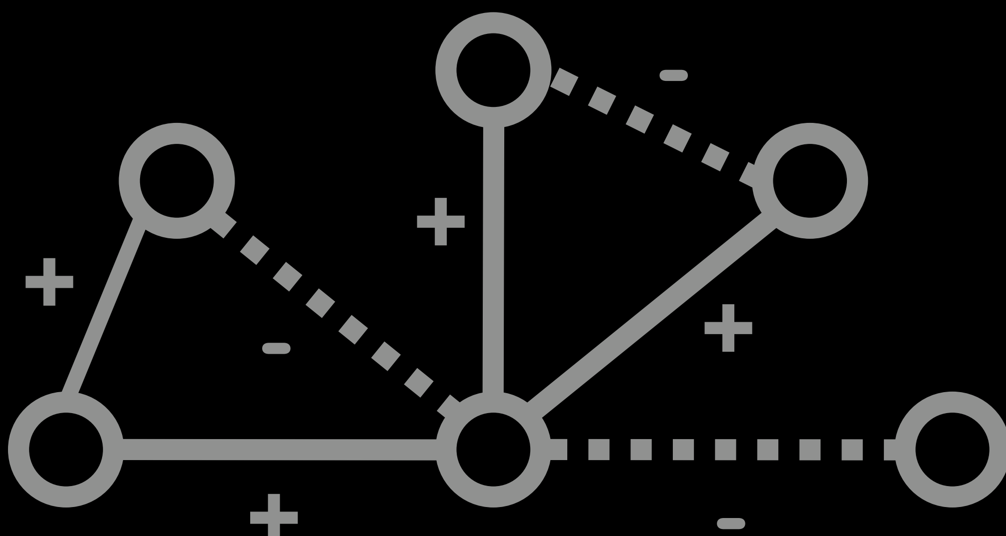
15-251

Great Theoretical Ideas in Computer Science

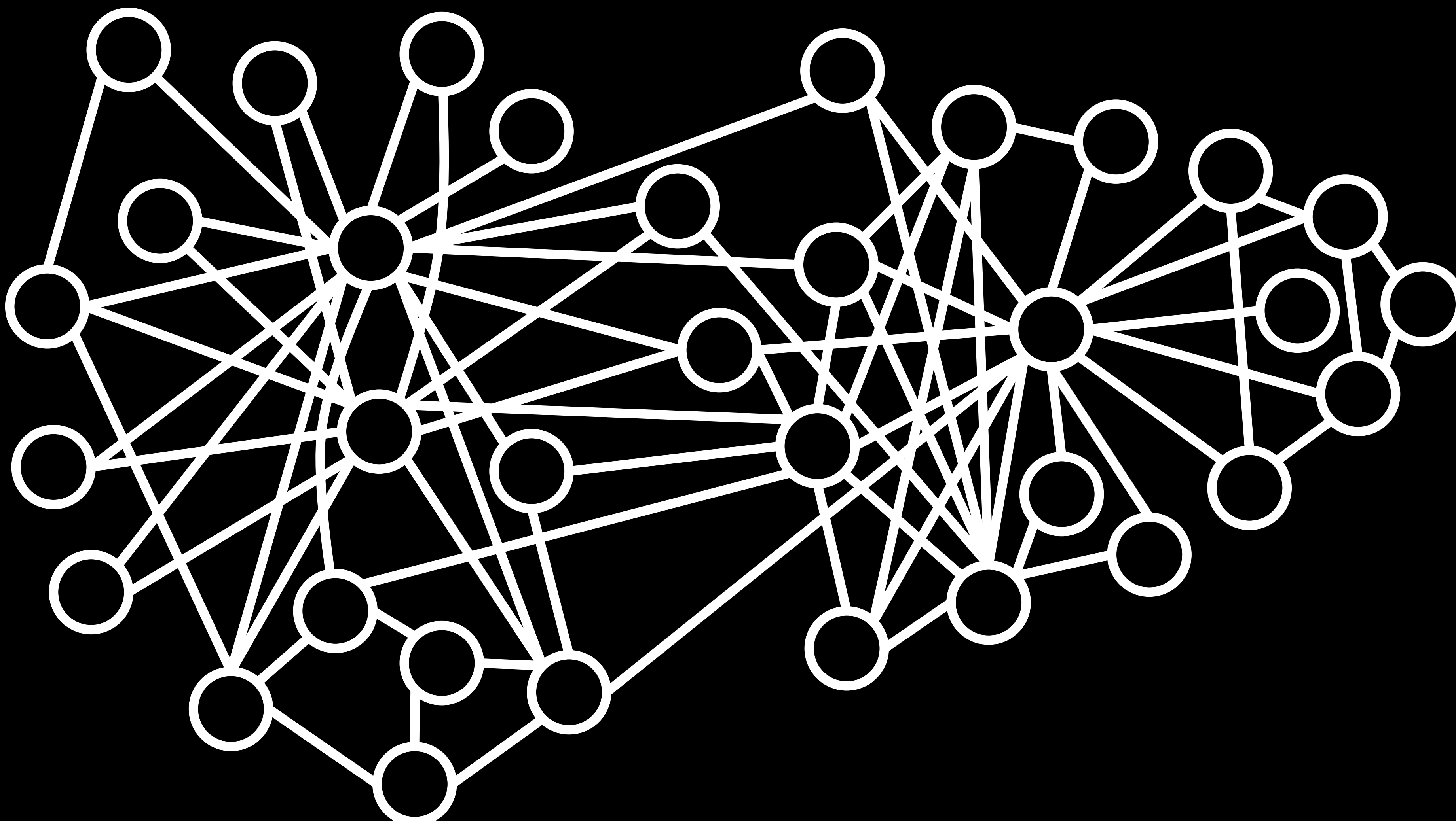
Social Networks



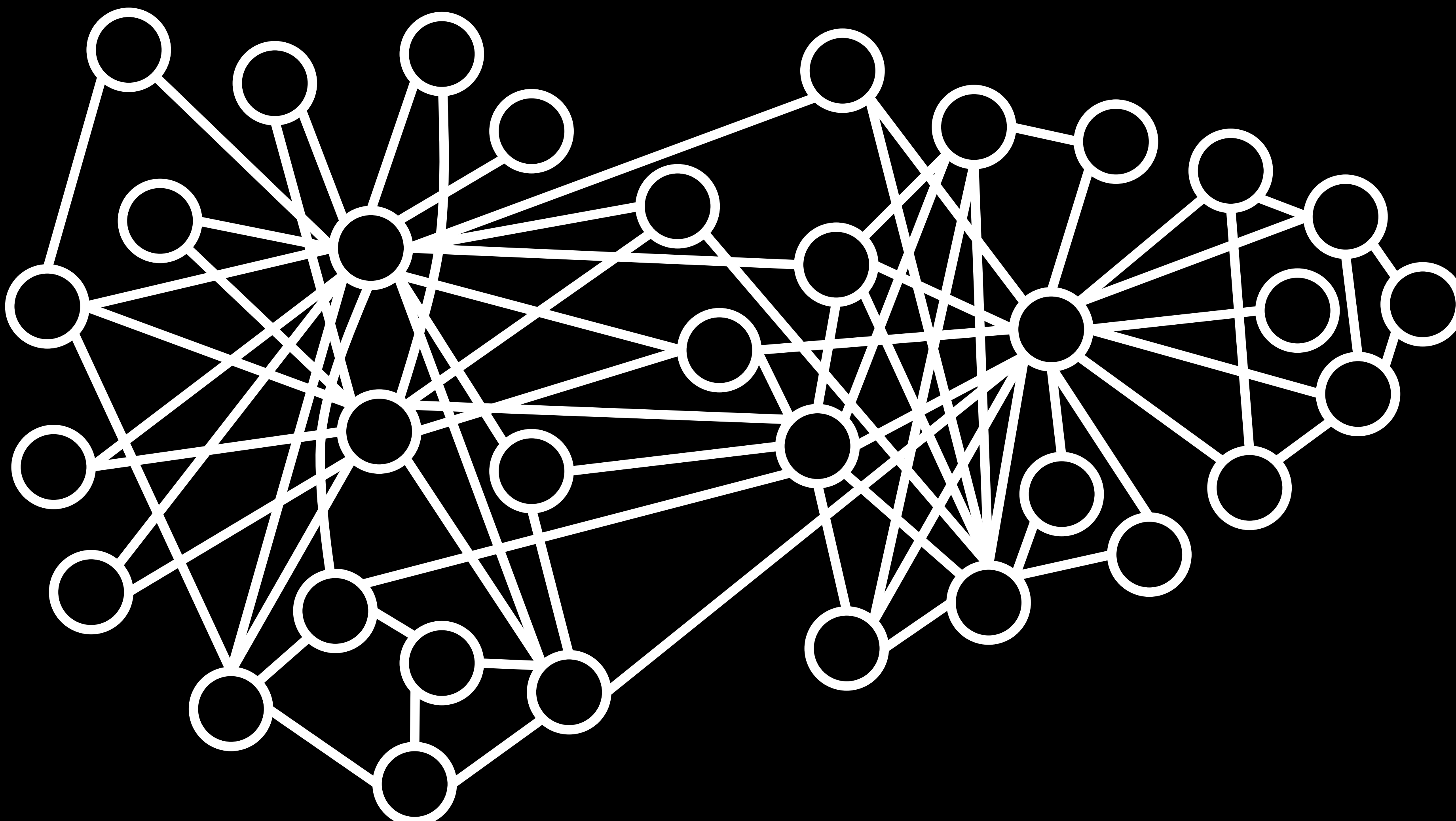
Brendan Meeder

March 26, 2009

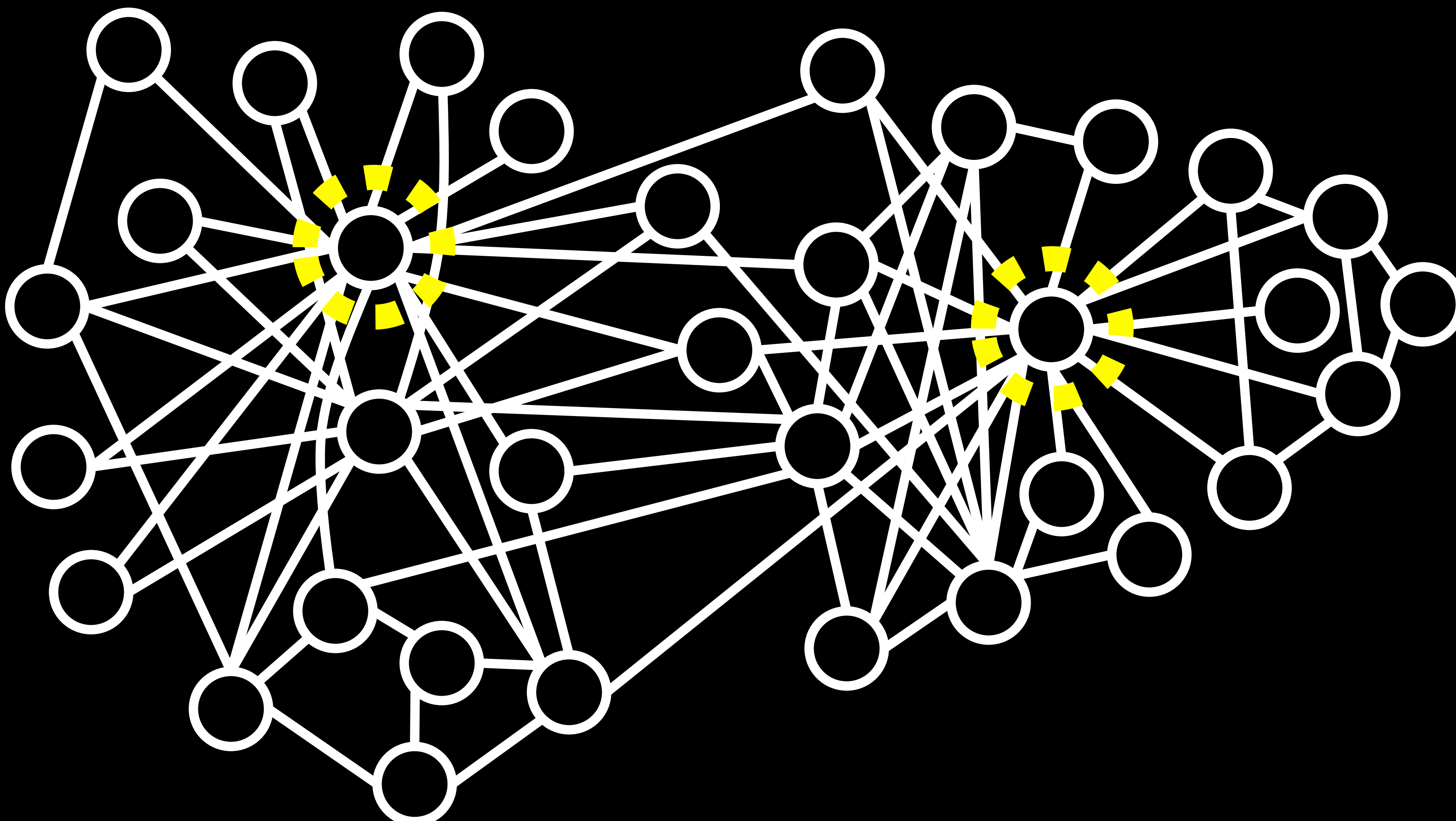
Social Networks (1977)



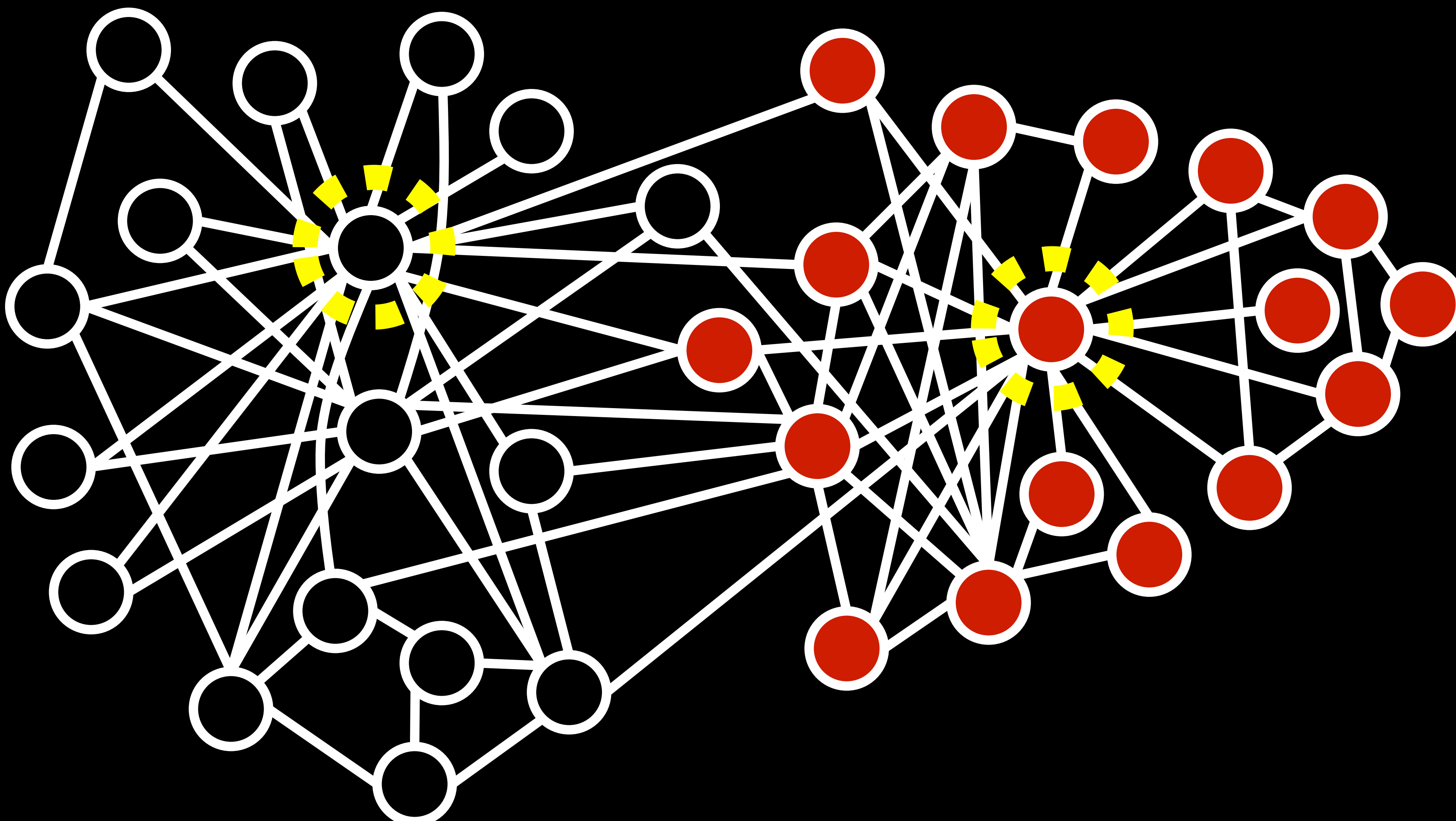
Friendships in Karate



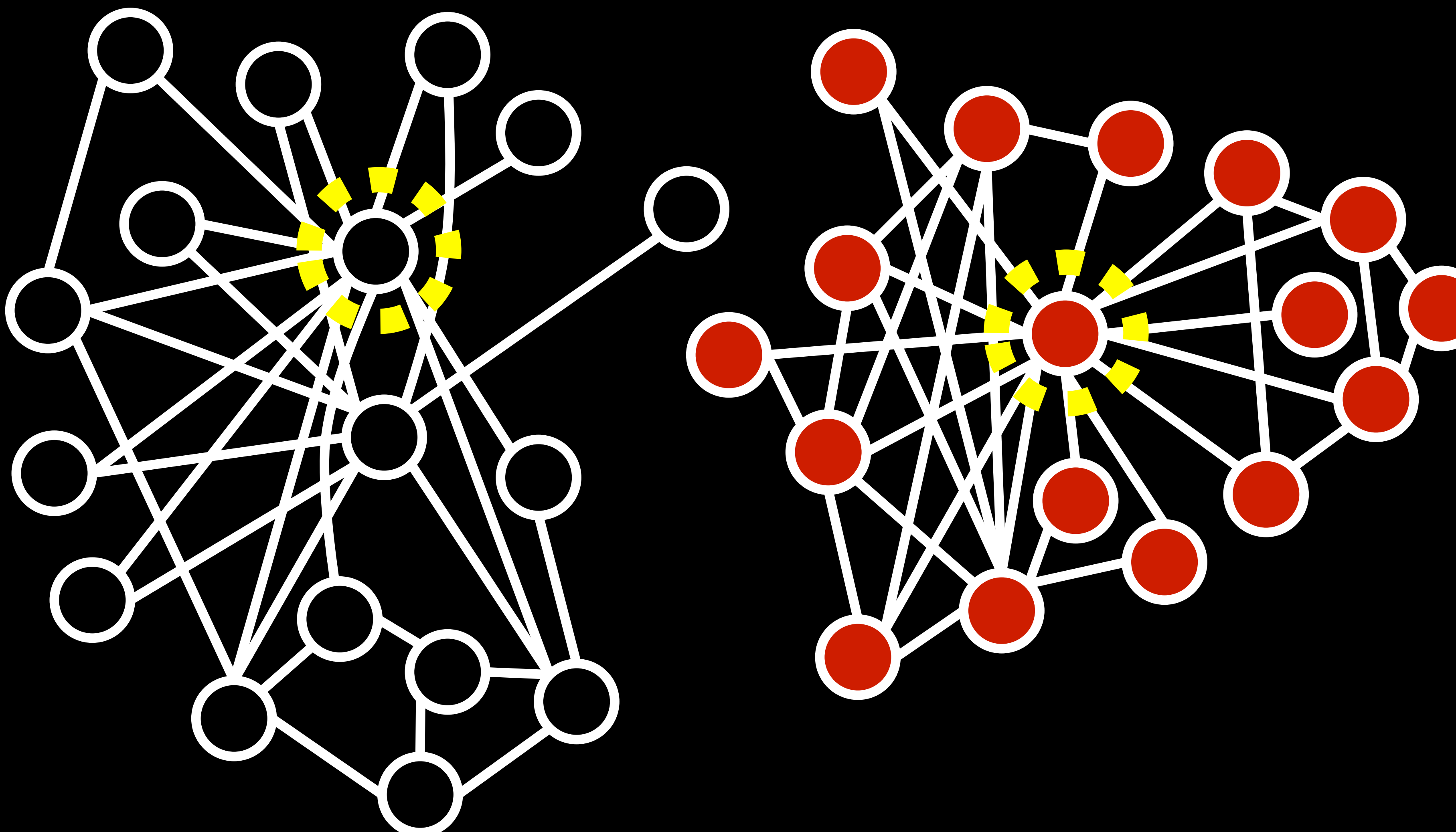
Friendships in Karate



Friendships in Karate



The Breakup



Mathematical Explanation

The split occurs along a minimum cut separating the two central figures

Individuals sided with the central figure they were closer to

Some Aspects of Social Networks

- Network structure

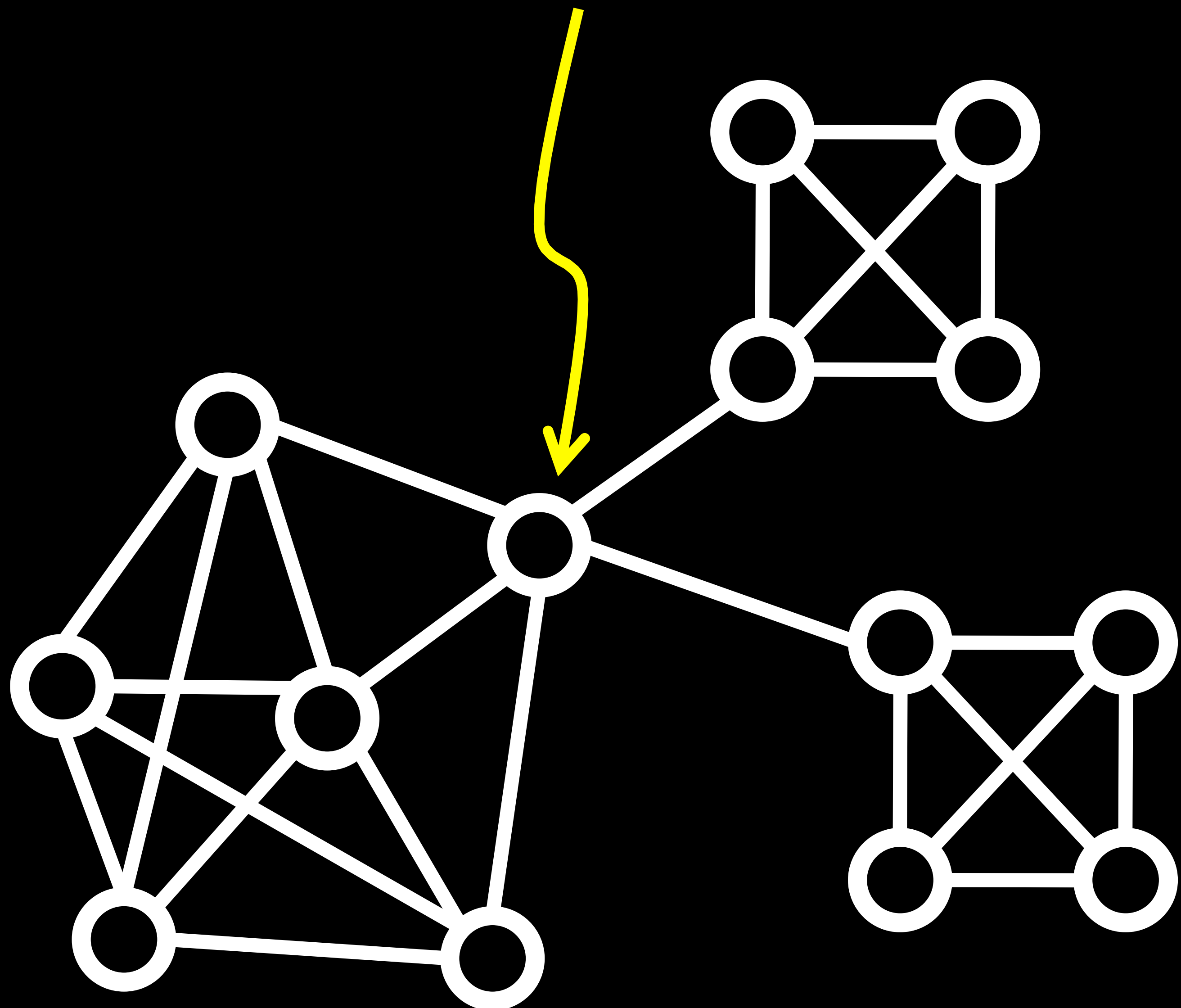
- Models of network structure
  - Random graphs, power laws, small-world phenomena

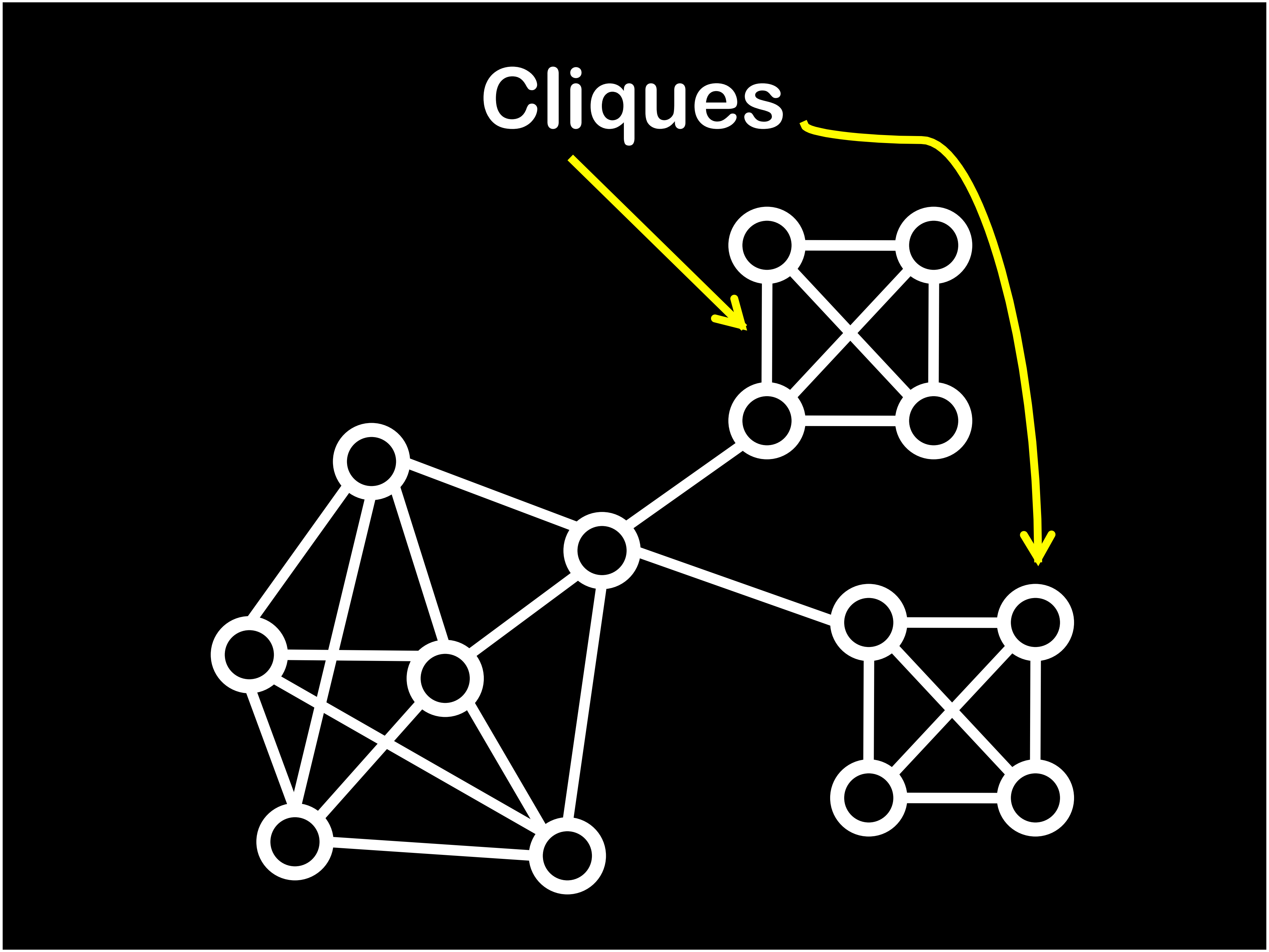
- Information flow in networks
  - Fads, rumors, diseases, distributed search

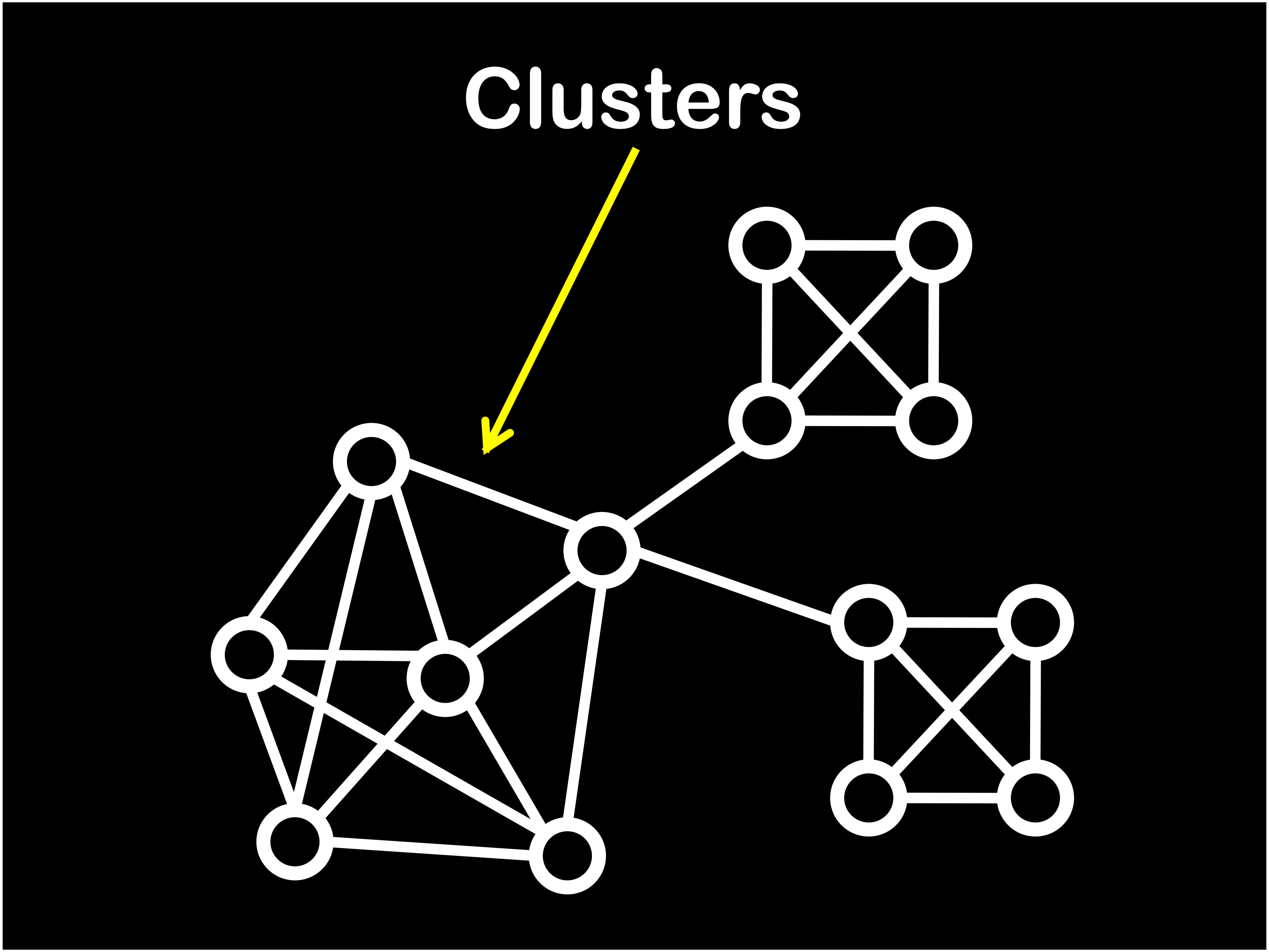
- The visual beauty of networks

Network Structures

Structural Holes

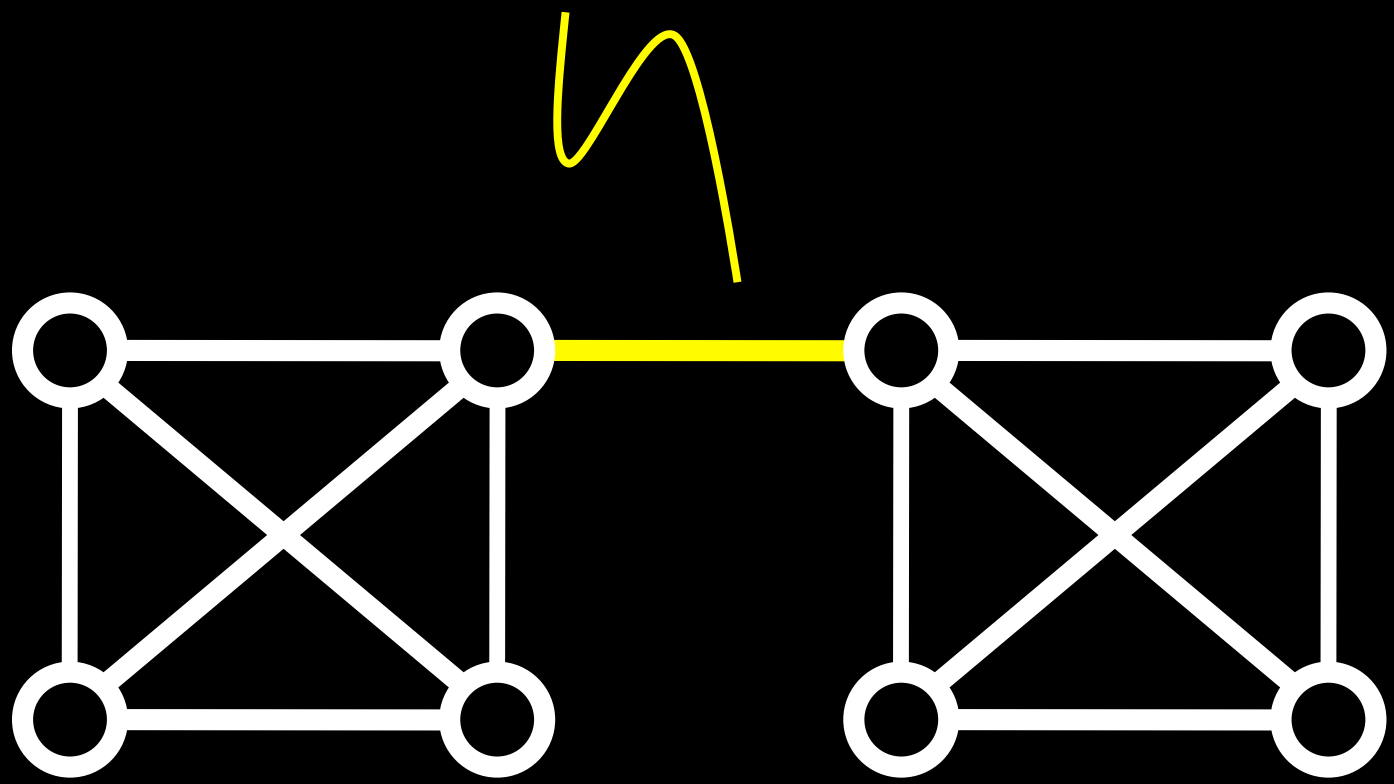



Cliques


Clusters

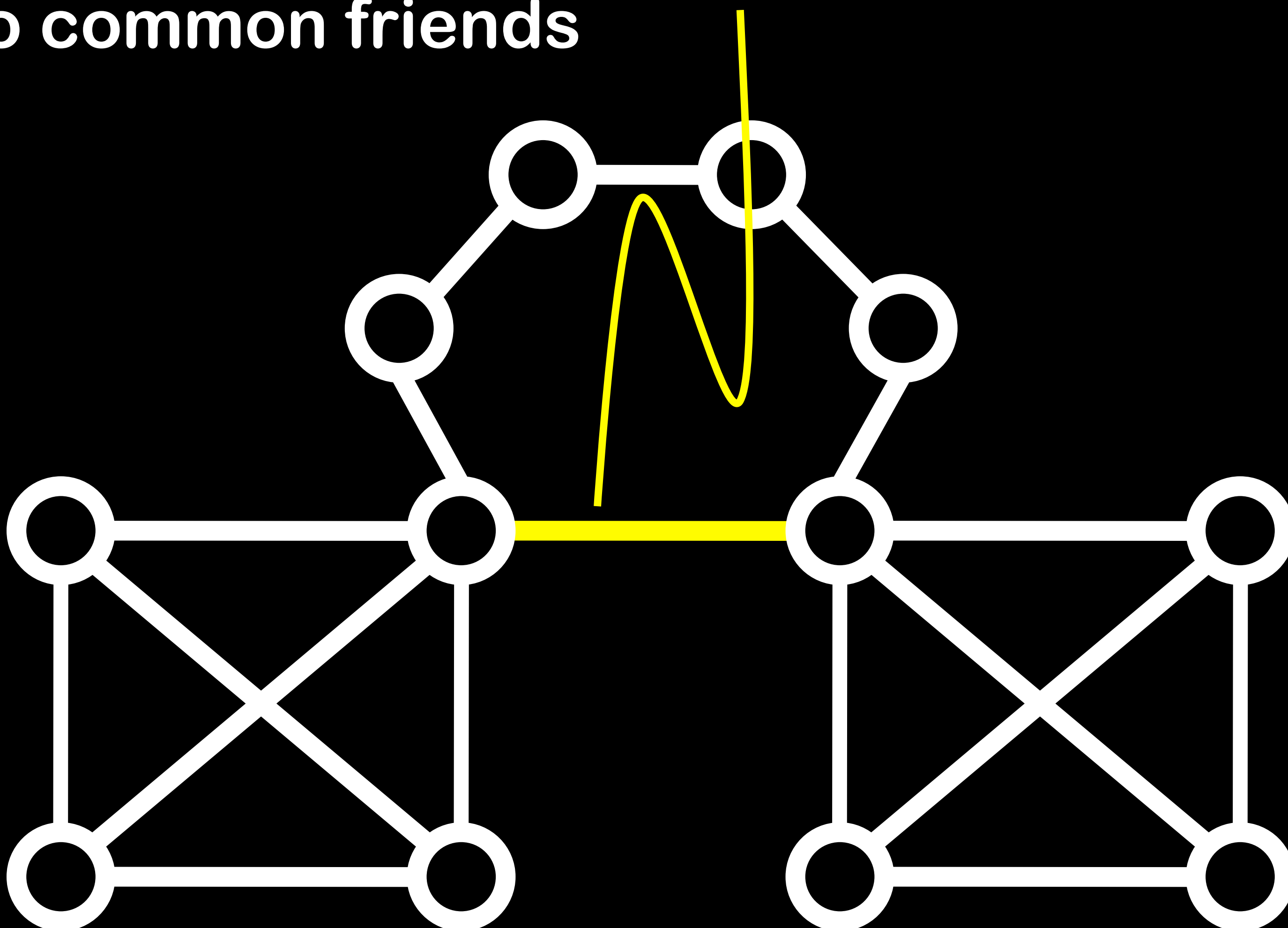
Bridges

An edge is a **bridge** if deleting it would cause its endpoints to lie in different components

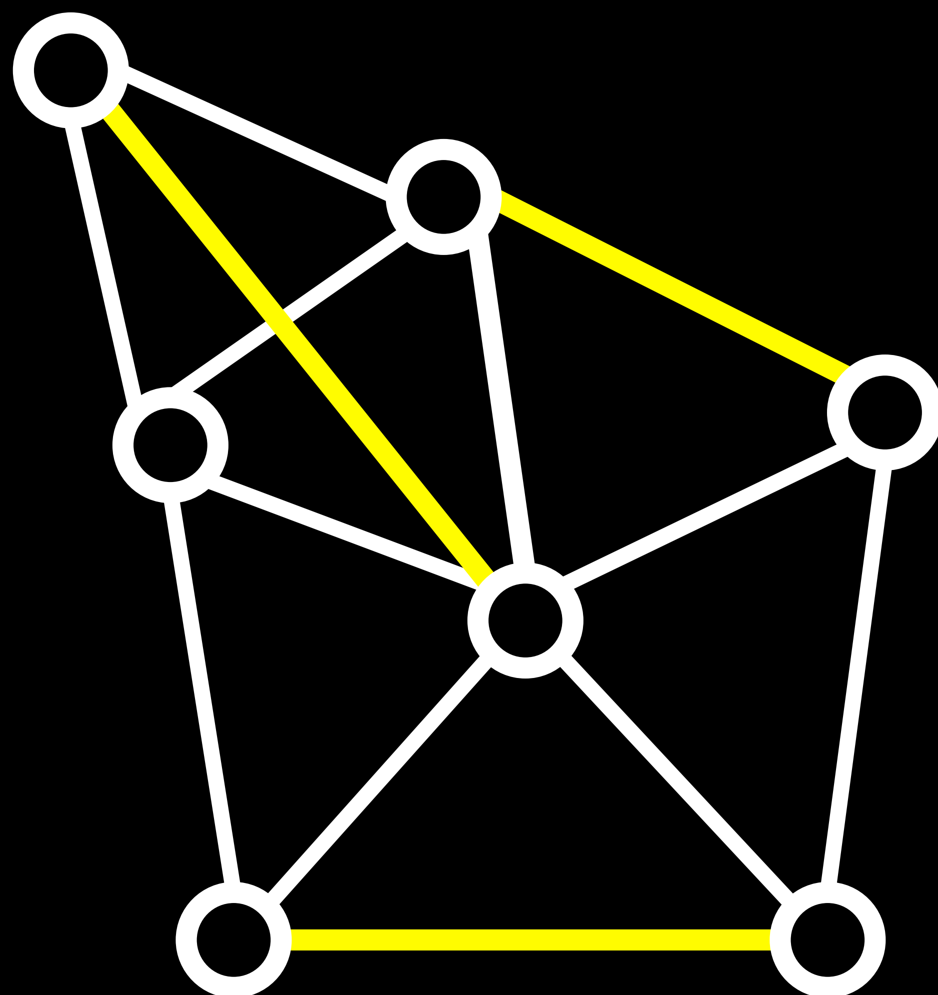


Local Bridges

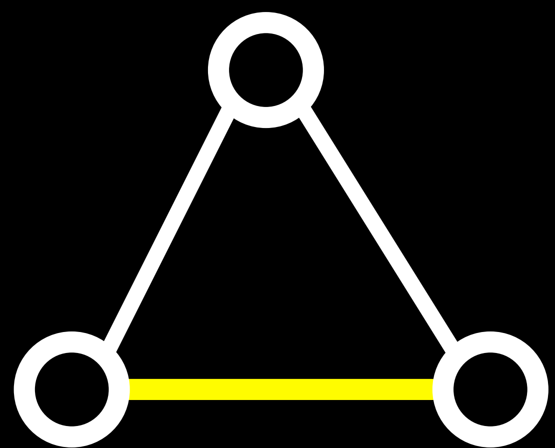
An edge is a **local bridge** if its endpoints have no common friends



How will this network evolve?

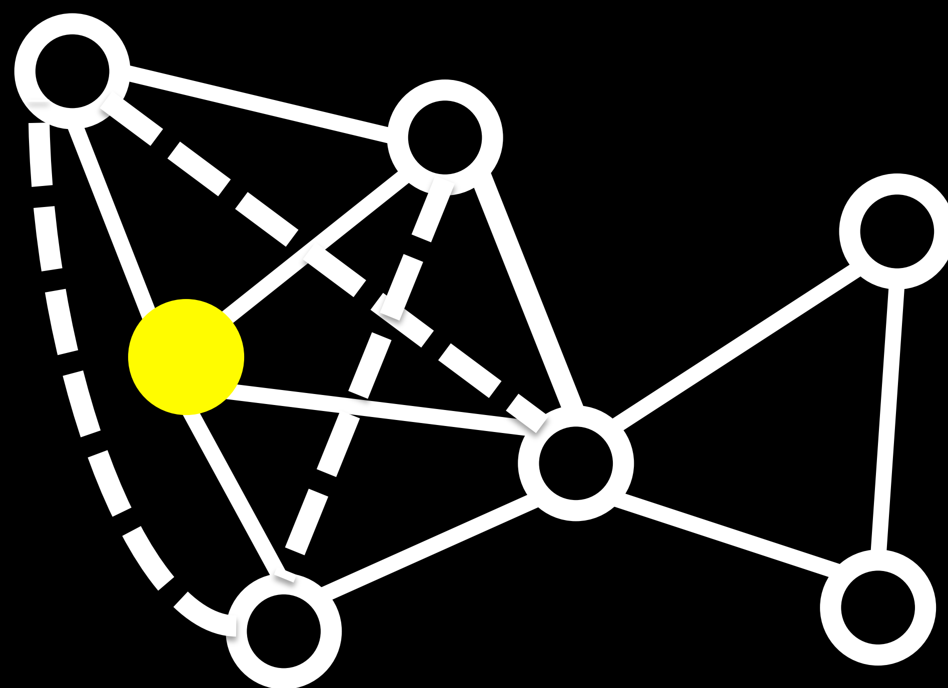


Triadic Closure: If two nodes have common neighbor, there is an increased likelihood that an edge between them forms



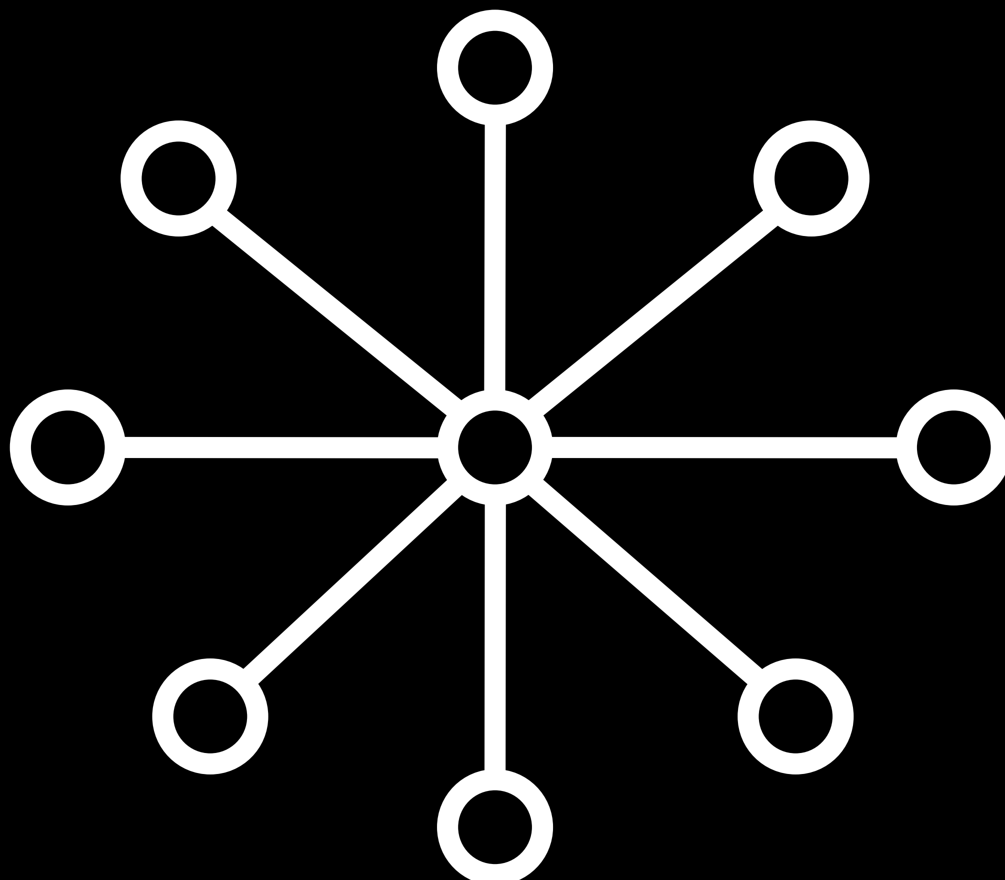
Definition: The clustering coefficient of a node v is the fraction of pairs of v's friends that are connected to each other by edges

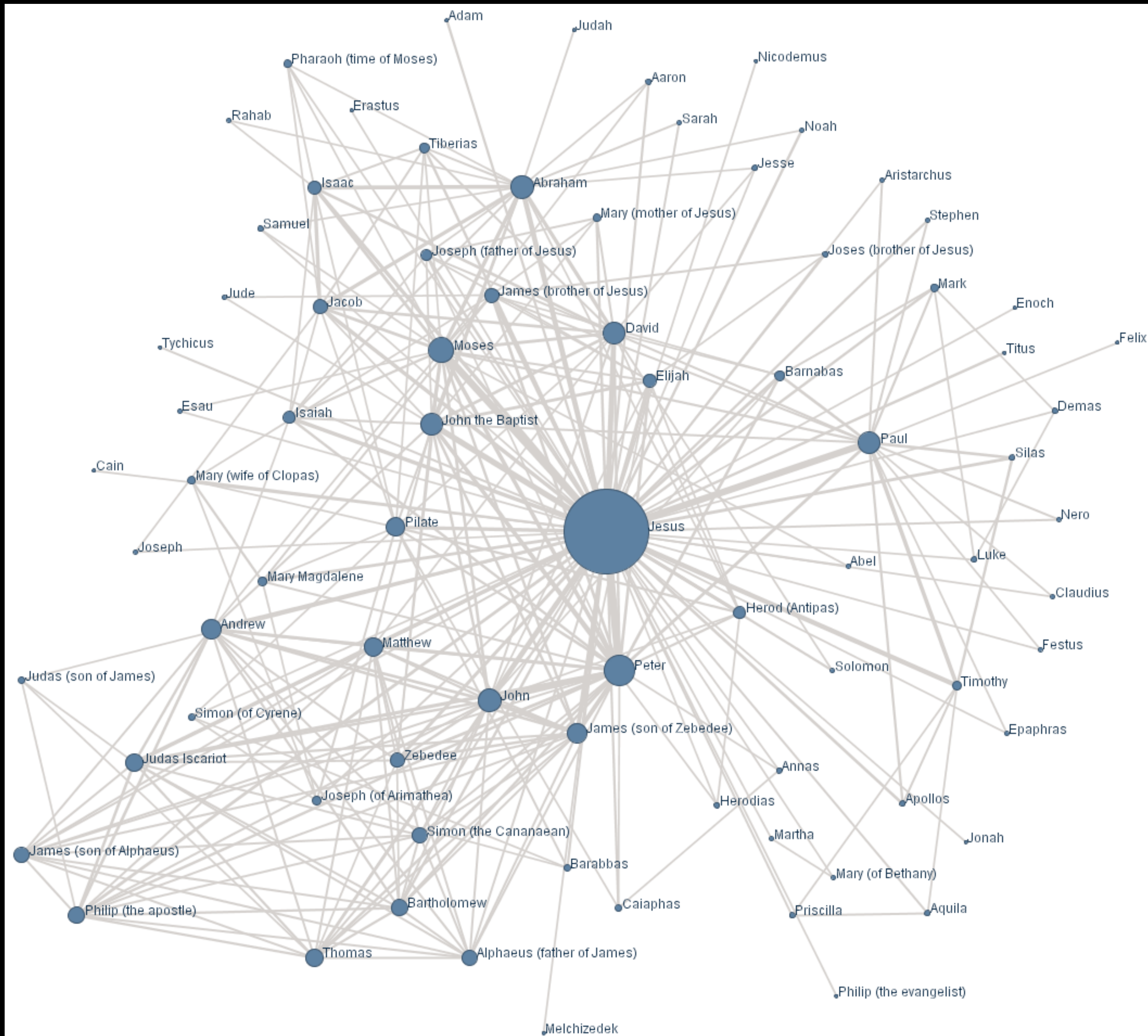
Clustering Coefficient = 1/2

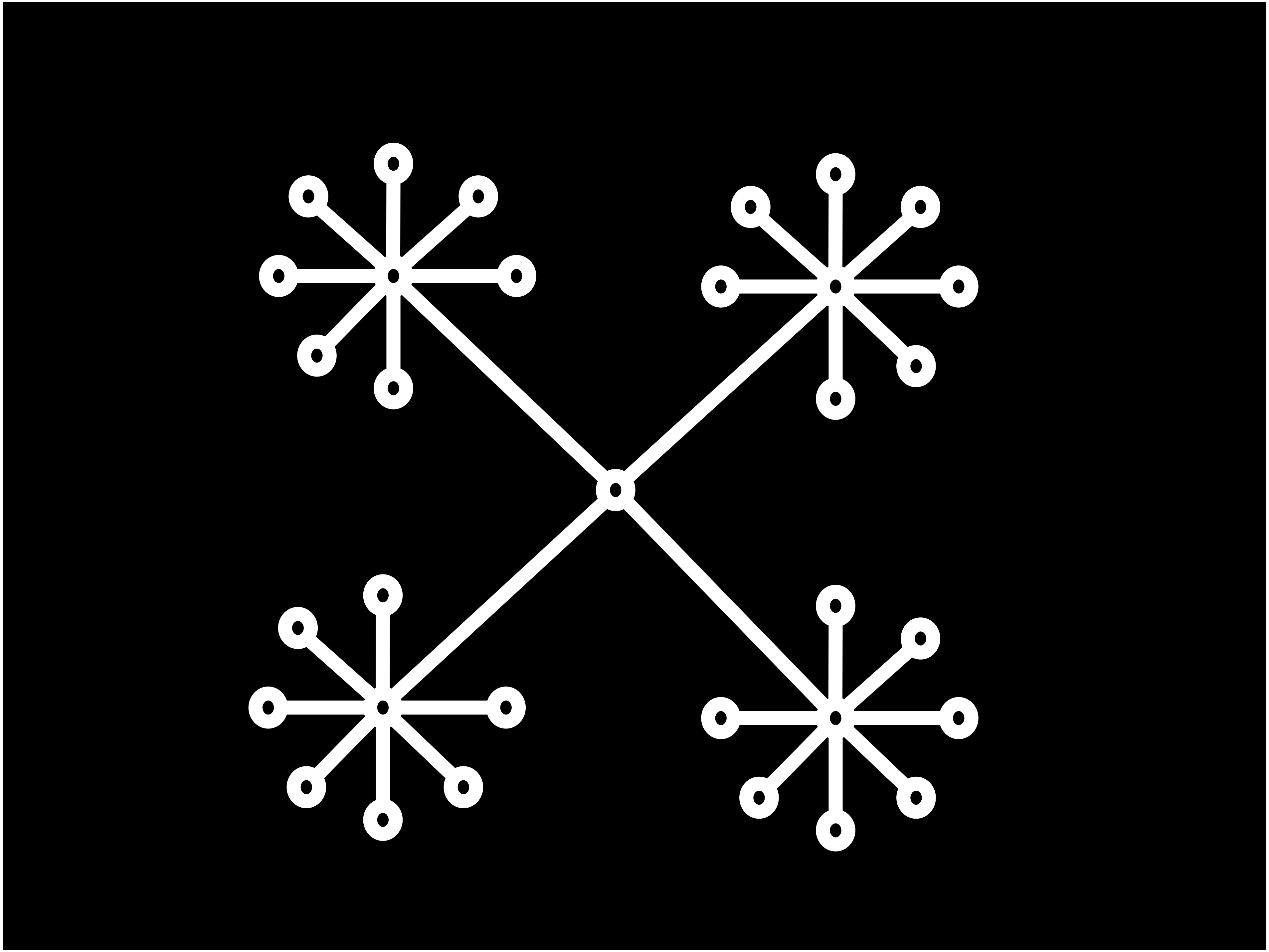


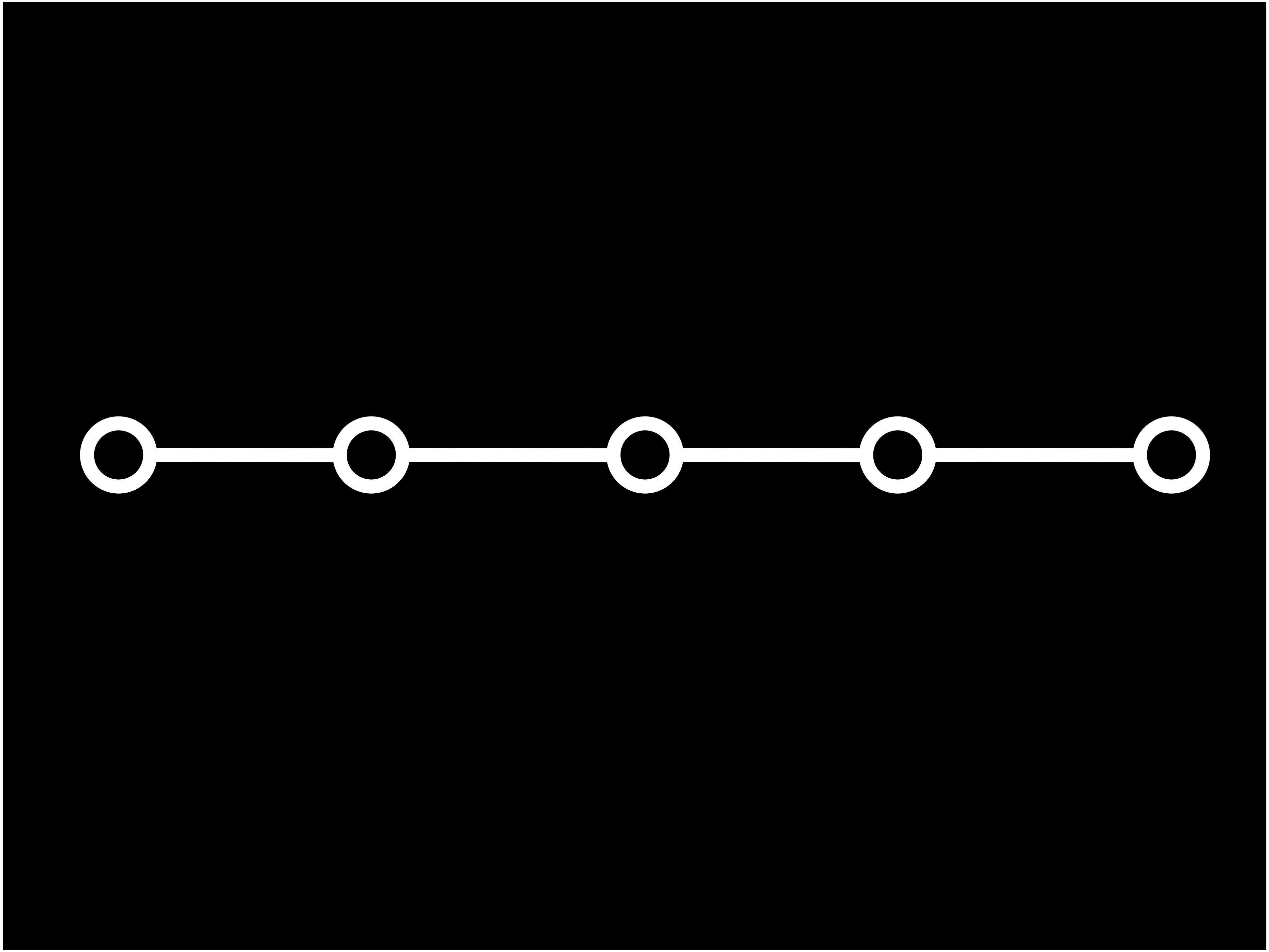
The higher the clustering coefficient of a node, the more strongly triadic closure is acting on it

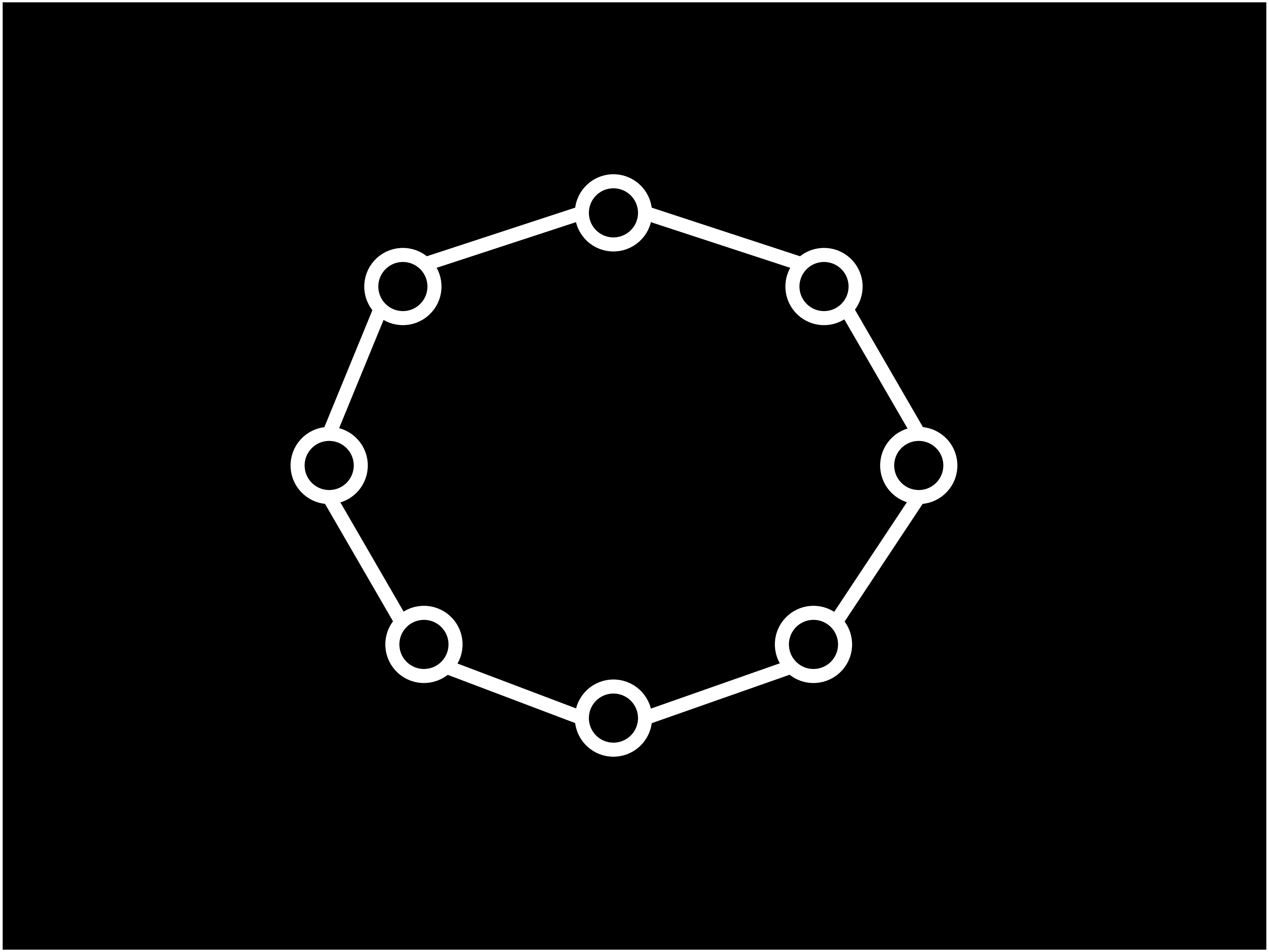
What’s the “most central node”?



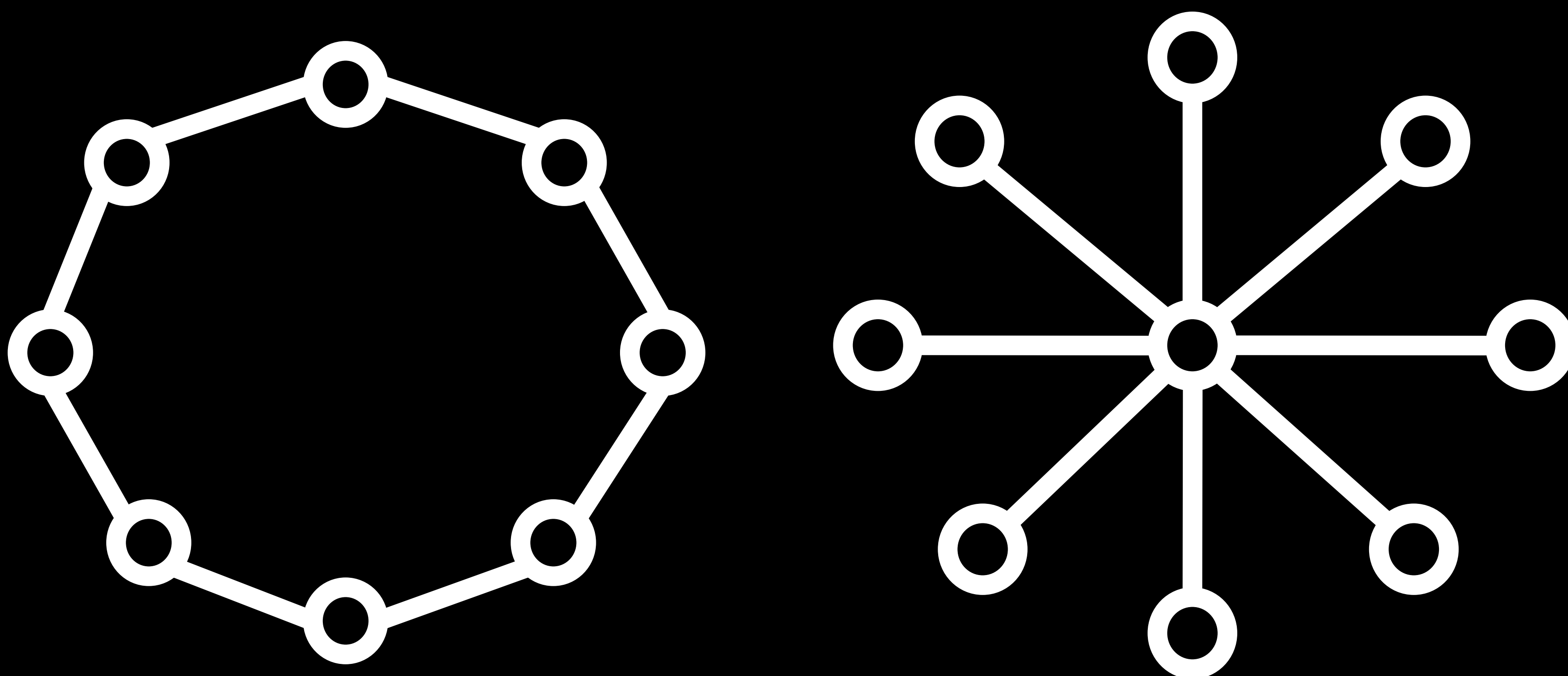

Adam
Judah
Pharaoh (time of Moses)
Nicodemus
Aaron
Erastus
Sarah
Rahab
Noah
Tiberias
Jesse
Aristarchus
Isaac
Abraham
Mary (mother of Jesus)
Stephen
Samuel
Joseph (father of Jesus)
Joses (brother of Jesus)
James (brother of Jesus)
Mark
Jude
Jacob
Enoch
David
Felix
Tychicus
Moses
Titus
Elijah
Barnabas
Esau
Isaiah
Demas
John the Baptist
Paul
Silas
Cain
Mary (wife of Clopas)
Pilate
Jesus
Nero
Joseph
Abel
Luke
Mary Magdalene
Claudius
Herod (Antipas)
Andrew
Matthew
Festus
Peter
Solomon
Judas (son of James)
Timothy
John
Simon (of Cyrene)
James (son of Zebedee)
Epaphras
Zebedee
Judas Iscariot
Annas
Joseph (of Arimathea)
Apollos
Herodias
Simon (the Cananaean)
Martha
Jonah
James (son of Alphaeus)
Barabbas
Mary (of Bethany)
Caiaphas
Philip (the apostle)
Bartholomew
Priscilla
Aquila
Thomas
Alphaeus (father of James)
Philip (the evangelist)
Melchizedek







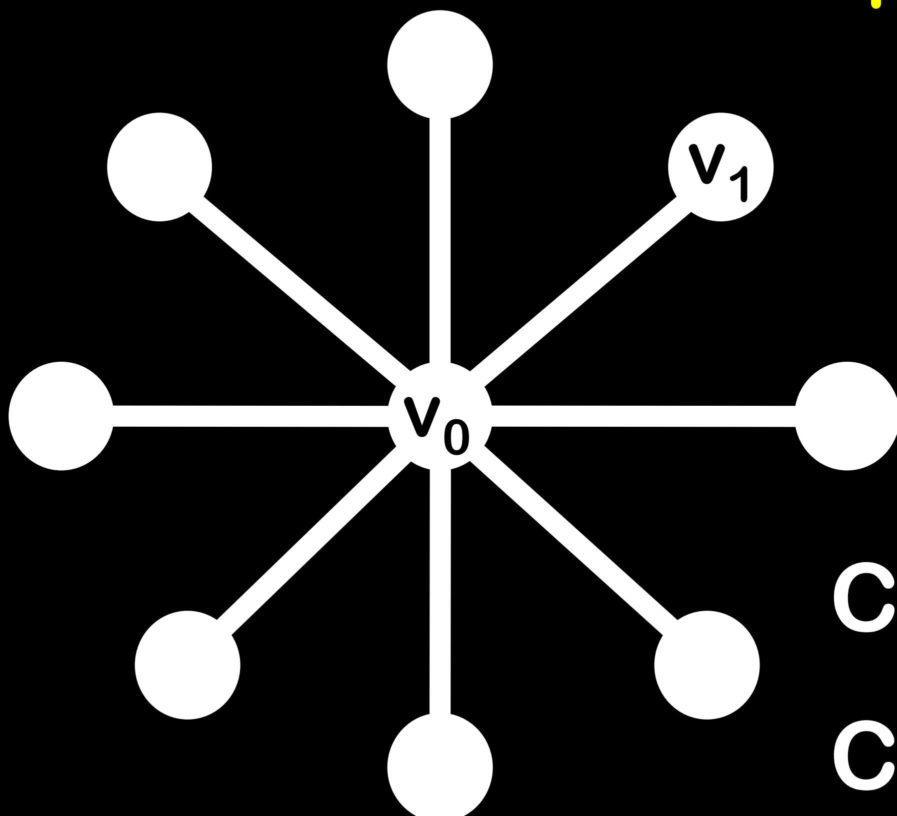
Which Graph is More "Centralized"?



Degree Centrality of a Node

$$C_D(v) = d(v)/(n-1)$$

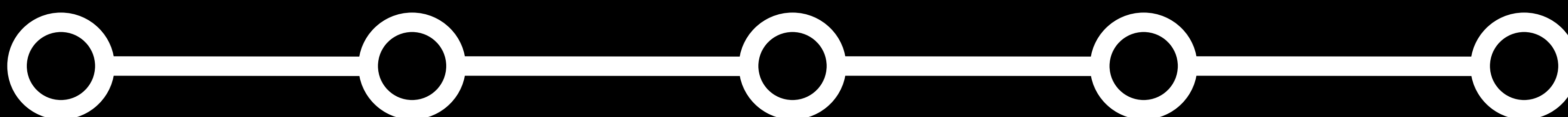
↑ Number of nodes



$C_D(v_0) = 1$

$C_D(v_1) = 1/8$

Degree centrality is easy to calculate, but it’s not a very good measure



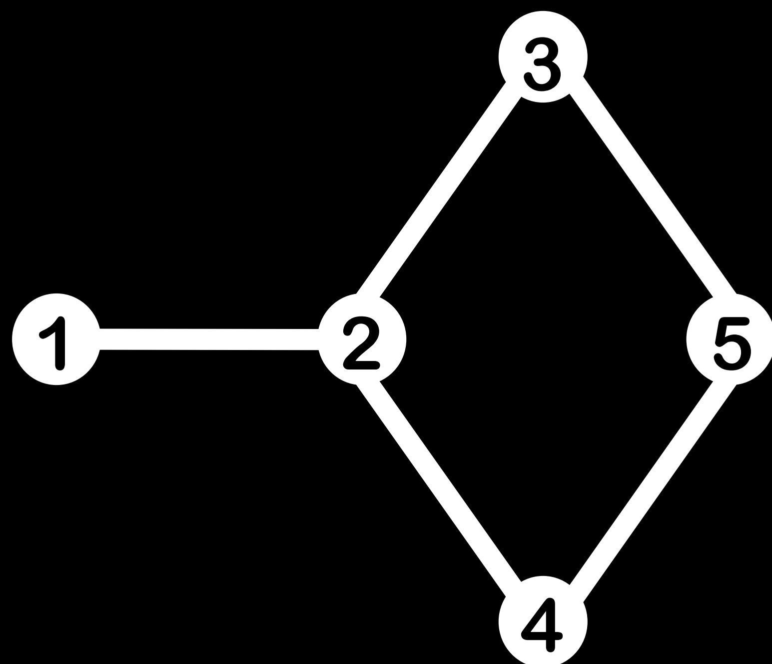
Betweenness Centrality of a Node

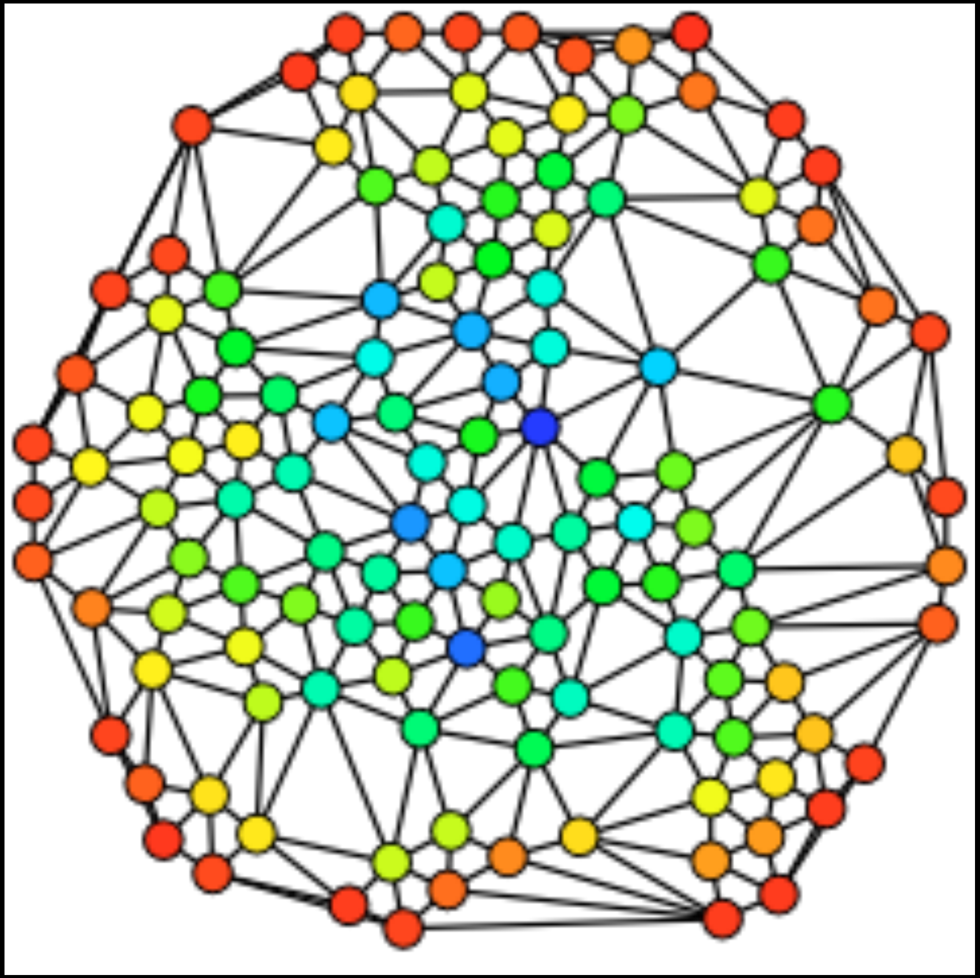
$Short_{jk}$ = Number of shortest paths from v_j to v_k

$Short_{jk}(v_i)$ = Number of shortest paths from v_j to v_k that pass through v_i

$$C_B(v_i) = \sum_{j<k} short_{jk}(v_i)/short_{jk}$$

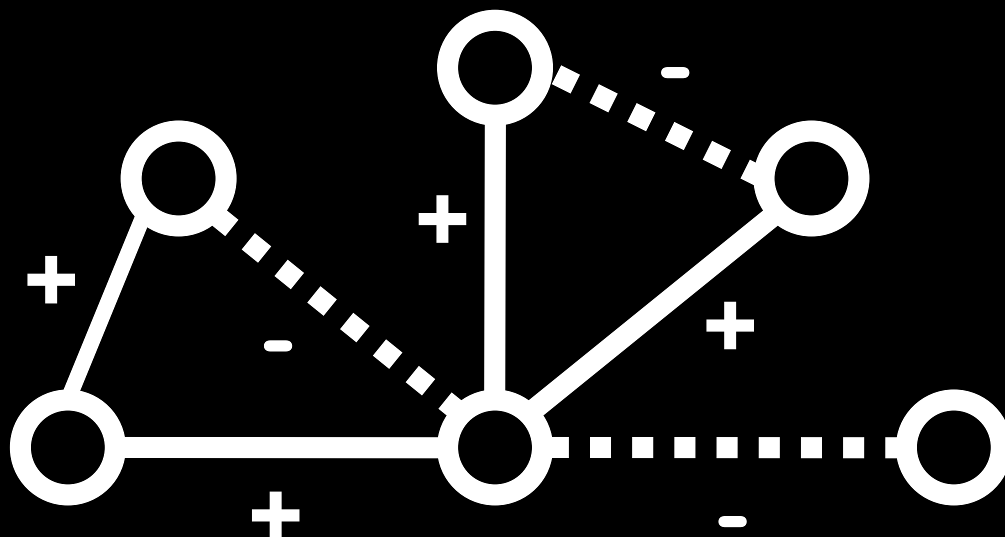
$$C_B(v_i) = \sum_{j<k} short_{jk}(v_i)/short_{jk}$$



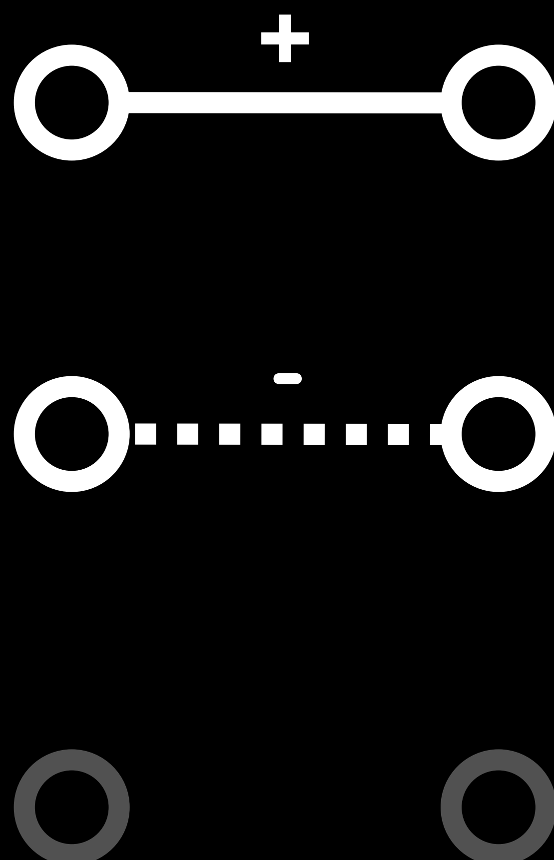


Networks with Extra Structure

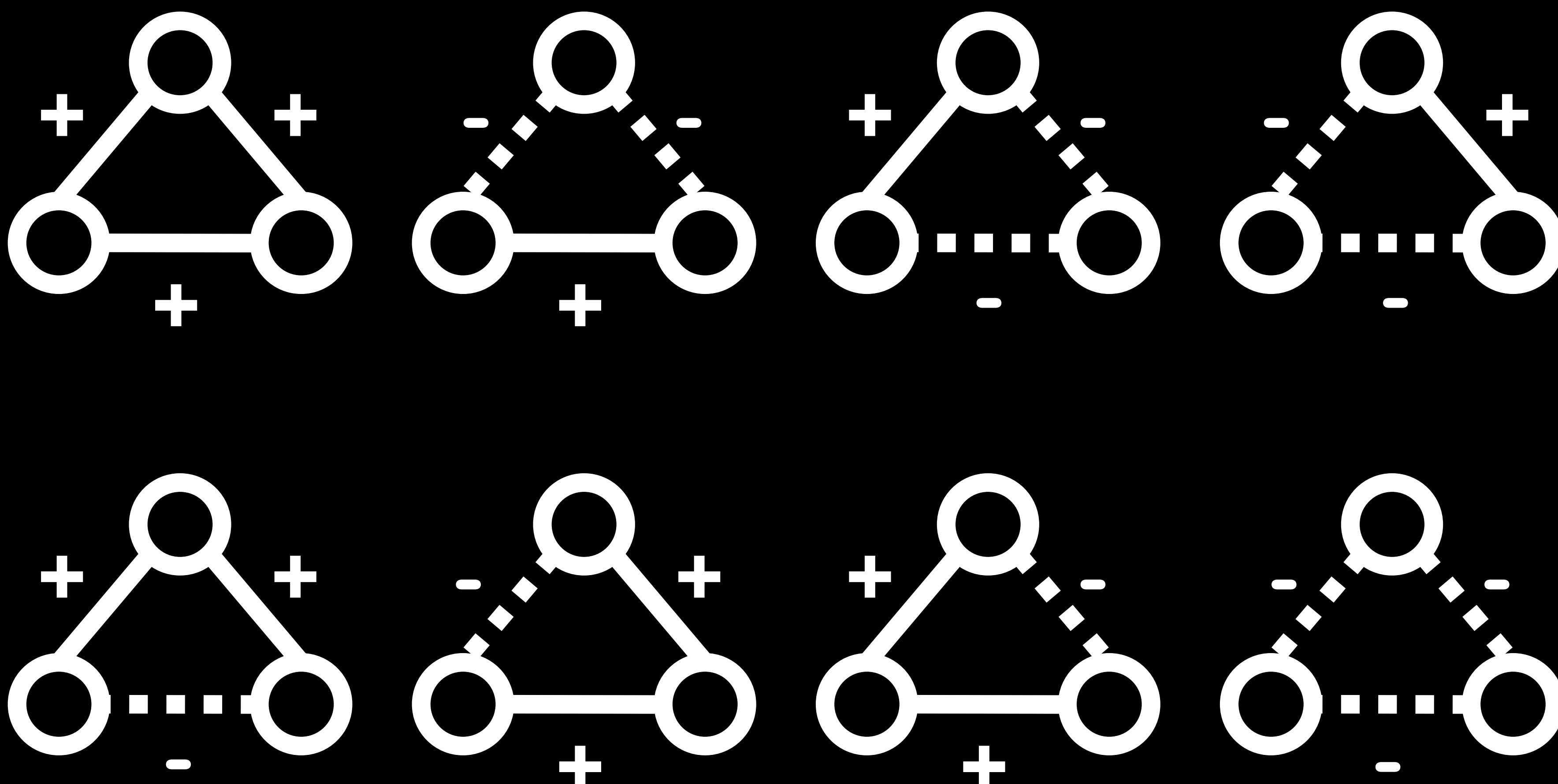
Signed Graphs

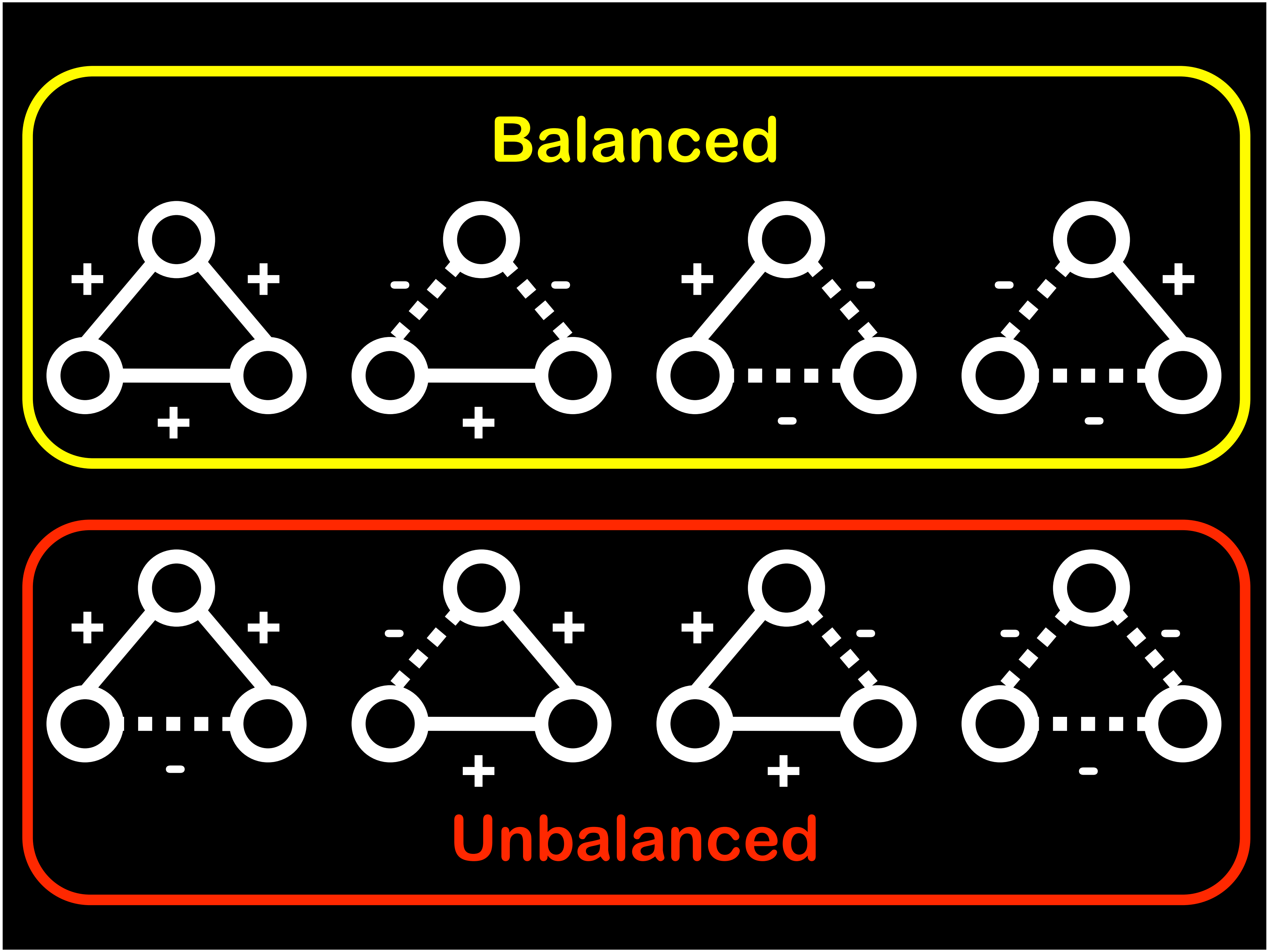


Two-Node Signed Graphs

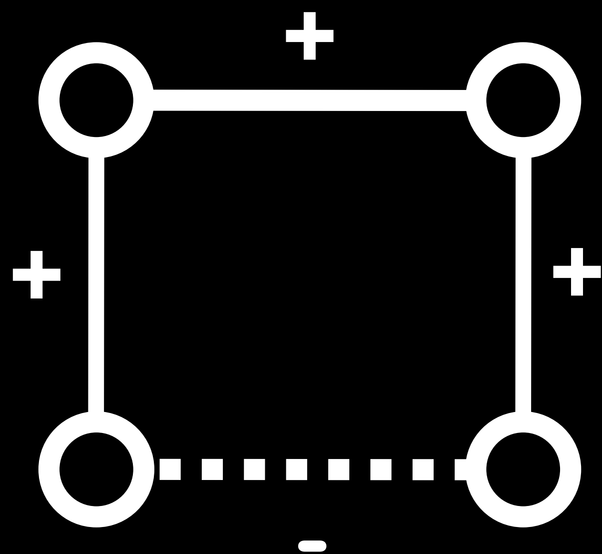


Complete Three-Node Signed Graphs

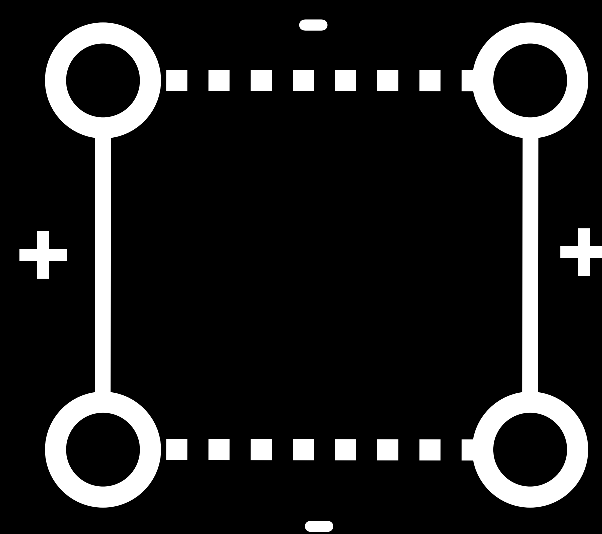



Balanced
+
+
+
-
-
+
+
-
-
-
+
-
+
+
-
-
-
+
+
-
+
+
-
-
-
Unbalanced

Four Node Cycles



Unbalanced

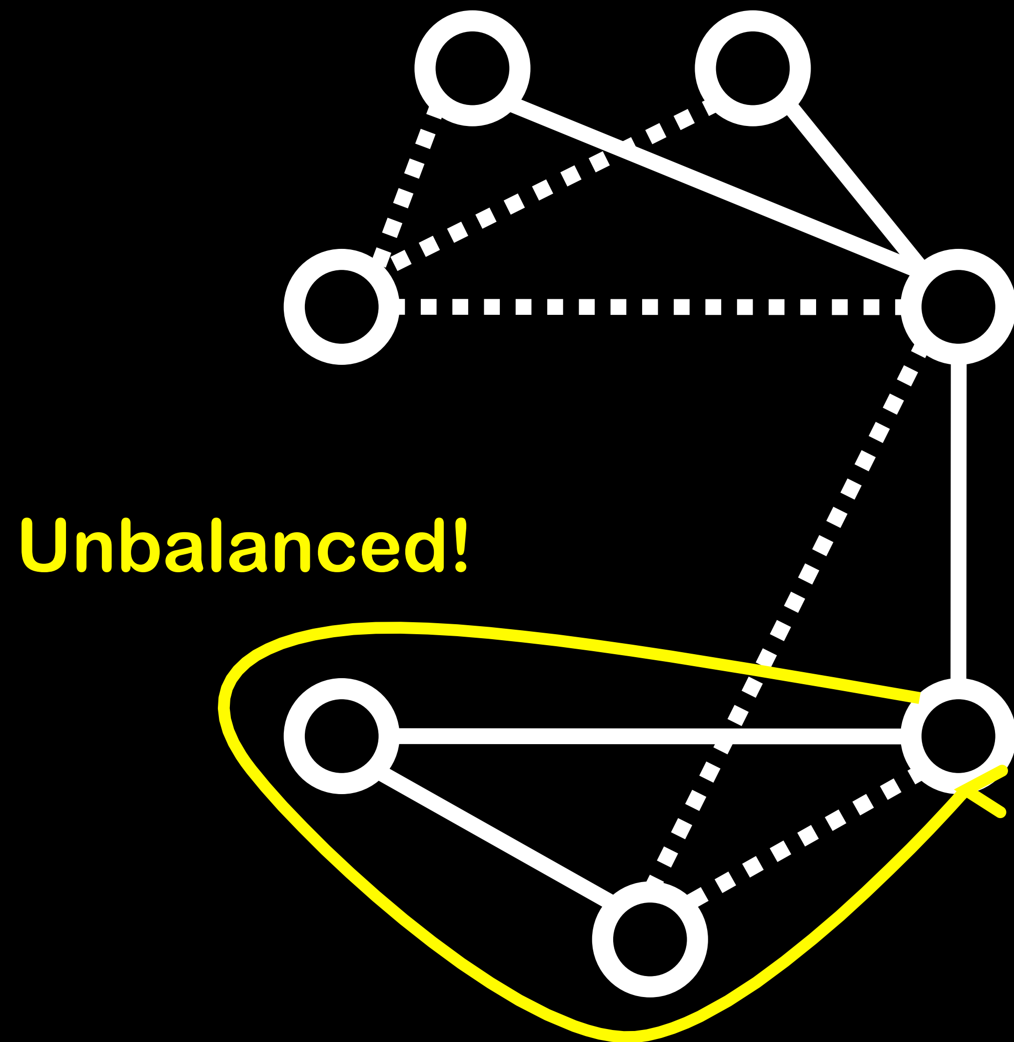


Balanced

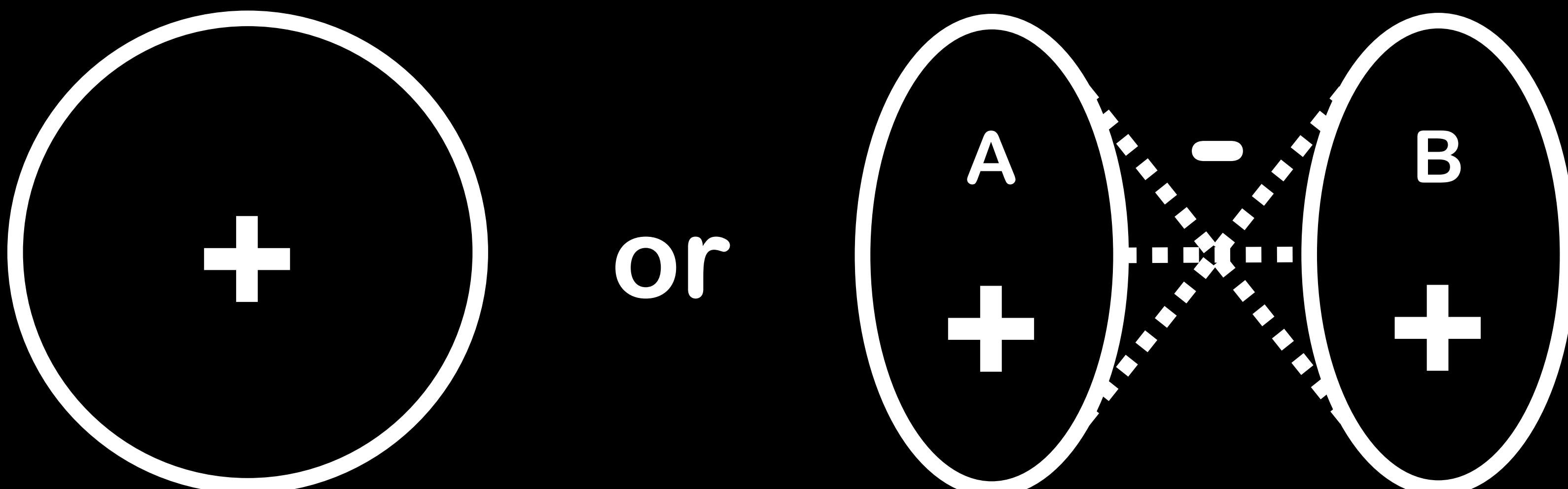
Definition: A cycle is balanced if the product of its signs is positive

Definition: A graph is balanced if all its cycles are balanced

Example



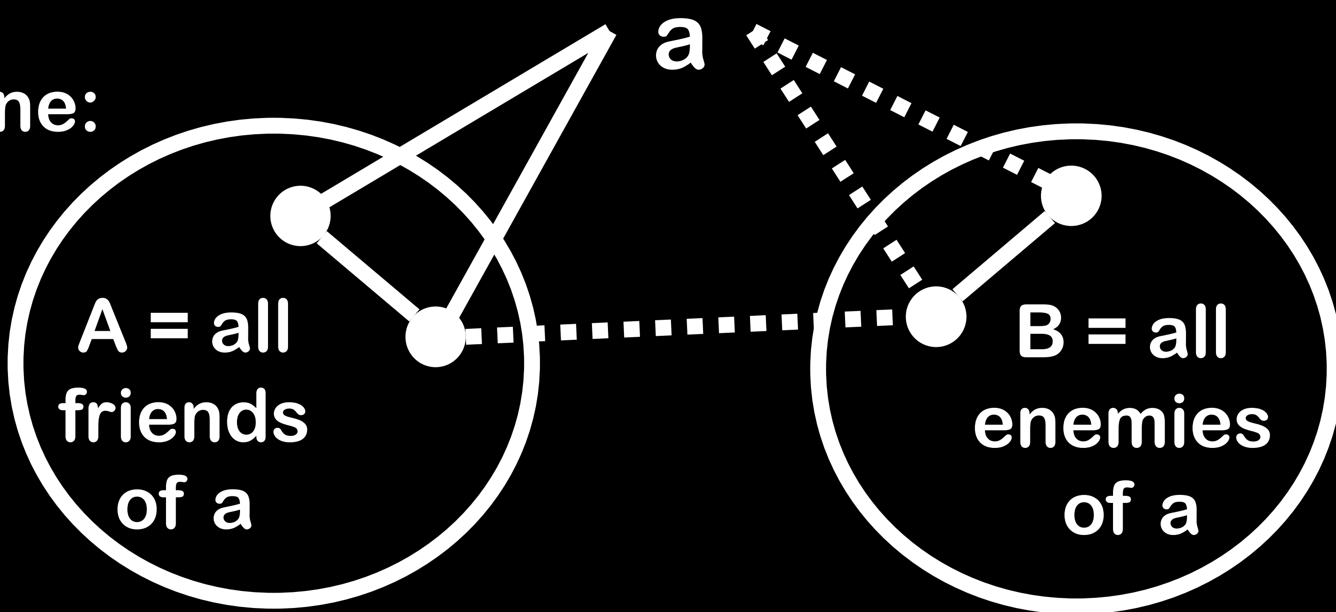
Theorem: If every 3-cycle in a signed complete graph is balanced, then either (1) all nodes are friends, or (2) the nodes can be divided into two groups, A and B, such that every pair of people in A like each other, every pair of people in B like each other, and everyone in A is the enemy of everyone in B.



Proof:

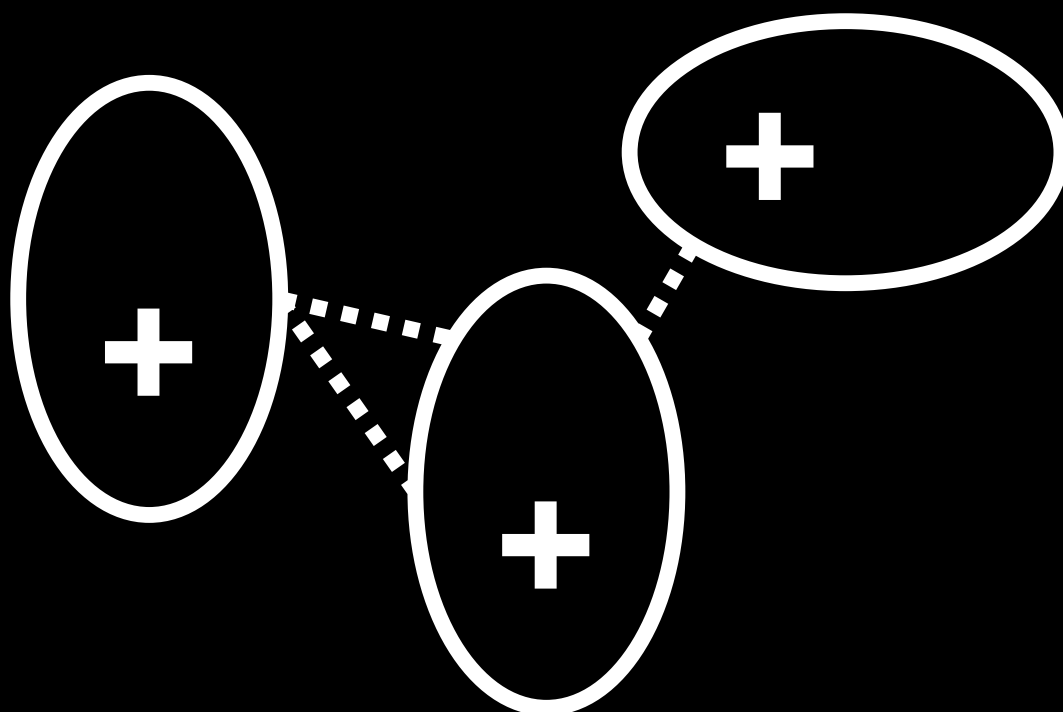
Pack any node:

Define:



1. Every two nodes in A are friends
2. Every two nodes in B are friends
3. Every node in A is an enemy of every node in B

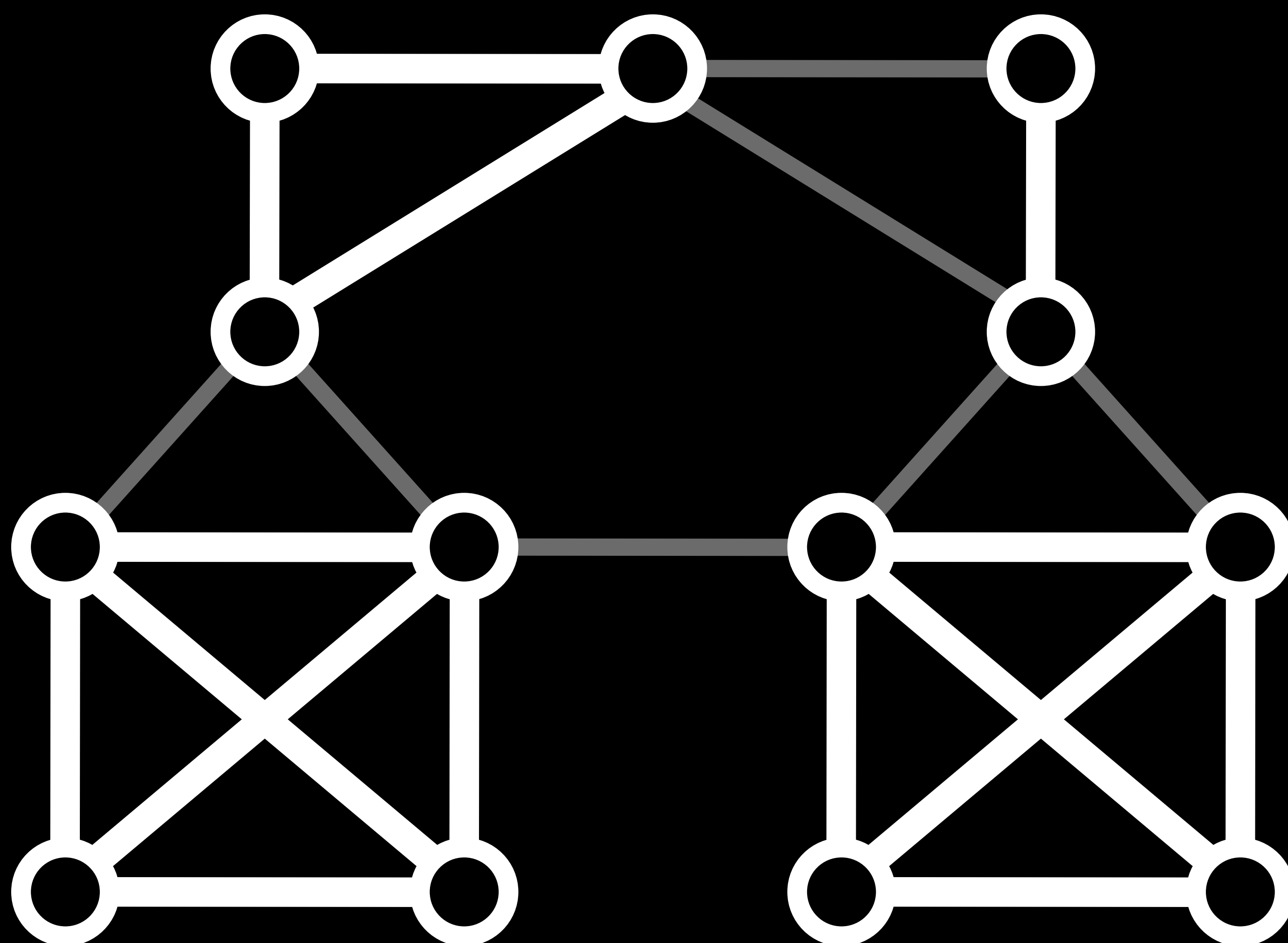
Definition: A signed graph is clusterable if the nodes can be partitioned into a finite number of subsets such every positive edge is between nodes of the same subset, and every negative edge is between nodes of different subsets



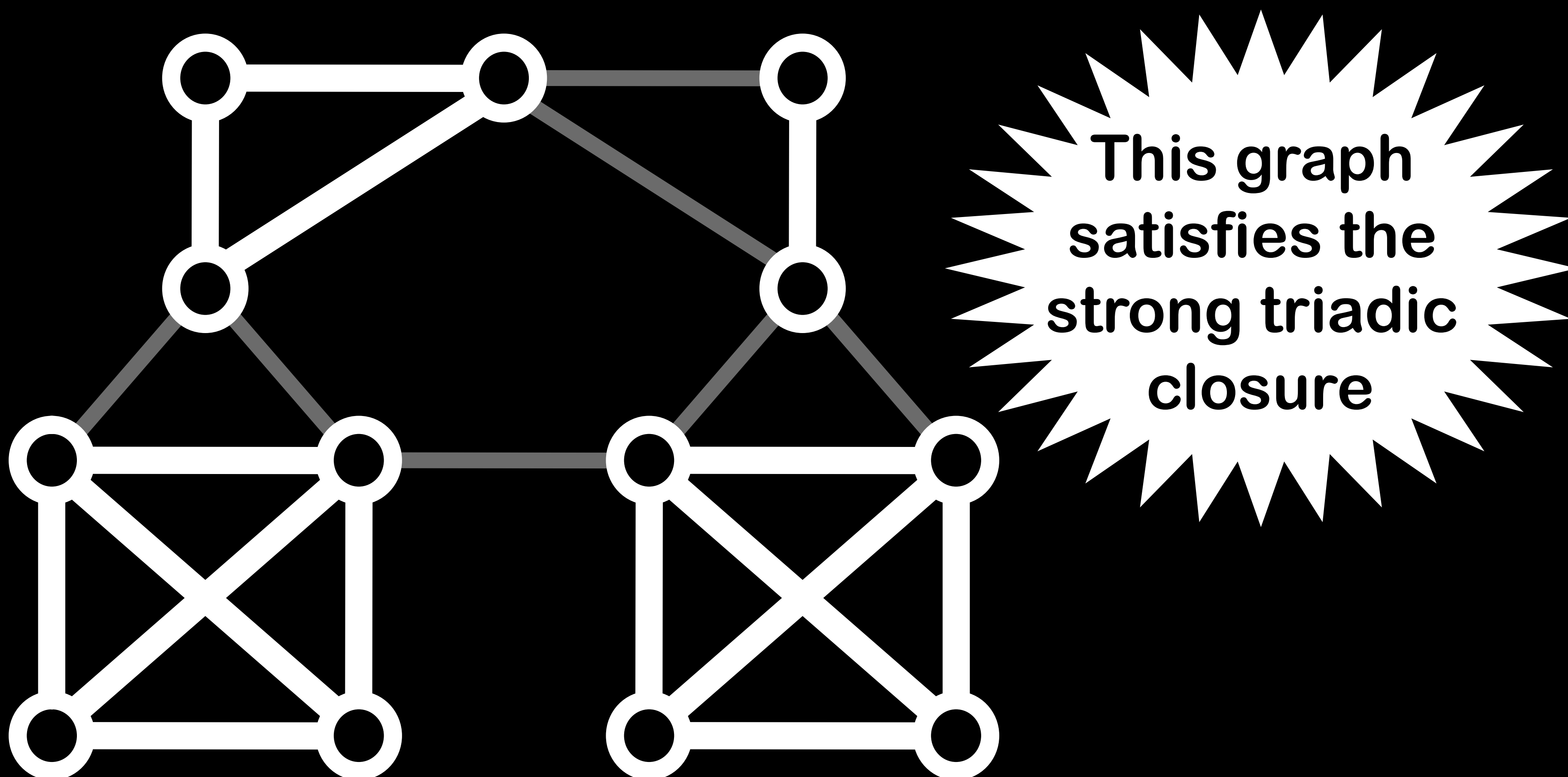
Theorem: A signed graph has a clustering if and only if the graph contains no cycles which have exactly one negative edge

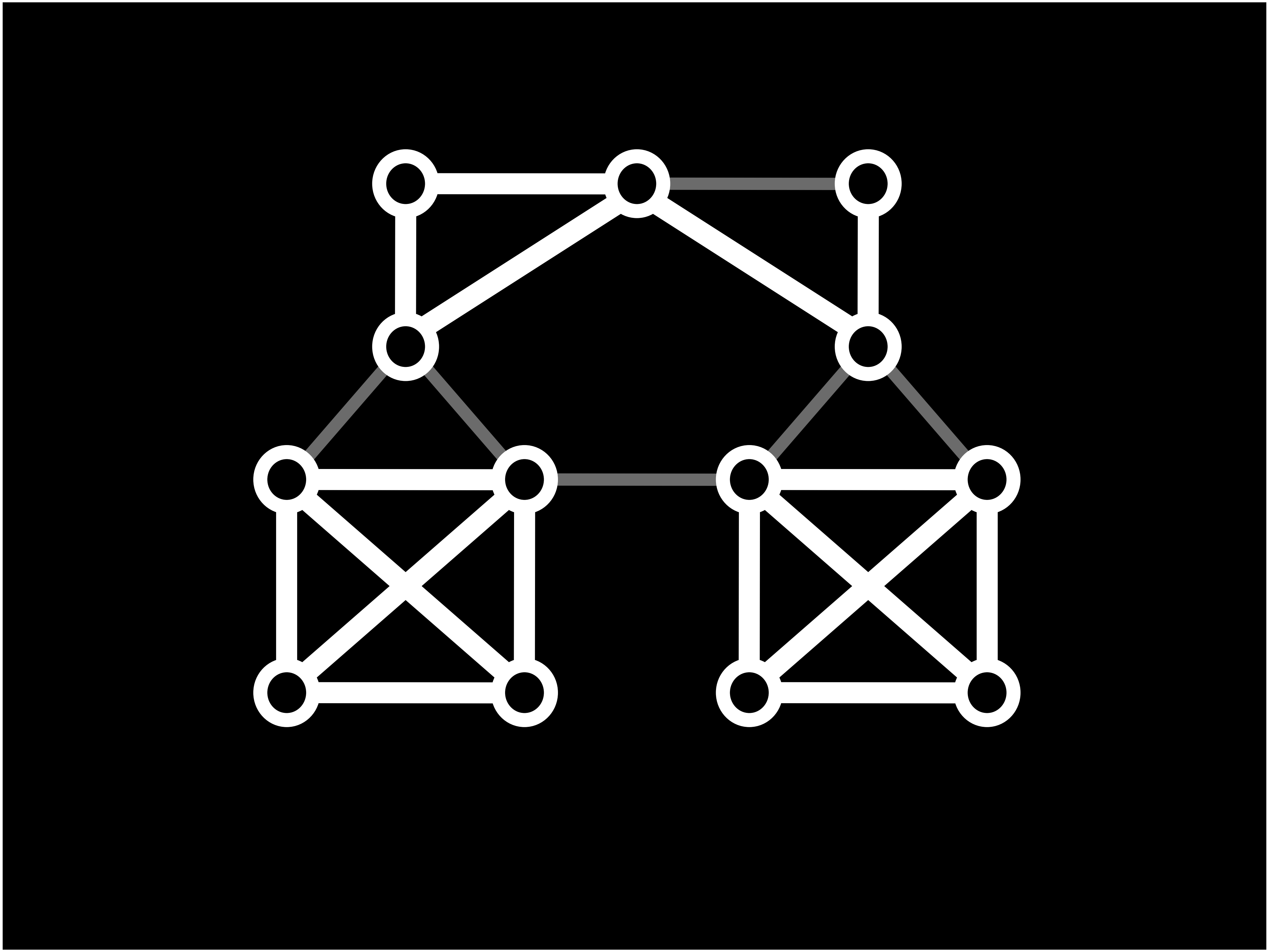
Where do the best job leads come from: your close friends or your acquaintances?

Weak Versus Strong Ties



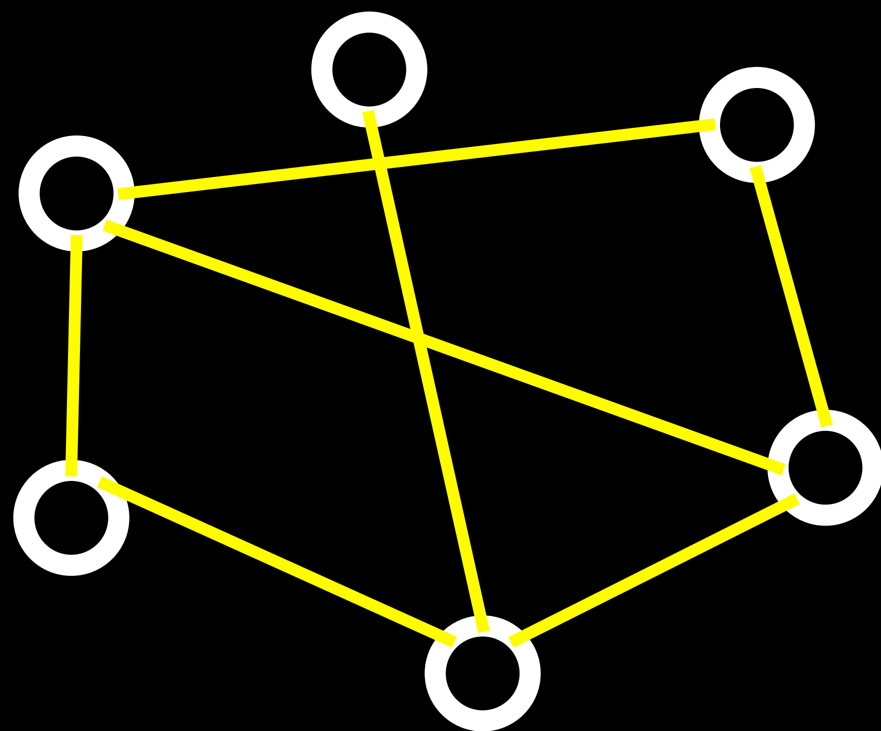
Definition: Node v satisfies the **Strong Triadic Closure** if, for any two nodes u and w to which it has strong ties, there is an edge between u and w (which can be either weak or strong)

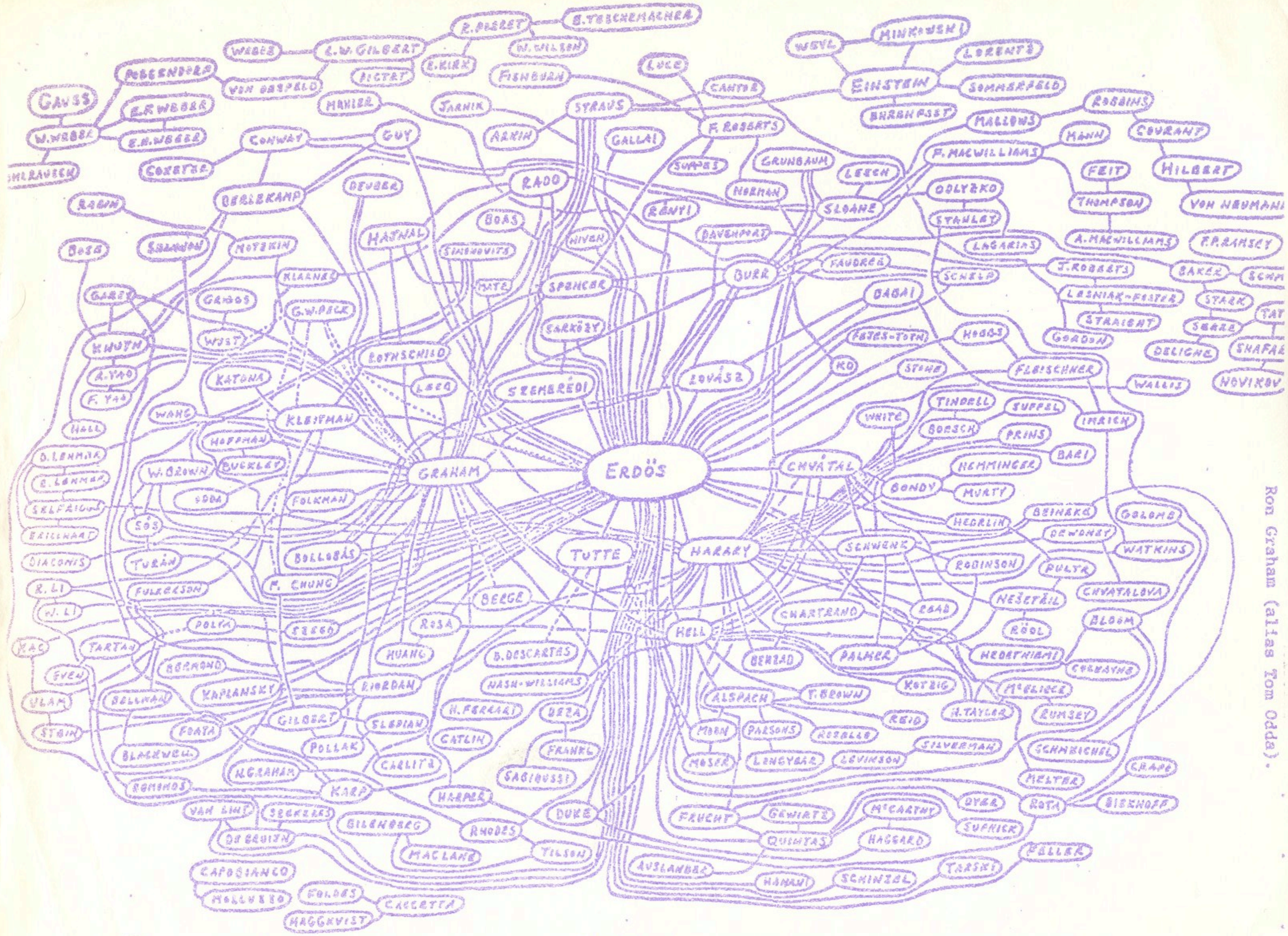




Models of Network Structure

Random Graphs





Ron Graham (alias Tom Odda).

Figure 1

To appear in Topics in Graph Theory (F. Harary, ed.) New York Academy of Sciences (1979).

Random Graphs

- Graph with N people
- For every pair (i,j) of people in the graph, add the edge (i,j) with probability p

- Called the Erdos-Renyi model G(n,p): n vertices, each possible edge occurs with probability p

Some Properties of G(n,p)

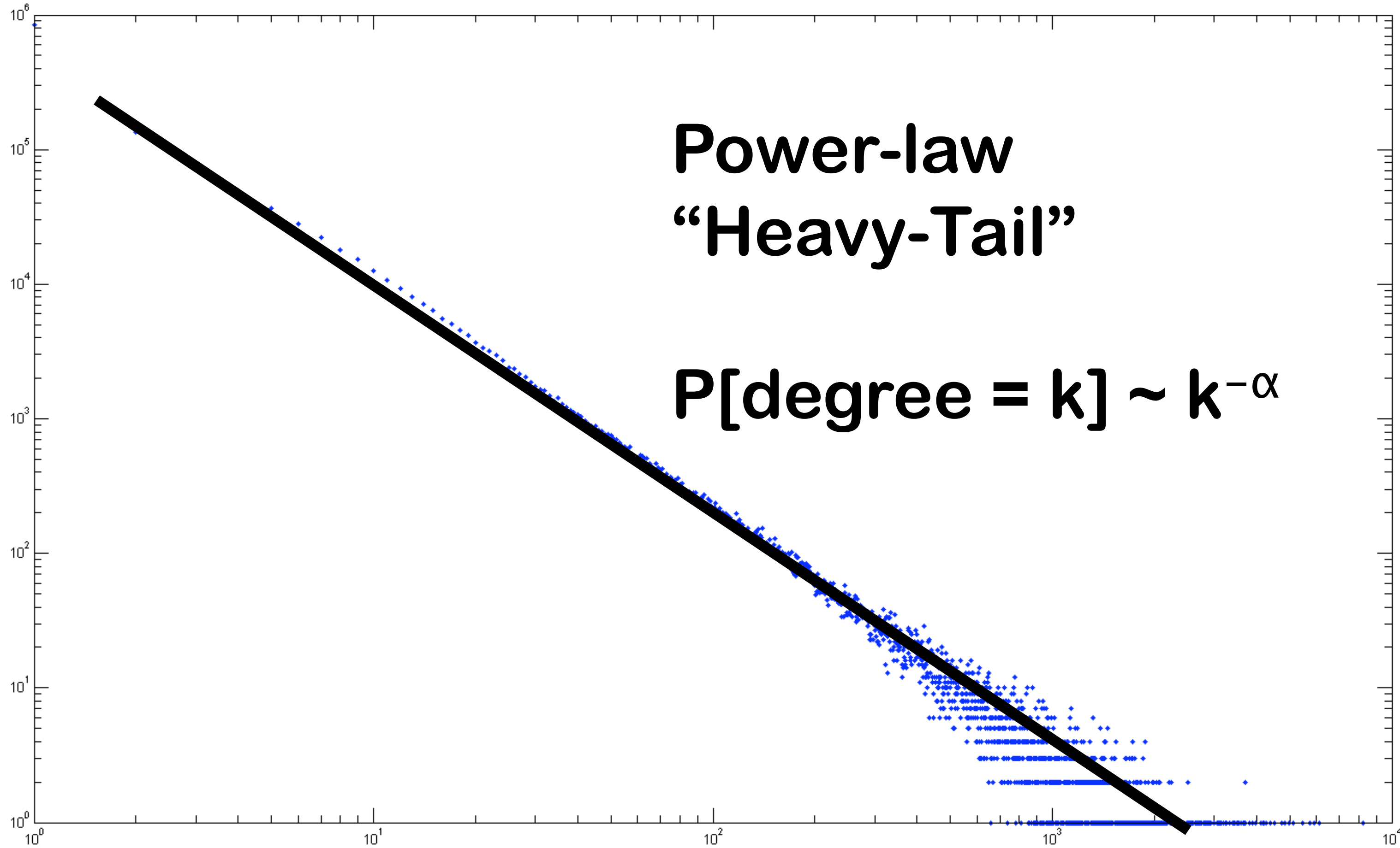
How many “clusters” (i.e. connected components with at least 5% of the population) does the global friendship graph have?

The **GIANT** Component

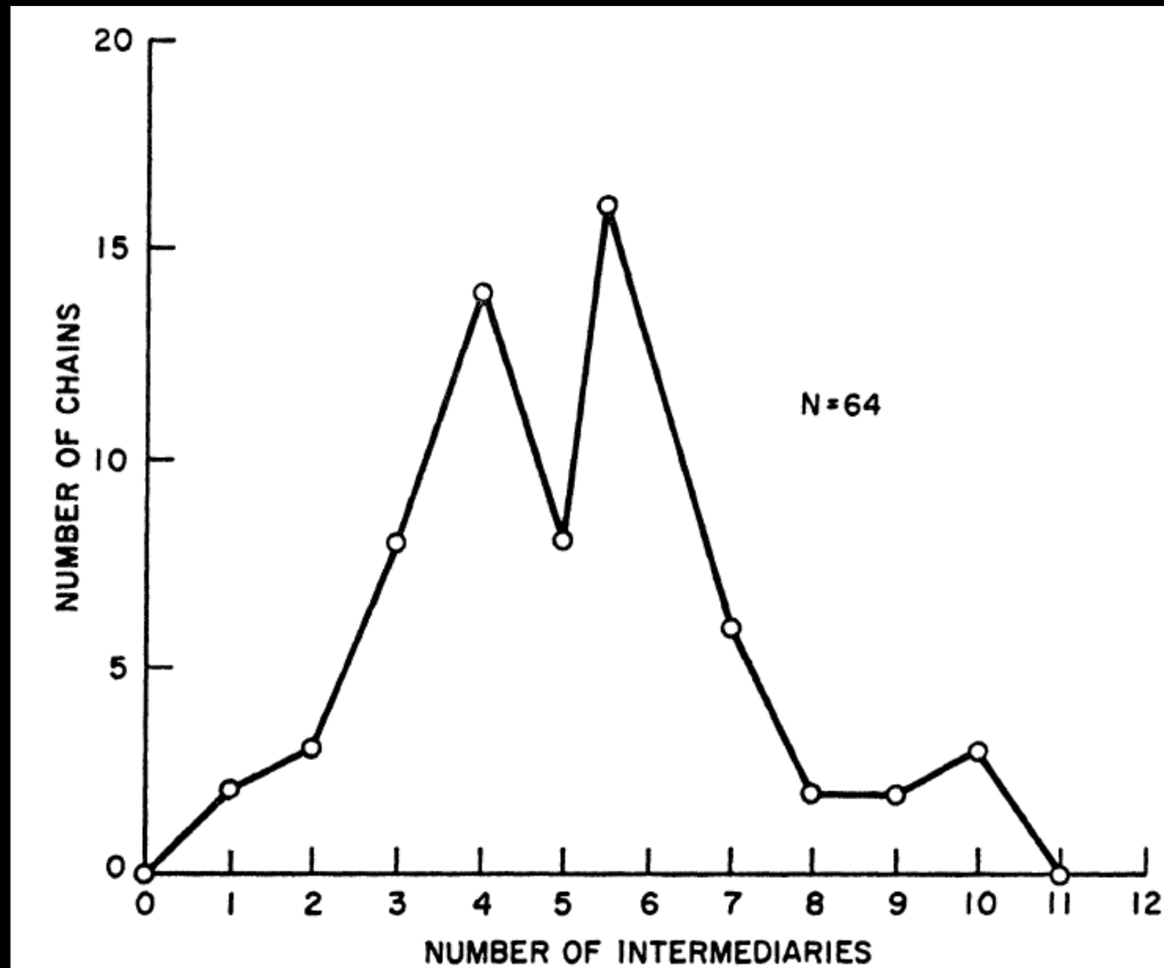
Many (most?) real-world networks don't follow the Erdos-Renyi model!

Real networks are often "scale-free"

Degree Distribution in Flickr

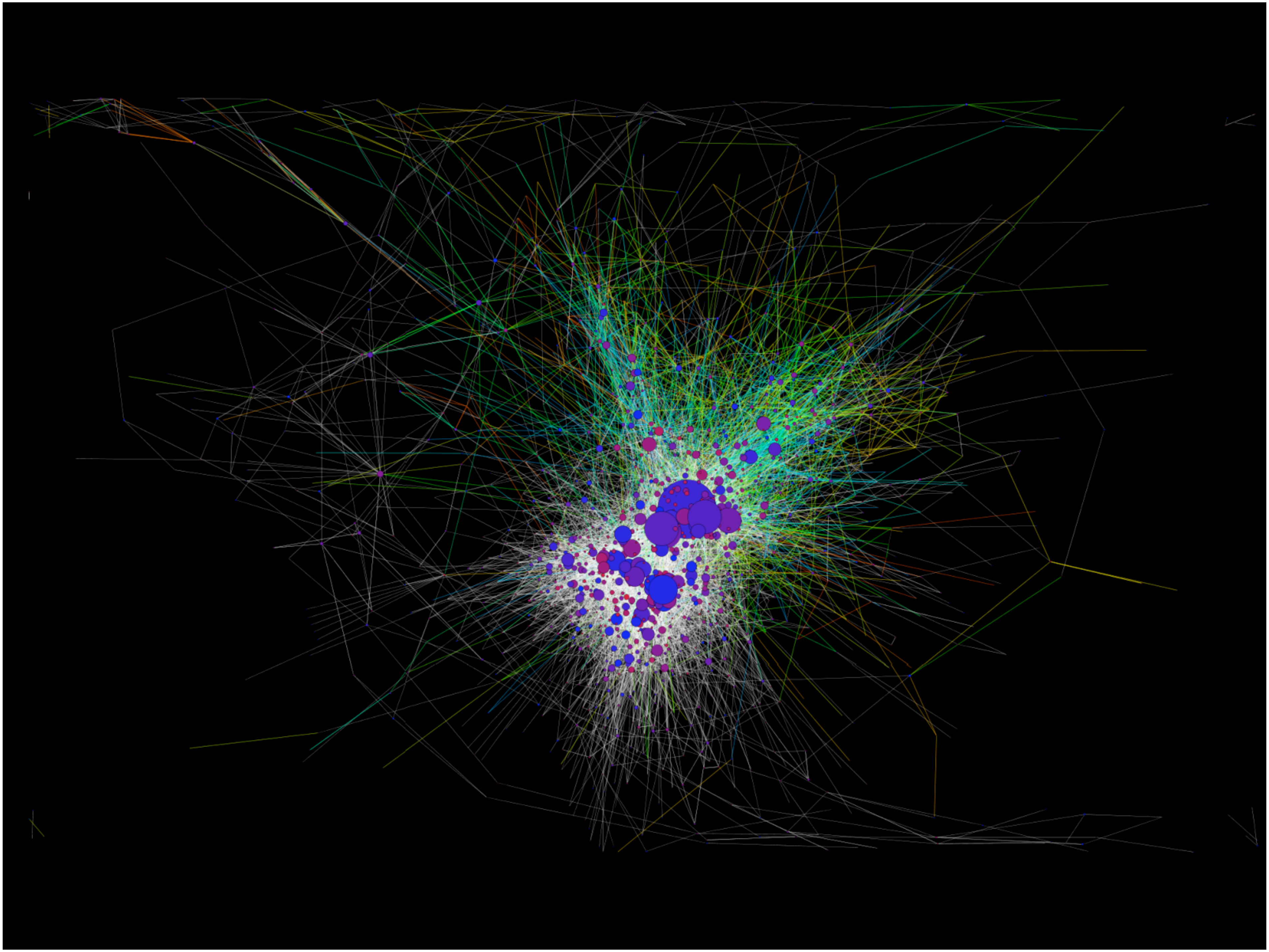


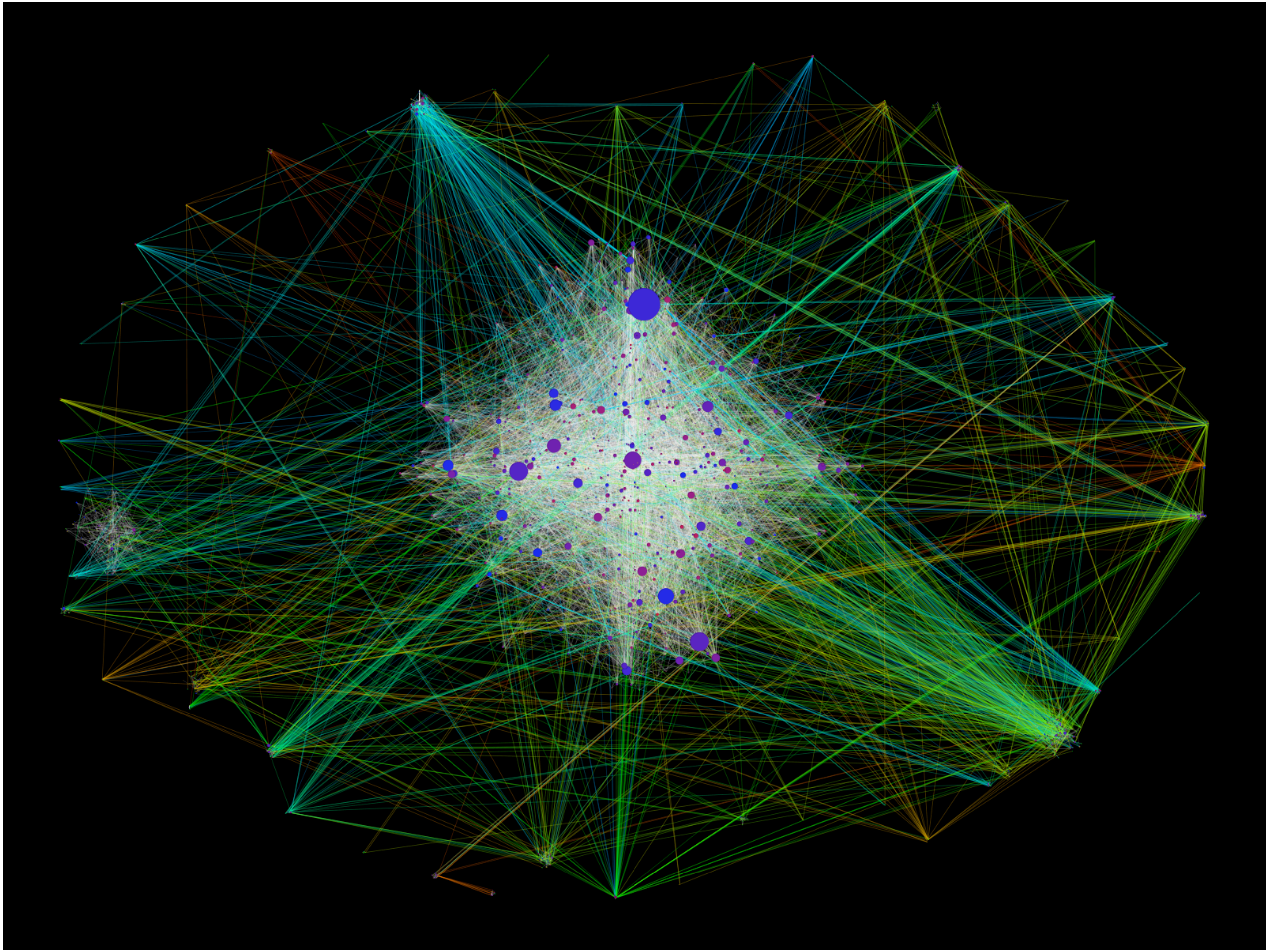
Milgram's Small World

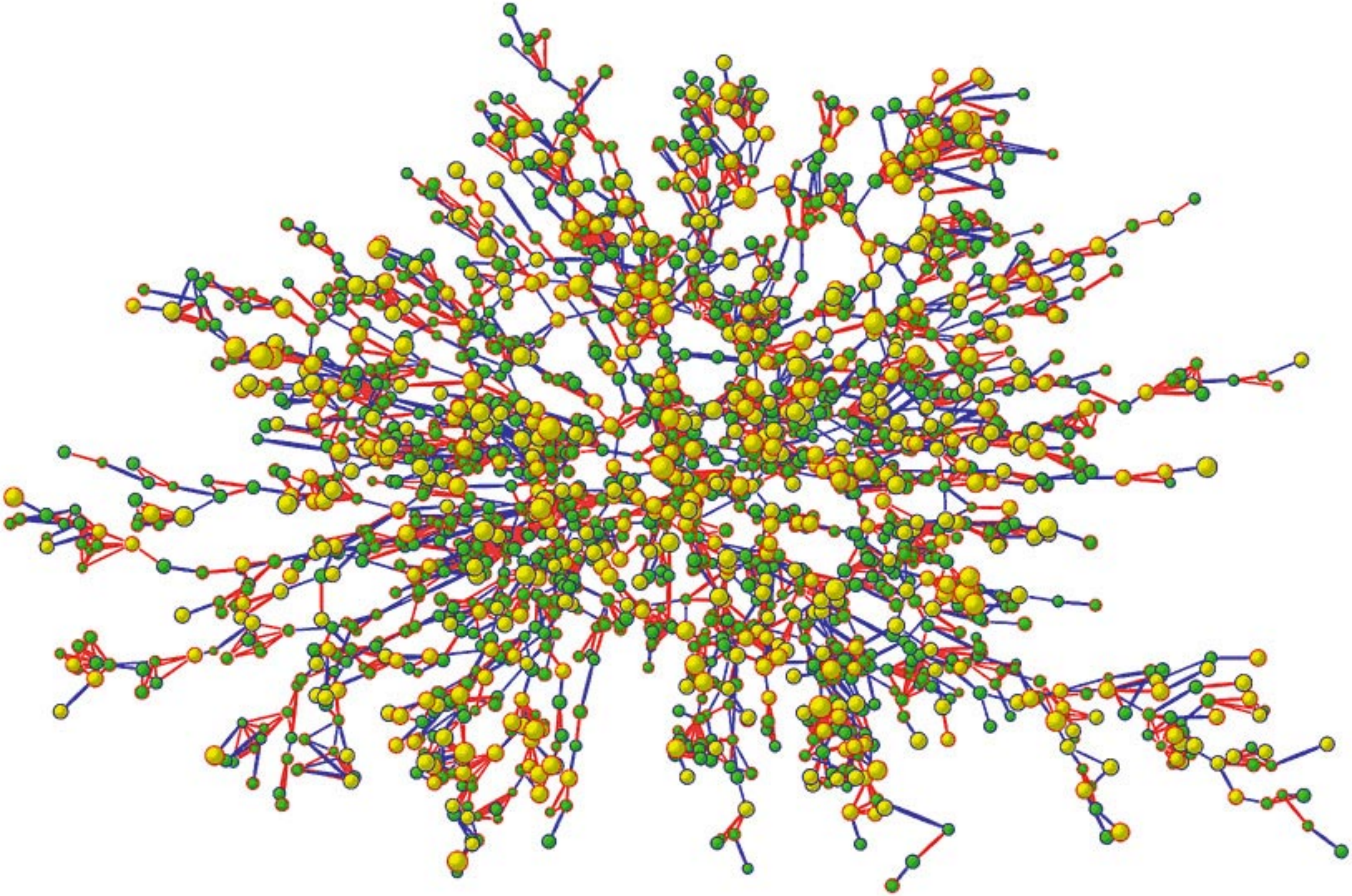


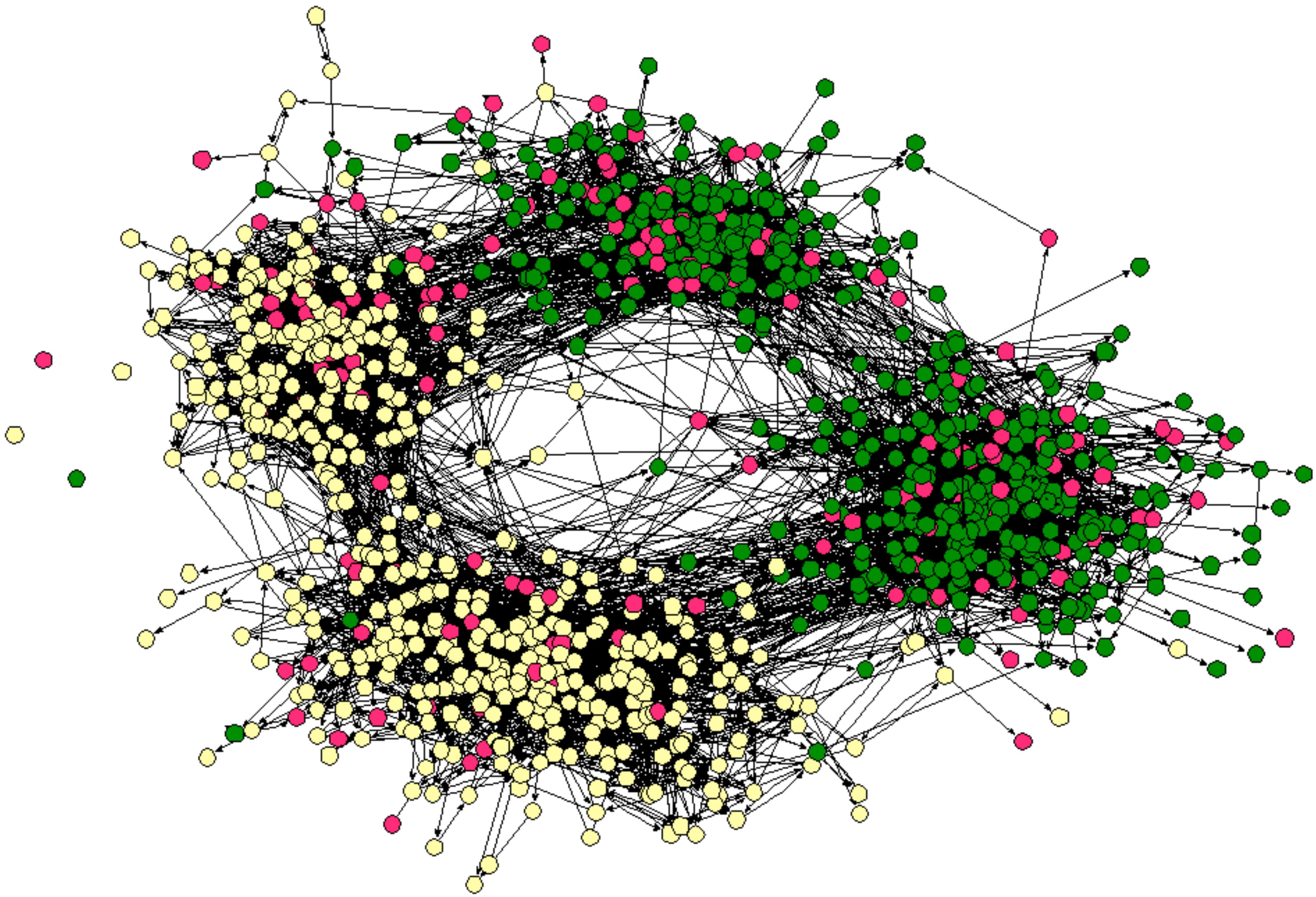
The reason the small world phenomenon is surprising is that the human social network is highly clustered

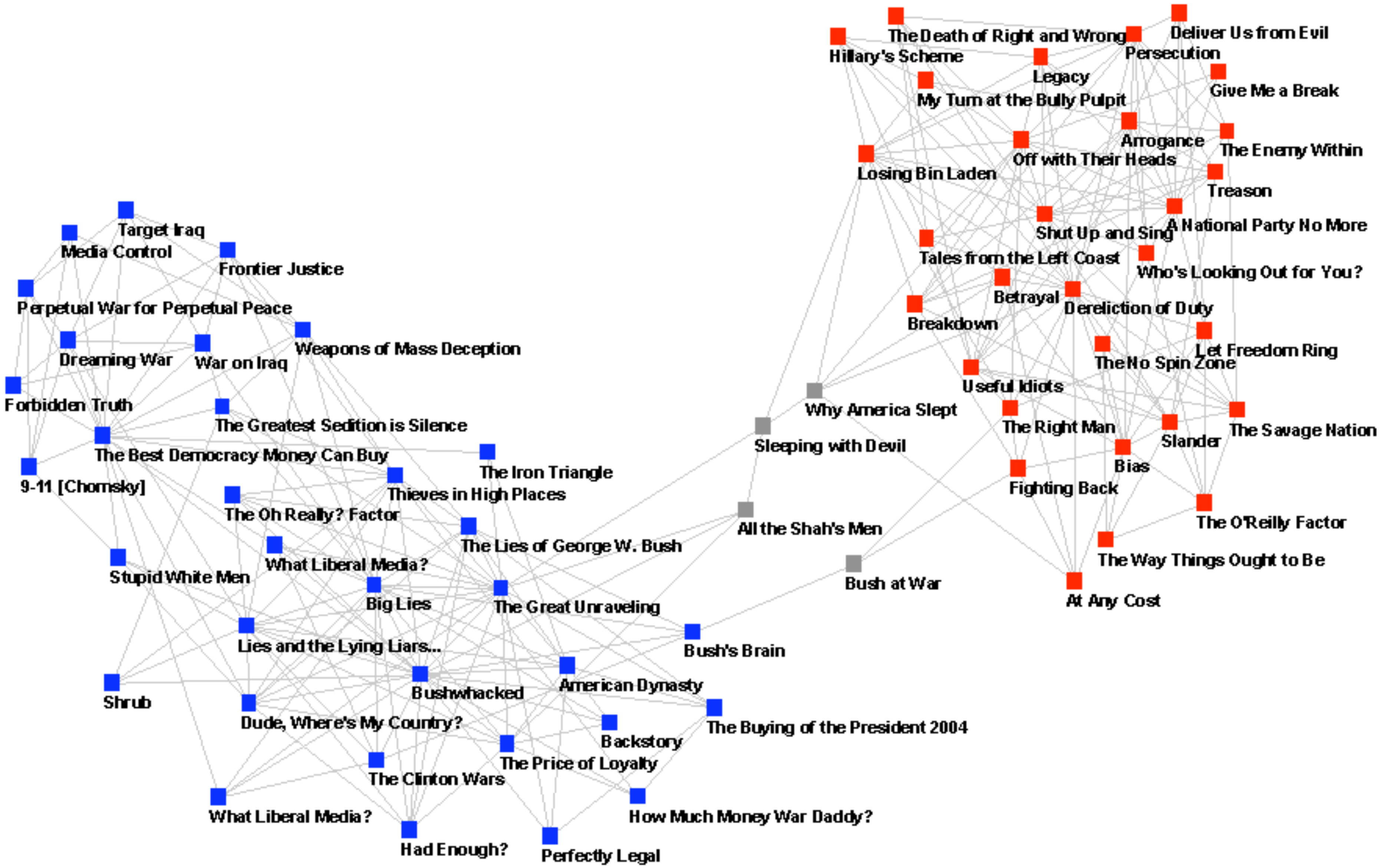
The Beauty of Networks



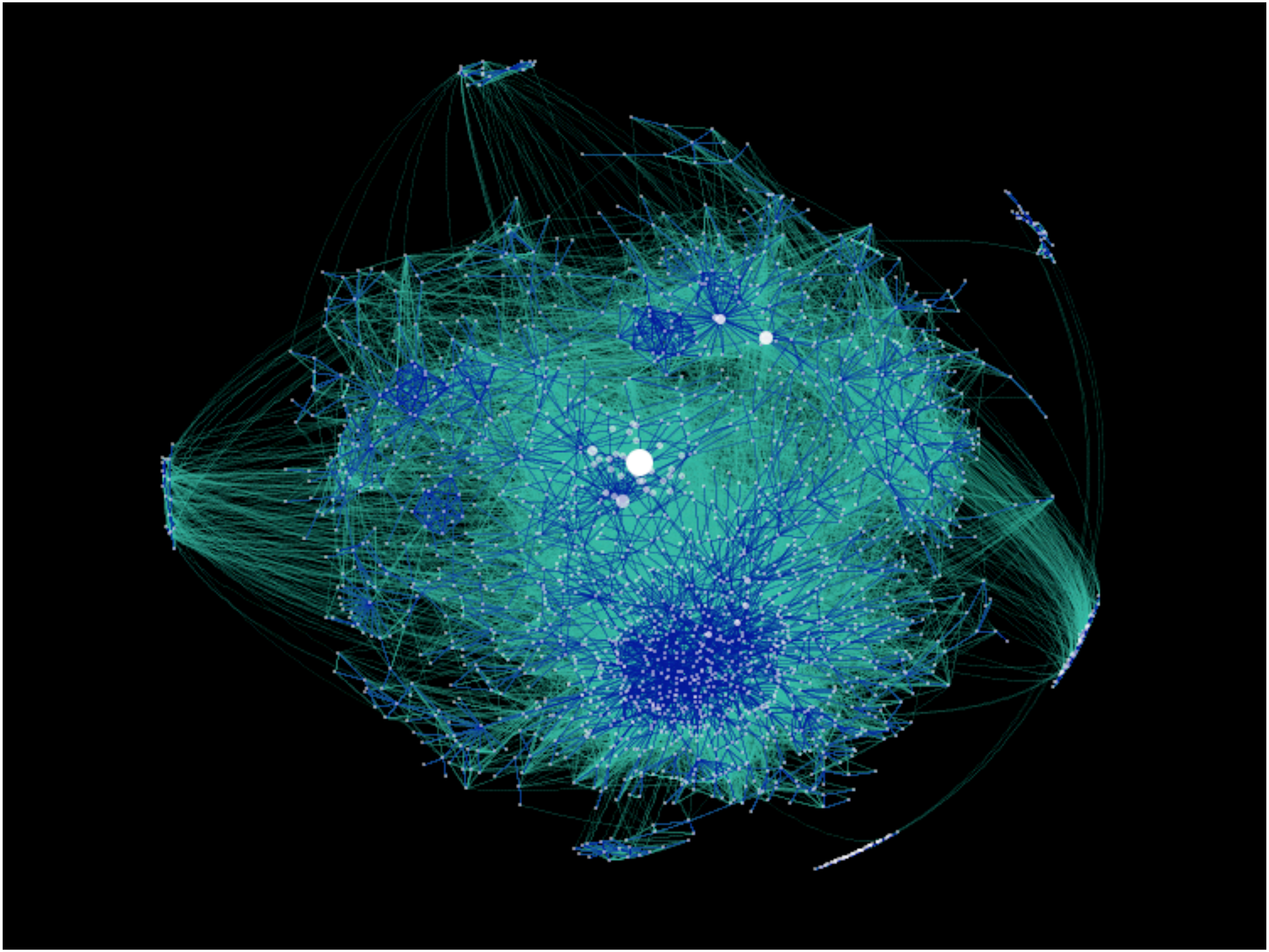


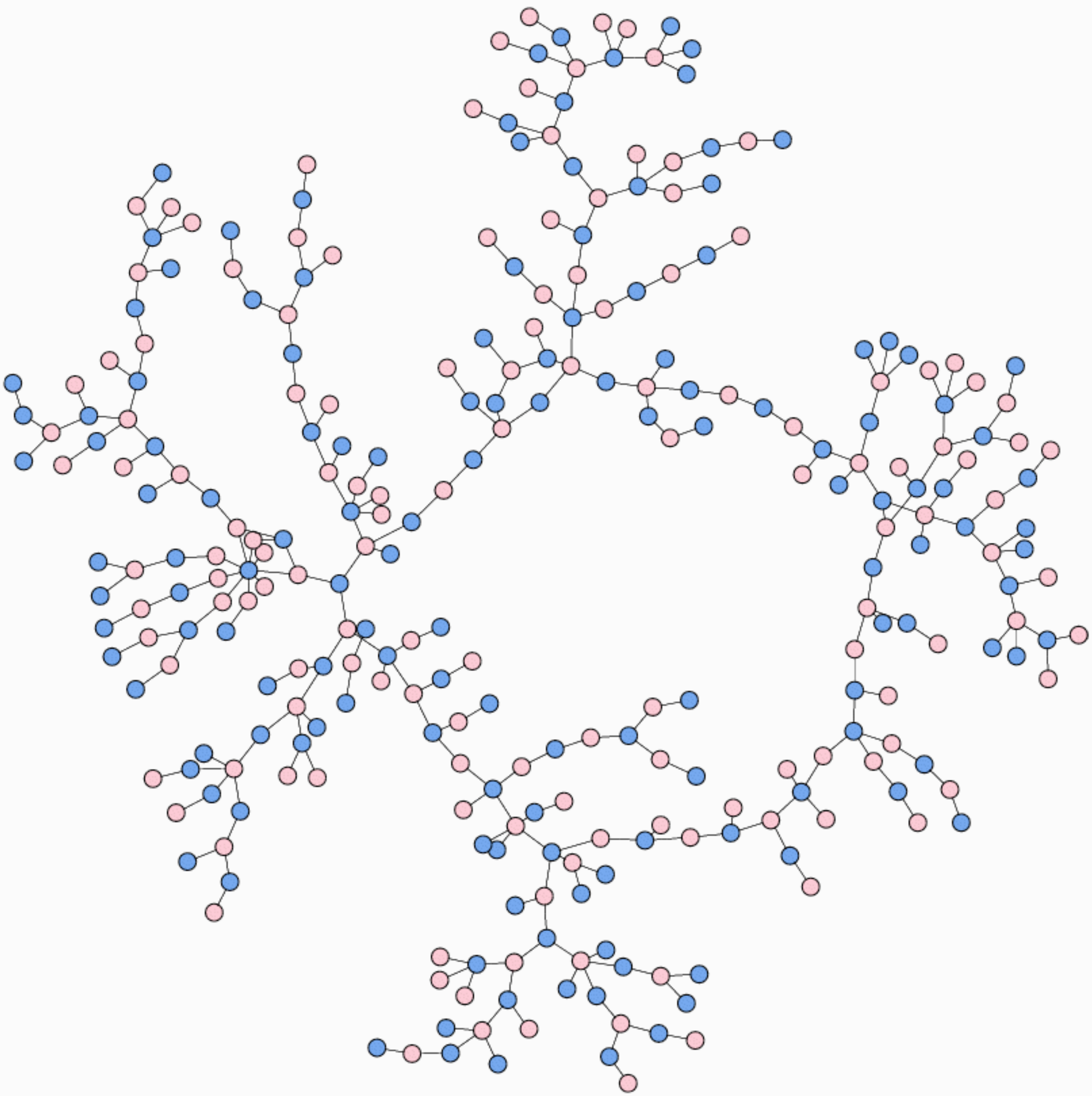


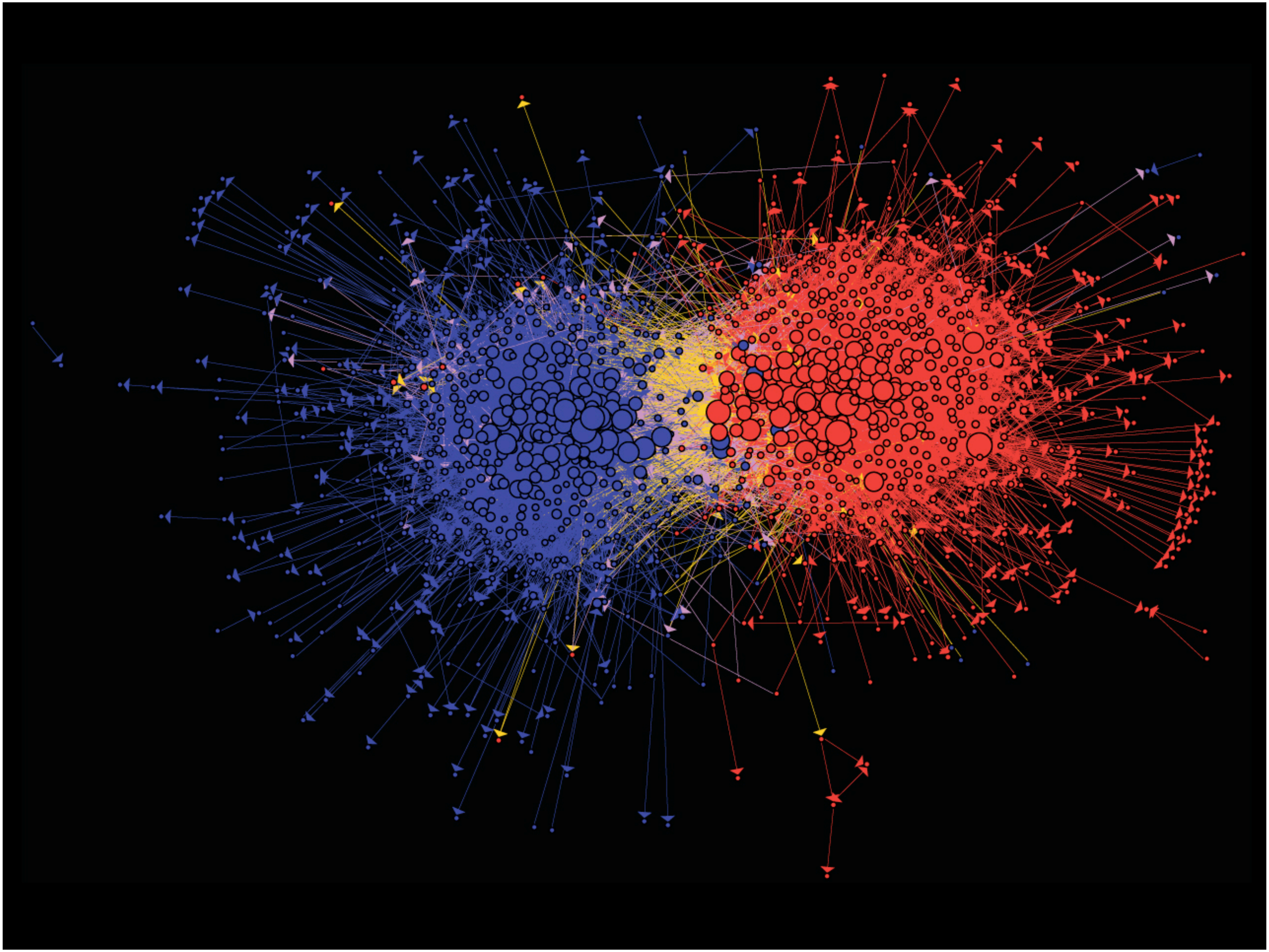


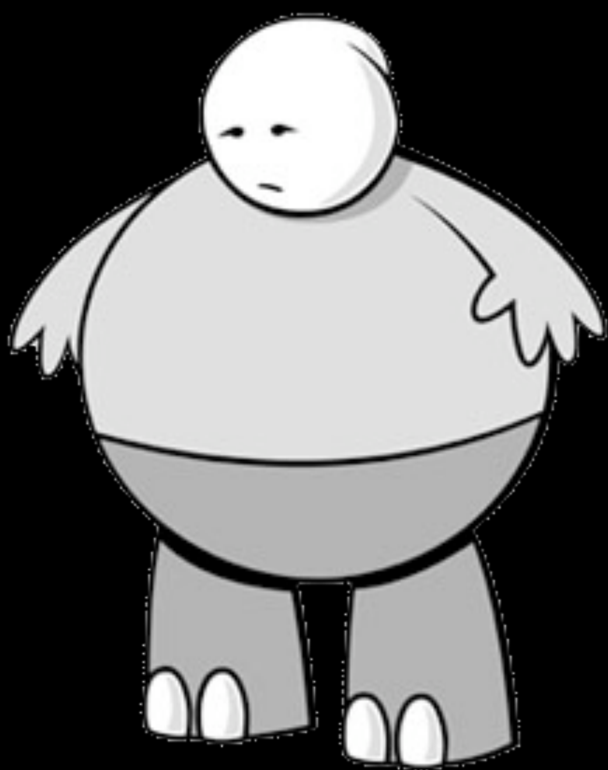


The Death of Right and Wrong
Deliver Us from Evil
Hillary's Scheme
Persecution
Legacy
Give Me a Break
My Turn at the Bully Pulpit
Arrogance
The Enemy Within
Off with Their Heads
Losing Bin Laden
Treason
Shut Up and Sing
A National Party No More
Tales from the Left Coast
Who's Looking Out for You?
Betrayal
Dereliction of Duty
Breakdown
Let Freedom Ring
The No Spin Zone
Useful Idiots
Why America Slept
The Right Man
The Savage Nation
Slander
Sleeping with Devil
Bias
Fighting Back
The O'Reilly Factor
All the Shah's Men
The Way Things Ought to Be
Bush at War
At Any Cost
Target Iraq
Media Control
Frontier Justice
Perpetual War for Perpetual Peace
Weapons of Mass Deception
Dreaming War
War on Iraq
Forbidden Truth
The Greatest Sedition is Silence
The Best Democracy Money Can Buy
The Iron Triangle
9-11 [Chomsky]
Thieves in High Places
The Oh Really? Factor
The Lies of George W. Bush
What Liberal Media?
Stupid White Men
Big Lies
The Great Unraveling
Bush's Brain
Lies and the Lying Liars...
American Dynasty
Bushwhacked
Shrub
The Buying of the President 2004
Dude, Where's My Country?
Backstory
The Price of Loyalty
The Clinton Wars
What Liberal Media?
How Much Money War Daddy?
Had Enough?
Perfectly Legal









Here's What You Need to Know…

Network Structures

• Structural holes, cliques, clusters, bridges, local bridges
• Balanced cycles and graphs
• Strong triadic closure

Network Measures

• Degree centrality, betweenness centrality

Graph Models

• Erdos-Renyi model
• Real-world networks have power-law degree distribution