15-451 Algorithms, Fall 2007

Homework # 7 **due: Thursday December 6, 2007**

Please hand in each problem on a separate sheet and put your **name** and **recitation** (time or letter) at the top of each sheet. You will be handing each problem into a separate box, and we will then give homeworks back in recitation. Remember: written homeworks are to be done **individually**. Group work is only for the oral-presentation assignments.

Problems:

(26 pts) 1. [NP-completeness and approximation algorithms]

Let $\mathcal{A}$ be the set of pairs (G, k) such that G is a graph with a vertex cover of size k or less. Let $\mathcal{C}$ be the set of pairs (G, k) such that G has a vertex cover of size $k/2$ or less. Notice that if $(G, k) \in \mathcal{C}$ then clearly $(G, k) \in \mathcal{A}$ also, so $\mathcal{A} \supseteq \mathcal{C}$. Determining whether a given input (G, k) belongs to $\mathcal{A}$ is NP-Complete (this is the Vertex-Cover problem), and also determining whether a given input (G, k) belongs to $\mathcal{C}$ is NP-complete (since this is really the same problem). Describe a set $\mathcal{B}$ such that $\mathcal{A} \supseteq \mathcal{B} \supseteq \mathcal{C}$ but membership in $\mathcal{B}$ can be decided in polynomial time. So this is just like the situation on Mini 5. Hint: think approximation algorithms.

(26 pts) 2. [Random-access[1] long division].

Give a polynomial time algorithm to find the Nth digit of the fraction A/B, where A, B and N are all given in binary.

Input: integers (A, B, N) in binary notation, where $A < B$.

Let $0.d_1 d_2 d_3 \cdots$ be the decimal expansion of the fraction $\frac{A}{B}$.

Output: d_N.

Note: the key thing here is that your algorithm's running time should be polynomial in $\log N$ (and $\log A$ and $\log B$). The standard way of doing long division would instead be polynomial in N. In particular, the standard long division would look like this:

> for $i = 1$ to N do:
> $\quad d_i = 10A$ div B;
> $\quad A = 10A$ mod B;

where "div" is integer division.

(48 pts) 3. [Review] Last year's final is attached to this assignment. We recommend that you complete the entire final for practice. For this homework, for 48 points, choose 4 problems out of $\{1, 2, 6, 7, 8, 9\}$ and turn in solutions to them. For the purpose of this assignment, they will be graded at 12 points apiece. (Problems 3 and 5 (and portions of 4) have already appeared in previous minis, tests, or recitation notes).

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[1] "Random access" as in random-access memory, i.e., as opposed to sequential-access. Not "random" as in probability.