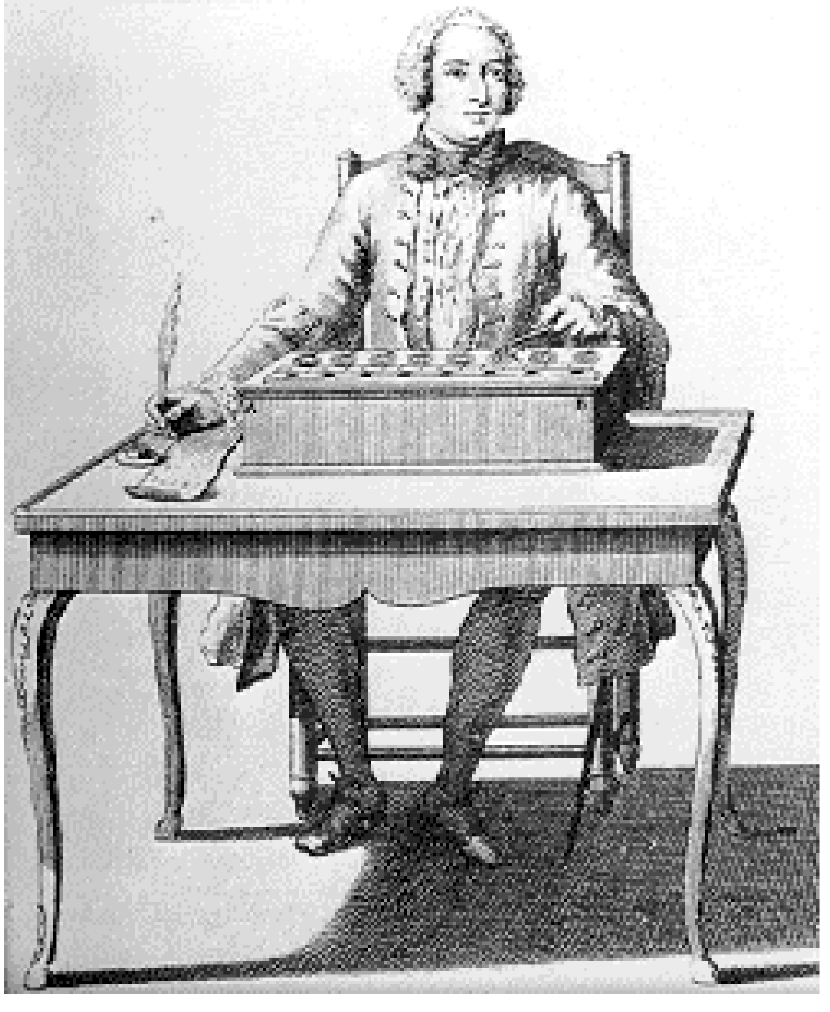
15-294 Rapid Prototyping Technologies:



The Pascaline

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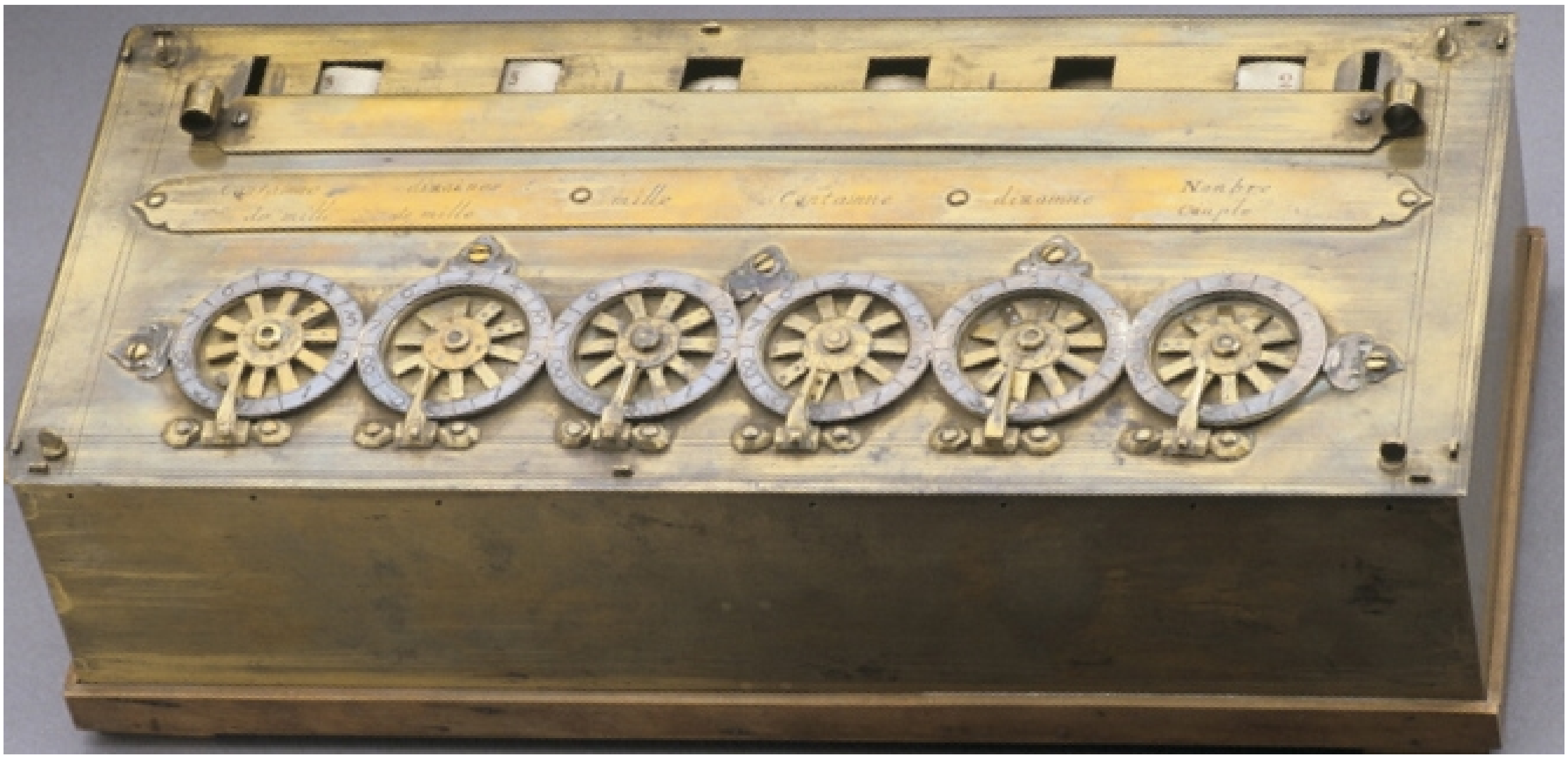
Blaise Pascal

- French mathematician, physicist, writer, philosopher.
- b. 1623 – d. 1662 (age 39)
- Father was a tax collector.
- Known for:
  - Theory of probability; Pascal's triangle
  - Study of fluids, pressure, and vacuum
  - Writings on philosophy and theology
  - First working mechanical calculator: the Pascaline

The Pascaline



Musée des arts et métiers-CNAM, Paris. Photo: J. C. Wetzel.

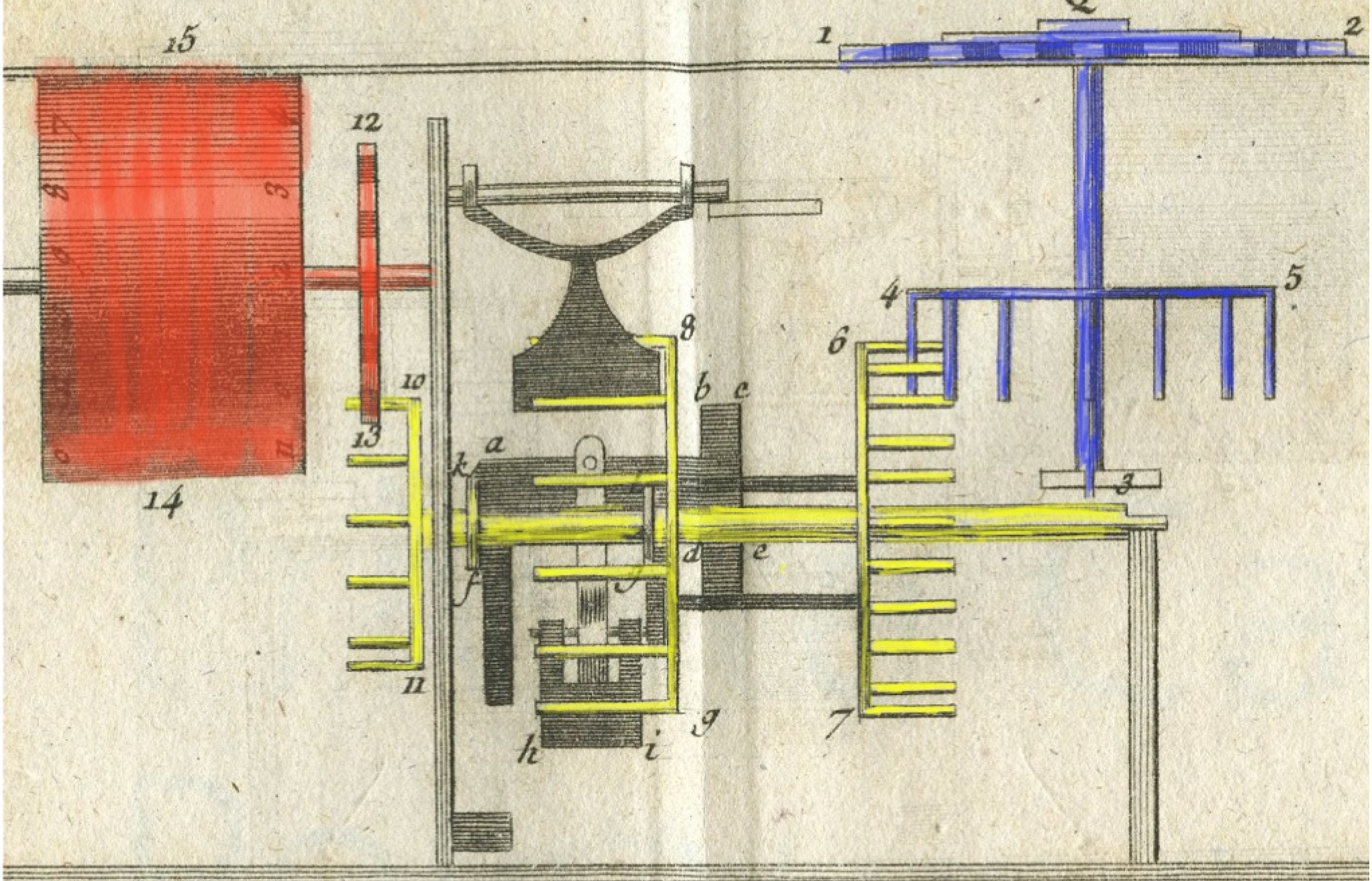
History of the Pascaline

- Pascal invented the device in 1642 (at age 19) to help his father with his tax computations.
- Widely viewed as the first mechanical calculator. Could add and (with a trick) subtract.
- Over 40 were built over several decades, in a variety of models.
- 9 survive today in museums or private collections.

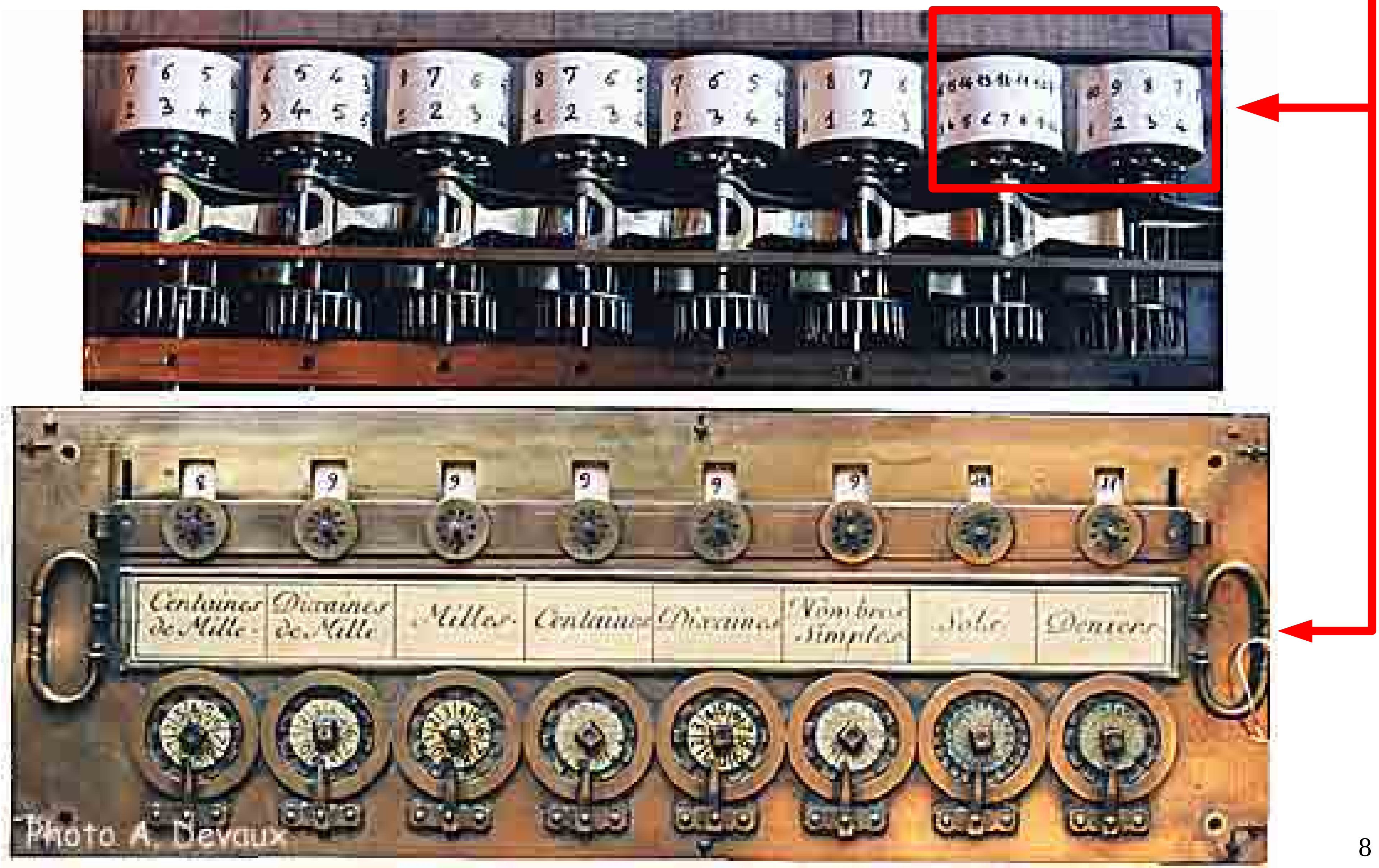
Input and Output

Input Close-Up

Fig. 3.



French Currency (17th century)

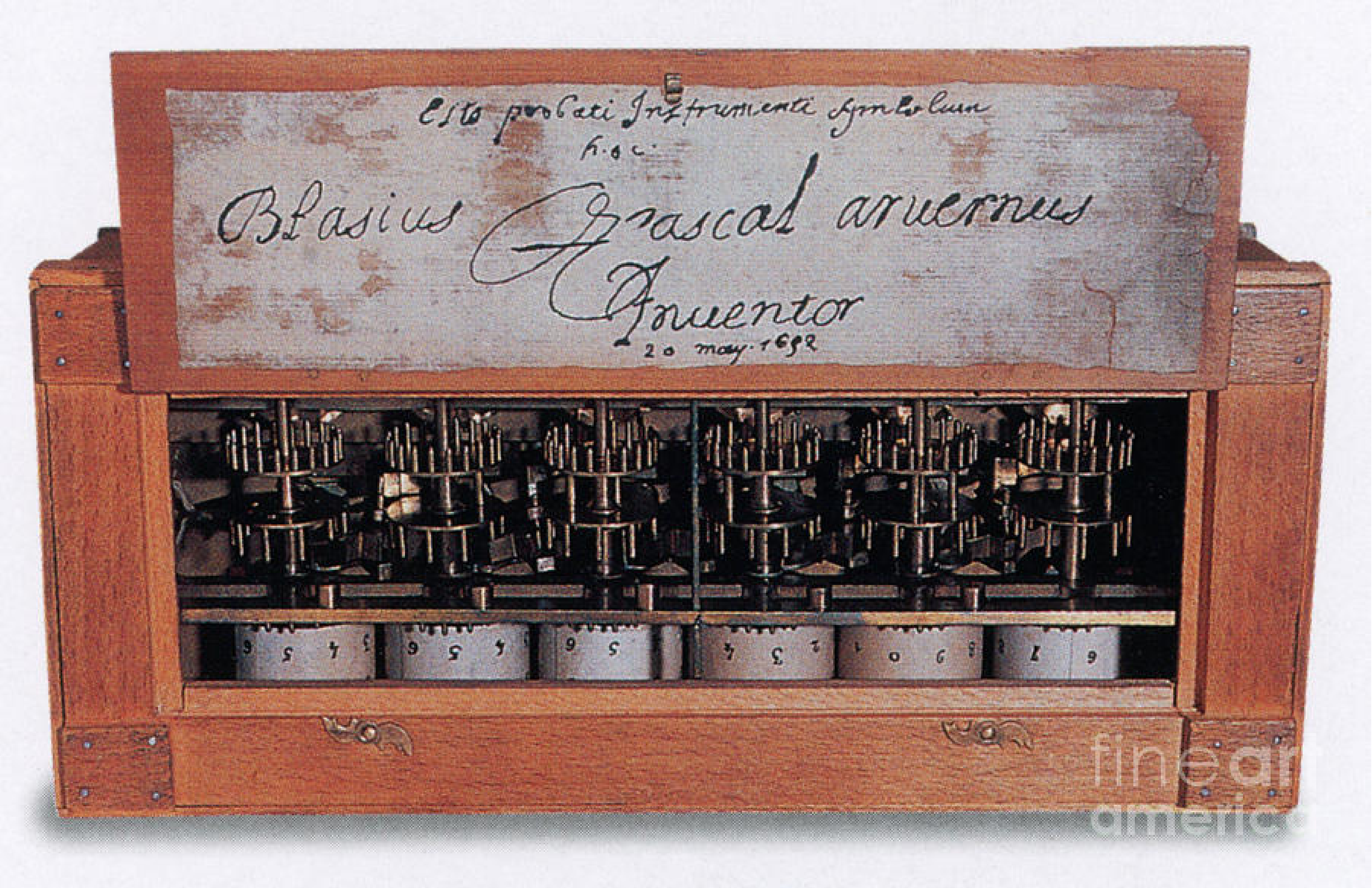


17th Cent. French Currency: Livres

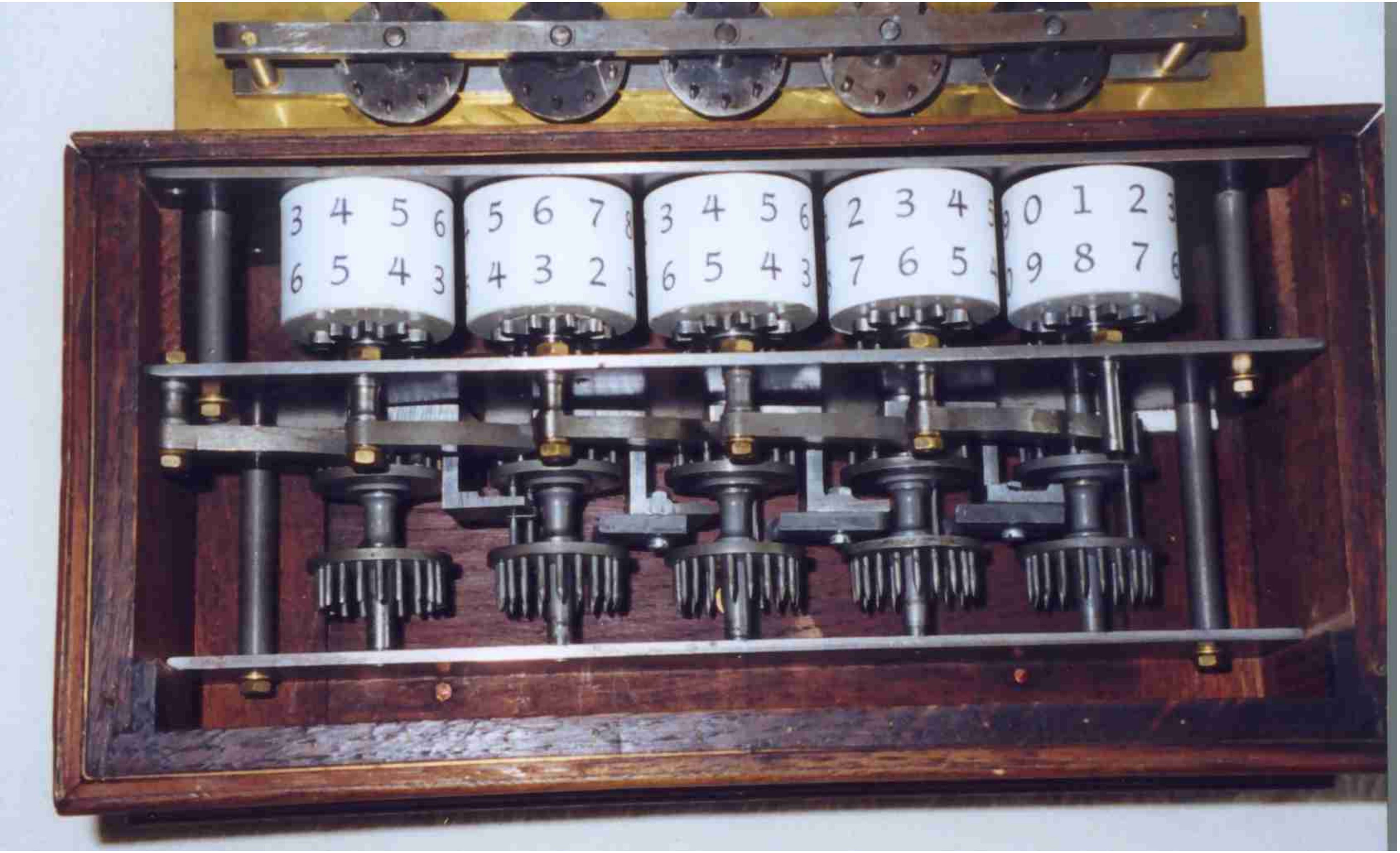
• Deniers	12 deniers = 1 sol
• Sols	20 sols = 1 livre
• Nombres Simples	“simple numbers”
• Dixaines	tens
• Centaines	hundreds
• Milles	thousands
• Dixaines de Mille	tens of thousands
• Centaines de Mille	hundreds of thousands

Under the Hood

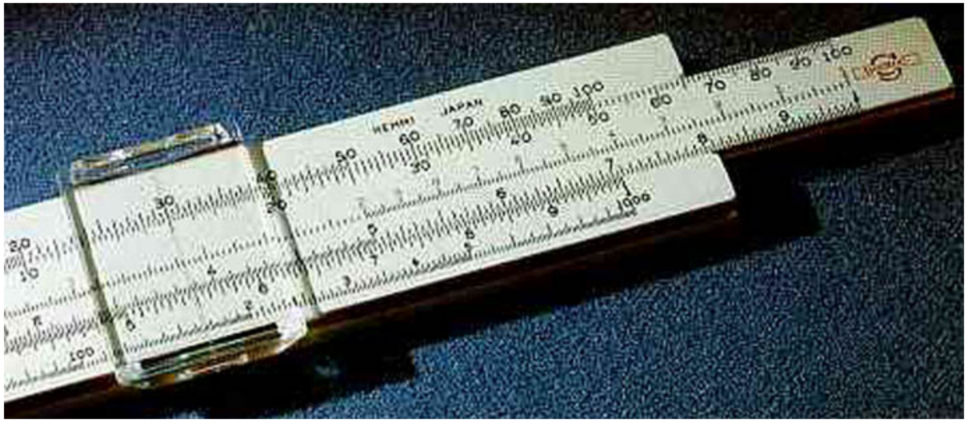
Early Version, From 1642



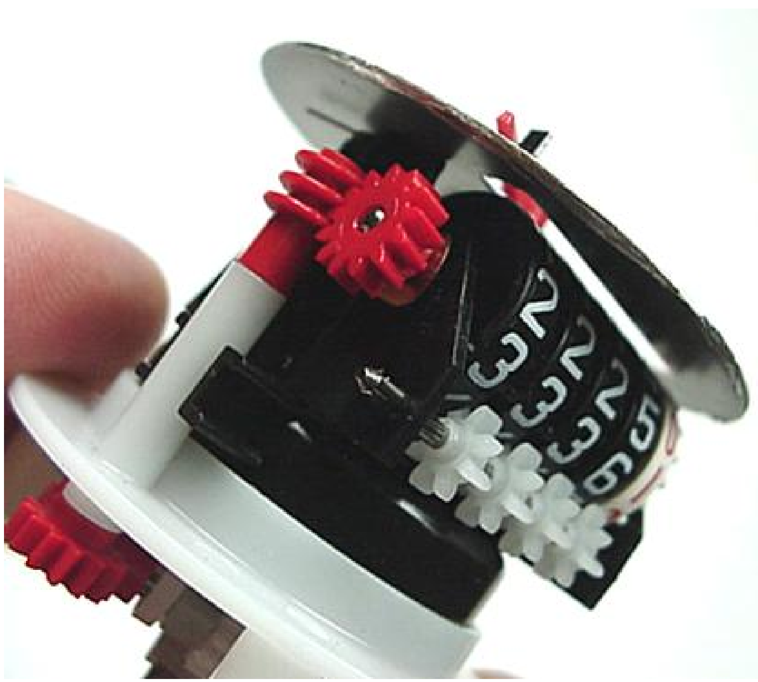
Why Do the Read-Out Wheels Have Two Sets of Numbers?



What Makes A Device "Digital"?

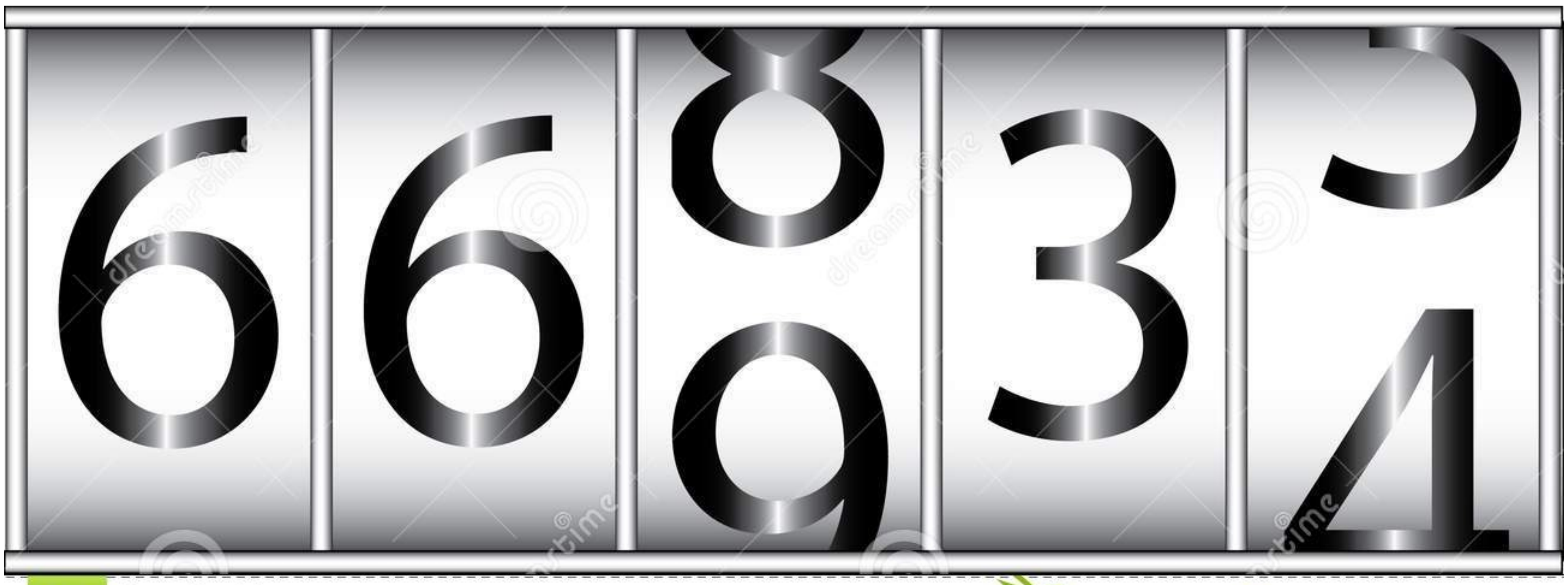


A slide rule is analog: it has a continuous state space, so an infinite number of states.

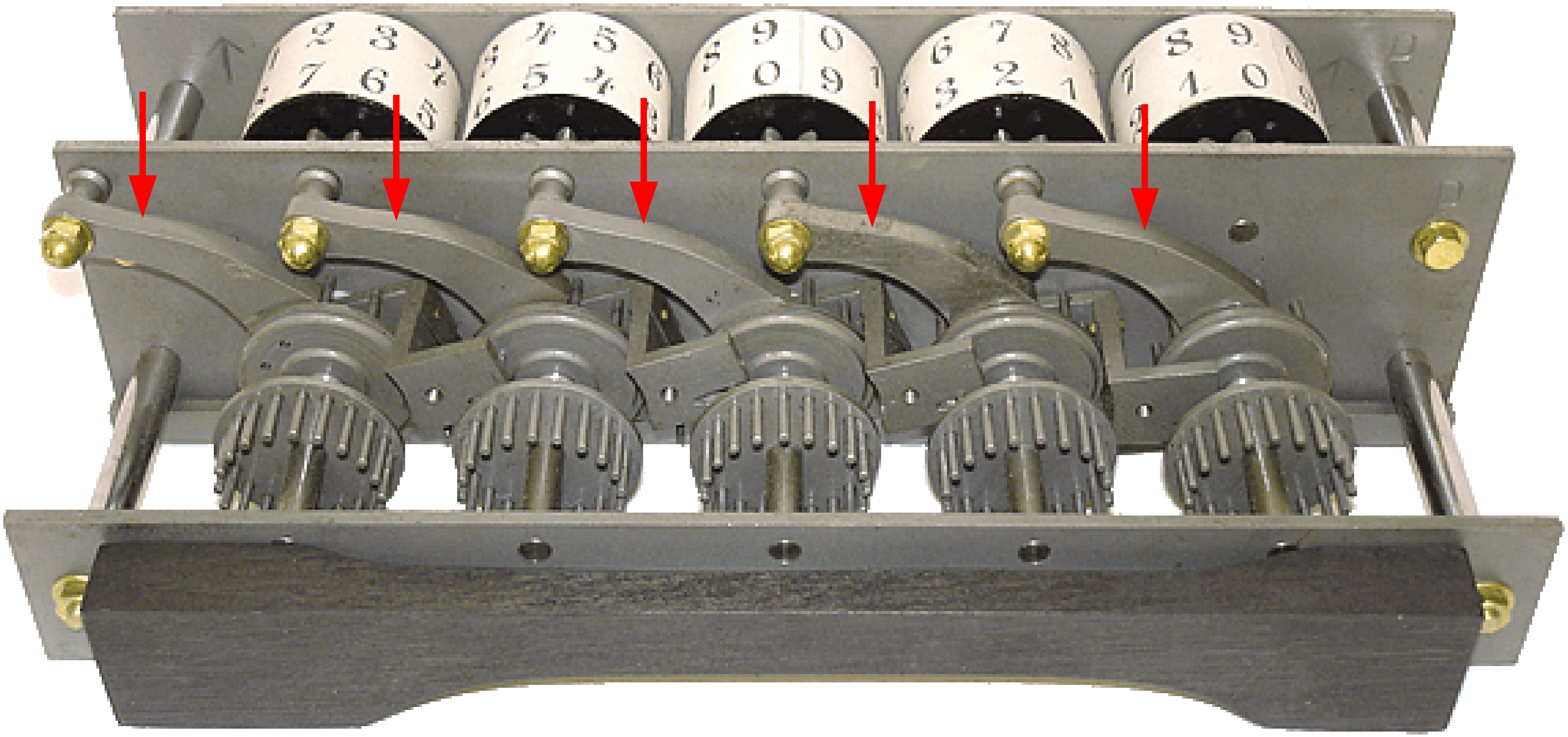


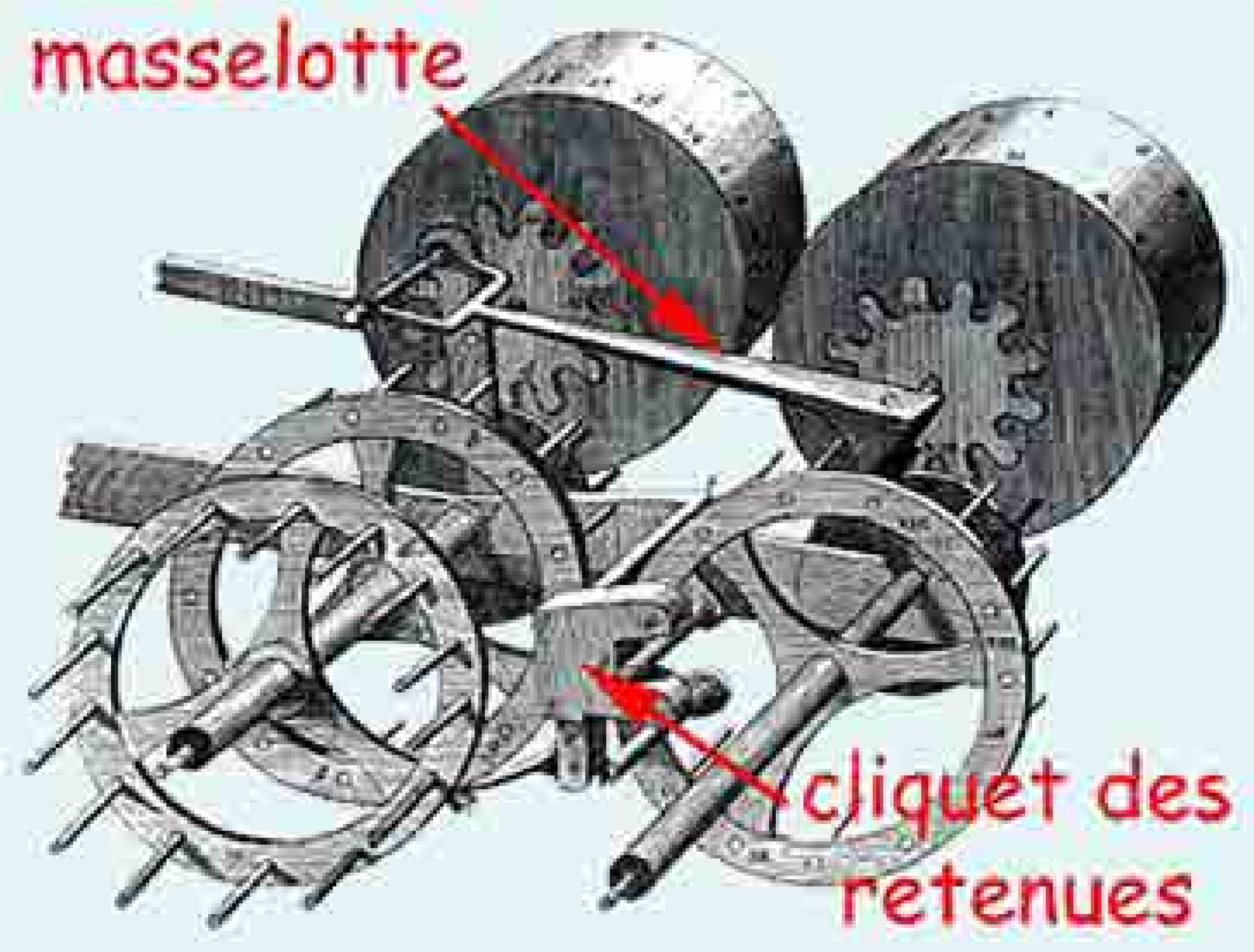
Digital devices have *discrete* state spaces, and a physical non-linearity to force clean transitions from one state to another.

Digits Don't Guarantee Discreteness



The Backstop Pawl

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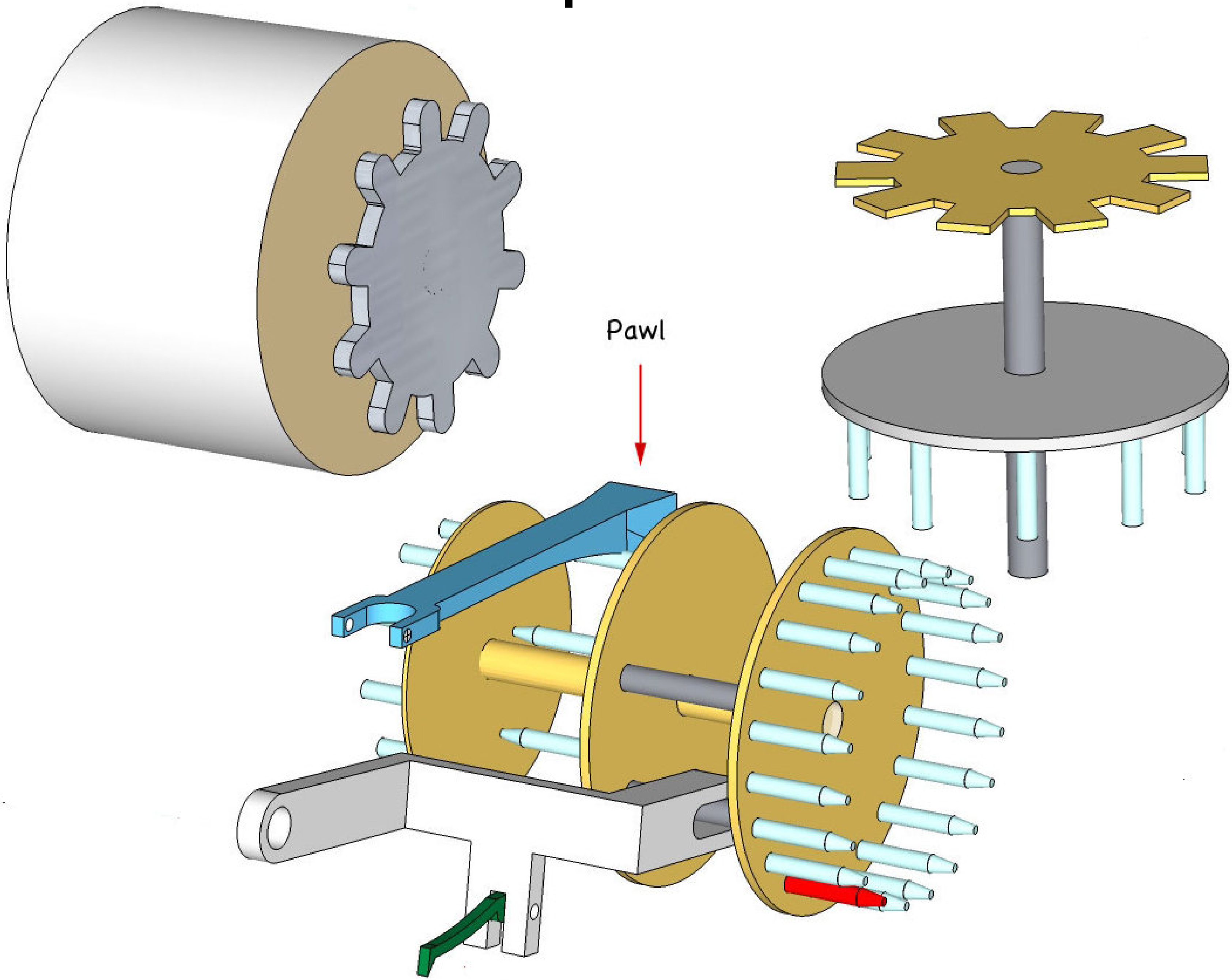
Backstop Pawl and Sautoir



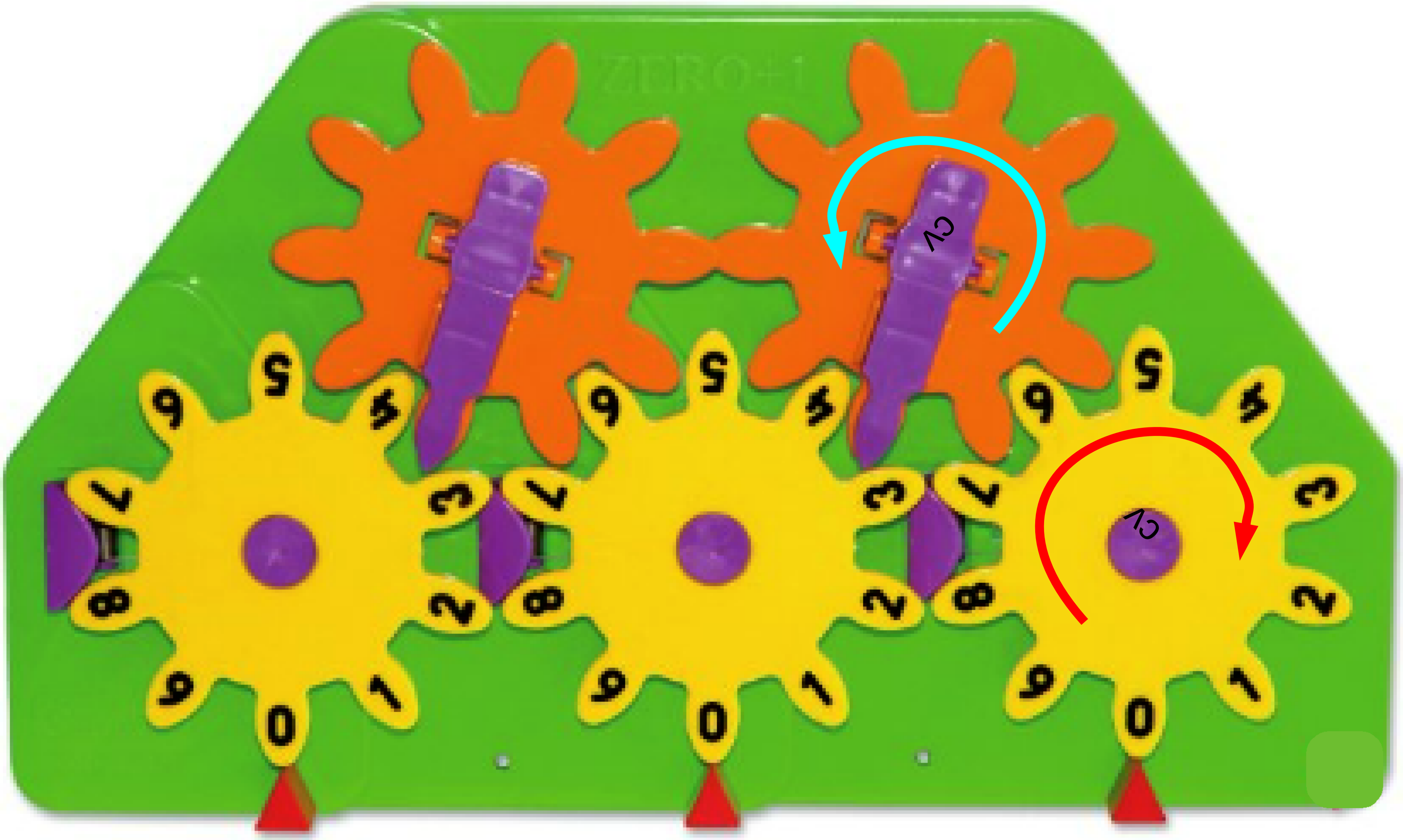
Carry Operations

- Carrying is what makes addition difficult.
- How do you get one digit to affect the digit next to it?
- Carry can propagate:

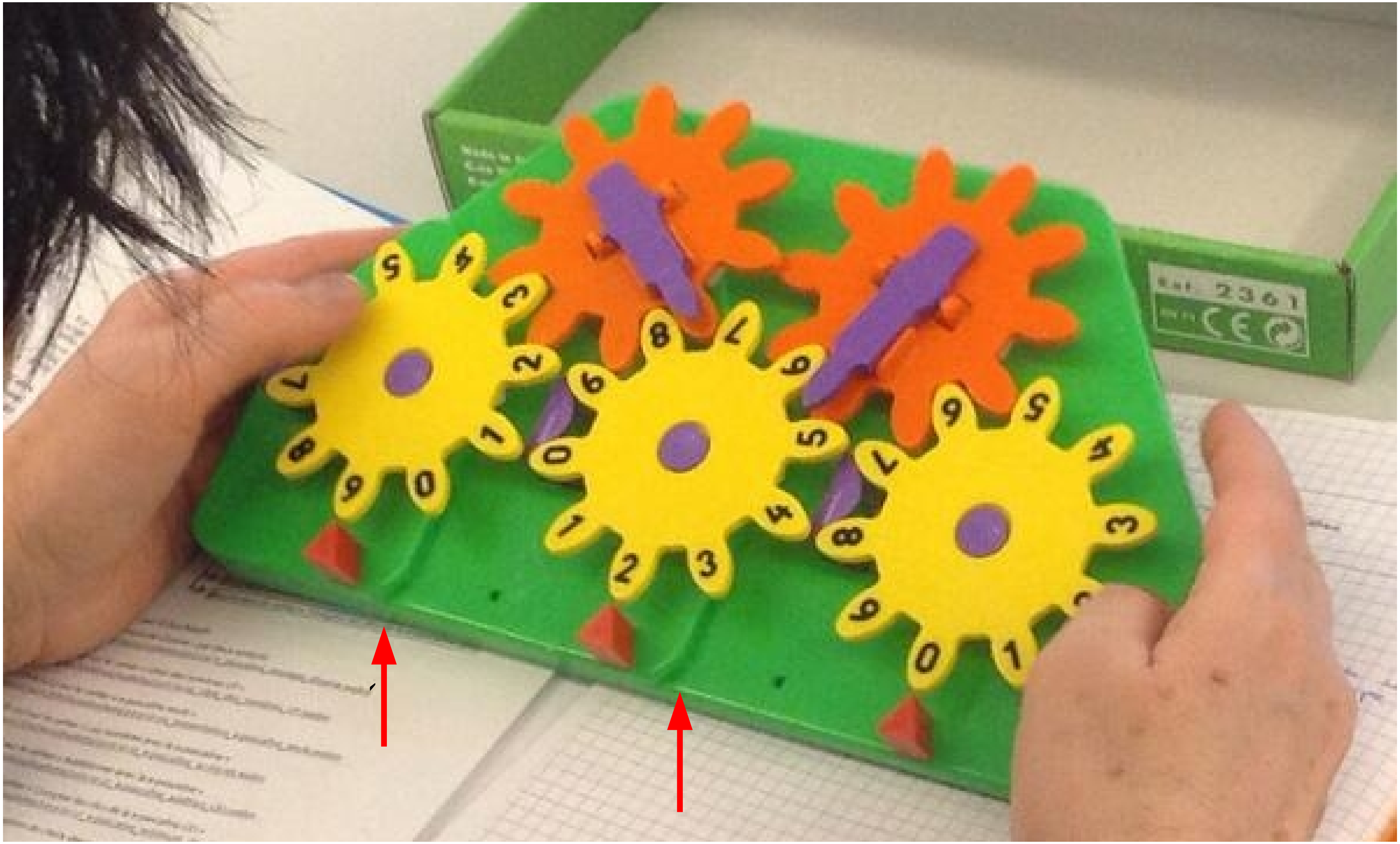
  099999 → 100000

- Mechanically, this is a nightmare.
- Two solutions:
  - One-toothed gear (doesn't chain well; can jam)
  - The sautoir (“jumper”) – Pascal's invention

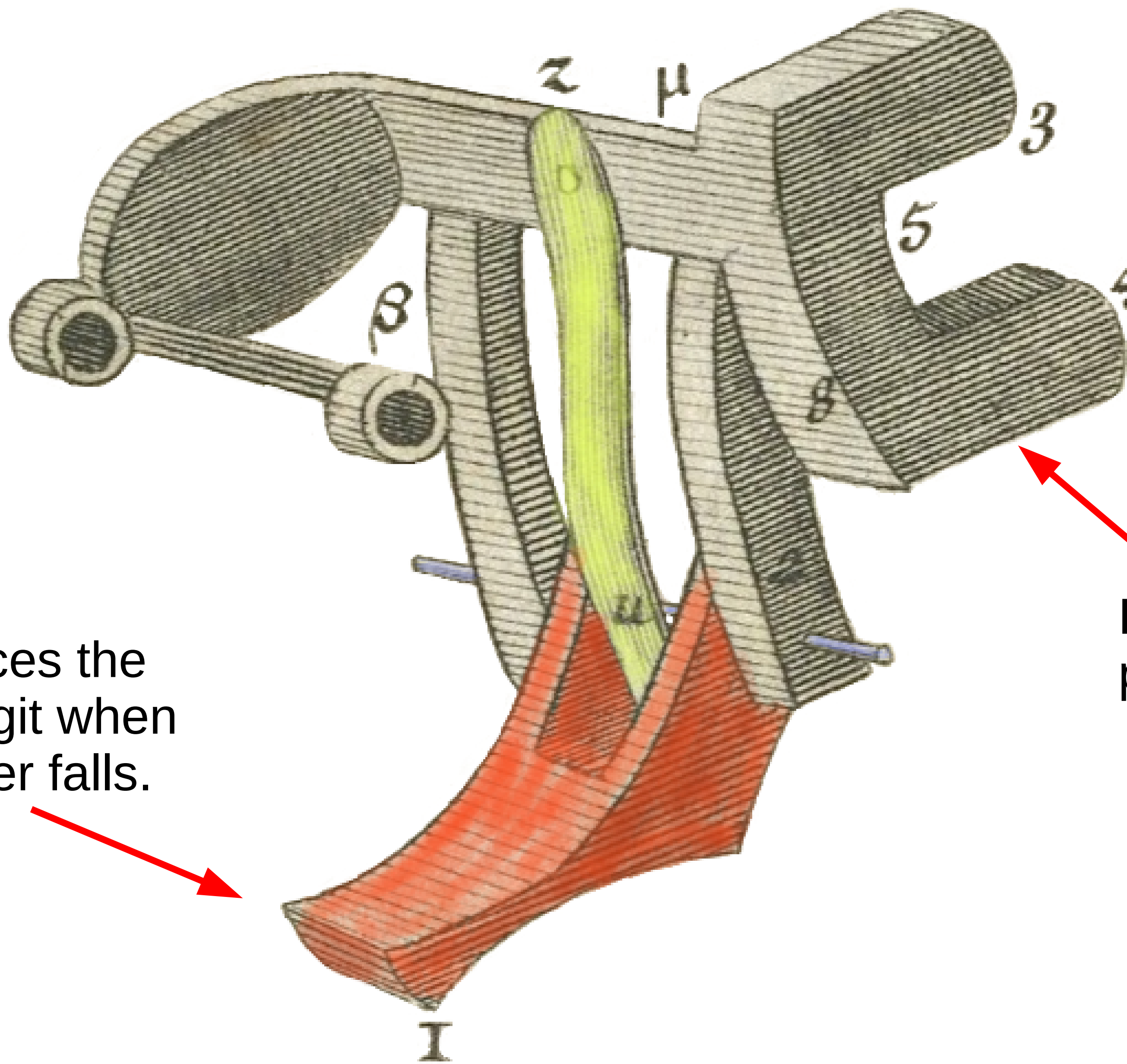
One-Tooth Gear (Purple) For Carry



Trick: Elevate Successive Digits



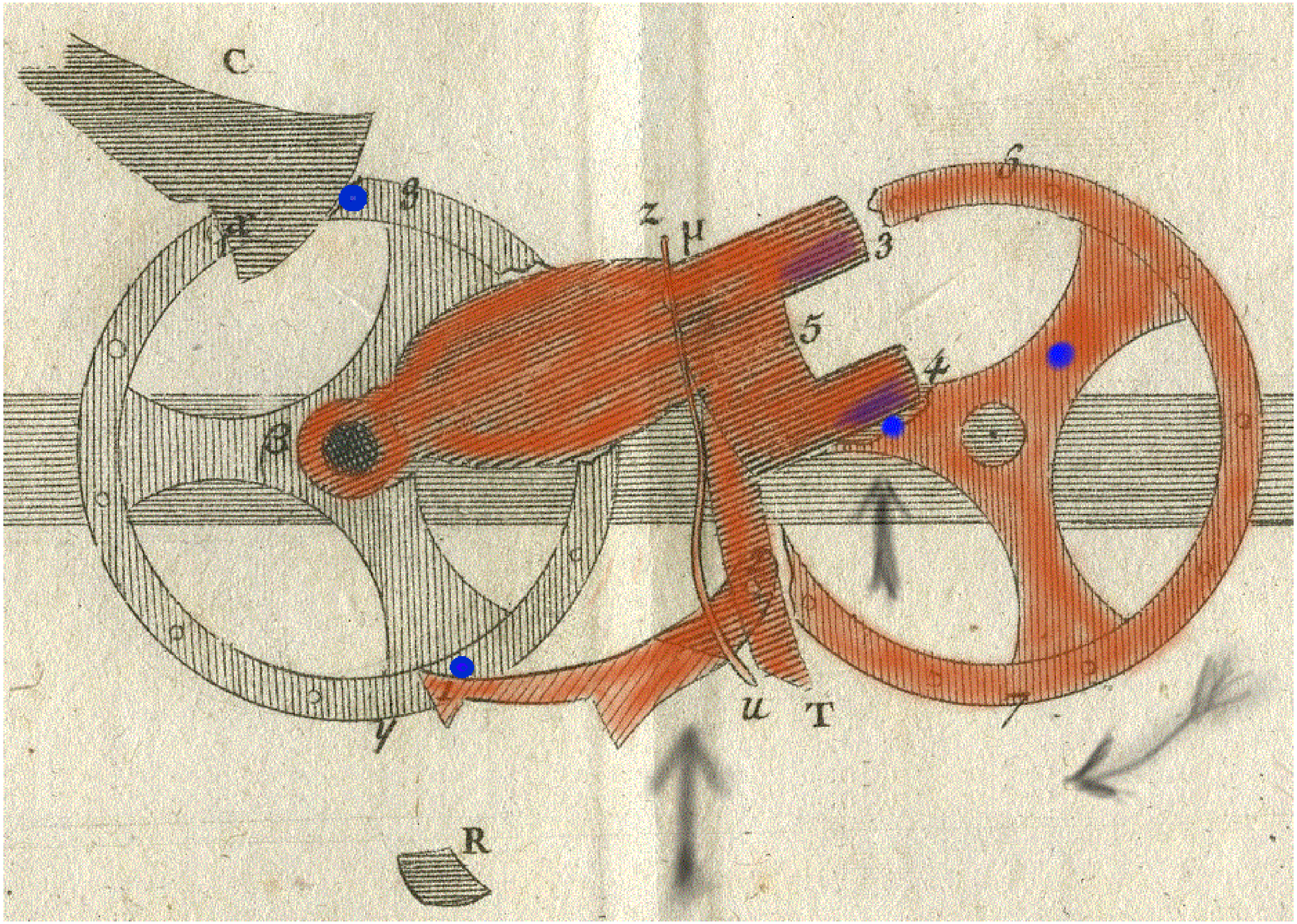
The Sautoir



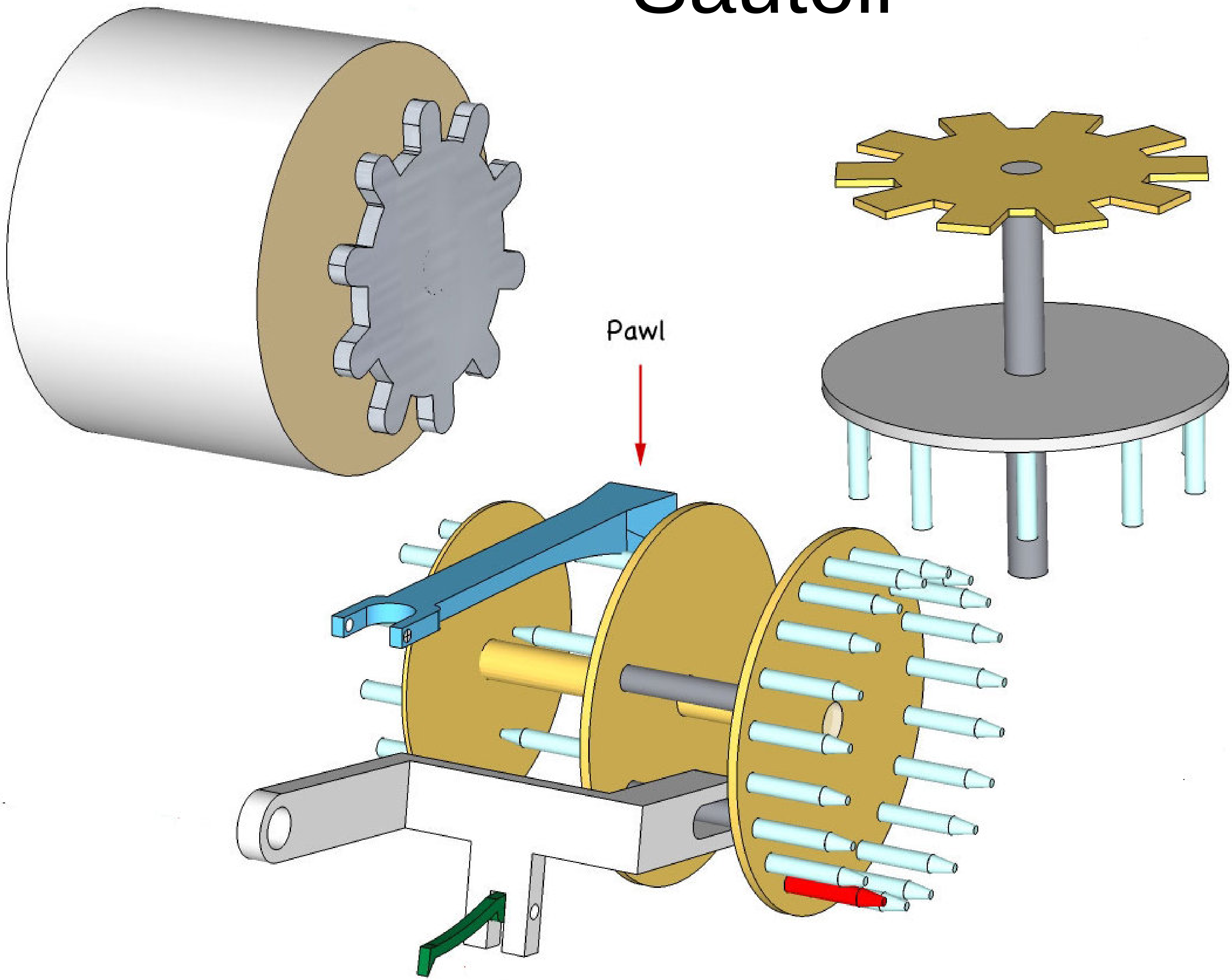
Advances the next digit when the lever falls.

Driven by the previous digit.

Sautoir



Sautoir



How To Add

1. Clear the machine:
   (a) Set all digits to “9”.
   (b) Add 1 to get all zeros: tests the ripple carry.
2. Dial in the digits of the first addend.
3. Dial in the digits of the second addend: this may cause carries to occur.
4. Read the result on the output wheels.

Representing Signed Numbers

- In a 3 digit machine 999 + 1 = 000.
- So “999” is also -1.
- Which means “998” is also -2.
- We can choose between signed and unsigned representations:
  - Unsigned: 0 to 999 represented as “000” to “999”.
  - Signed: -500 to +499 represented as
    “500” to “999” followed by “000” to “499”.

Converting Positive to Negative

- Subtract the value from 999, then add 1.
- Example: how do we represent -14?

“999”
 - 14
 ——
“985” + 1 = “986” is -14

Nines' Complement

- To form the nines' complement of a number, subtract every digit from 9.
- Note: there is never any borrowing or carrying, so this can be computed very quickly.
- Denote the machine representation of the nines' complement as C(n).
- Verify for yourself:

  C(C(n)) = n   because 9-(9-x) = (9-9)+x = 0+x = x

- Denote the machine representation of negation as:

  Neg(n) = C(n) + 1

Tens' Complement Subtraction

$a - b \quad = \; a + -b$

$\qquad\quad = \; a + \mathrm{Neg}(b)$

$\qquad\quad = \; a + C(b) + 1$ **three operations**

But we can simplify this by taking the nines' complement of both sides.

Nines' Complement Subtraction

C(a-b) = “9999” – (a – b)

= (“9999” – a) + b *associative property*

= C(a) + b **two operations**

Nines' Complement of a Sum

Proof that $C(a+b) = C(a) + C(b) + 1$:

$$Neg(a+b) = C(a+b) + 1$$

$$\begin{aligned} Neg(a+b) &= Neg(a) + Neg(b) \\ &= [C(a) + 1] + [C(b) + 1] \\ &= C(a) + C(b) + 2 \end{aligned}$$

So $C(a+b) + 1 = C(a) + C(b) + 2$

Therefore $C(a+b) = C(a) + C(b) + 1$.

Nines' Complement Subtraction

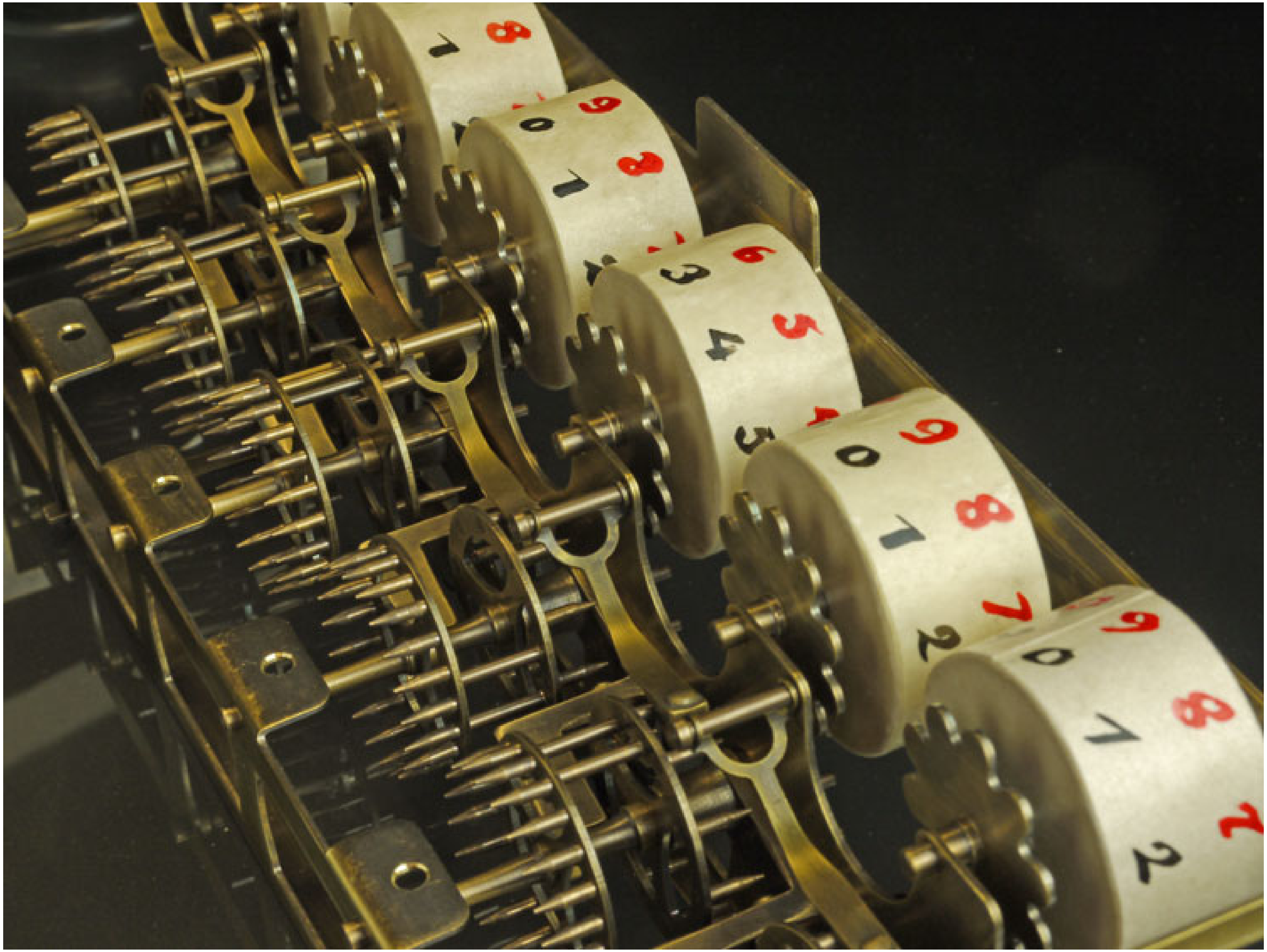
$$
\begin{aligned}
C(a\text{-}b) &= C(a + Neg(b)) \\
&= C(a + C(b) + 1) \\
&= C((a+1) + C(b)) \\
&= C(a+1) + C(C(b)) + 1 \\
&= C(a+1) + b + 1 \\
&= [C(a) + C(1) + 1] + b + 1 \\
&= C(a) + \text{“9998”} + 1 + b + 1 \\
&= C(a) + -2 + 1 + b + 1 \\
&= C(a) + b \quad \textbf{two operations}
\end{aligned}
$$

Subtraction in the Pascaline

Compute a – b as C(C(a) + b):

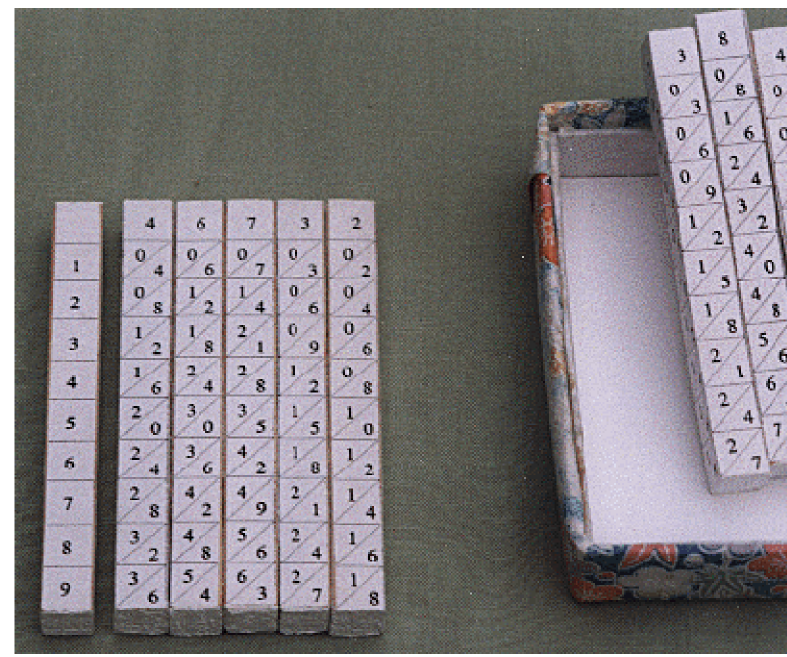
1. Clear the machine.
2. Enter C(a) using complement digit marks.
3. Add in b using the regular digit marks.
   – This gives C(a) + b
4. Read the result on the complement number readout instead of the regular readout.
   – This gives C(C(a) + b)

Pascaline Replicas (Many)

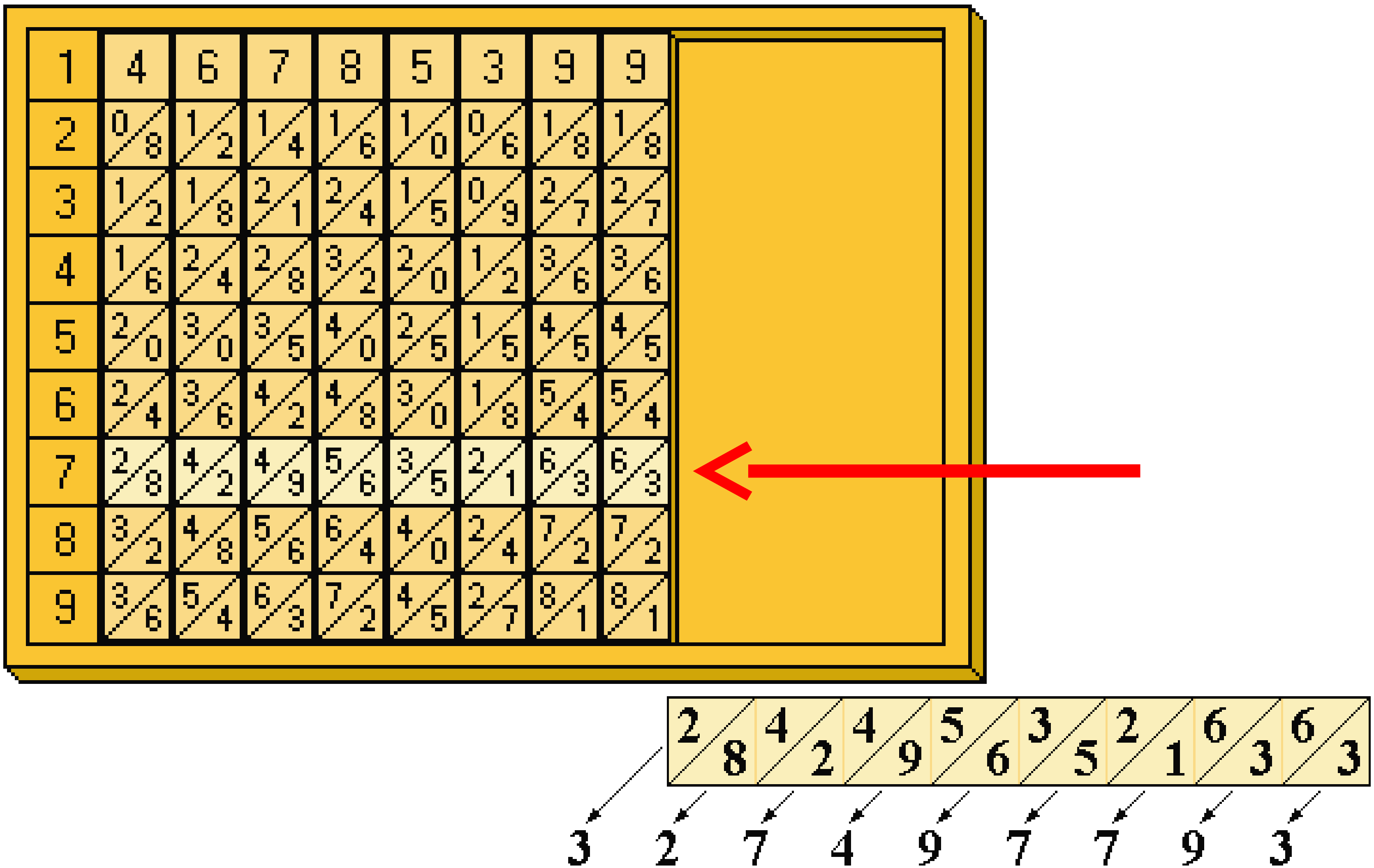


John Napier

- Scottish mathematician (1550-1617)
- Invented “Napier's bones”, used to perform multiplication using only addition.
- Napier's bones were very successful and widely used in Europe until the mid-1960s.
- Napier is also the inventor of logarithms.

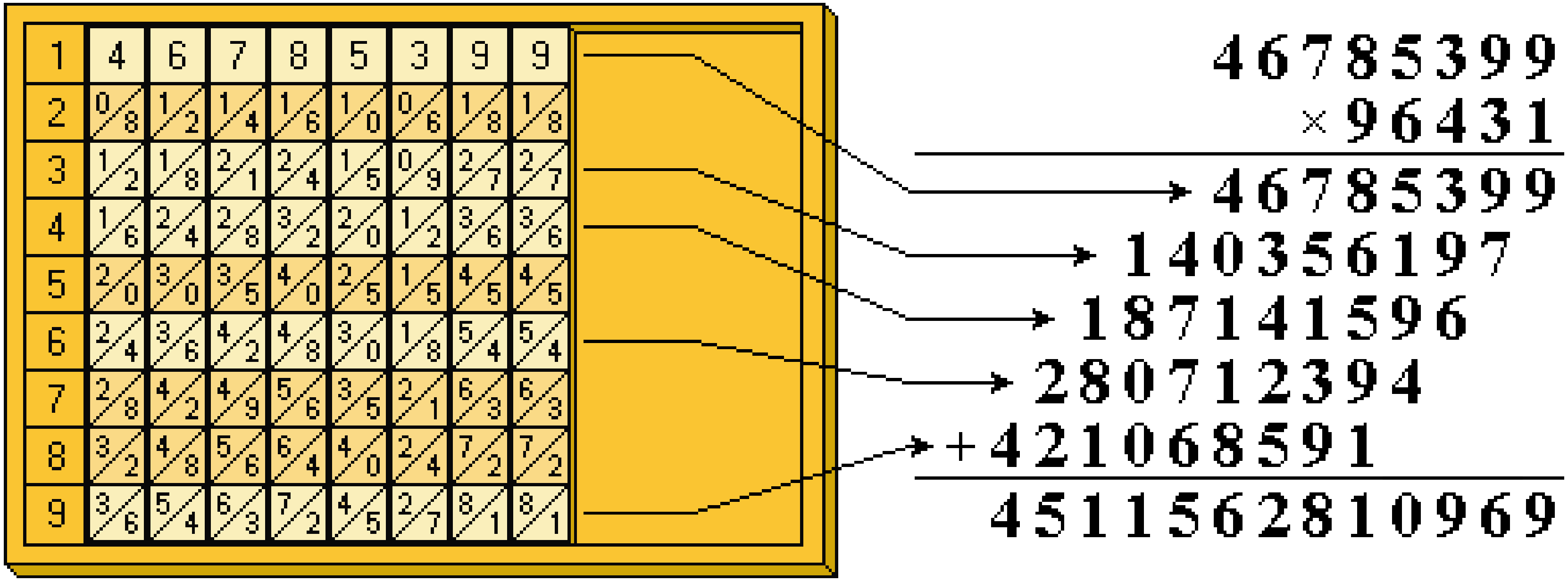


7 x 46785399 = ?



Multiplying Multi-Digit Numbers

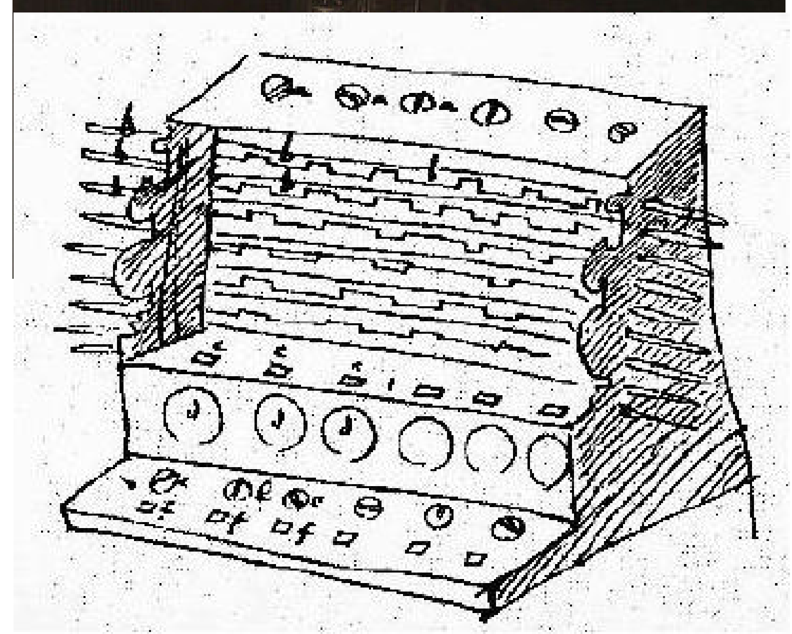
- Do single digit multiplications, shift, and add:



As with the abacus, humans do most of the work.

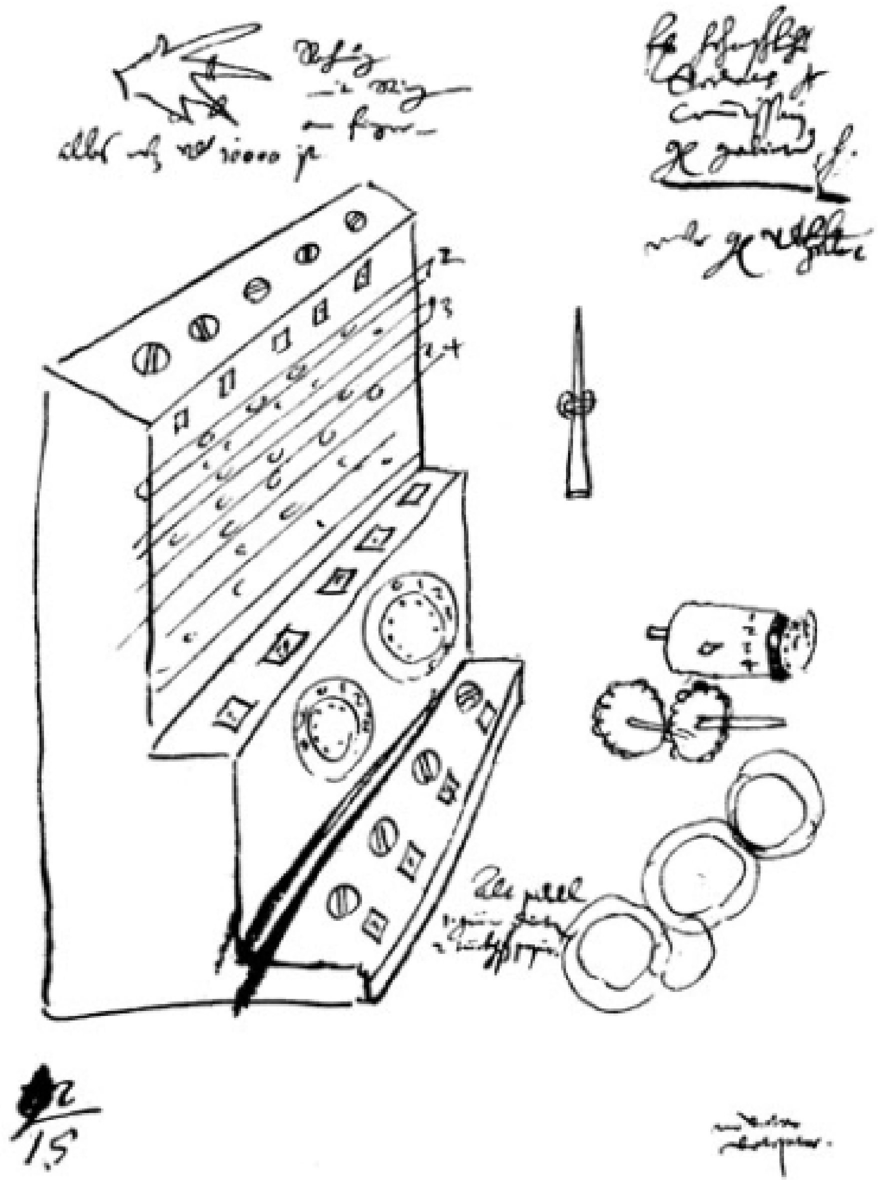
Wilhelm Schickard (1592 - 1635)

- Described a “calculating clock” in letters to his friend Johannes Kepler in 1623 (the year Pascal was born) and 1624, with sketches included.
- Claims the prototype worked, but it has not survived. Second, professionally-built version was destroyed in a fire before delivery.
- Addition by rotating wheels.
- Subtraction by moving wheels in opposite direction.
- Multiplication via Napier's bones (lookup table).



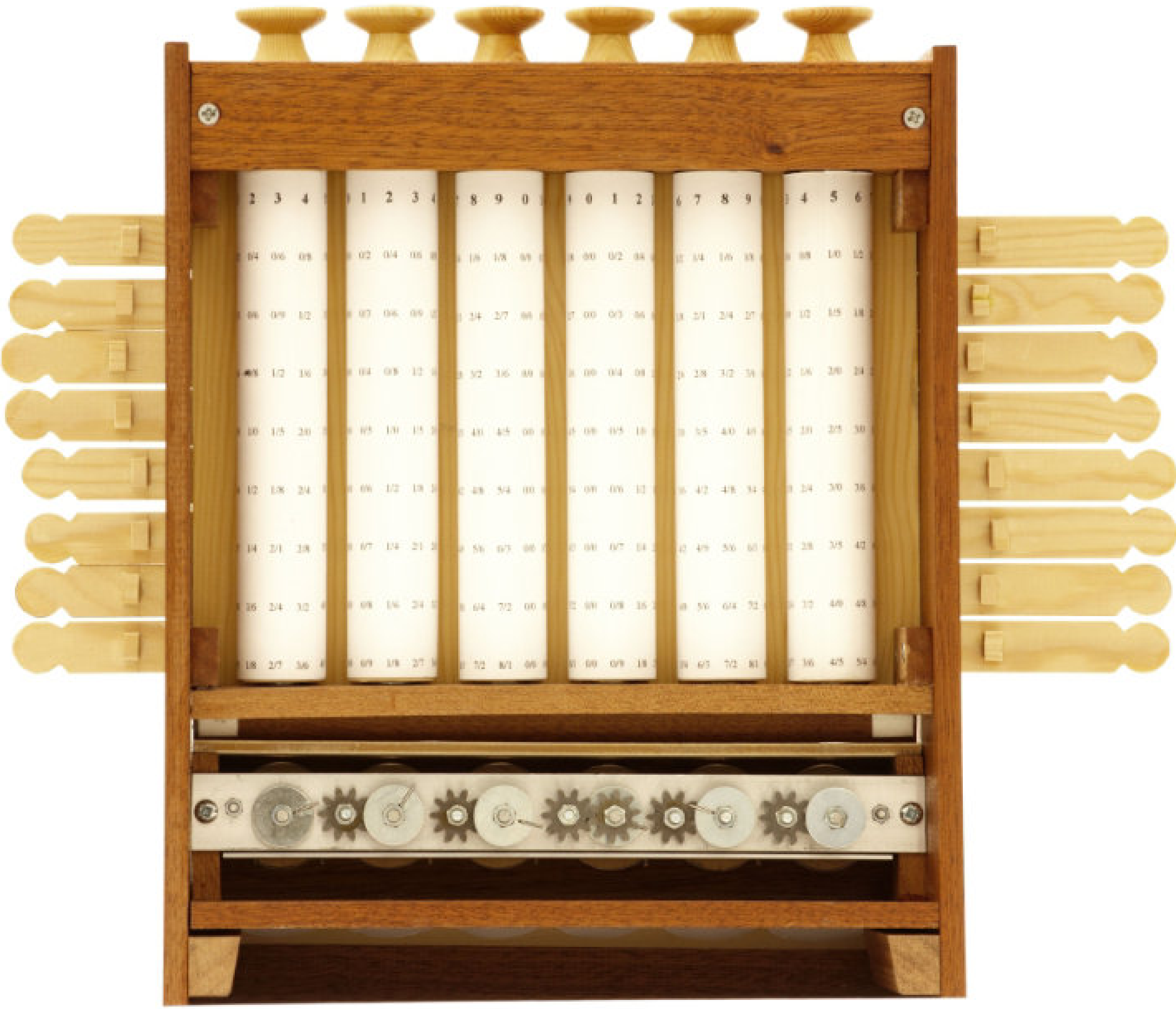
Schickard's Calculating Clock

- The surviving notes don't describe a fully functional machine.
- Requires additional wheels and springs. Did he add them?
- Used single tooth carry gear, which doesn't work for many-digit carries.



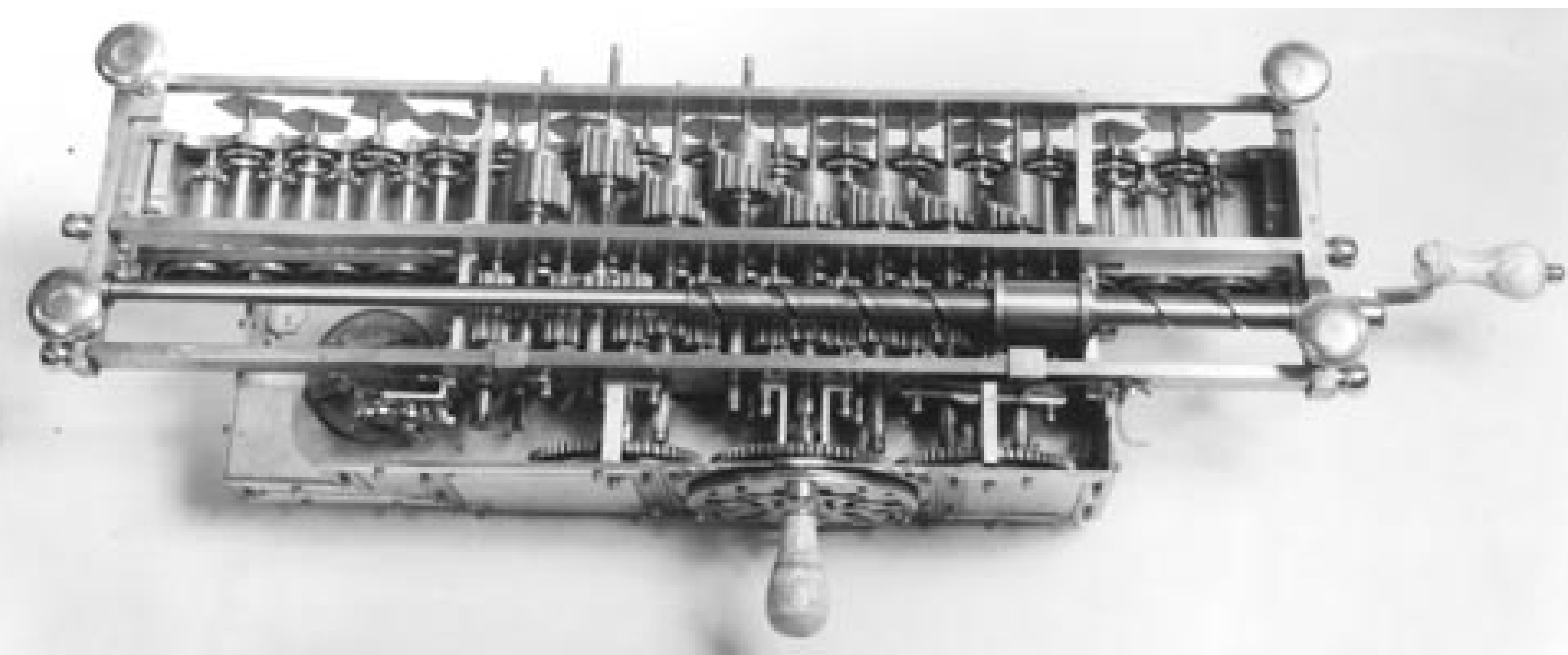
Schickard Replica

Back Side of the Replica



Leibniz Step Reckoner

- Successor to the Pascaline. Designed in 1673, completed in 1694.
- Could add and subtract automatically.
- Multiply and divide by shifting the carriage.



History of Computing

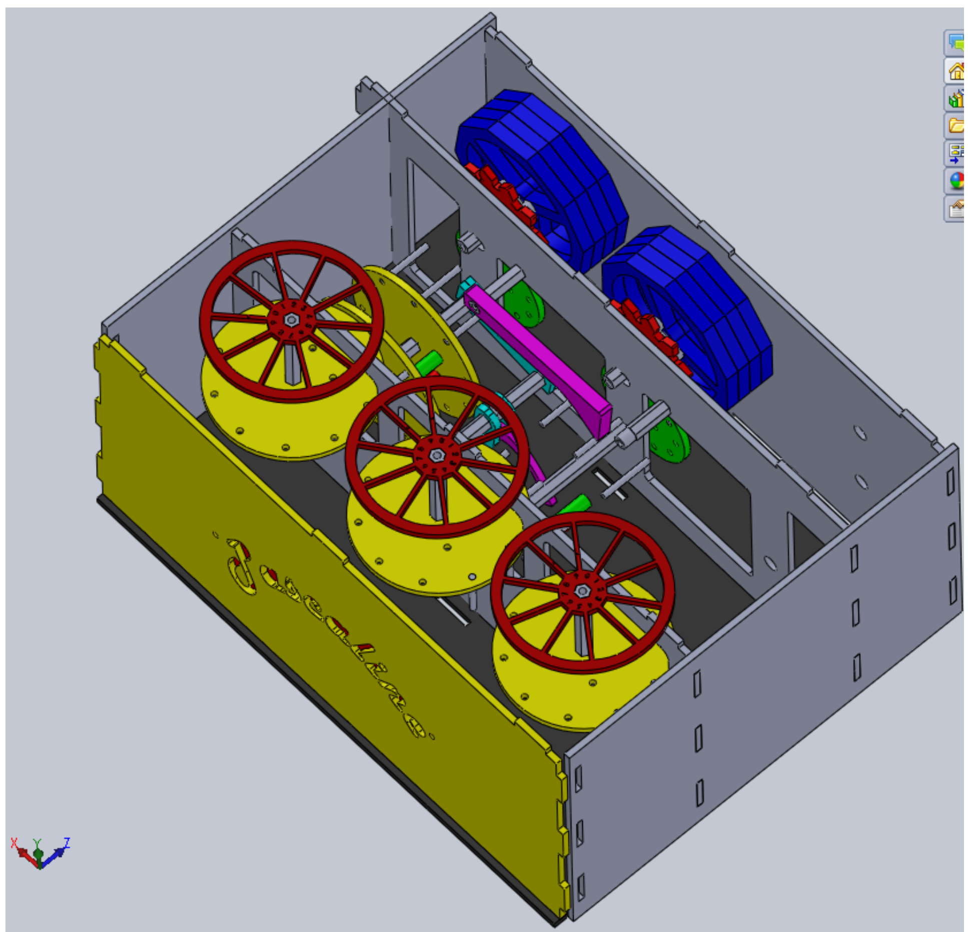
To learn more about the history of computing:

- Take Tom Cortina's mini-course:
  15-292 History of Computing

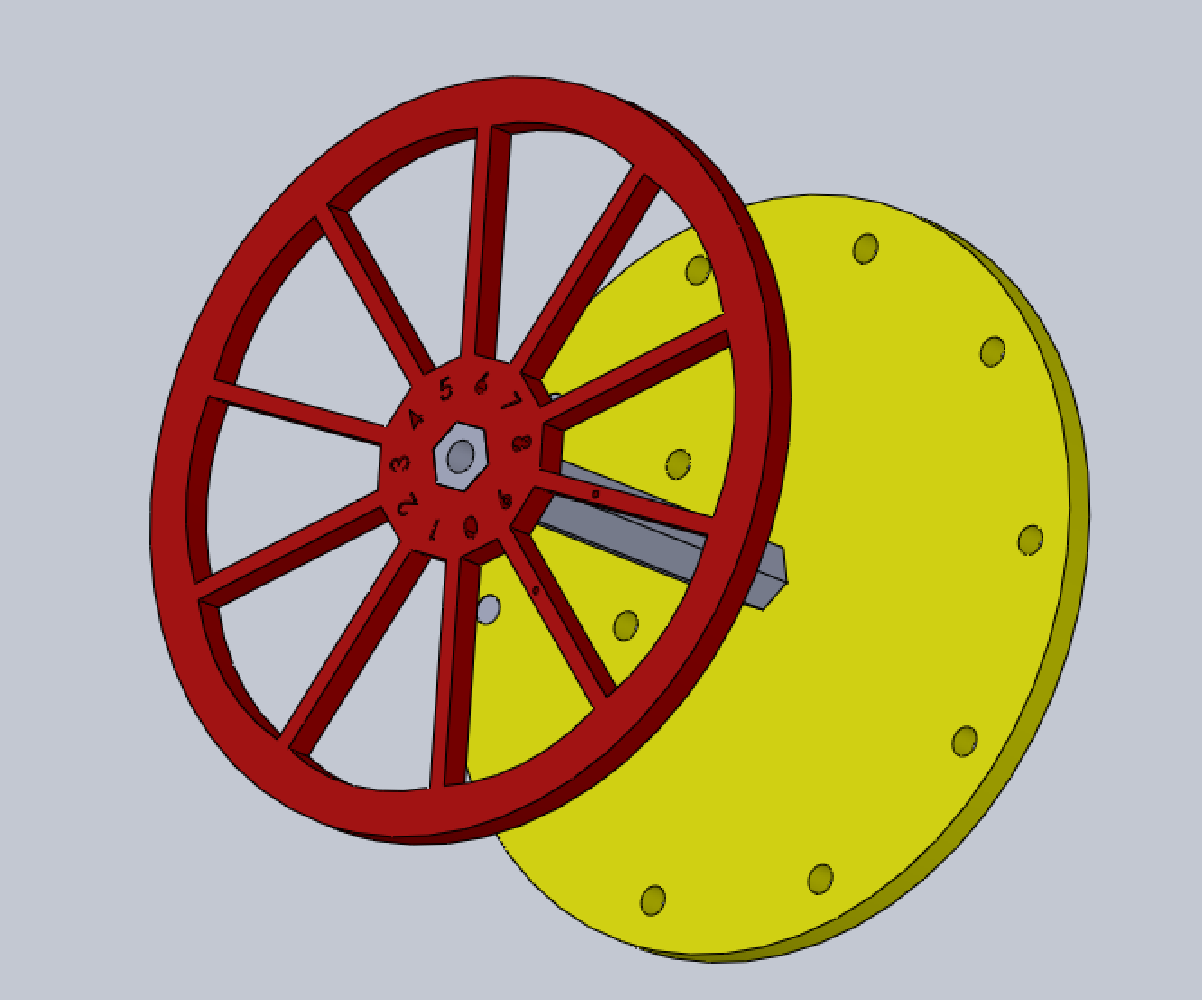
- Visit the Computer History Museum at www.computerhistory.org or in person in Mountain View, California.

Building Our Own Pascaline

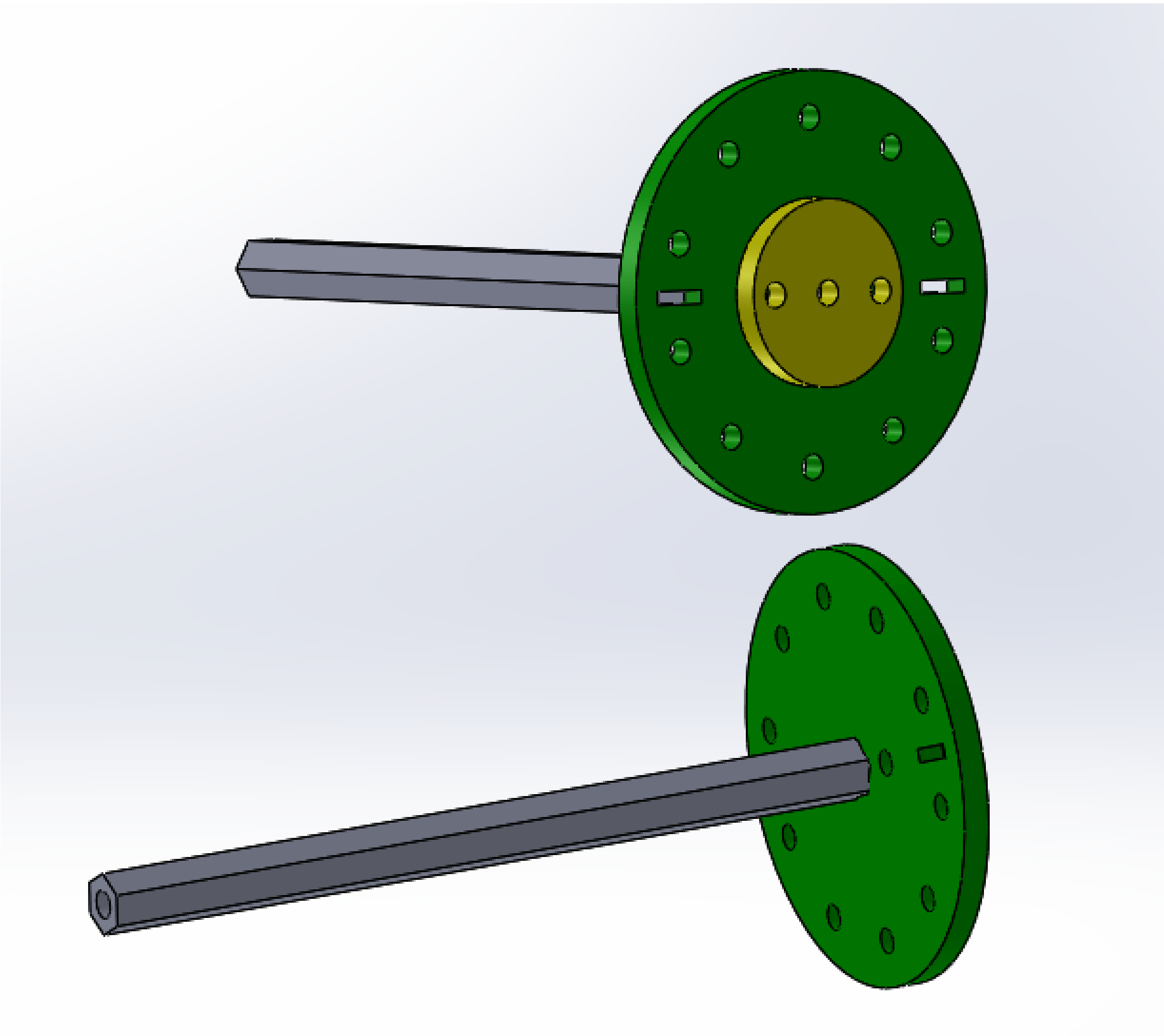
- Just laser-cut parts plus metal fasteners.
- No sawing. No drilling. No glue. Assemble with a screwdriver and pliers.

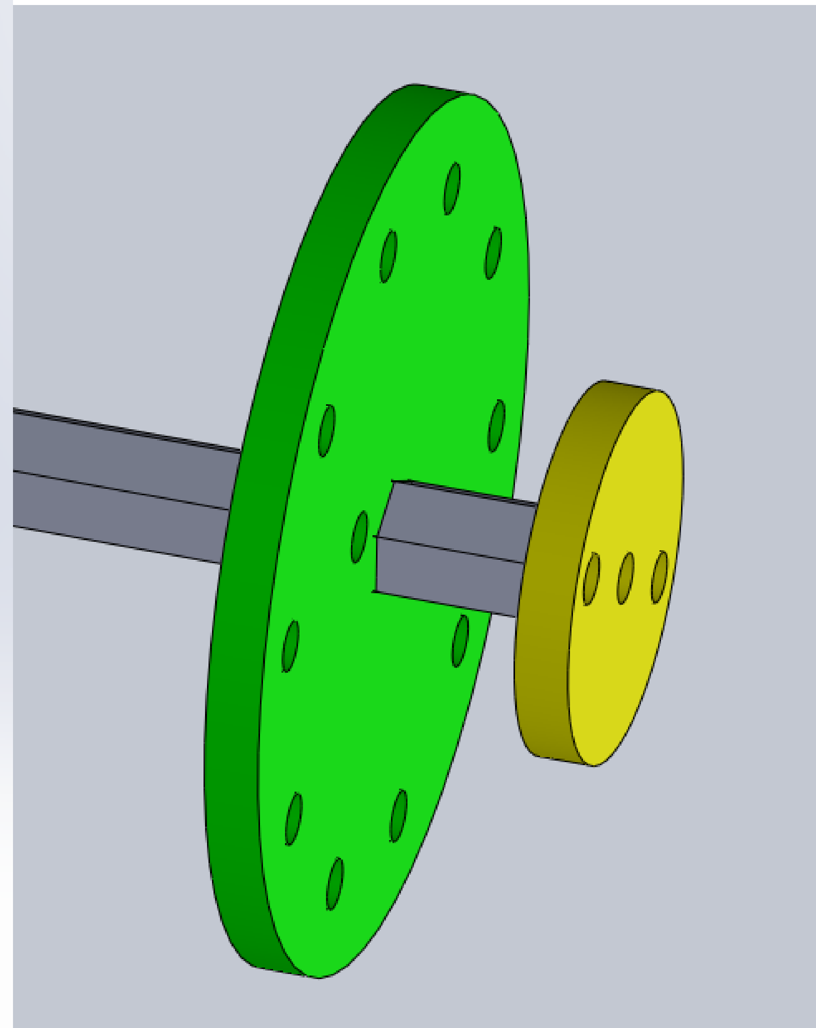


Hex Standoffs Prevent Rotation

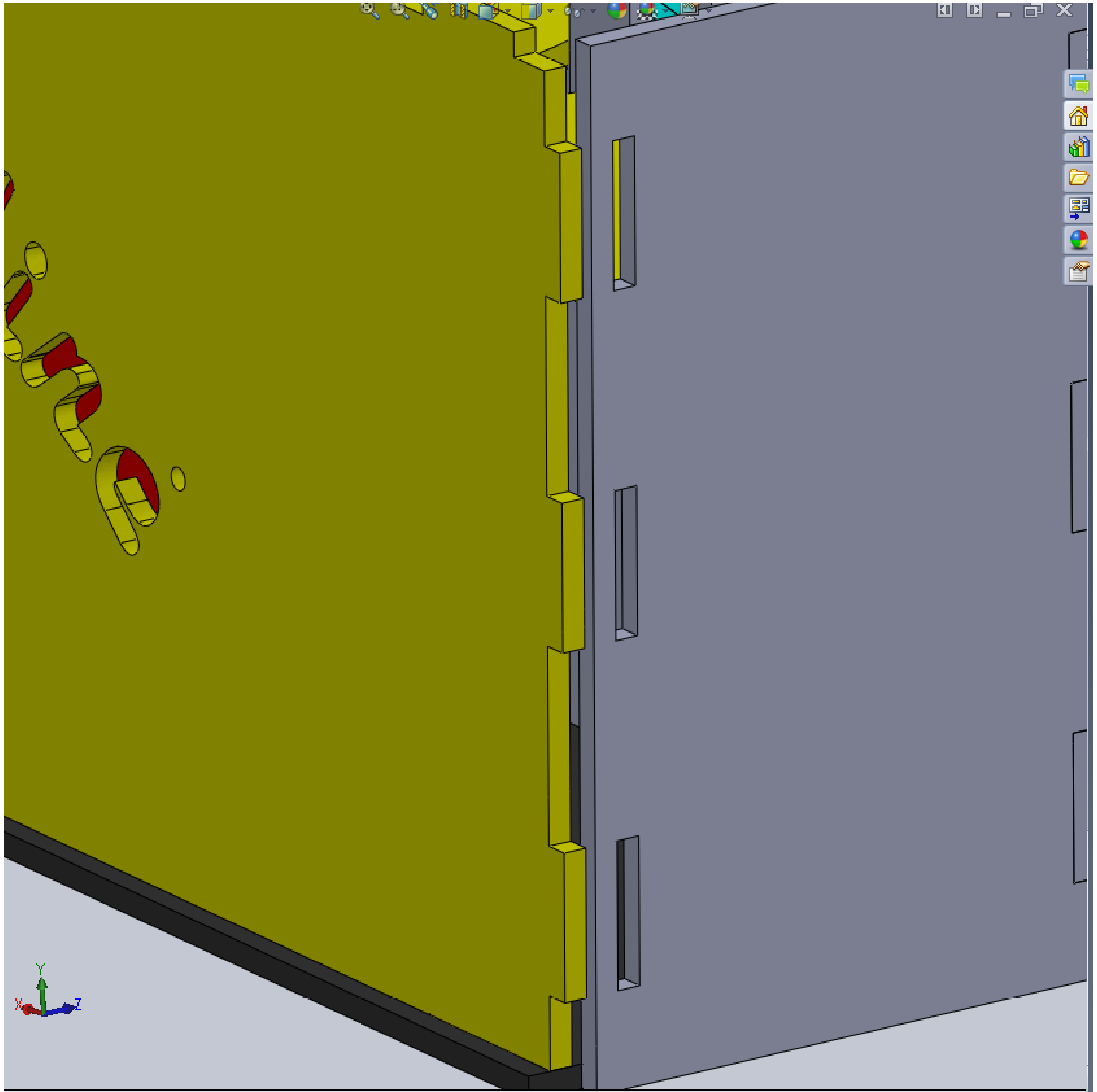


End-Cap Holds Gear on Shaft



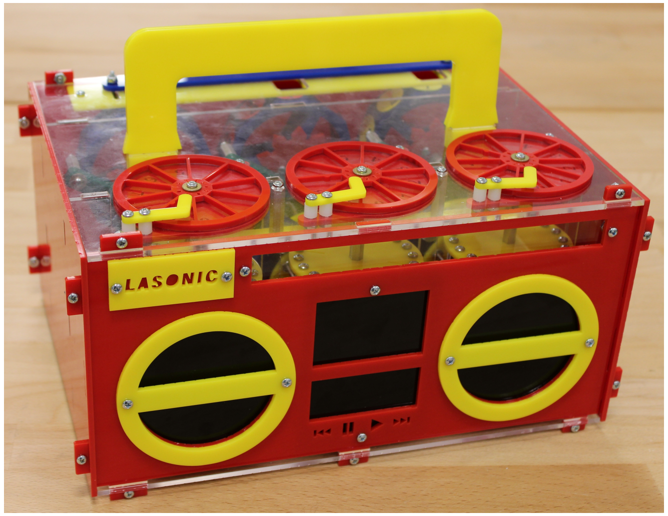


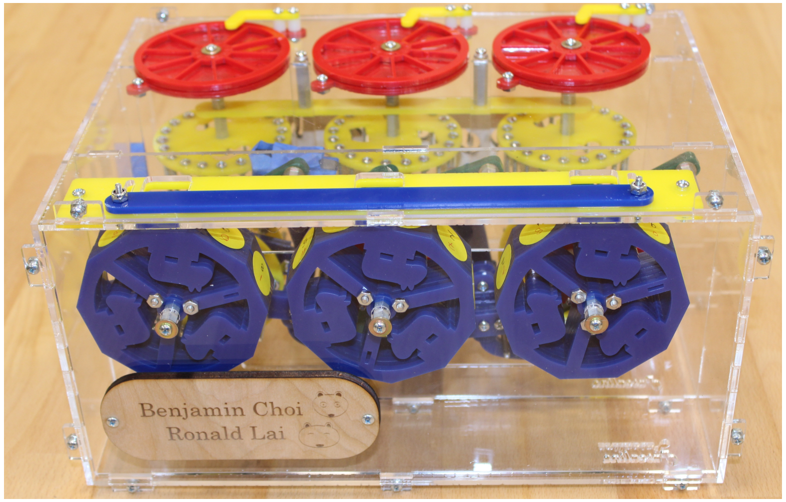
Slot and Tab Box Construction

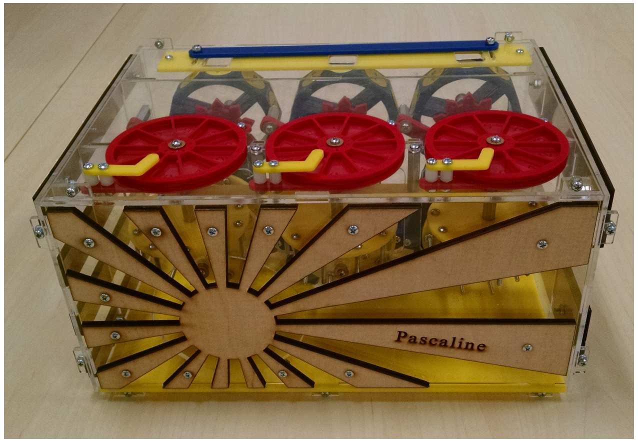


Goal

- Make a Pascaline kit.
- Open source.
- Distribute via:
  - TechShop?
  - Instructables?
  - [your suggestions here]







Assignment 3: The Pascaline

- Will be done in groups of 2.
- Files due in a week.
- Pascaline “checkpoint” during recitation.
- Assembled machines will be tested for grading.
- What you need to do today:
  - Find a partner.
  - Email your pairings to Dave by Wednesday.