

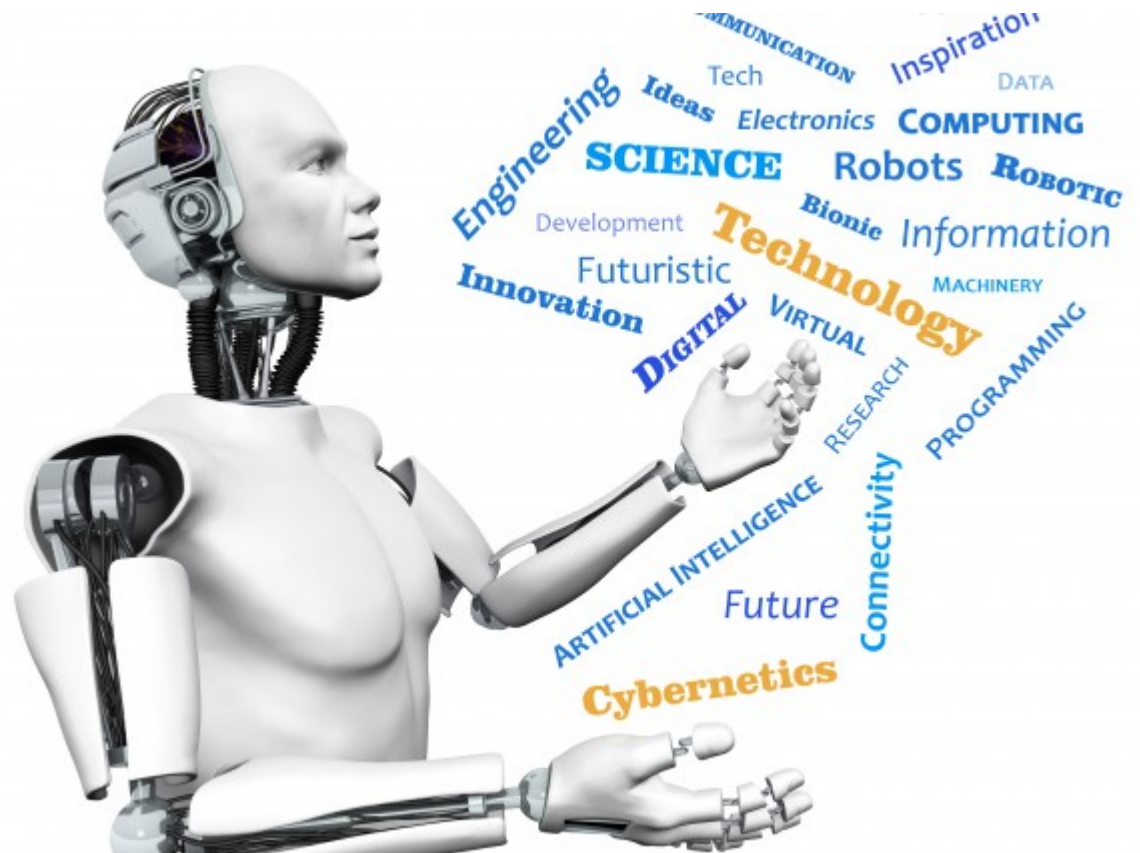
# 15-494/694: Cognitive Robotics

Dave Touretzky

Lecture 9:

Path Planning with  
Rapidly-exploring  
Random Trees

Navigating with the Pilot



# Outline

- How is path planning used in robotics?
- Path planning as state space search
- RRTs: Rapidly-exploring Random Trees
- The RRT-Connect algorithm
- Collision detection
- Smoothing
- Path planning with constraints
- Navigating with the Pilot

# Path Planning in Robotics

## 1. Navigation path planning

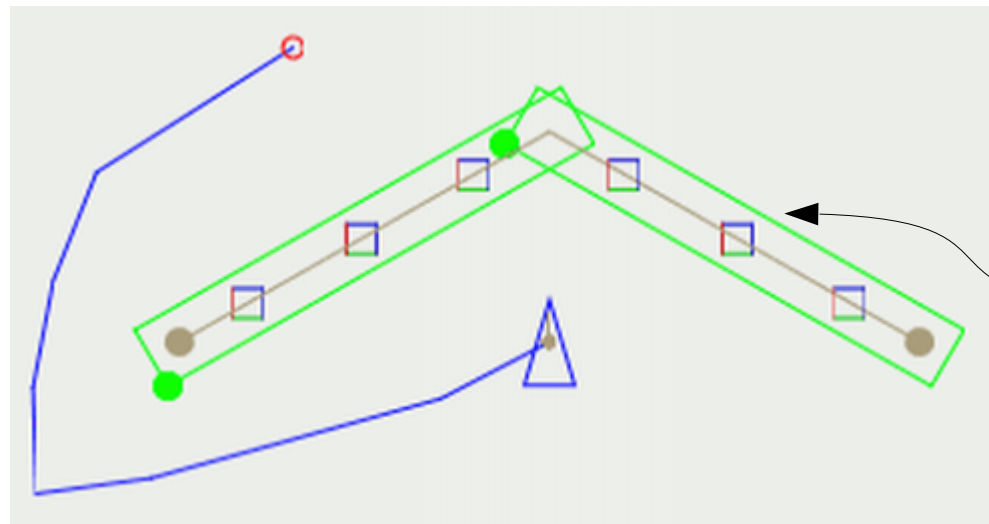
- How to get from the robot's current location to a goal.
- Avoid obstacles.
- Provide for localization.

## 2. Manipulation path planning

- Move an arm to grasp and manipulate an object.
- Avoid obstacles.
- Obey constraints (e.g., don't spill the coffee).

# Navigation Planning

- 2D state space:  $(x,y)$  coordinates of the robot
  - Treat the robot as a point or a circle.



- 3D state space:  $(x,y,\theta)$  pose of the robot
  - Heading matters when the robot is asymmetric
  - Heading matters when the robot's motion is constrained

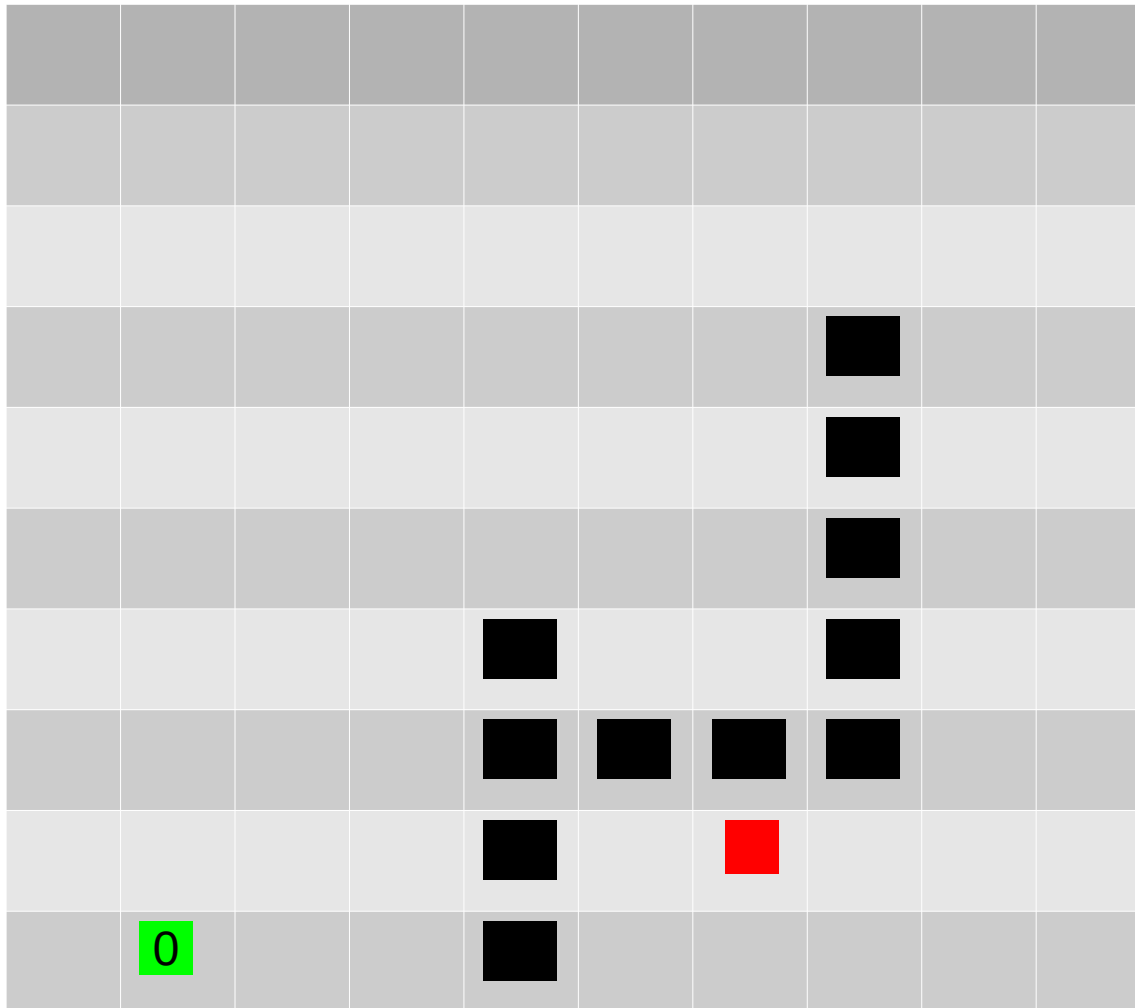
# Grid-Based Path Planning

- Discretizes the environment into a 2D grid.
- Wavefront algorithm: propagate from the start location.
- Can also use best-first or A\* search.
- Works okay in small spaces.

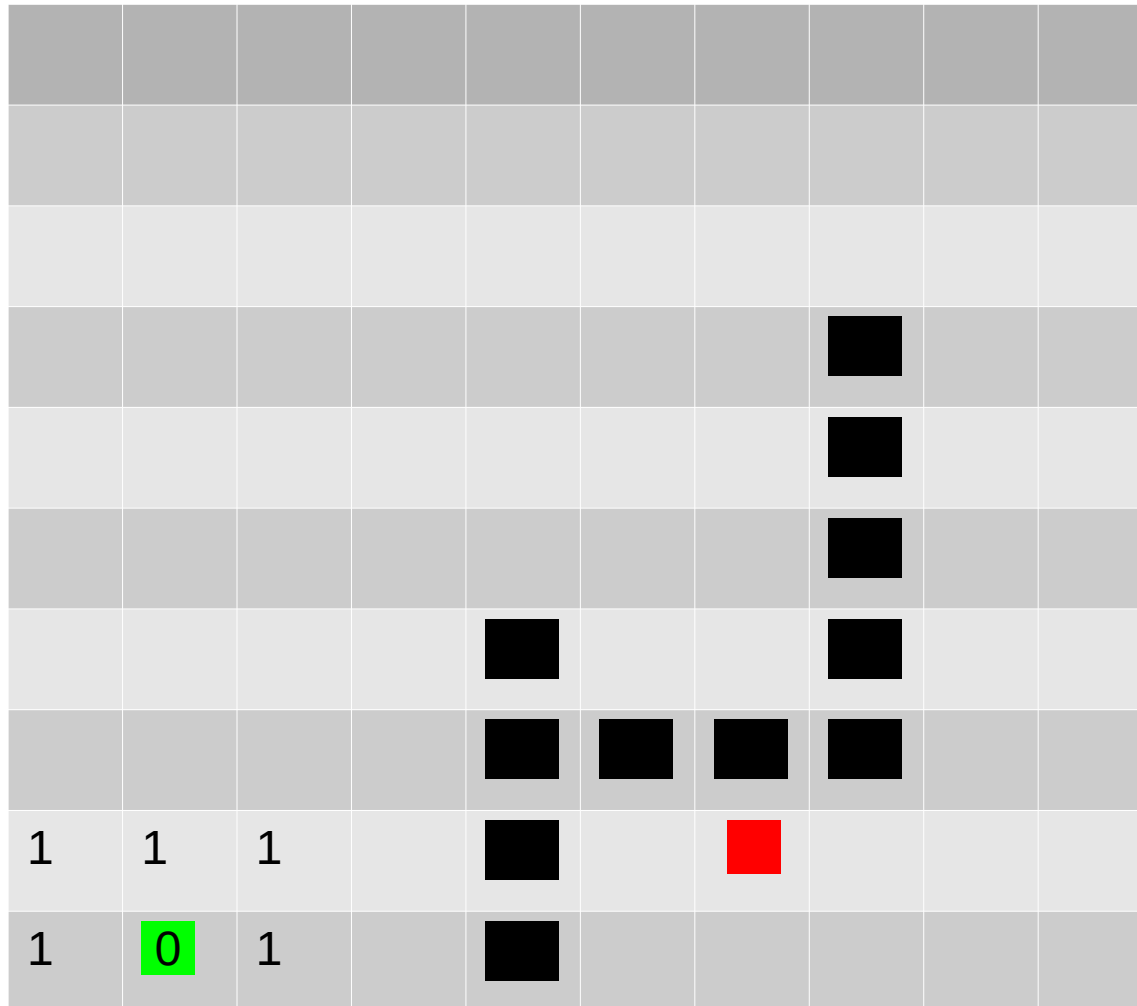
But it has its drawbacks:

- Treats the robot as a point. Unrealistic!
- Not efficient in higher dimensional state spaces.

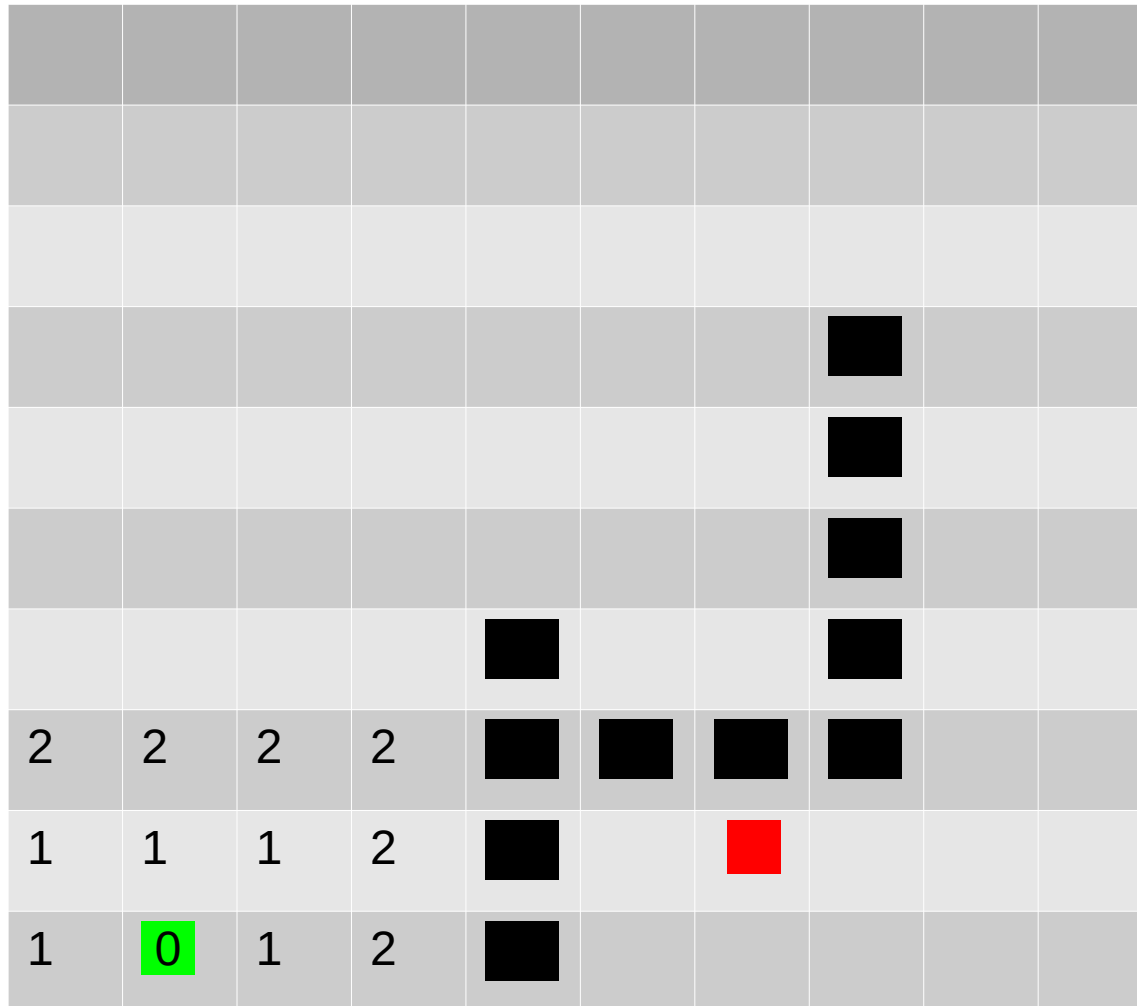
# Wavefront Algorithm



# Wavefront Algorithm

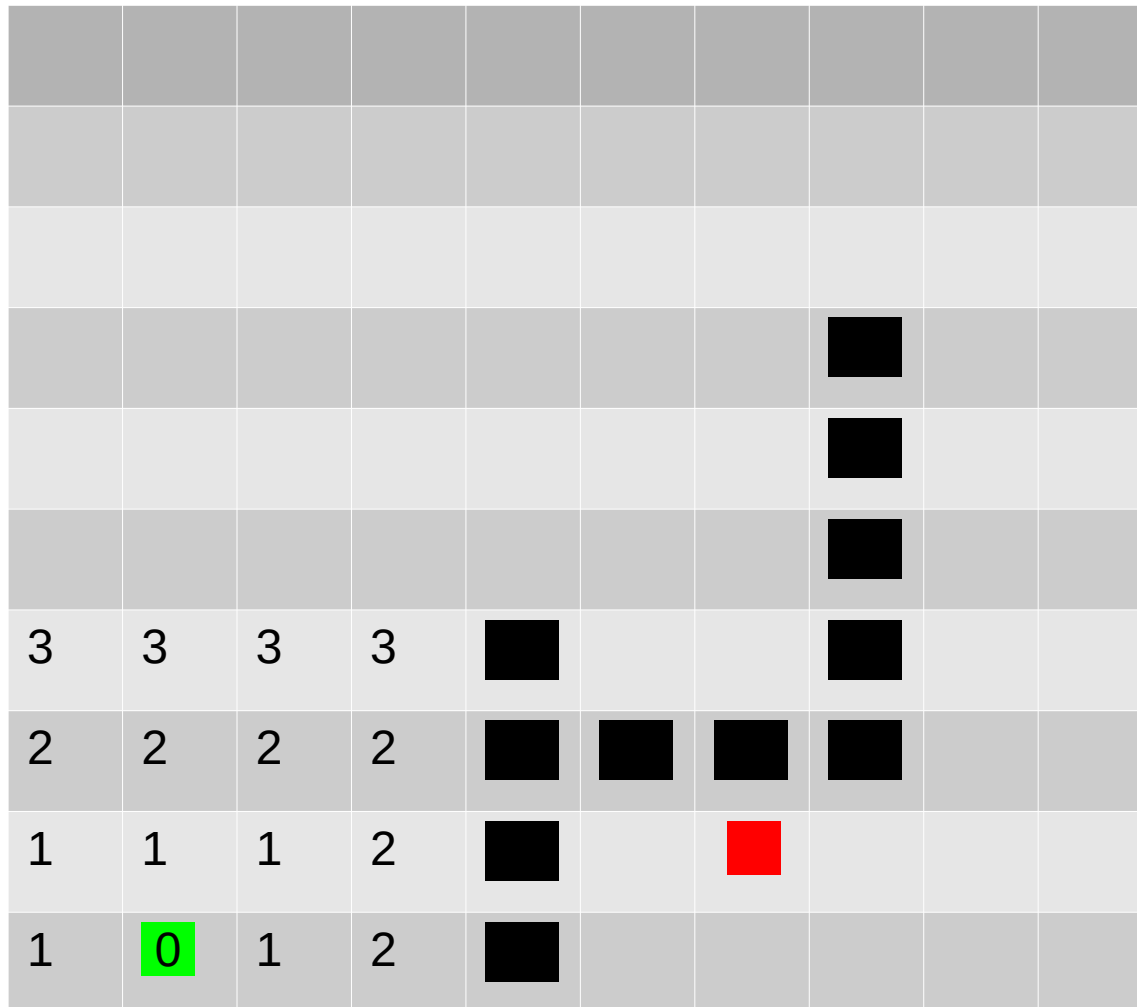


# Wavefront Algorithm

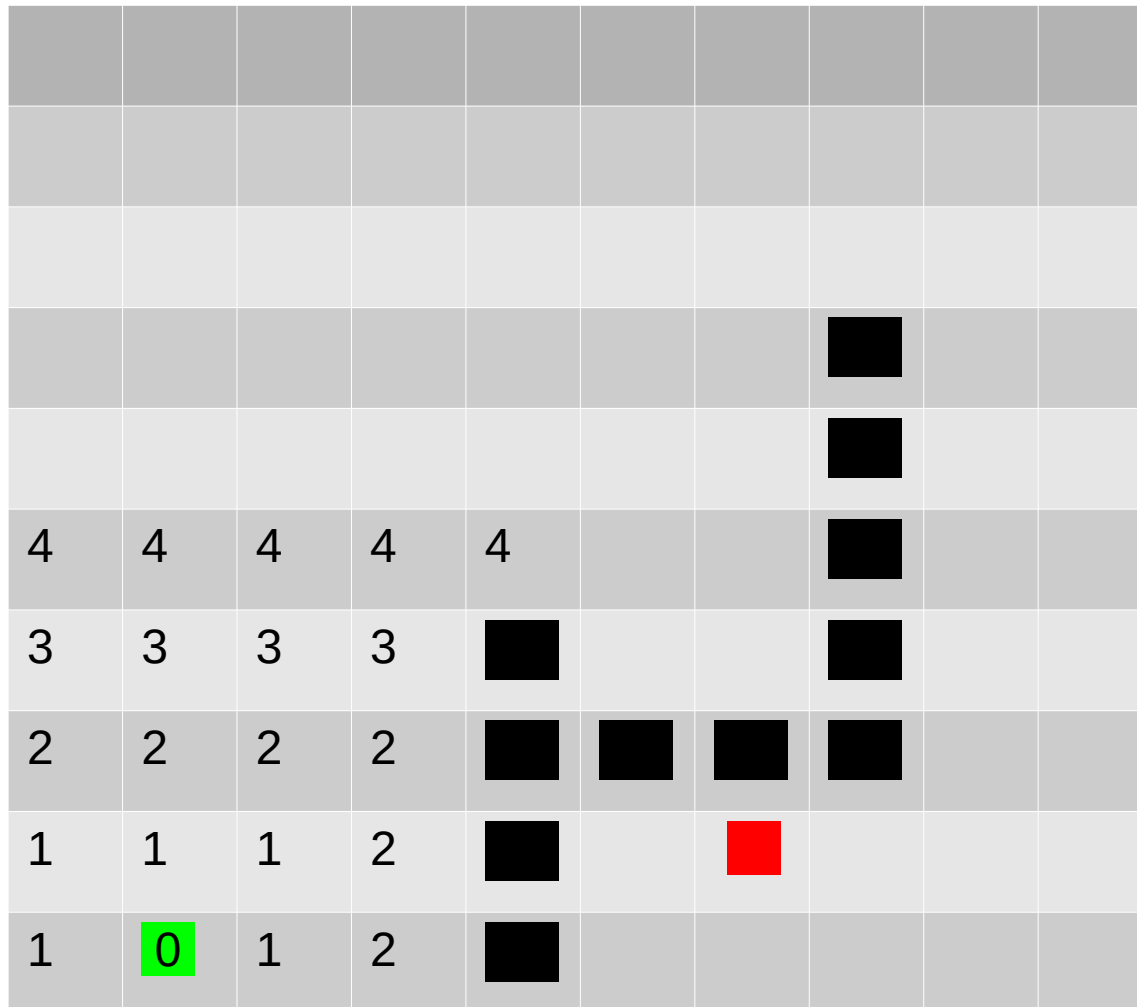




# Wavefront Algorithm



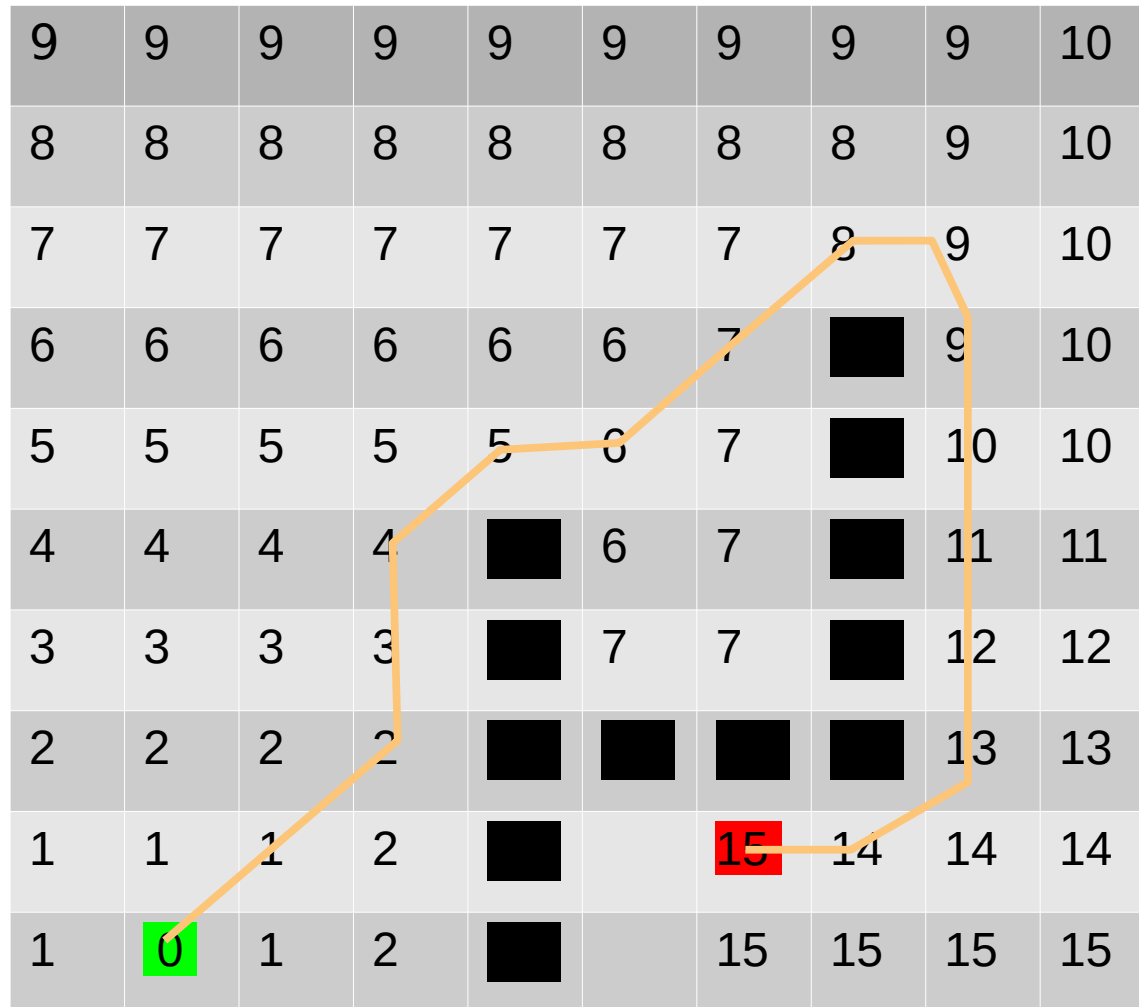
# Wavefront Algorithm



# Wavefront Algorithm

|   |   |   |   |   |   |    |    |    |    |
|---|---|---|---|---|---|----|----|----|----|
| 9 | 9 | 9 | 9 | 9 | 9 | 9  | 9  | 9  | 10 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8  | 8  | 9  | 10 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7  | 8  | 9  | 10 |
| 6 | 6 | 6 | 6 | 6 | 6 | 7  | ■  | 9  | 10 |
| 5 | 5 | 5 | 5 | 5 | 6 | 7  | ■  | 10 | 10 |
| 4 | 4 | 4 | 4 | ■ | 6 | 7  | ■  | 11 | 11 |
| 3 | 3 | 3 | 3 | ■ | 7 | 7  | ■  | 12 | 12 |
| 2 | 2 | 2 | 2 | ■ | ■ | ■  | ■  | 13 | 13 |
| 1 | 1 | 1 | 2 | ■ |   | 15 | 14 | 14 | 14 |
| 1 | 0 | 1 | 2 | ■ |   | 15 | 15 | 15 | 15 |

# Wavefront Algorithm



# Best-First or A\* Search

- Works okay in small spaces.

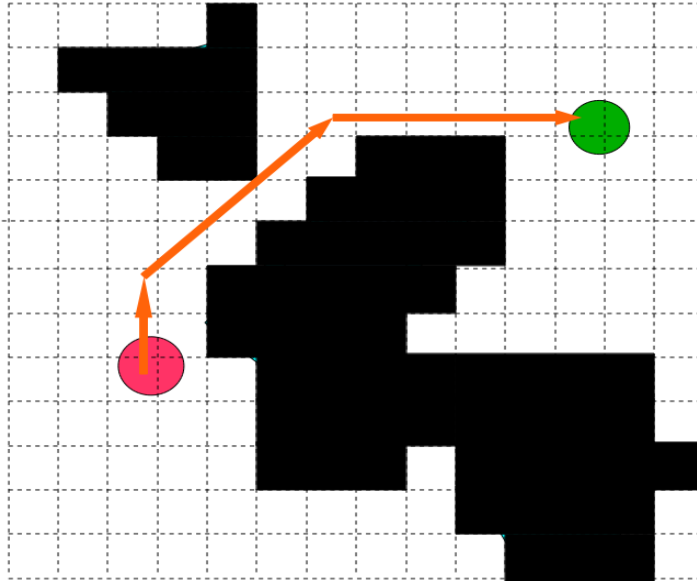


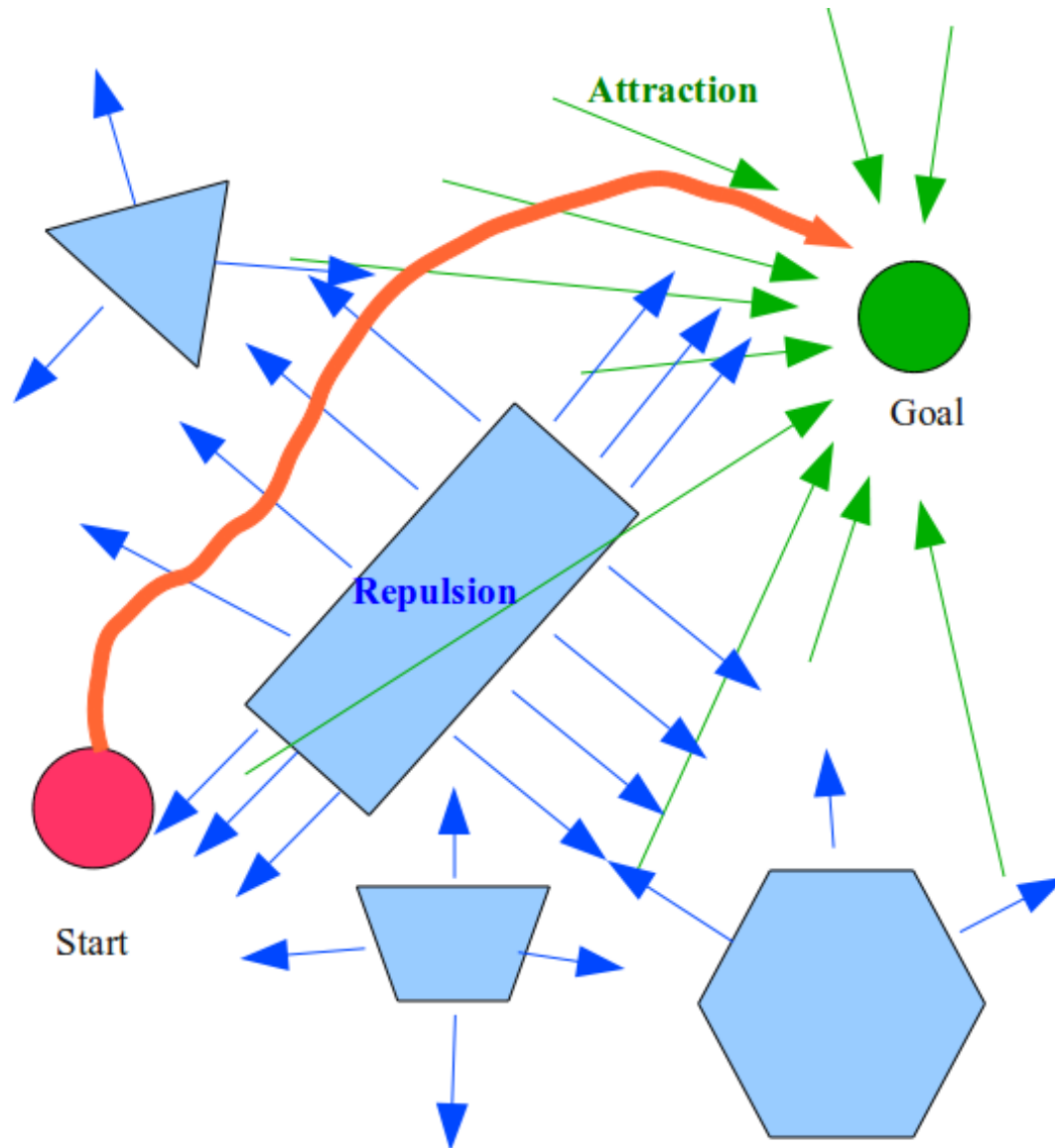
Figure from  
[http://www.gamasutra.com/blogs/MattKlingensmith/20130907/199787/Overview\\_of\\_Motion\\_Planning.php](http://www.gamasutra.com/blogs/MattKlingensmith/20130907/199787/Overview_of_Motion_Planning.php)

Same drawbacks as wavefront:

- Treats the robot as a point. Unrealistic!
- Not efficient in higher dimensional state spaces.



# Potential Field Path Planning



- Can fail due to local minima in the potential function.
- Consider a U-shaped obstacle.
- Requires careful tuning.

# Cspace Transform

- The area around an obstacle that would cause a collision with the robot.

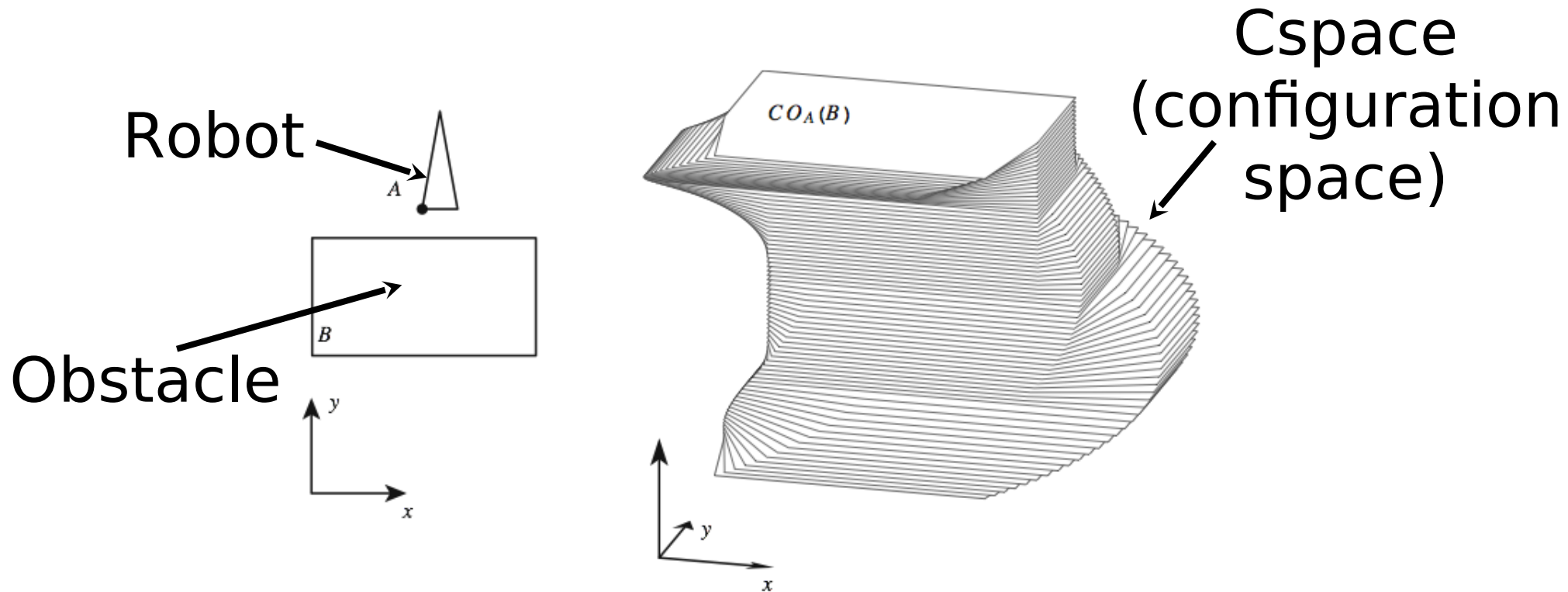


Figure 4.4 - Mason, Mechanics Of Robotic Manipulation



# Arm Path Planning

- Cspace transform blocks out regions of joint space

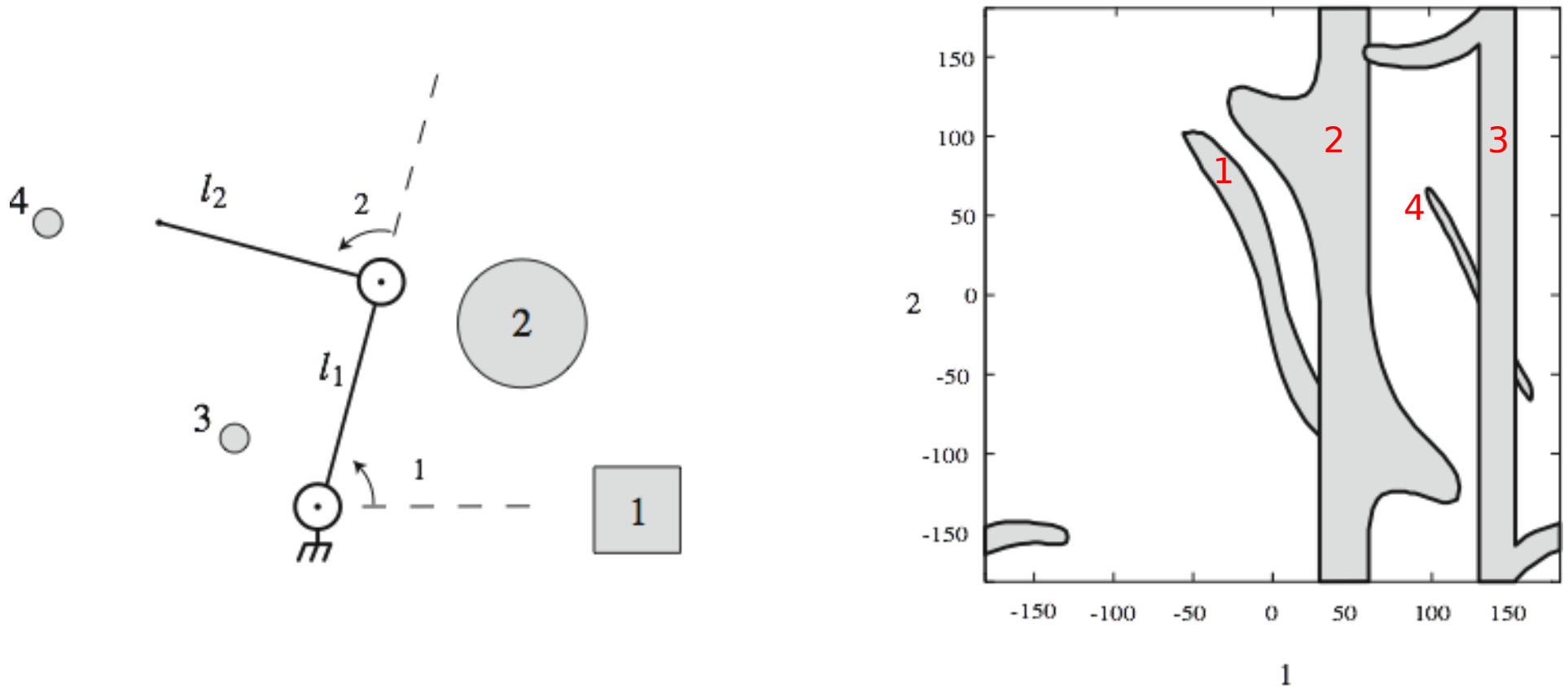


Figure 4.5 - Mason, Mechanics Of Robotic Manipulation

# State Space Search

The path planning problem:

Given an n-dimensional state space and

- a start state  $S = \langle s_1, s_2, \dots, s_n \rangle$
- a goal state  $G = \langle g_1, g_2, \dots, g_n \rangle$
- an admissibility predicate  $P$  (collision test + constraints)

find a path from  $S$  to  $G$  such that every state on the path satisfies  $P$ .

# Rapidly-exploring Random Trees

- Described in LaValle (1998), Kuffner & LaValle (2000)
- Create a tree with initial state  $S$  as the root.
- Repeat up to  $K$  times:

Pick a point  $\mathbf{q}_{\text{rand}}$  in configuration space:

- Sometimes  $\mathbf{q}_{\text{rand}}$  is really random
- Sometimes  $\mathbf{q}_{\text{rand}}$  is the goal  $G$
- Find  $\mathbf{q}_{\text{nearest}}$ , the closest node to  $\mathbf{q}_{\text{rand}}$
- Add a new node  $\mathbf{q}_{\text{new}}$  by extending  $\mathbf{q}_{\text{nearest}}$  some distance  $\Delta$  toward  $\mathbf{q}_{\text{rand}}$ .
- If  $\mathbf{q}_{\text{new}}$  is close enough to the goal  $G$ , return.

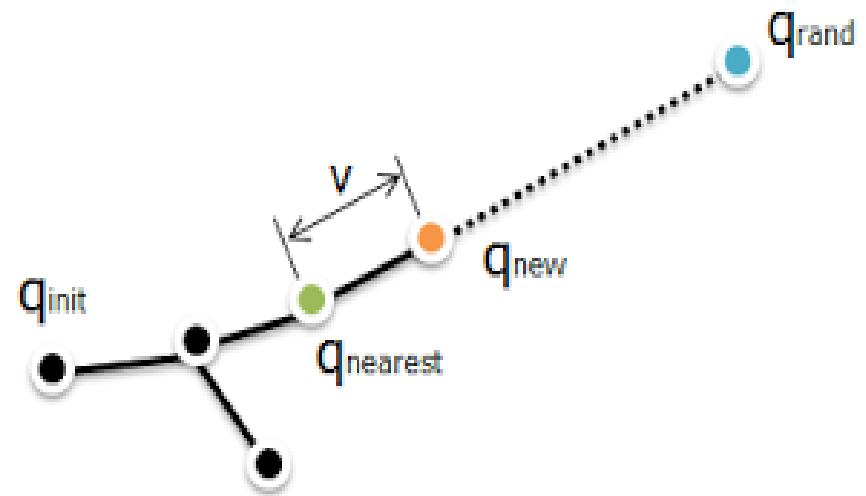
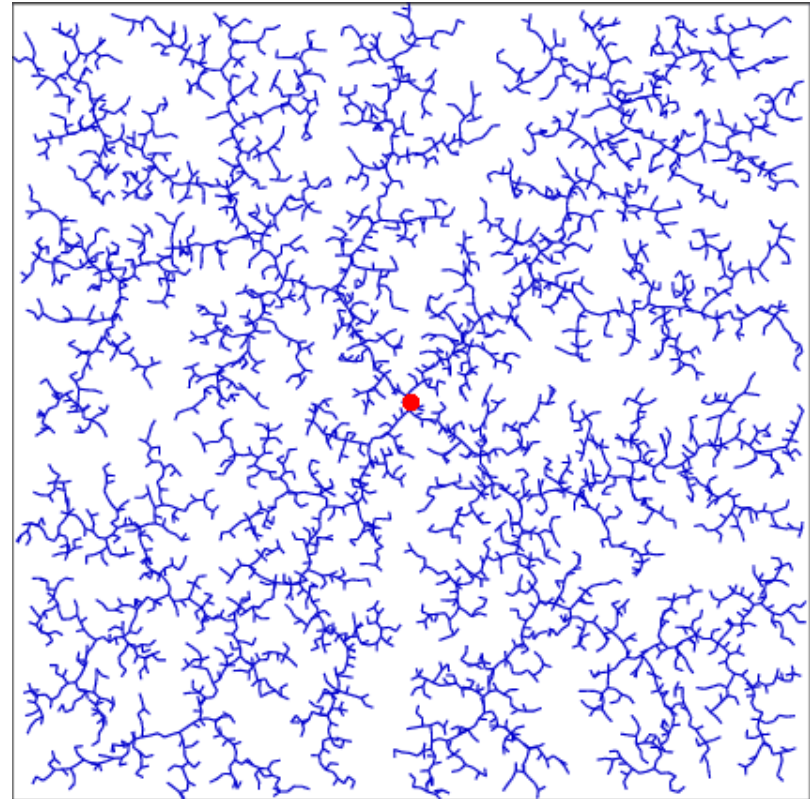


Image from  
<http://joonlecture.blogspot.com/2011/02/improving-optimality-of-rrt-rrt.html>

# RRT Algorithm

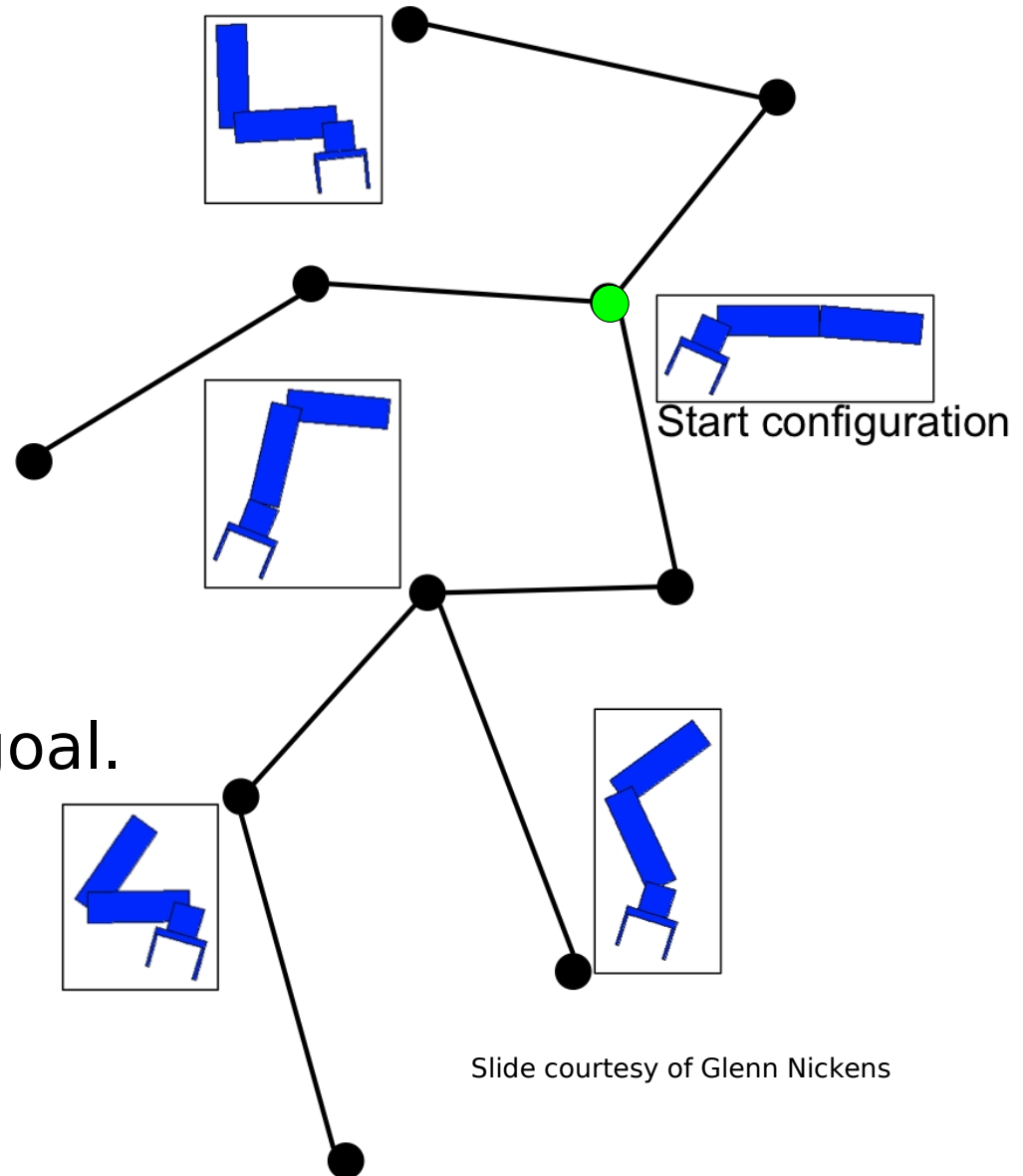
- Rapidly samples the state space.
- Cannot get trapped in local minima.
- Works well in high-dimensional spaces.
- Does not generate smooth paths.
- Can't tell when no solution exists; only quits when it exceeds the iteration limit  $K$ .



<http://msl.cs.uiuc.edu/rrt/treemovie.gif>

# RRTs for Arm Path Planning

- Each node encodes an arm configuration in joint space.
- Only add nodes that don't cause collisions (with self or obstacles).
- Alternately (i) extend the tree in random directions and (ii) move toward the goal.



# Implementation Notes

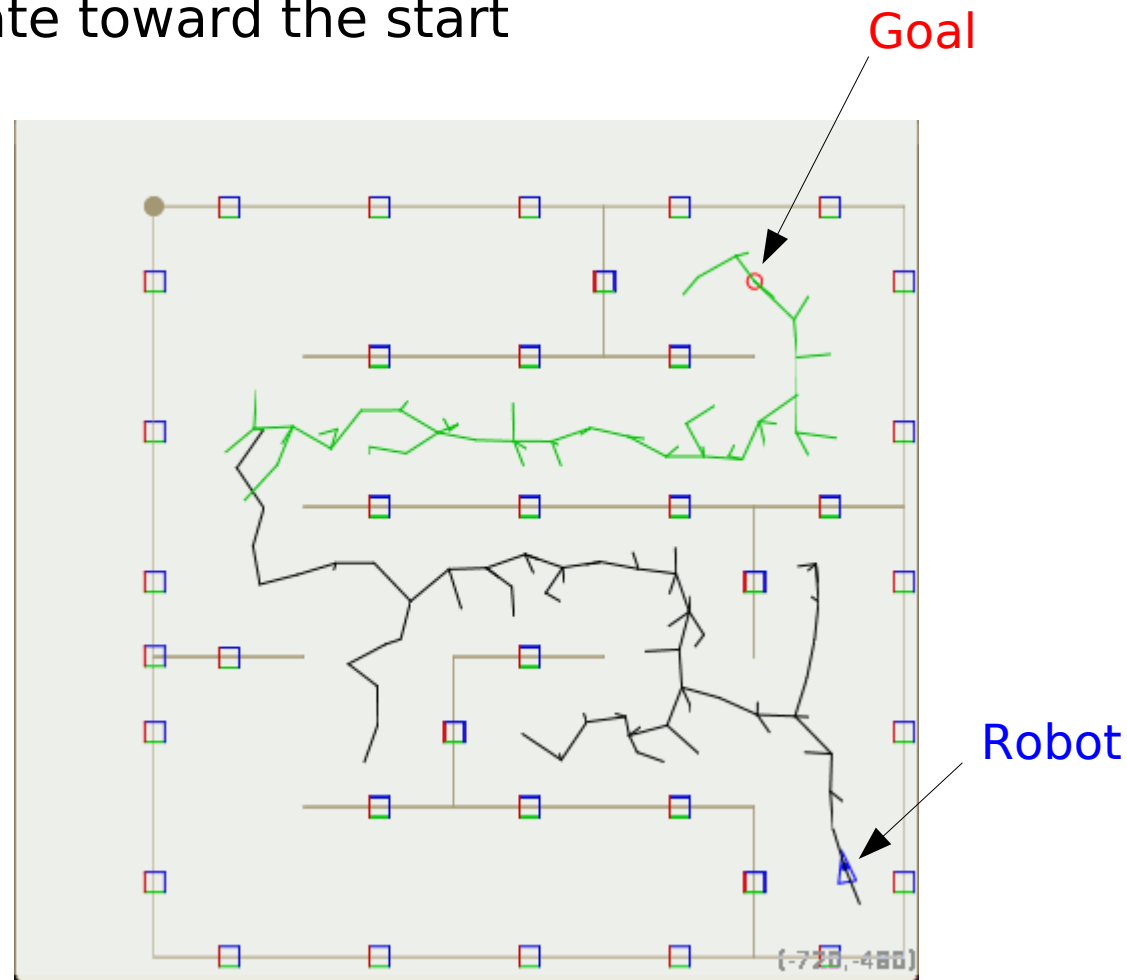
- Finding  $\mathbf{q}_{\text{nearest}}$ , the nearest node in the tree to  $\mathbf{q}_{\text{rand}}$ , is the most expensive part of the algorithm.
  - Use K-D trees to efficiently find  $\mathbf{q}_{\text{nearest}}$ ?
  - In practice, K-D trees are slower unless you have a huge number of nodes (several thousand).
- Why only go a distance  $\Delta$  toward the goal state  $G$ ? Why not go as far as we can, in steps of  $\Delta$ ?
  - With no obstacles, this reaches the goal very quickly, but random search will get there nearly as quickly as we keep extending the nearest node to the goal.
  - But when obstacles are present, this can waste time filling out branches that will ultimately fail.
  - Generating lots of extra nodes bloats the tree, which slows down the algorithm.

# RRT-Connect Algorithm

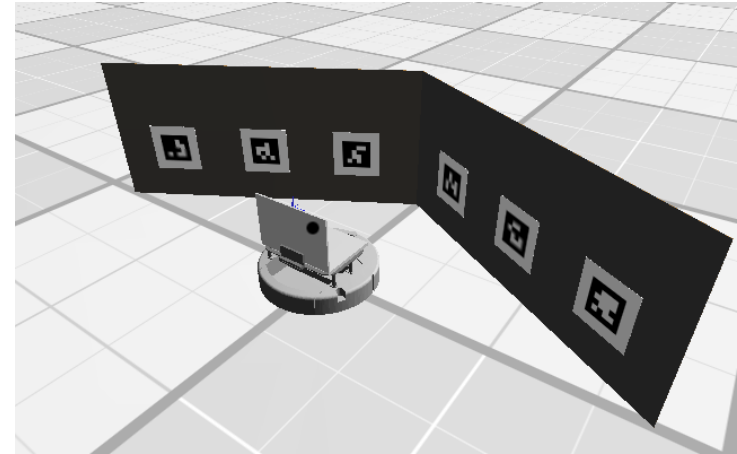
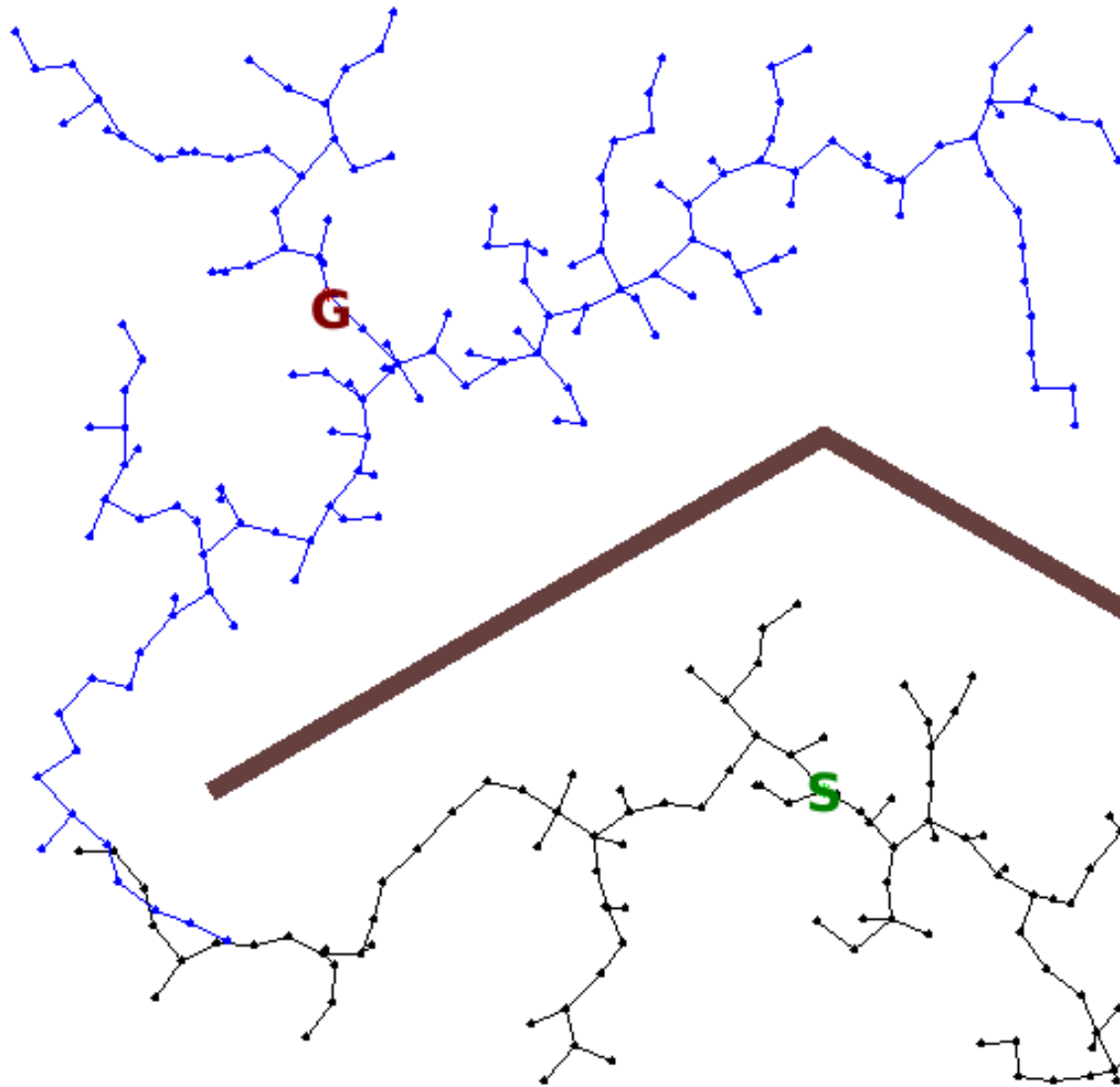
- Variant of RRT that grows two trees:
  - one from the start state toward the goal
  - one from the goal state toward the start

- When the two trees connect, a solution has been found.

- Not guaranteed to be better than RRT, but often helps.



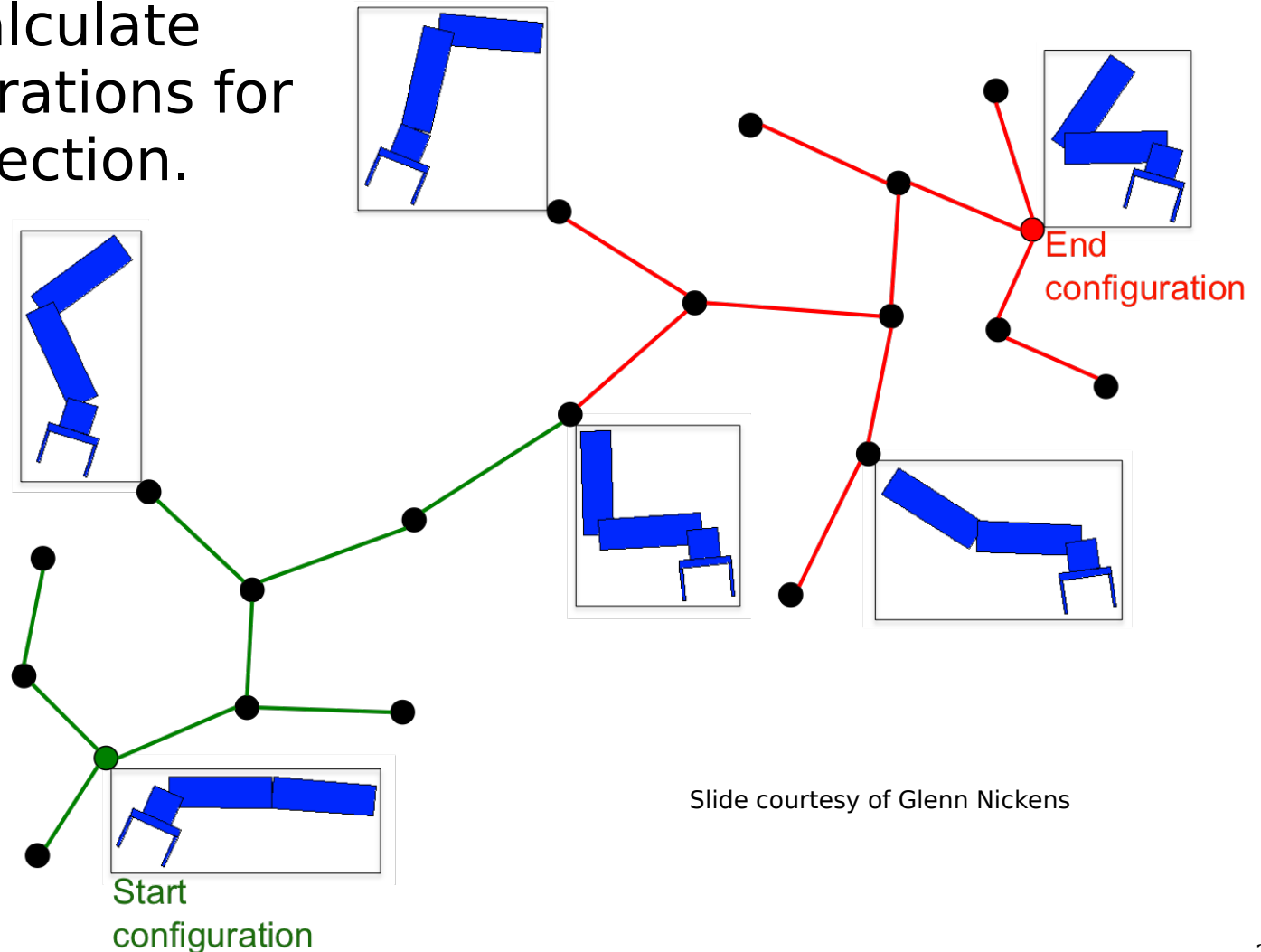
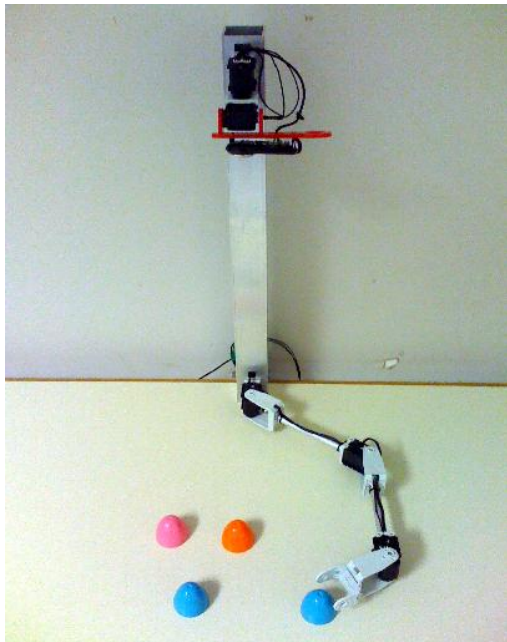
# RRTs in An Open Field





# RRT-Connect For Arms

- Use IK to calculate the goal configuration.
- Use FK to calculate arm configurations for collision detection.



Slide courtesy of Glenn Nickens

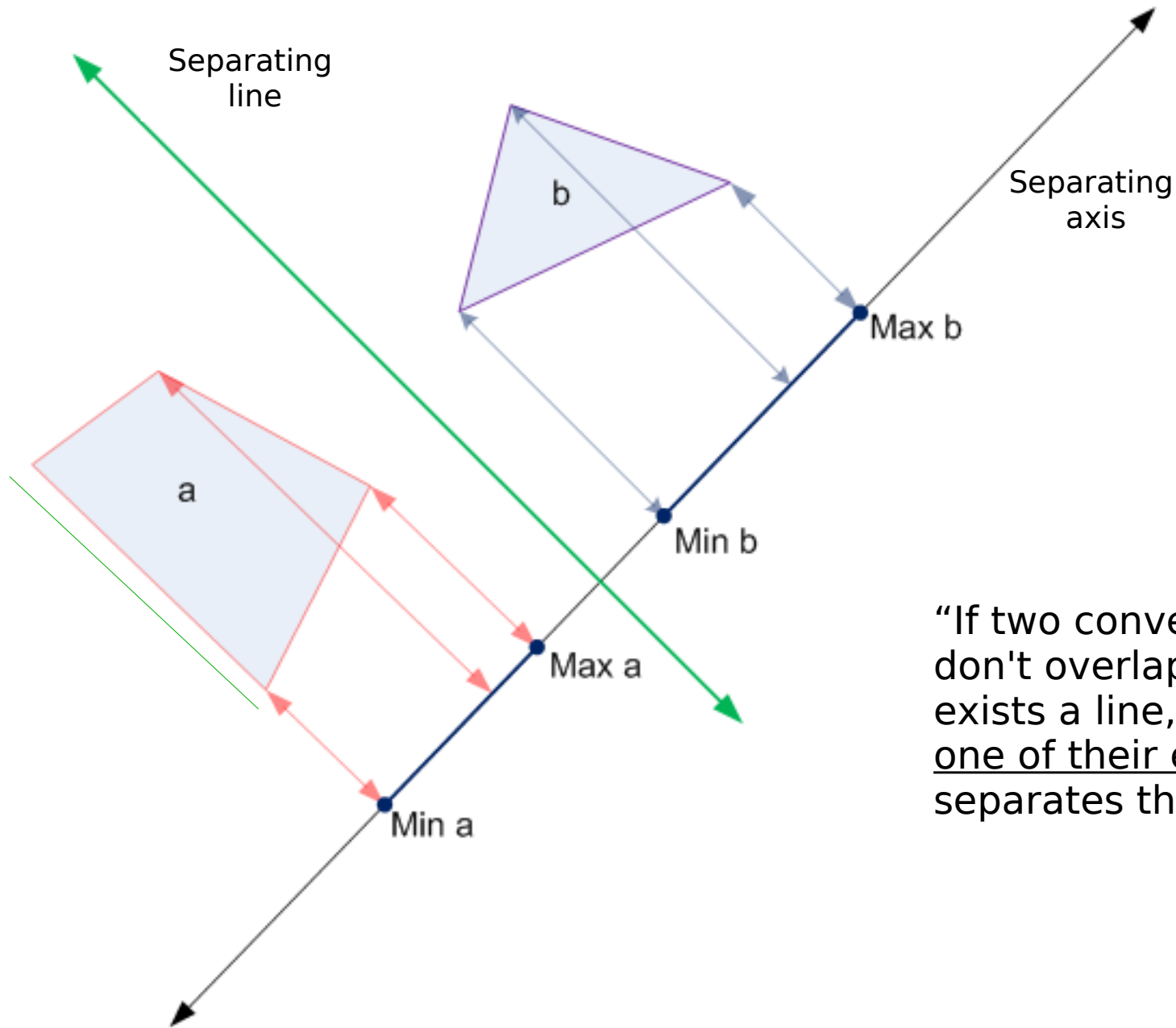
# Collision Detection

- Represent the robot and the obstacles as **convex polygons**.
- In 2D, use the Separating Axis Theorem to check for collisions.
  - Easy to code
  - Fast to compute
- In 3D, things get more complex.
  - Could use the GJK (Gilbert-Johnson-Keerthi) algorithm, used in many physics engines for video games.

# Algorithm to Apply the SAT

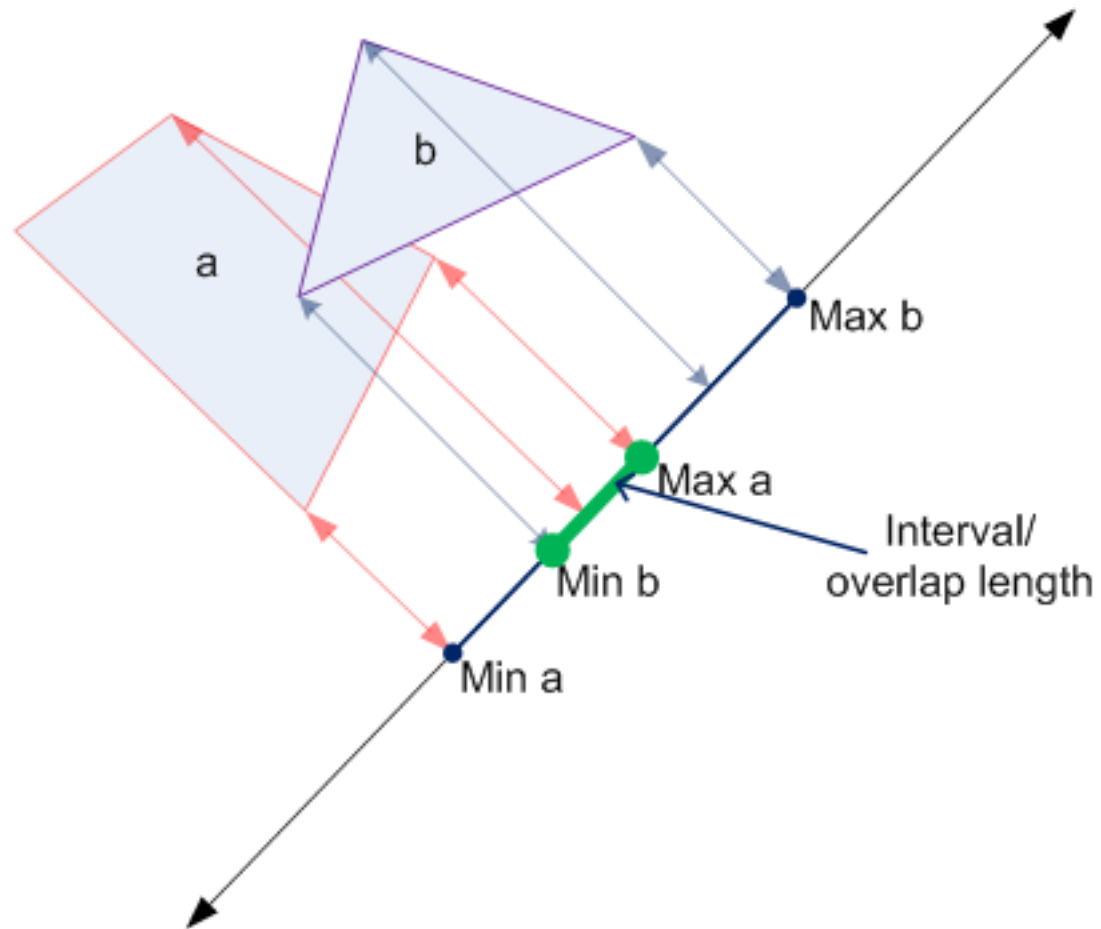
- For every edge of polygon A and of polygon B:
  - Project all the vertices onto the line normal to that edge.
  - Calculate the min and max coordinates for each polygon
  - If  $\min A < \min B$  and  $\max A > \min B$  OR  
if  $\min B < \min A$  and  $\max B > \min A$   
then the polygons collide.
- If you find any edge projection in which the ranges don't overlap, the polygons do not collide.

# Separating Axis Theorem



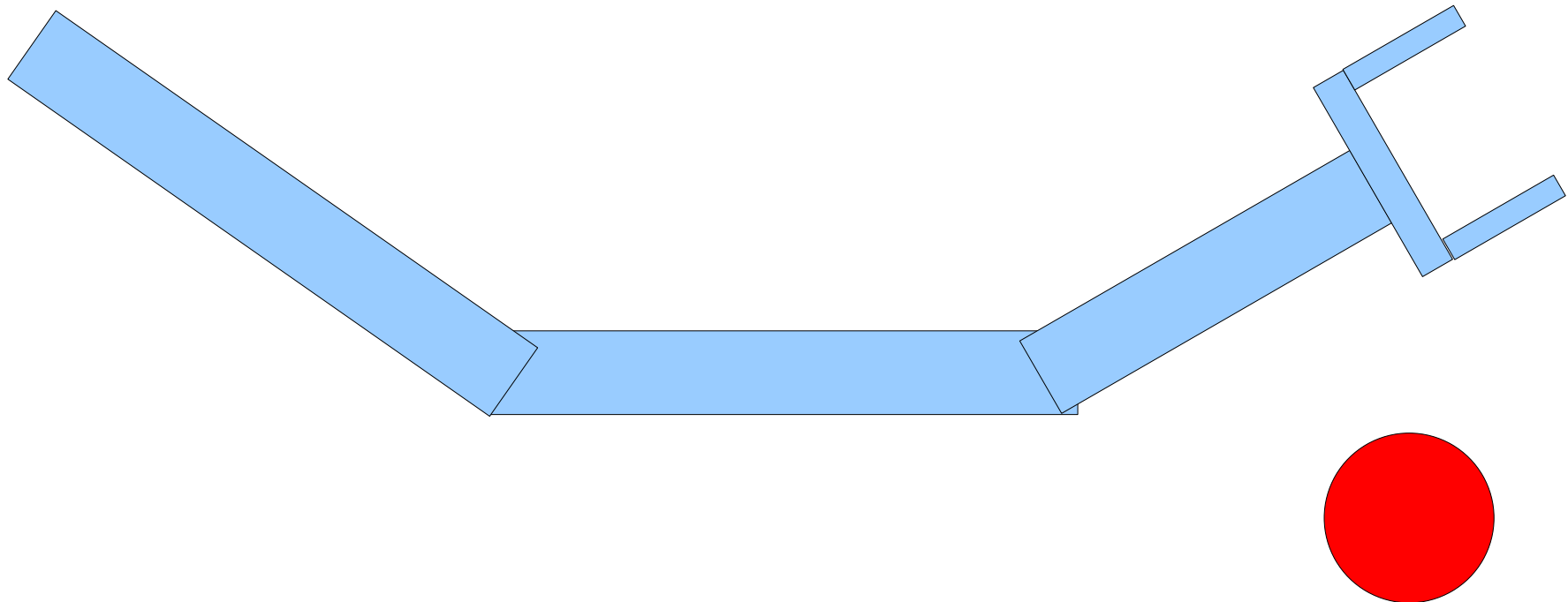
“If two convex polygons don't overlap, then there exists a line, parallel to one of their edges that separates them.”

# Separating Axis Theorem



# Arm Collision Detection

- Represent each link as a separate polygon.
- Check for:
  - Self-collisions other than link  $n$  with link  $n+1$
  - Collisions of a link with an obstacle

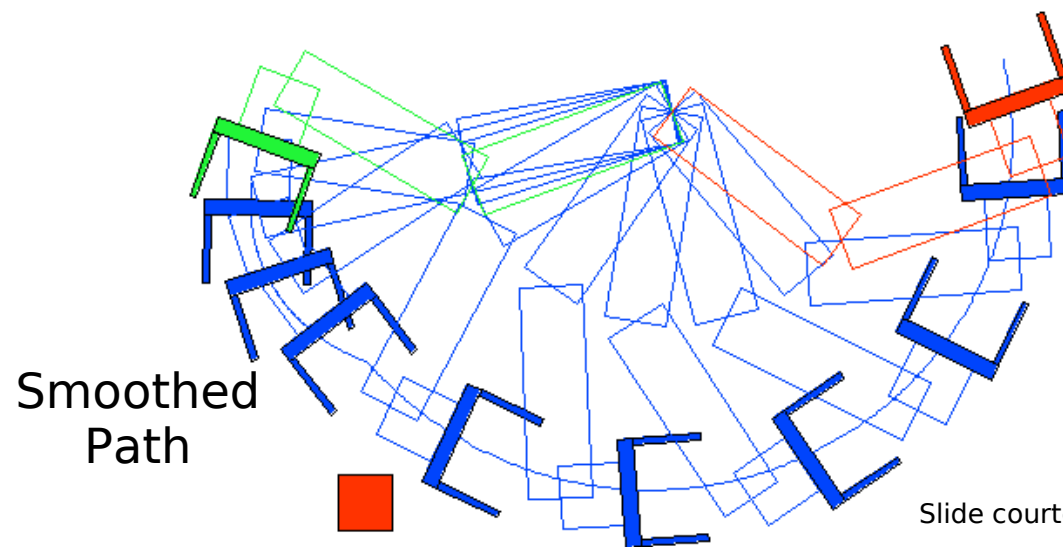
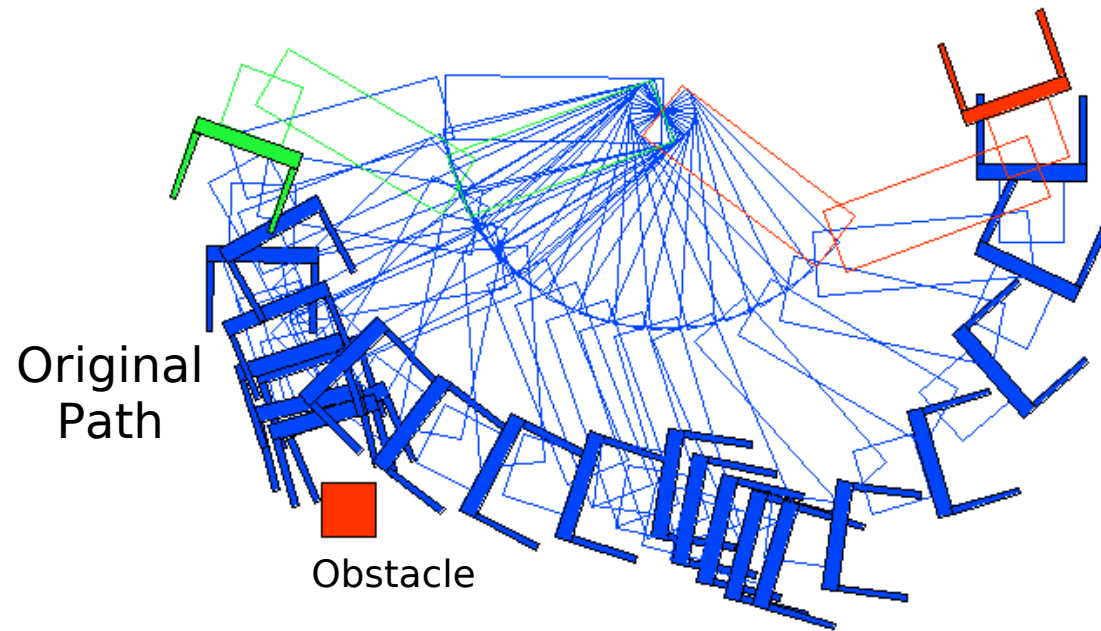


# Path Smoothing

- The random component of RRT-Connect search often results in a jerky and meandering solution.
- Solution: apply a path smoothing algorithm.
- Repeat N times:
  - Pick two points on the path at random
  - See if we can linearly interpolate between those points without collisions.
  - If so, then snip out that segment of the path.

# Smoothing An Arm Trajectory

- Start state
- Intermed. states
- End state

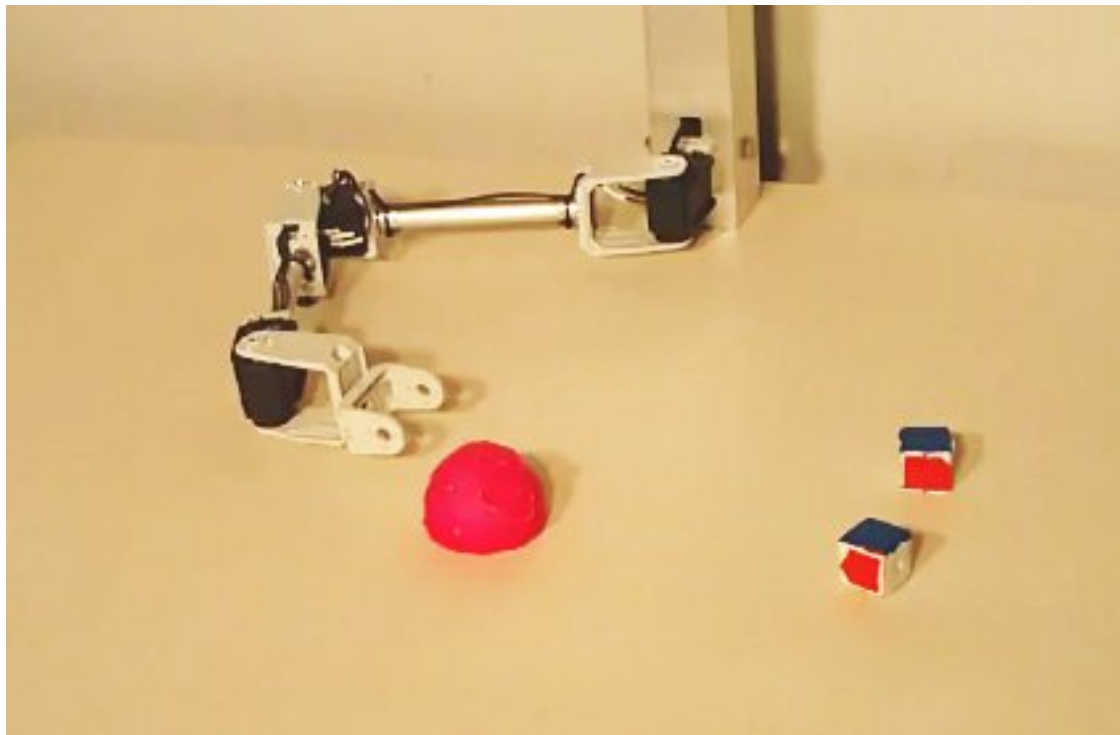


Slide courtesy of Glenn Nickens



# Path Planning With Constraints

- With no closeable fingers, arm motion is constrained to be within about  $60^\circ$  of finger direction or we'll lose the object.



(video)

<http://www.youtube.com/watch?v=9oDQ754YVoc>

# Implementing Constraints

- Each time we generate a new state  $\mathbf{q}_{\text{new}}$  :
  - Check to see if  $\mathbf{q}_{\text{new}}$  obeys the constraint.
  - For finger motion constraint, check if the direction of motion from parent state  $\mathbf{q}_{\text{nearest}}$  to new state  $\mathbf{q}_{\text{new}}$  is **within 60°** of the finger direction.
- What if  $\mathbf{q}_{\text{new}}$  doesn't obey the constraint?
  - Reject it and pick a new  $\mathbf{q}_{\text{rand}}$  from which we'll generate a new  $\mathbf{q}_{\text{new}}$ .
  - Or try to “fix”  $\mathbf{q}_{\text{new}}$  by perturbing its value slightly so as to satisfy the constraint.

# Path Planning Failure

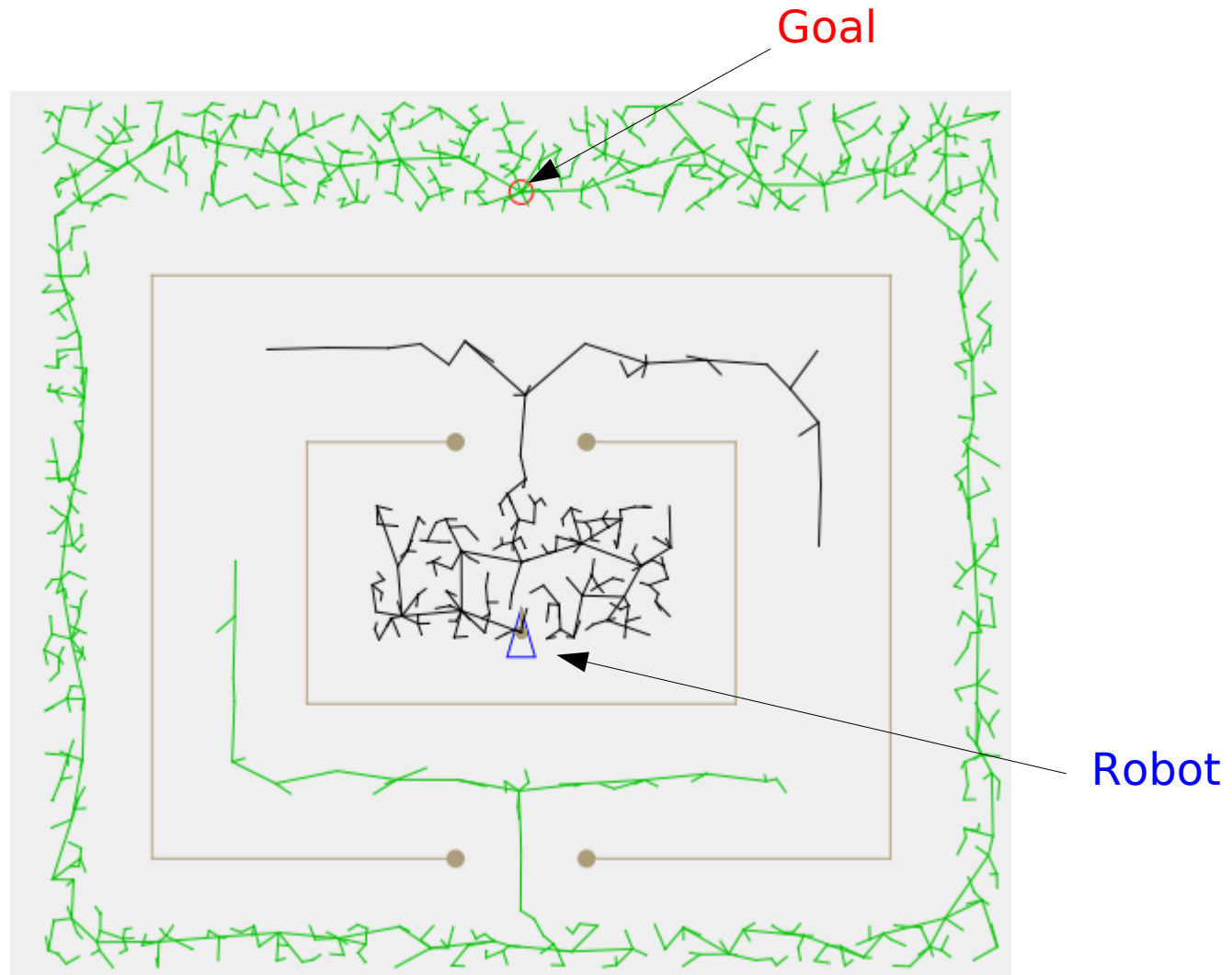
RRT path planning can legitimately fail if:

- There is no route to the goal due to obstacles blocking every path from start to goal.
- The paths to the goal don't lie entirely within the allowed world bounds (world map too small).

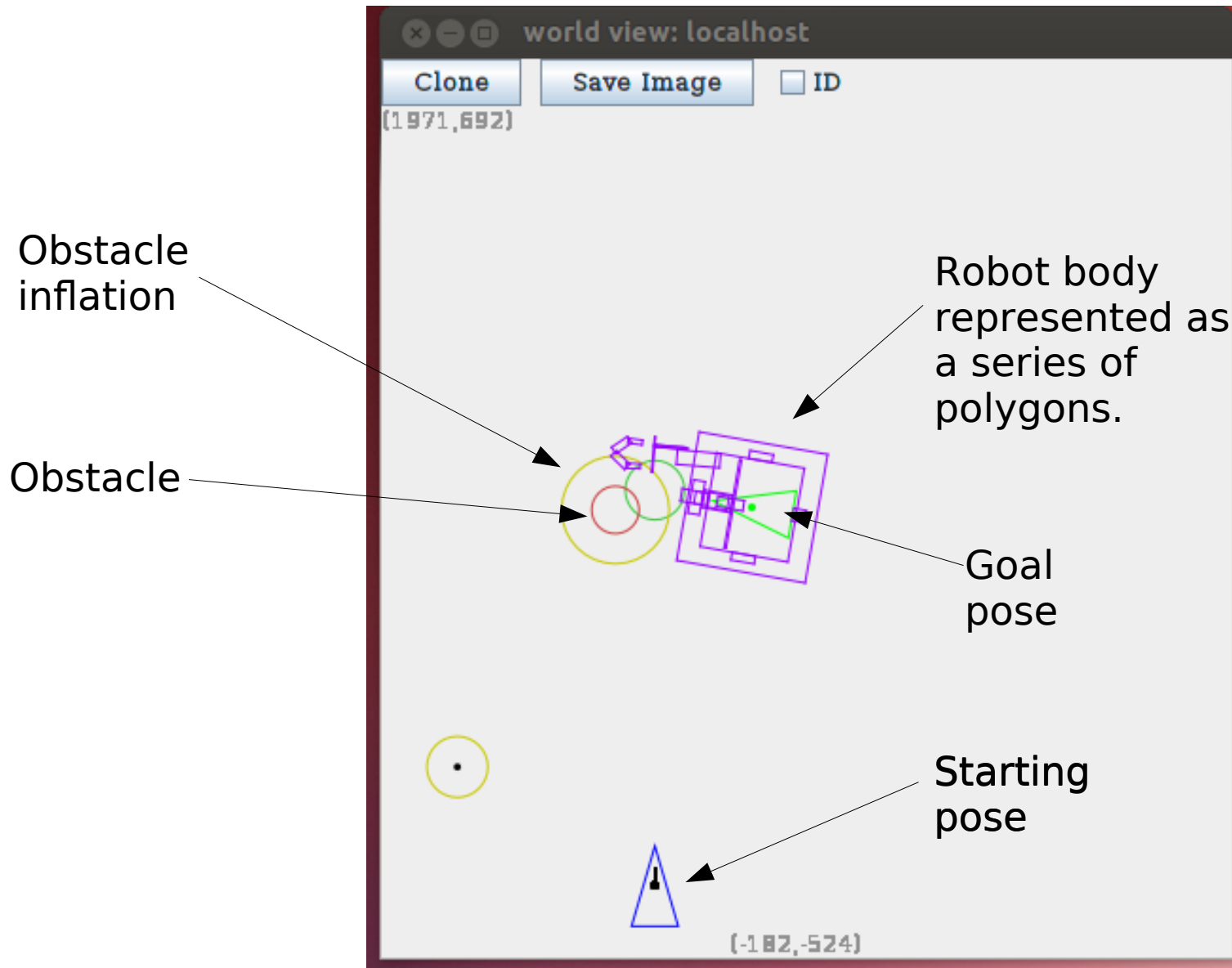
But it can also fail if:

- The iteration limit was set too low.
- The start state is already in collision with something.
- The goal state is in collision with something.

# Running Out of Iterations



# Path Planning Failure: Goal State Is In Collision



# Full 3D Path Planning: The Piano Movers Problem

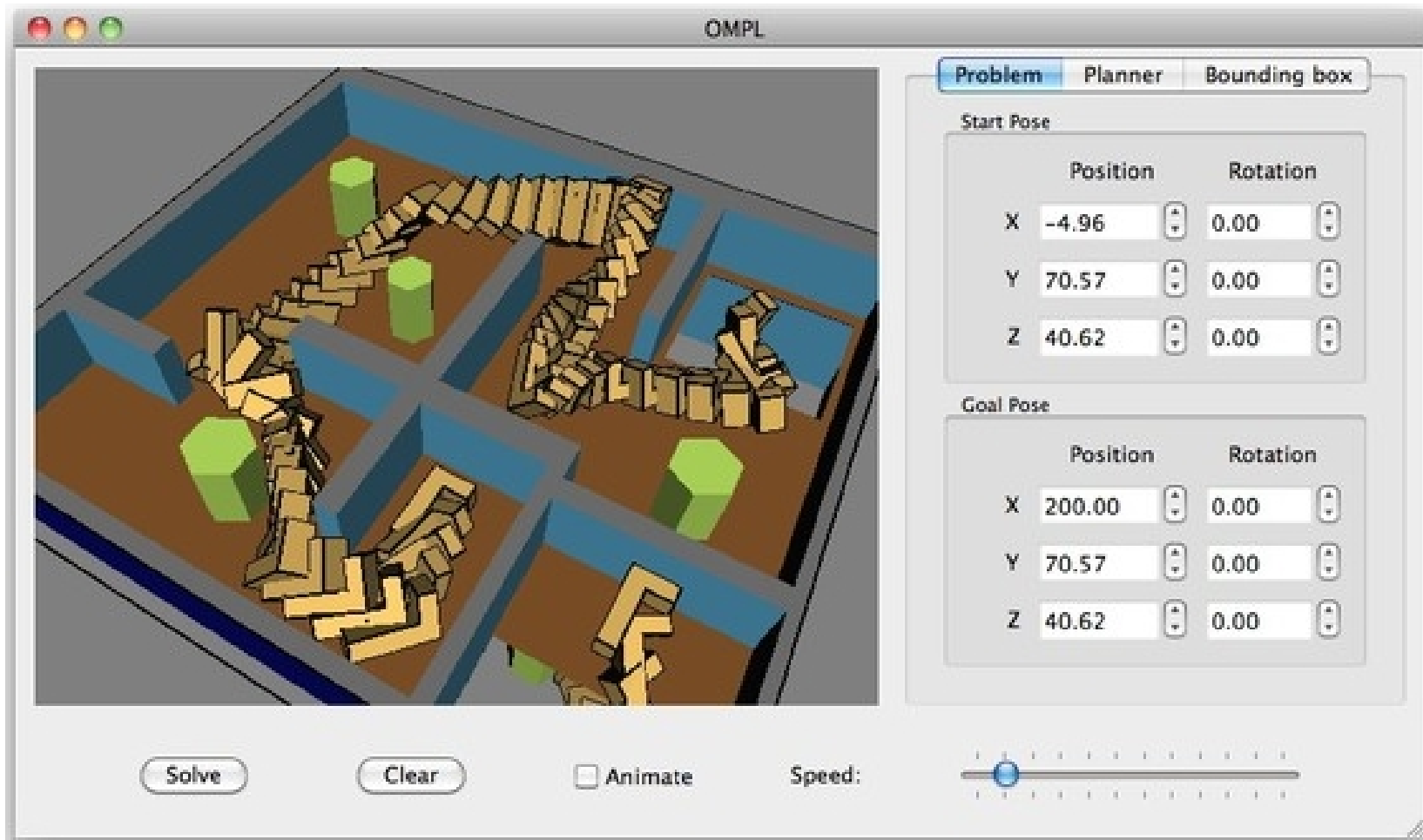


Figure from  
[http://www.gamasutra.com/blogs/MattKlingensmith/20130907/199787/Overview\\_of\\_Motion\\_Planning.php](http://www.gamasutra.com/blogs/MattKlingensmith/20130907/199787/Overview_of_Motion_Planning.php)

Open Motion Planning Library:  
<http://ompl.kavrakilab.org>

# The Pilot

- Navigation utility defined in `cozmo_fsm/pilot.fsm`
- How to go from A to B:
  - Generate obstacle list from current world map.
  - Use RRT-Connect to plan a path from A to B?
    - Good in open spaces; has trouble with doorways.
  - Formulate a navigation plan to follow the path.
    - Straight segments
    - Turns
    - Arcs
    - Landmark checks
  - Execute the navigation plan, correcting as necessary.
  - Report success or failure.

# PilotToPose Node

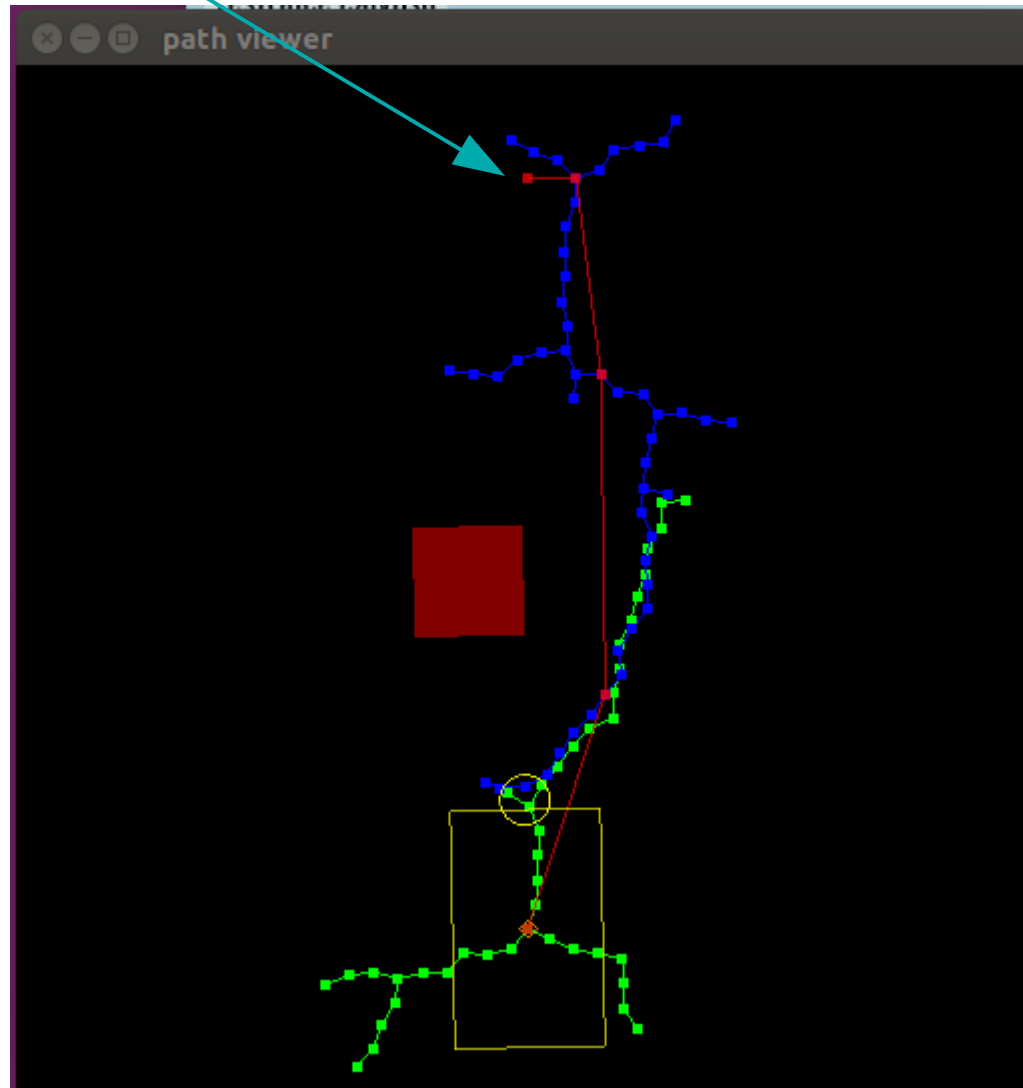
- State node for invoking the Pilot.
- Tell it where you want to go, and (optionally) the desired heading at the destination.
- Use a heading of NaN if you don't care.
- =PILOT=> transition can check for errors.

```
go: PilotToPose(Pose(500, 0, 0, angle_z=degrees(90)))  
go =C=> Say("Success")  
go =PILOT(StartCollides)=> Say("Start collides!")
```



# Path Viewer

```
PilotToPose(Pose(300, 0, 0, angle_z=degrees(90)))
```



# Hybrid Path-Planner

- cozmo-tools now uses a hybrid path planner.
- Check for StartCollides condition and use RRT to find a maneuver that disengages from the obstacle, e.g., move away from a wall.
- Wavefront algorithm finds a route to the goal using large obstacle inflation. Easily goes through doorways.
- RRT post-processing:
  - Check for collisions using actual robot shape and less obstacle inflation
  - Generate a condensed path with fewer steps
- Check for doorway crossing and insert “doorpass” steps in the navigation plan.