr:elvtr
      - LFr:knee
  - RFr:rotor
  - LMd:sweep
  - RMd:sweep
  - LBk:sweep
  - RBk:sweep

theta	0
alpha	0
d	0
r	93.777
qOffset	0
min	-1.745
max	1.745
mass	0.225
q (angle)	
type	revolute
mesh	Chiara/Wrist
model scale X	1
model scale Y	1
model scale Z	1

DH Wizard



- BaseFrame
  - Shoulder
    - Elbow1
      - Elbow2
        - Elbow3
          - Pan
            - Tilt
  - Leg:S
    - Leg:E1
  - Leg:S
    - Leg:E1
      - Leg2:E2
  - Leg:S
    - Leg:E1
      - Leg3:E2

theta	0
alpha	0
d	0
r	210
qOffset	0
min	-3.15
max	3.15
mass	0
q (angle)	
type	revolute
mesh	HandEye/Link1
model scale X	1
model scale Y	1
model scale Z	1

Now, The Math...

- How do we represent transformations from one reference frame to the next in a kinematic chain?
  - Homogeneous coordinates
  - Transformation matrices

- How do we perform these calculations in C++?
  - The fmat package

- How do I get Tekkotsu to do the work for me?
  - Forward kinematics solver

Homogeneous Coordinates

- Represent a point in N-space by an (N+1)-dimensional vector. Extra component is an inverse scale factor.
  - In “normal” form, last component is 1.

$$\vec{\mathrm{v}} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  - Points at infinite distance: last component is 0.
- Allows us to perform a variety of transformations using matrix multiplication:

Rotation, Translation, Scaling

- Tekkotsu uses 3D coordinates (so 4-dimensional vectors) for everything.

Transformation Matrices

- Let θ be rotation angle in the x-y plane.
  Let dx, dy, dz be translation amounts.
  Let 1/s be a scale factor.

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & dx \\ -\sin\theta & \cos\theta & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & s \end{bmatrix}$$

$$T\ \vec{\mathrm{v}} = \begin{bmatrix} x\cos\theta + y\sin\theta + dx \\ -x\sin\theta + y\cos\theta + dy \\ z + dz \\ s \end{bmatrix} = \begin{bmatrix} (x\cos\theta + y\sin\theta + dx)/s \\ (-x\sin\theta + y\cos\theta + dy)/s \\ (z + dz)/s \\ 1 \end{bmatrix}$$

Transformations Are Composable

- To rotate about point p, translate p to the origin, rotate, then translate back.

$$Translate(p) = \begin{bmatrix} 1 & 0 & 0 & p.x \\ 0 & 1 & 0 & p.y \\ 0 & 0 & 1 & p.z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rotate(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RotateAbout(p,\theta) = Translate(p) \cdot Rotate(\theta) \cdot Translate(-p)$$

fmat

- Tekkotsu uses the fmat package to represent coordinates and transformation matrices.
- fmat is optimized for efficient representation of small, fixed-size matrices and vectors.

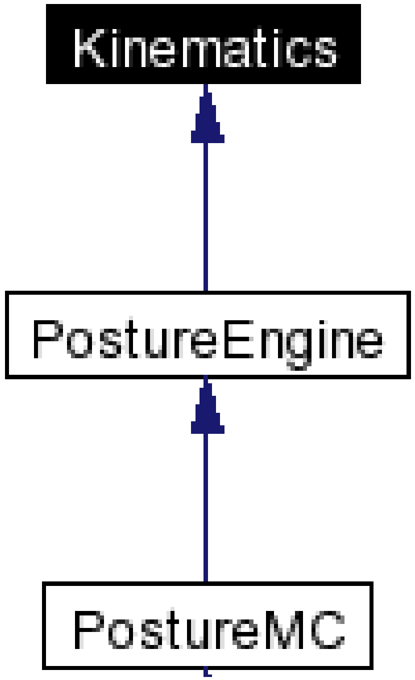
```
fmat::Column<4> v, w;
v = fmat::pack(5.75, 30.0, 115, 1);
w = fmat::pack(17, -4.2f, 100, 1);

fmat::Matrix<4,4> T;
T = v * w.transpose();
```

fmat::Transform

- Transformation matrices using homogenous coordinates are 4×4.
- But the last row is always [0 0 0 1].
- So fmat eliminates the last row and overloads the arithmetic operators to make the math work correctly.
- fmat::Transform is really a Matrix<3,4>

The Kinematics Class

- Tekkotsu contains its own kinematics engine for kinematics calculations, modeled after ROBOOP.
- The Kinematics class provides access to basic functionality for forward kinematics.
- Defined in Tekkotsu/Motion/Kinematics.h
- Global variable **kine** holds a special Kinematics instance:
  - Joint values reference WorldState.
- PostureEngine is a child of Kinematics so it can do kinematics calculations too.



Converting Between Reference Frames

- Most common conversions are between the base frame (body coordinates) and a limb or camera frame.
- Conversion requires computing a transformation matrix.
- Specify the frame with an unsigned int (a joint offset).

```
fmat::Transform linkToBase(unsigned int link)

fmat::Transform baseToLink(unsigned int link)

fmat::Transform linkToLink(unsigned int ilink,
                           unsigned int olink)
```

Reference Frame Conversion 1

- Transform Base to Base:

```
fmat::Transform T = kine->linkToBase(BaseFrameOffset);
cout << T.fmt(“%8.3f”) << endl;
```

- Result:

```
1.000   0.000   0.000   0.000
0.000   1.000   0.000   0.000
0.000   0.000   1.000   0.000
0.000   0.000   0.000   1.000
```

Reference Frame Conversion 2

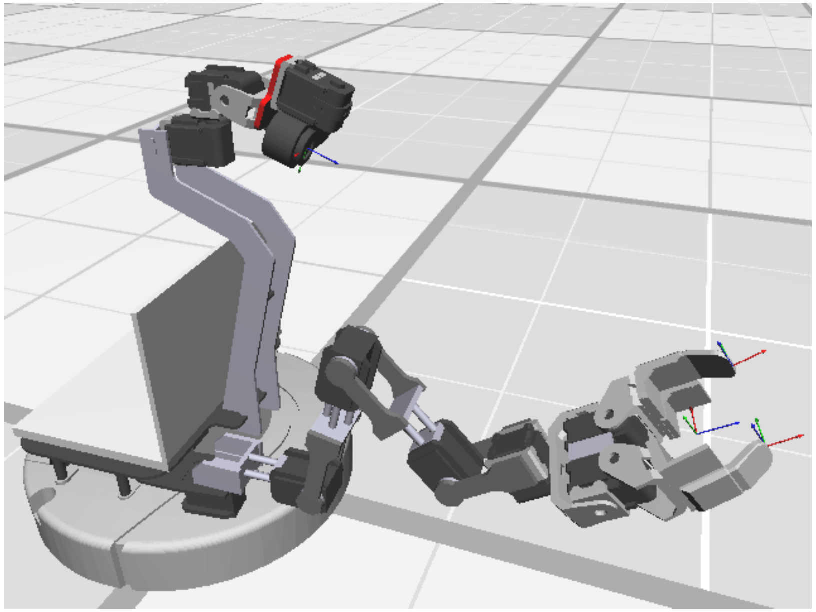
Translate Calliope head pan frame to base frame:

```
const float headpan = state->outputs[HeadOffset+PanOffset];
cout << "Head pan  is " << headpan * 180/M_PI
     << " degrees." << endl;

fmat::Transform TpanL = kine->linkToBase(HeadOffset+PanOffset);

cout << "pan linkToBase=\n" << TPanL.fmt(“%8.3f”) << endl;
```

At ~Zero Degree Pan Angle

```
Head pan is 0.0016182 degrees.
```

```
pan linkToBase=
[  1.000  -0.000   0.000  75.230
   0.000   1.000   0.000   0.000
   0.000   0.000   1.000 383.916 ]
```



At ~ 30 Degree Pan Angle

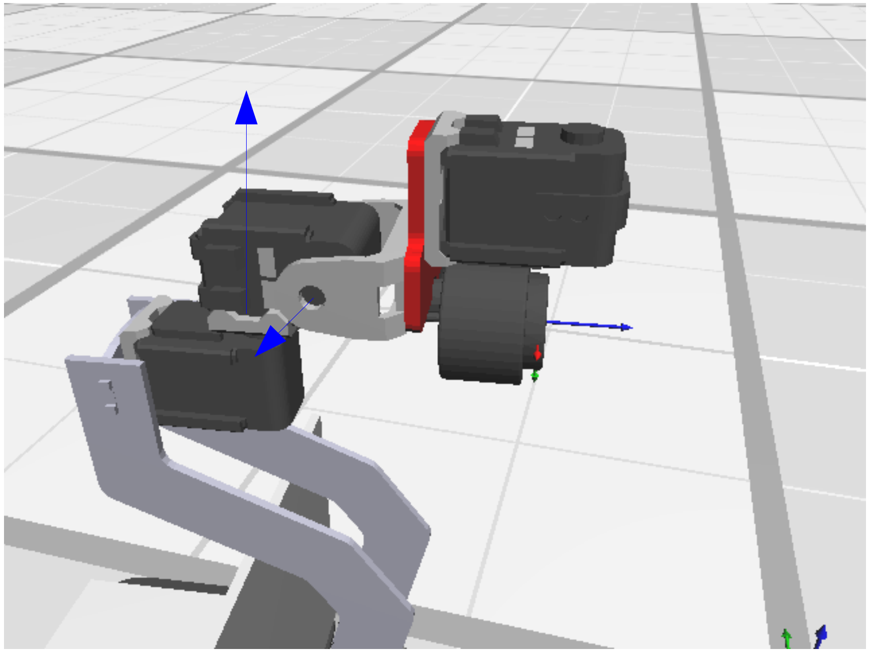
```
Head pan is 32.7 degrees.


pan linkToBase=
[  0.846  -0.534   0.000  75.230
   0.534   0.846  -1.000   0.000
   0.000   0.000   0.000 383.916 ]
```

$\cos(30^{o}) = 0.866$

$\sin(30^{o}) = 0.500$

How About Tilt w/Head Centered?

```
Head pan is -0.001547 degrees.

pan linkToBase=
[  1.000  -0.000   0.000  75.230
   0.000   1.000   0.000   0.000
   0.000   0.000   1.000 383.916 ]
```



```
Head tilt is 0.009223 degrees.

tilt linkToBase=
[  1.000  -0.000  -0.000  97.730
  -0.000  -0.000   1.000  -0.001
   0.000   1.000  -0.000 422.916 ]
```

Forward Kinematics: Measure Distance From Wrist to Arm Base

```
$nodeclass ComputeDistance : StateNode : doStart {

  fmat::Transform wrist =
    kine->linkToBase(ArmWristOffset);
  fmat::Column<3> wristPos = wrist.translation();

  fmat::Transform armbase =
    kine->linkToBase(ArmBaseOffset);
  fmat::Column<3> armbasePos = armbase.translation();

  float dist = (wristPos-armbasePos).norm();

  cout << “Distance is “ << setw(5) < dist << “ mm.” << endl;

}
```

```
startnode: ComputeDistance =T(1000)=> startnode
```

Inverse Kinematics

- Inverse kinematics finds the joint angles to put an effector at a particular point in space.
- Hard problem:
  - solution space can be discontinuous
  - can be highly nonlinear
  - multiple solutions may be possible
  - maybe no solution (so find closest approximation)

- Example: lookAtPoint(x,y,z)
  - point described in base frame coordinates
  - calculates head joint angles

CameraTrackGripper Demo

Root Control > Framework Demos > Kinematics Demos > CameraTrackGripper

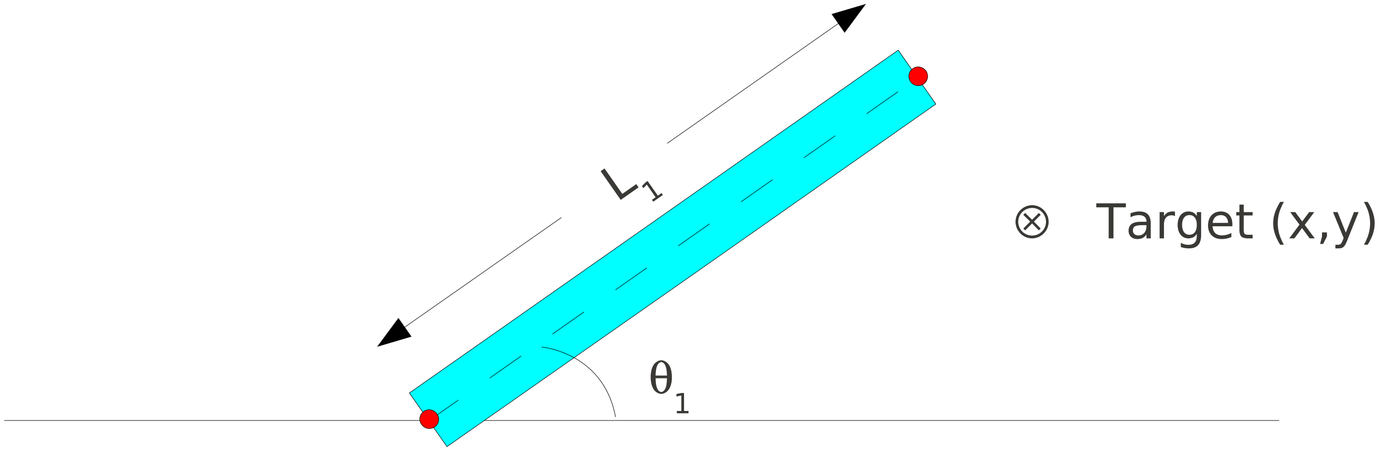
```
$nodeclass CameraTrackGripper : StateNode :  {

  $nodeclass HeadMover : HeadPointerNode : doStart {
    fmat::Transform Tgripper =
        kine->linkToBase(GripperFrameOffset);

    fmat::Column<3> Pgripper = Tgripper.translation();

    std::cout << "Transform:\n"
              << Tgripper.fmt("%8.3f") << std::endl;

    getMC()->lookAtPoint(Pgripper[0], Pgripper[1], Pgripper[2]);
  }
```

CameraTrackGripper (2)

```
virtual void setup() {
  MotionManager::MC_ID headmc =
    addMotion(MotionPtr<HeadPointerMC>());

 $statemachine{

   startnode: StateNode =N=> {headmover, unrelaxed}

   headmover: HeadMover[setMC(headmc)]
        =E(sensorEGID)=> headmover

   unrelaxed: SpeechNode("arm not relaxed")
                 =B(GreenButOffset)=> armrelaxer

   armrelaxer: SpeechNode("arm is relaxed")
      =N=> PIDNode(ArmOffset, ArmOffset+NumArmJoints, 0.f)
        =B(GreenButOffset)=> unrelaxed
 }

}
```

Initializer expression

Solving the 1-Link Arm



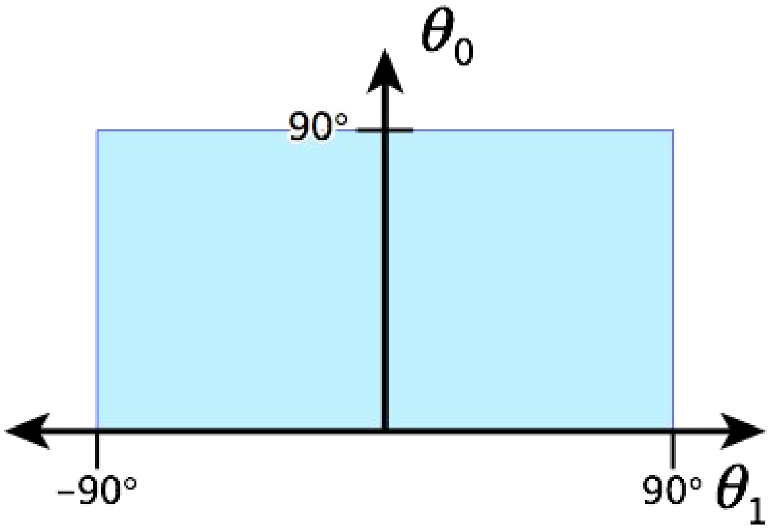
Reachable if: $L_1 = \sqrt{x^2 + y^2}$

Solution: $\theta_1 = \text{atan2}(y, x)$

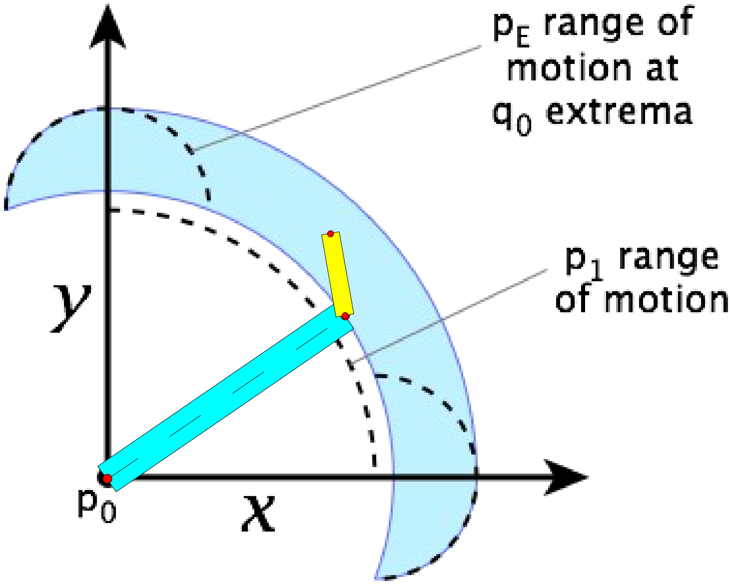
Configuration Space vs. Work Space

Consider a 2-link arm, with joint constraints

$0° < \theta_0 < 90°, \qquad -90° < \theta_1 < 90°$

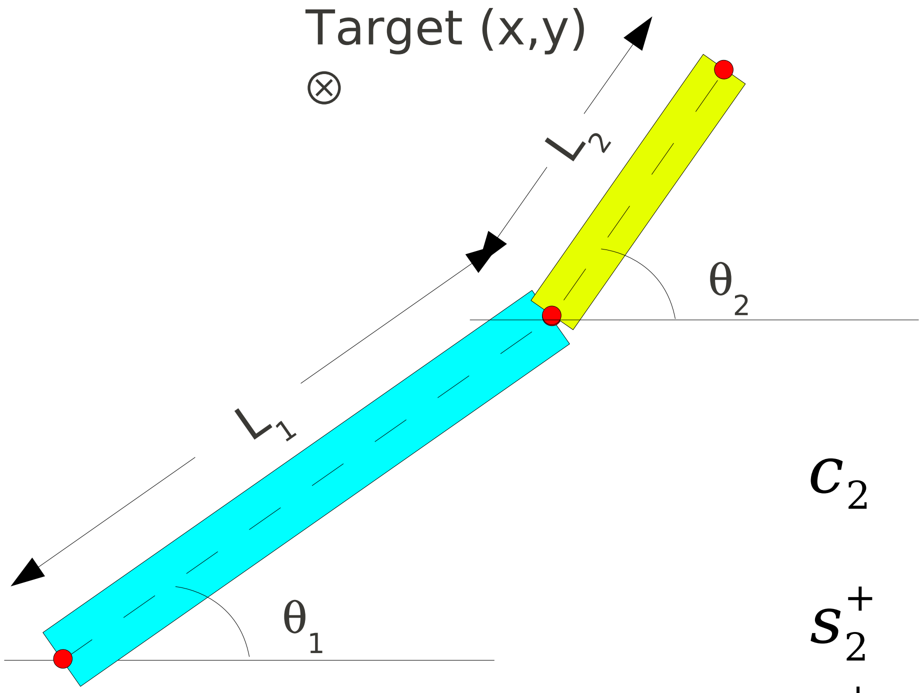


Configuration Space: robot's internal state space (e.g. joint angles)



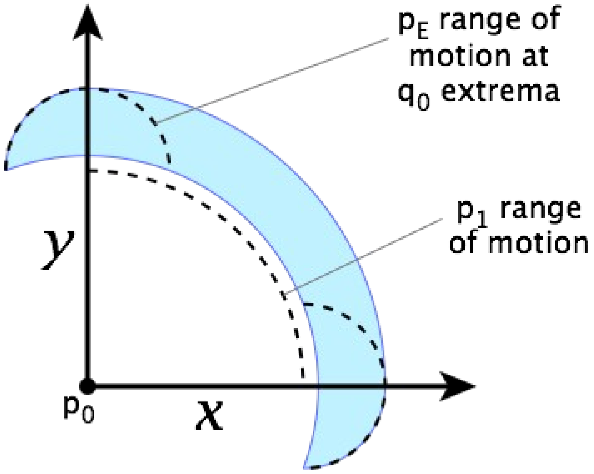
Work Space: set of all possible end-effector positions

Solving the 2-Link Planar Arm

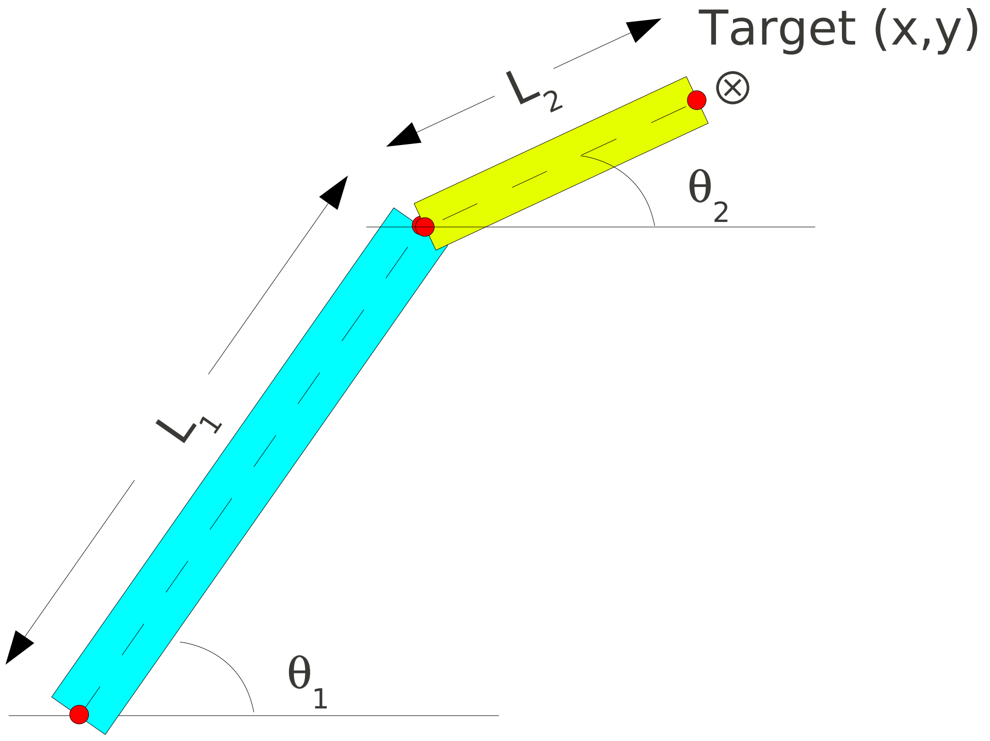


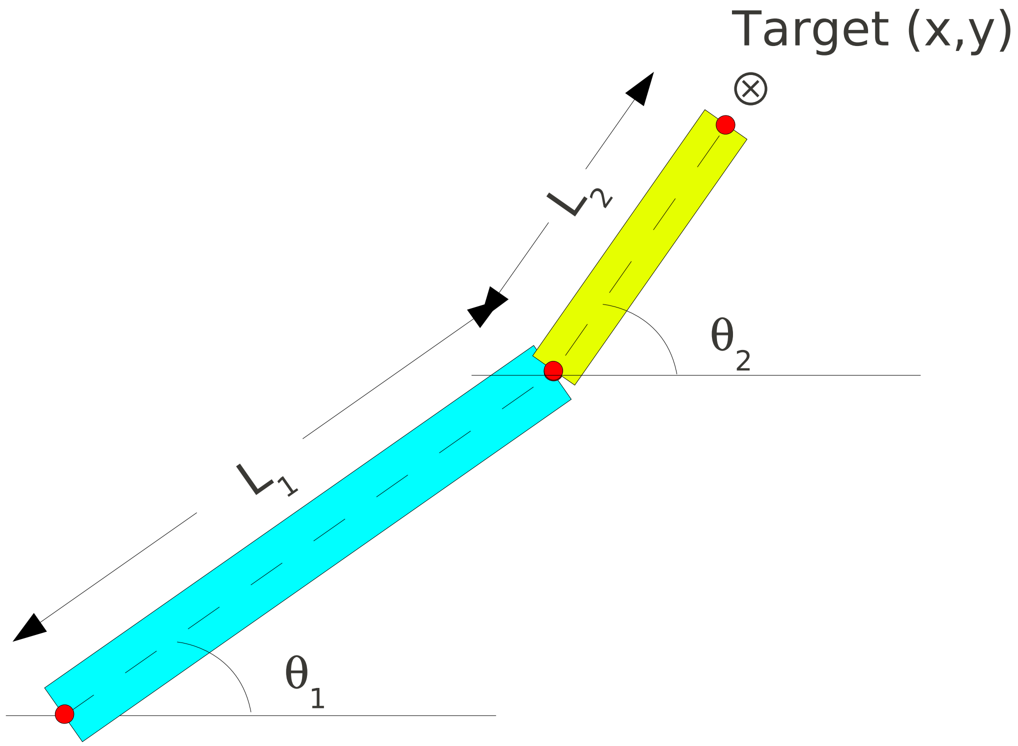
$$
\begin{aligned}
c_2 &= \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2} \\
s_2^+ &= \sqrt{1-c_2^2} \\
\theta_2^+ &= \operatorname{atan2}(s_2^+, c_2) \\
K_1 &= L_1 + c_2 L_2 \\
K_2 &= s_2^+ L_2 \\
\theta_1 &= \operatorname{atan2}(y, x) - \operatorname{atan2}(K2, K1)
\end{aligned}
$$

Reachable if: $c_2^2 \leq 1$



Two Possible Solutions





$$s_2^- = -\sqrt{1-c_2^2}$$
$$\theta_2^- = \text{atan2}(s_2^-, c_2)$$

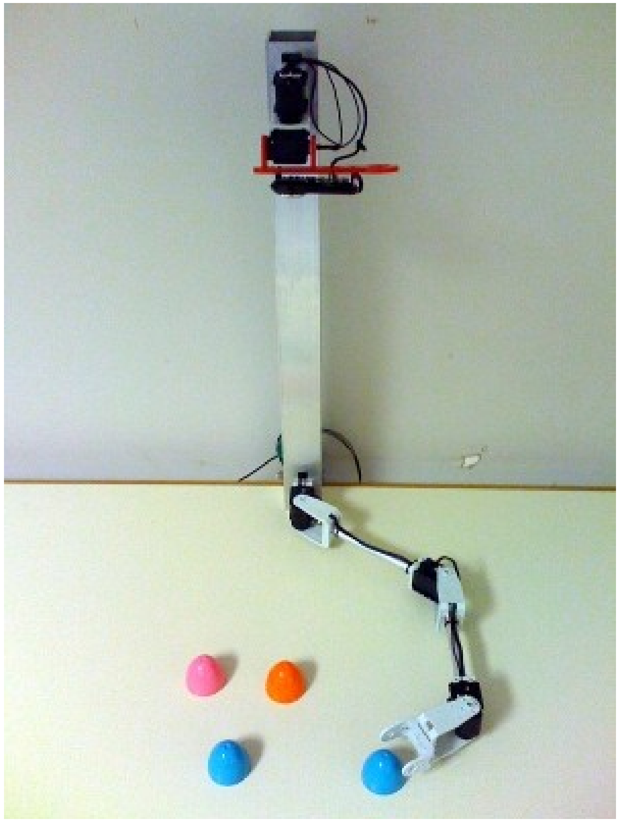
$$s_2^+ = \sqrt{1-c_2^2}$$
$$\theta_2^+ = \text{atan2}(s_2^+, c_2)$$

“Elbow up”

“Elbow down”

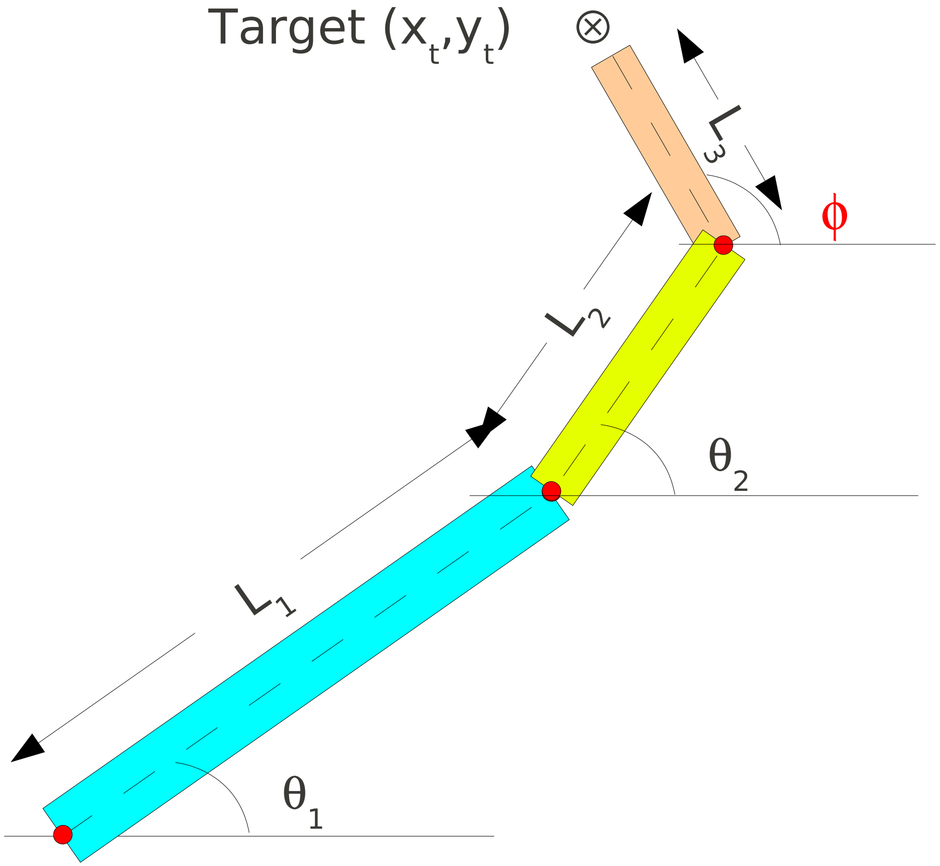
How Many Degrees of Freedom Are Enough?

- With 2 dof you can put the end effector at any point in the workspace.
- But you can't control end-effector orientation.
  - What if the arm is holding a screwdriver?



- With 3 dof in the same plane you can control both position and orientation.

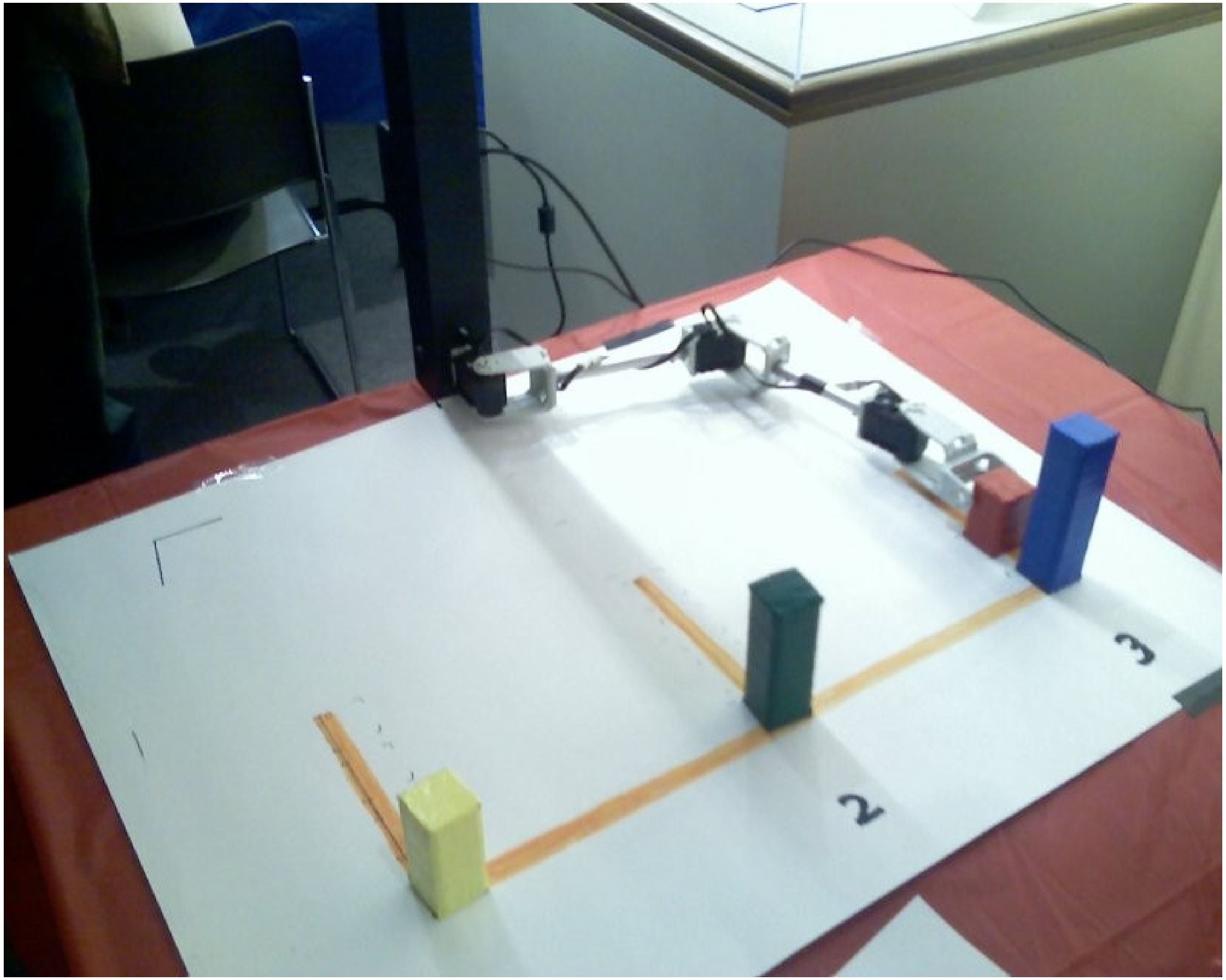
Solving the 3-Link Planar Arm



- Choose tool angle ϕ
- Given target position x_t, y_t, calculate wrist position: x_w and y_w
- Solve 2-link problem to put wrist at x_w, y_w.

If you don't know ϕ, pick an arbitrary value and search from there until you find a solution that works.

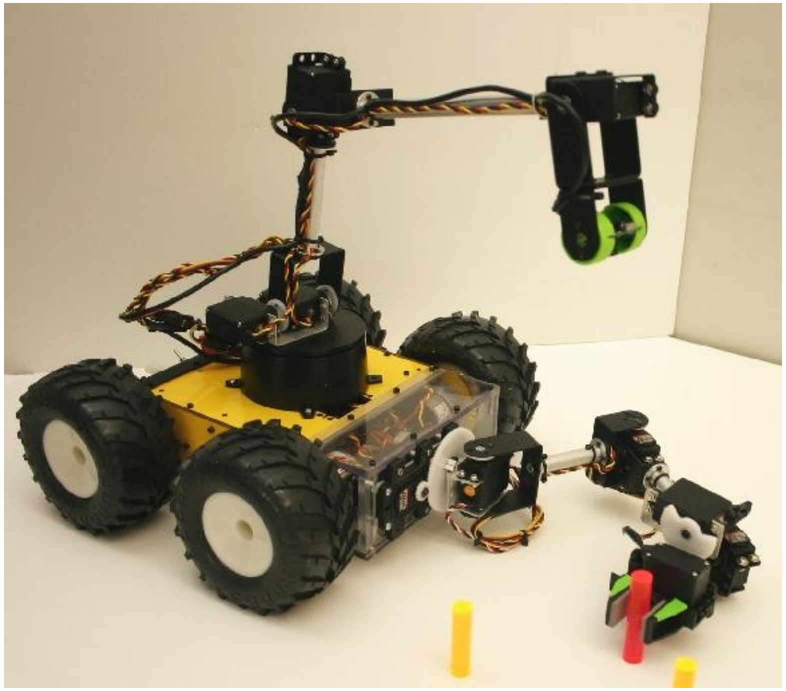
Towers of Hanoi in the Plane



Video by Michel Brudzinski and Evan Patton at RPI.

Customized Kinematics Solvers

- For some simple kinematic chains, such as a pan/tilt, we can write analytical solutions to the IK problem.
- For the general case, must use gradient descent search.



See IK videos.

Inverse Kinematics Functions

- Inverse kinematics solver included in PostureEngine:

```
solveLinkPosition(const fmat::Column<3> &Ptgt,
                  unsigned int link,
                  const fmat::Column<3> &Peff)
```

  - Ptgt is the target point to move to (in base frame coordinates)
  - link is the index of some effector on the body, e.g., GripperFrameOffset
  - Peff is a point on the effector that is to be moved to Ptgt, in the reference fame of that effector.

- Returns true if a solution was found. False if no solution exists (e.g., joint limits exceeded, distance too far, etc.)
- Solution is stored in the PostureEngine as joint values.

GripperTrackCamera

```
$nodeclass GripperTrackCamera : StateNode {

  $nodeclass ArmMover : PostureNode : doStart {

    fmat::Column<3> targetInCam = fmat::pack(0, 0, 100);
    fmat::Column<3> targetInBase =
      kine->linkToBase(CameraFrameOffset) * targetInCam;
    fmat::Column<3> noOffset = fmat::pack(0, 0, 0);

    getMC()->solveLinkPosition(targetInBase,
                               LeftFingerFrameOffset,
                               noOffset);
  }
```

GripperTrackCamera (2)

```
virtual void setup() {

  MotionManager::MC_ID armmc =
    addMotion(MotionPtr<PostureMC>());

  $statemachine{
    startnode: ArmMover[setMC(armmc)]
       =E(sensorEGID)=> startnode
  }

}
```

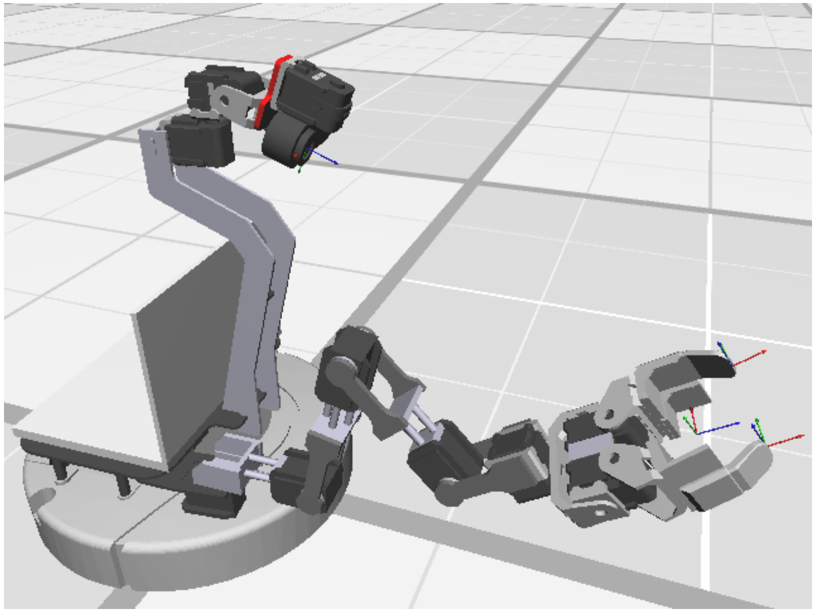
Additional IK Functions

PostureEngine provides:

- solveLinkPosition(...)
- solveLinkVector(...)
- solveLinkOrientation(...)
- solveLink(...)

The actual IK calculations for Calliope are done in Tekkotsu/Motion/IKCalliope.cc

Calliope's 5-dof ARM



- Only one degree of freedom in the horizontal plane:
  - ARM:base
- Three degrees of freedom in a vertical plane:
  - ARM:shoulder, ARM:elbow, ARM:wrist
- An additional degree of freedom in an orthogonal plane:
  - ARM:wristrot
- Conclusion: can only partially control the 3D pose of the end-effector.
  - What kinds of motions can this arm not make?