CMU 15-451/651 lecture 11/15/12

An Algorithms-based Intro to Machine Learning

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[Based on portions of intro lectures in 15-859(B) Machine Learning Theory, and on a talk given at the National Academy of Sciences "Frontiers of Science" symposium. This material will not be on the final.]

Plan for today

- Machine Learning intro: models and basic issues
- An interesting algorithm for "combining expert advice"

Machine learning can be used to...

- recognize speech,
- identify patterns in data,
- steer a car,
- play games,
- adapt programs to users,
- improve web search, ...

From a scientific perspective: can we develop models to understand learning as a computational problem, and what types of guarantees might we hope to achieve?

A typical setting

- Imagine you want a computer program to help filter which email messages are spam and which are important.
- Might represent each message by n features. (e.g., return address, keywords, spelling, etc.)
- Take sample S of data, labeled according to whether they were/weren't spam.
- Goal of algorithm is to use data seen so far produce good prediction rule (a "hypothesis") $h(x)$ for future data.

The concept learning setting

E.g.,

money	pills	Mr.	bad spelling	known-sender	spam?

The concept learning setting

E.g.,

	money	pills	Mr.	bad spelling	known-sender	spam?
	Y	N	Y	Y	N	Y
	N	N	N	Y	Y	N
a positive example	N	Y	N	N	N	Y
	Y	N	N	N	Y	N
a negative example	N	N	Y	N	Y	N
	Y	N	N	Y	N	Y
	N	N	Y	N	N	N
	N	Y	N	Y	N	Y

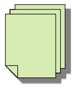
Given data, some reasonable rules might be:

- Predict SPAM if ¬known AND (money OR pills)
- Predict SPAM if money + pills – known > 0.
- ...

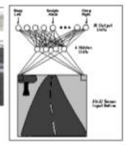
Power of basic paradigm

Many problems solved by converting to basic "concept learning from structured data" setting.

- E.g., document classification
  - convert to bag-of-words
  - Linear separators do well



- E.g., driving a car
  - convert image into features.
  - Use neural net with several outputs.



Big questions

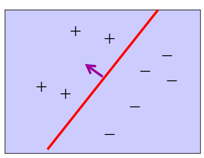
(A) How might we automatically generate rules that do well on observed data?
[algorithm design]

(B) What kind of confidence do we have that they will do well in the future?
[confidence bound / sample complexity]

for a given learning alg, how much data do we need...

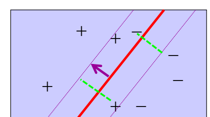
Algorithm design portion

- How about this problem of learning a linear separator?
  - Want to solve for weight vector w such that $w \cdot x \geq 1$ for all positive x, and $w \cdot x \leq -1$ for all negative x.
- Any ideas?



Algorithm design portion

- How about this problem of learning a linear separator?
  - Want to solve for weight vector w such that $w \cdot x \geq 1$ for all positive x, and $w \cdot x \leq -1$ for all negative x.
- Any ideas?
- Additional issues: no perfect separator, margins.
- "Support Vector Machine":
  - $w \cdot x^{(i)} \geq 1 - \epsilon_i$ for positive ex i.
  - $w \cdot x^{(i)} \leq -1 + \epsilon_i$ for negative ex i.
  - Minimize $\sum_i \epsilon_i + c|w|^2$ (convex optimization)



Now, for the confidence question, we'll need some connection between future data and past data.

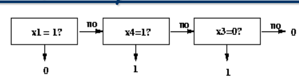
Natural formalization (PAC)

Email msg | Spam or not?

- We are given sample $S = \{(x,y)\}$.
  - View labels y as being produced by some target function f.
- Alg does optimization over S to produce some hypothesis (prediction rule) h.
- Assume S is a random sample from some probability distribution D. Goal is for h to do well on new examples also from D.

I.e., $\Pr_{x\sim D}[h(x)\neq f(x)] < \varepsilon$.

Example of analysis: Decision Lists



Say we suspect there might be a good prediction rule of this form.

1. Design an efficient algorithm **A** that will find a consistent DL if one exists.
2. Show that if S is of reasonable size, then Pr[exists consistent DL h with err(h) > ε] < δ.
3. This means that **A** is a good algorithm to use if f is, in fact, a DL.

(a bit of a toy example since would want to extend to "mostly consistent" DL)

How can we find a consistent DL?

x_1	x_2	x_3	x_4	x_5	label
1	0	0	1	1	+
~~0~~	~~1~~	~~1~~	~~0~~	~~0~~	~~−~~
~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~+~~
~~0~~	~~0~~	~~0~~	~~1~~	~~0~~	~~−~~
~~1~~	~~1~~	~~0~~	~~1~~	~~1~~	~~+~~
1	0	0	0	1	−

if (x_1=0) then -, else
if (x_2=1) then +, else
if (x_4=1) then +, else -

Decision List algorithm

- Start with empty list.
- Find if-then rule consistent with data.
  (and satisfied by at least one example)
- Put rule at bottom of list so far, and cross off examples covered. Repeat until no examples remain.

If this fails, then:
- No rule consistent with remaining data.
- So no DL consistent with remaining data.
- So, no DL consistent with original data.

OK, fine. Now why should we expect it to do well on future data?

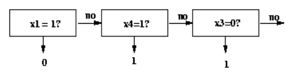
Confidence/sample-complexity

- Consider some DL h with err(h)>ε, that we're worried might fool us.
- Chance that h survives |S| examples is at most $(1-\varepsilon)^{|S|}$.
- Let |H| = number of DLs over n Boolean features. |H| < (4n+2)!. (really crude bound)

So, Pr[some DL h with err(h)>ε is consistent] $\leq |H|(1-\varepsilon)^{|S|}$.

- This is <0.01 for |S| > $(1/\varepsilon)$[ln(|H|) + ln(100)]
  or about $(1/\varepsilon)$[n ln n + ln(100)]

Example of analysis: Decision Lists



Say we suspect there might be a good prediction rule of this form.

1. Design an efficient algorithm **A** that will find a consistent DL if one exists. DONE
2. Show that if |S| is of reasonable size, then Pr[exists consistent DL h with err(h) > ε] < δ. DONE
3. So, if f is in fact a DL, then whp **A**'s hypothesis will be approximately correct. "PAC model"

Confidence/sample-complexity

- What's great is there was nothing special about DLs in our argument.
- All we said was: "if there are not *too* many rules to choose from, then it's unlikely one will have fooled us just by chance."
- And in particular, the number of examples needs to only be proportional to log(|H|).
  (big difference between 100 and e^{100}.)

Occam's razor

William of Occam (~1320 AD):

"entities should not be multiplied unnecessarily" (in Latin)

Which we interpret as: "in general, prefer simpler explanations".

Why? Is this a good policy? What if we have different notions of what's simpler?

Occam's razor (contd)

A computer-science-ish way of looking at it:

- Say "simple" = "short description".
- At most 2^s explanations can be < s bits long.
- So, if the number of examples satisfies:

Think of as 10x #bits to write down h. → $m > (1/\varepsilon)[s \ln(2) + \ln(100)]$

Then it's unlikely a bad simple explanation will fool you just by chance.

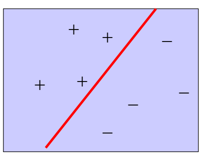
Occam's razor (contd)^2

Nice interpretation:

- Even if we have different notions of what's simpler (e.g., different representation languages), we can both use Occam's razor.
- Of course, there's no guarantee there will be a short explanation for the data. That depends on your representation.

Further work

- Replace log(|H|) with "effective number of degrees of freedom".



  - There are infinitely many linear separators, but not that many really different ones.
- Kernels, margins, more refined analyses….

Online learning

- What if we don't want to make assumption that data is coming from some fixed distribution? Or any assumptions on data?
- Can no longer talk about past performance predicting future results.
- Can we hope to say anything interesting at all??

Idea: regret bounds.

➢Show that our algorithm does nearly as well as best predictor in some large class.

Using "expert" advice

Say we want to predict the stock market.

- We solicit n "experts" for their advice. (Will the market go up or down?)
- We then want to use their advice somehow to make our prediction. E.g.,

Expt 1	Expt 2	Expt 3	neighbor's dog	truth
down	up	up	up	up
down	up	up	down	down
…	…	…	…	…

Basic question: Is there a strategy that allows us to do nearly as well as best of these in hindsight?

["expert" = someone with an opinion. Not necessarily someone who knows anything.]

Simpler question

- We have n "experts".
- One of these is perfect (never makes a mistake). We just don't know which one.
- Can we find a strategy that makes no more than lg(n) mistakes?

Answer: sure. Just take majority vote over all experts that have been correct so far.

➢Each mistake cuts # available by factor of 2.

➢Note: this means ok for n to be very large.

What if no expert is perfect?

Intuition: Making a mistake doesn't completely disqualify an expert. So, instead of crossing off, just lower its weight.

Weighted Majority Alg:

- Start with all experts having weight 1.
- Predict based on weighted majority vote.
- Penalize mistakes by cutting weight in half.

					prediction	correct
weights	1	1	1	1		
predictions	Y	Y	Y	N	Y	Y
weights	1	1	1	.5		
predictions	Y	N	N	Y	N	Y
weights	1	.5	.5	.5		

Analysis: do nearly as well as best expert in hindsight

- M = # mistakes we've made so far.
- m = # mistakes best expert has made so far.
- W = total weight (starts at n).
- After each mistake, W drops by at least 25%. So, after M mistakes, W is at most $n(3/4)^M$.
- Weight of best expert is $(1/2)^m$. So,

$$(1/2)^m \leq n(3/4)^M$$
$$(4/3)^M \leq n2^m$$
$$M \leq 2.4(m + \lg n)$$

So, if m is small, then M is pretty small too.

Randomized Weighted Majority

$2.4(m + \lg n)$ not so good if the best expert makes a mistake 20% of the time. Can we do better? Yes.

- Instead of taking majority vote, use weights as probabilities. (e.g., if 70% on up, 30% on down, then pick 70:30) Idea: smooth out the worst case.
- Also, generalize ½ to 1- ε.

Solves to: $M \leq \frac{-m\ln(1-\varepsilon)+\ln(n)}{\varepsilon} \approx (1+\varepsilon/2)m + \frac{1}{\varepsilon}\ln(n)$

M = expected #mistakes

$M \leq 1.39m + 2\ln n \quad \leftarrow \varepsilon = 1/2$

$M \leq 1.15m + 4\ln n \quad \leftarrow \varepsilon = 1/4$

$M \leq 1.07m + 8\ln n \quad \leftarrow \varepsilon = 1/8$

Analysis

- Say at time t we have fraction F_t of weight on experts that made mistake.
- So, we have probability F_t of making a mistake, and we remove an εF_t fraction of the total weight.
  - $W_{final} = n(1-\varepsilon F_1)(1 - \varepsilon F_2)\ldots$
  - $\ln(W_{final}) = \ln(n) + \sum_t [\ln(1 - \varepsilon F_t)] \leq \ln(n) - \varepsilon \sum_t F_t$ (using $\ln(1-x) < -x$)

$= \ln(n) - \varepsilon M.$ ($\sum F_t$ = E[# mistakes])

- If best expert makes m mistakes, then $\ln(W_{final}) > \ln((1-\varepsilon)^m)$.
- Now solve: $\ln(n) - \varepsilon M > m \ln(1-\varepsilon)$.

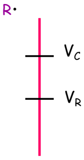
$$M \leq \frac{-m\ln(1-\varepsilon)+\ln(n)}{\varepsilon} \approx (1+\varepsilon/2)m + \frac{1}{\varepsilon}\log(n)$$

What can we use this for?

- Can use for repeated play of matrix game:
  - Consider a matrix where all entries 0 or -1.
  - Rows are different experts. Start at each with weight 1.
  - Pick row with prob. proportional to weight and update as in RWM.
  - Analysis shows do nearly as well as best row in hindsight!
  - In fact, analysis applies for entries in [-1,0], not just {-1,0}.
  - In fact, gives a proof of the minimax theorem…

Nice proof of minimax thm (sketch)

- Suppose for contradiction it was false.
- This means some game G has $V_C > V_R$:
  - If Column player commits first, there exists a row that gets the Row player at least V_C.
  - But if Row player has to commit first, the Column player can make him get only V_R.
- Scale matrix so payoffs to row are in [-1,0]. Say $V_R = V_C - \delta$.



Proof sketch, contd

- Now, consider randomized weighted-majority alg, against Col who plays optimally against Row's distrib.
- In T steps,
  - Alg gets $\geq (1-\varepsilon/2)$[best row in hindsight] – $\log(n)/\varepsilon$
  - BRiH $\geq T \cdot V_C$ [Best against opponent's empirical distribution]
  - Alg $\leq T \cdot V_R$ [Each time, opponent knows your randomized strategy]
  - Gap is δT. Contradicts assumption if use $\varepsilon=\delta$, once $T > 2\log(n)/\varepsilon^2$.

Other models

Some scenarios allow more options for algorithm.

- "Active learning": have large unlabeled sample and alg may choose among these.
  - E.g., web pages, image databases.

Other models

- A lot of ongoing research into better algorithms, models that capture additional issues, incorporating Machine Learning into broader classes of applications.