Natural dynamics of spring-like running: Emergence of selfstability

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ABSTRACT

In this study, the stability of conservative spring-mass running using a fixed angle of attack is addressed. For a given initial condition, different angles of attack result in periodic movement patterns. We apply a simple stride-to-stride analysis to prove the stability. Spring-like leg operation together with angle of attack control enables selfstabilised running.

1 INTRODUCTION

Legged locomotion aims to move an object in an intended direction. To achieve this goal, the system must provide enough energy to compensate losses within the system and with the environment. The system may reach a steady state movement, which is then characterised by a specific forward velocity and corresponding total mechanical energy. For instance, in human walking, a leg absorbs a certain amount of energy during the early stance phase and ‘refills’ this energy before leaving the ground (active plantar flexion). The overall energy after one stride remains approximately balanced.

A substantial requirement for locomotion is stability. The movement must not interrupt when a small obstacle occurs. For instance, a reduced or increased forward speed should be compensated to maintain the desired movement. This could be realized by controlling the system energy. Thus, stability requires a control of the mechanical system energy. However, even with a sufficiently adjusted energy, forward locomotion may fail. For instance, if a leg is not properly aligned with respect to ground, or the ground is higher than expected, the system might stumble and consequently fall. This indicates that there might be another quality of stability, which could lead to a robust locomotion without an energy-based control.

In this study we aim to investigate the influence of the leg adjustment on the robustness of running. Therefore, we describe running using a biologically inspired spring-mass template

(1). Stability is analysed assuming a given system energy according to the conservative nature of the considered model.

2 METHODS

Model. Running is represented using a simple planar spring-mass model (figure 1). During the flight phase, the leg is characterised by a fixed angle of attack α_0 and a nominal leg length ($\ell_0 = 1$ m). The centre of mass (m = 80 kg) trajectory is determined by a constant gravitational acceleration ($g = 9.81$ m/s^2). During the stance phase, the dynamic behaviour is a consequence of the leg spring (stiffness $k_{LEG} = 20$ kN/m) and the gravitation.

Periodicity. To investigate the periodic behaviour of spring-mass running, we seek an infinite number of successful steps, starting at a given initial condition (apex height $y_0 = 1$ m, forward speed $v_{X,0} = 5$ m/s). A step is defined as a sequence from one apex to the next in two subsequent flight phases including one stance phase. The simulation is stopped if (i) the centre of mass reaches the ground level ($y = 0$), or (ii) the forward velocity reaches zero ($v_X = 0$).

Stability. The stability of running is analysed using a return map $y_{i+1}(y_i)$ of the apex height y_{APEX}. As the spring-mass model is conservative, for a given total system energy the state vector at apex $(x, y, v_X, v_Y)_{APEX}$ is uniquely determined by the apex height y_{APEX}. This holds as at this instant the vertical velocity vanishes ($v_{Y,APEX} = 0$) and the horizontal position x_{APEX} has no influence on the further system dynamics. For stability, two conditions must be fulfilled: (i) the existence of a fixed point $y_{FP} = y_{i+1} = y_i$, and (ii) a slope of $y_{i+1}(y_i)$ within (-1, 1) in a neighbourhood of this fixed point y_{FP}.

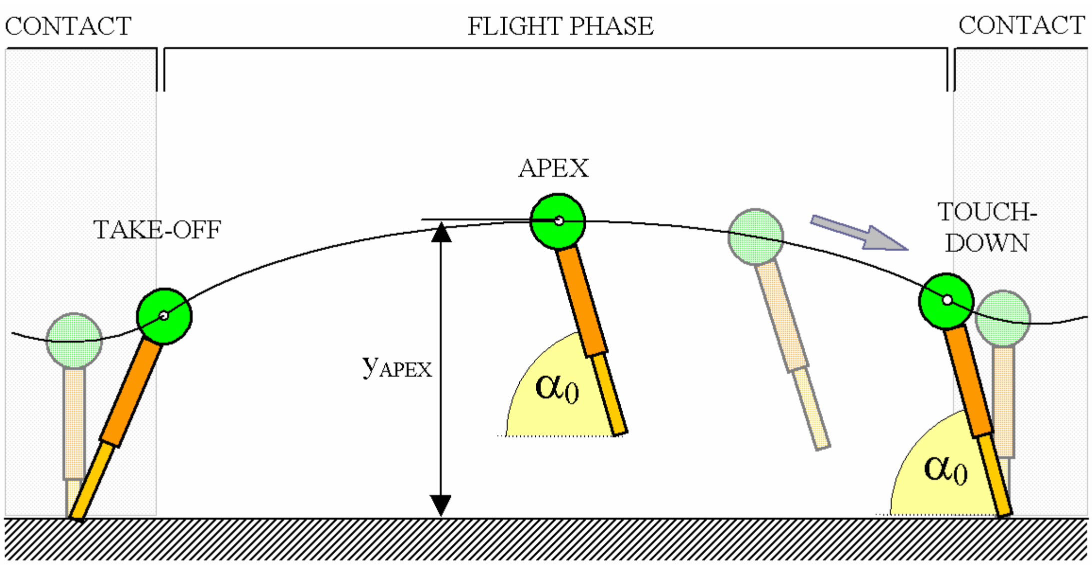

Figure 1. Running with elastic legs. The leg angle before landing is kept constant (angle of attack α_0): $\alpha(t) = \alpha_0$ = const.

3 RESULTS

For different angles of attack ($\alpha_0 = 66, 67, 68$ and $69°$), the vertical excursions of the centre of mass are shown in figure 2. For $\alpha_0 = 67$ and $68°$ periodic movement patterns are observed, whereas for $\alpha_0 = 66°$ the system successively increases the vertical excursions (which consequently reduces the forward velocity), and for $\alpha_0 = 69°$ the centre of mass falls on the ground (as it can not reach sufficient height after the first ground contact to land a second time).

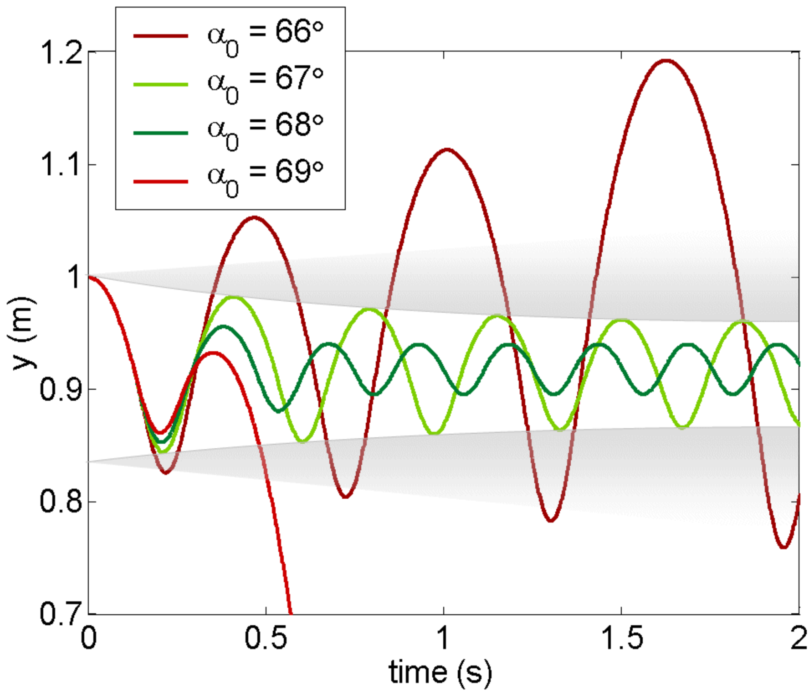

Figure 2. Influence of the chosen angle of attack ($\alpha_0 = 66, 67, 68, 69°$) on the trajectory in spring-mass running (k = 20 kN/m) starting at an initial forward velocity of 5 m/s and 1 m apex height. Different angles of attack (67, 68°) may result in periodic solutions. Steeper angles bring the system to fall; flatter angles successively increase the vertical excursions.

In the case of periodic solutions, the spring-mass system attracts different initial apex heights to a final steady-state height. To check this observation, we analyse the stability using the return map $y_{i+1}(y_i)$ of the apex height y_{APEX} for different angles of attack α_0 (figure 3).

For the flattest angle in figure 2 ($\alpha_0 = 66°$), no intersection with diagonal ($y_{i+1} = y_i$) is found (figure 3B). However, due to the close alignment with the diagonal, a limited number of steps with increasing apex heights are possible. In the case of the steepest angle of attack ($\alpha_0 = 69°$), a fixed point with $y_{i+1} = y_i$ exists. But, as the slope of $y_{i+1}(y_i)$ at this point is somewhat smaller than -1, the fixed point is *not stable*.

There are two return maps (for $\alpha_0 = 67, 68°$) with *stable* fixed points. Starting at different initial apex heights (e.g. 0.93 and 1 m), the return map predicts the attraction to the stable fixed point (a shown in figure 3B for $\alpha_0 = 67°$).

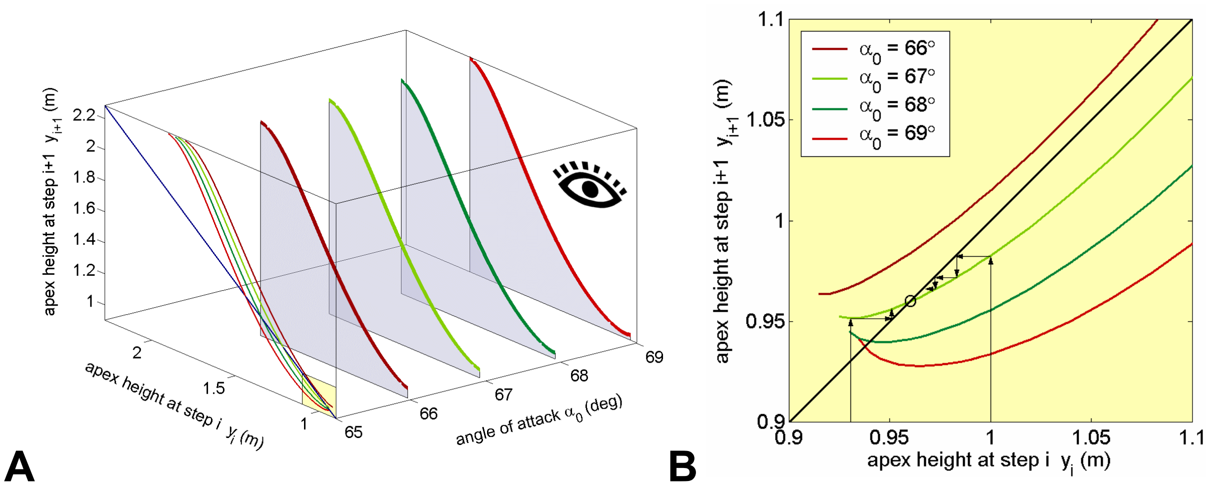

Figure 3. (A) Selfstabilisation of spring-mass running can be shown in the return map of the apex height $y_{i+1}(y_i)$ for different angles of attack. A small region of the left (y_{i+1}, y_i)-plane in the 3D plot is magnified in section (B) to prove stability.

4. DISCUSSION

Spring-mass running with a fixed angle of attack is selfstabilising if the leg parameters are properly adjusted. For a given total system energy, even different angles of attack (at the same leg stiffness) can result in periodic movements, whereby the steady-state apex height is dependent on the selected leg angle α_0.

The same holds for the opposite situation, where the leg stiffness is varied and the angle of attack is kept constant (2). Consequently, there is a variable and adjustable distribution of the total system energy to the forward running speed and the vertical oscillation (figure 2). For instance, a higher apex height reduces the forward speed and vice versa.

Spring-mass running does not incorporate dissipation or energy production and, therefore, can not address the mechanisms of stabilising the total system energy as usually studied in classical mechanics. In contrast to the traditional understanding of stability, here the system selects a favourite energy distribution and even resists to disturbances (of this energy distribution). This observation is called selfstabilisation of a conservative system.

A similar finding was obtained in a study addressing the stability of cockroach running in the horizontal plane (3). The supporting tripod was represented by a massless leg spring with a fixed leg angle with respect to the body centreline at touch-down. The alternate contact of the two leg springs resulted in a periodic movement, which proved to be robust with respect to lateral perturbations.

Selfstabilisation of spring-mass running requires a minimum total system energy if a fixed angle of attack α_0 is used (2). Then, a proper adjustment of leg stiffness and angle of attack can guarantee robust running. These requirements can be considered as a movement criterion for running.

To illustrate the consequences of selfstability, an example of running on a bumpy surface is given in figure 4. Here, the system can handle perturbations in ground level of up to 20 cm if an angle of attack (between 66 and 67°) resulting in high vertical excursions is used.

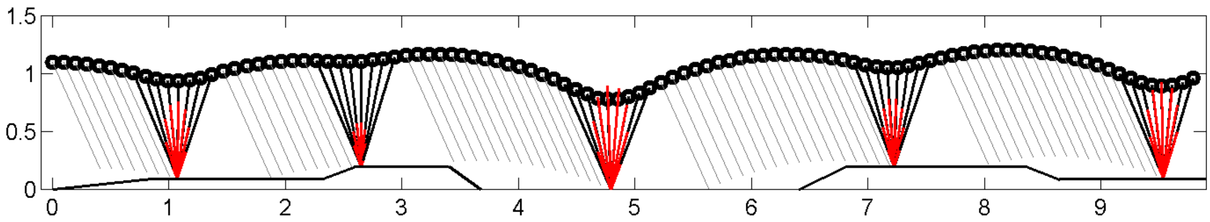

Figure 4. Running on uneven ground. Using a constant angle of attack α_0, considerable changes in ground level can be successfully managed by the spring-mass runner.

The emergence of asymptotic stability in piecewise holonomic, conservative systems might be an important feature for the control of locomotion (4). However, the underlying mechanisms are only barely understood and need further investigation. In a second presentation (5), a method optimising selfstability in such systems (e.g. spring-mass runner) is proposed. In contrast to the simple control strategy used here (fixed angle of attack), which requires a precise adjustment of the leg angle during the flight phase, in (5) the rotational control of the leg during the swing phase will be taken into account.

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