15-494/694: Cognitive Robotics

Dave Touretzky

Lecture 6:

Robot Kinematics

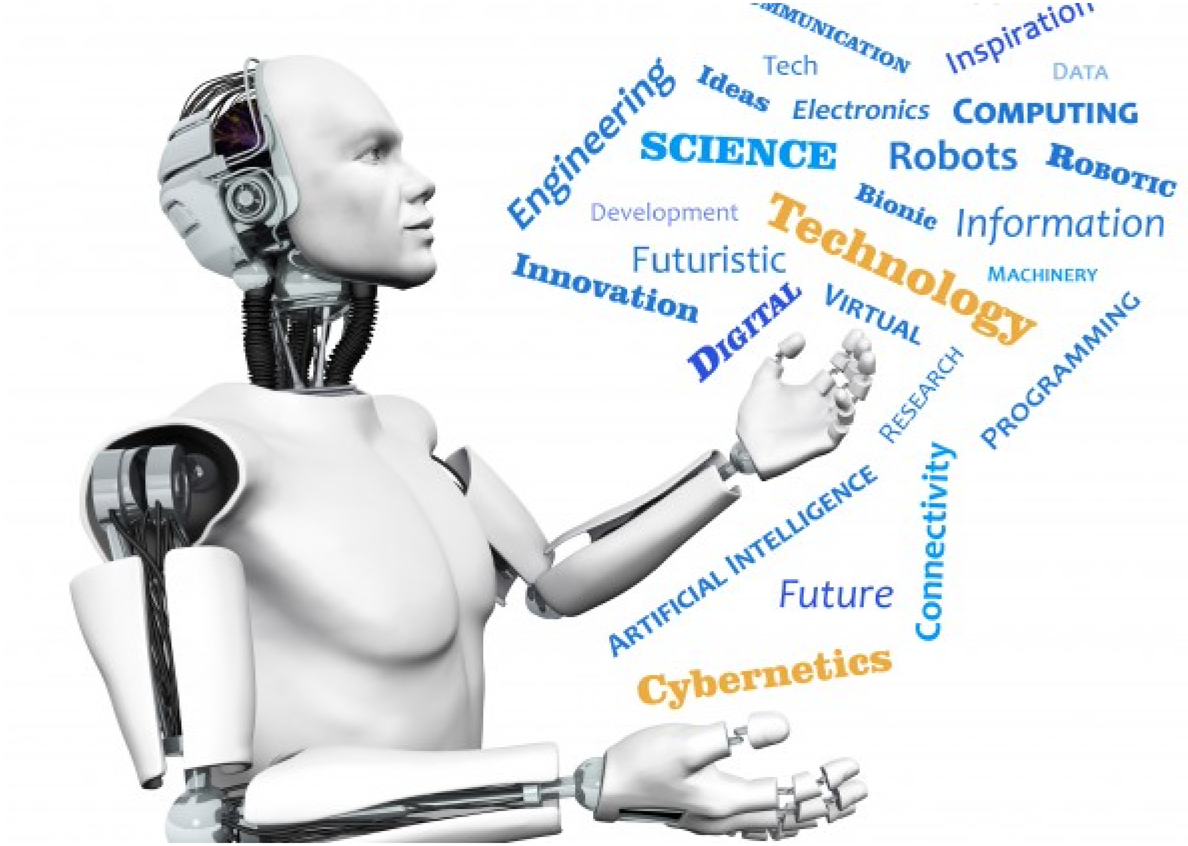
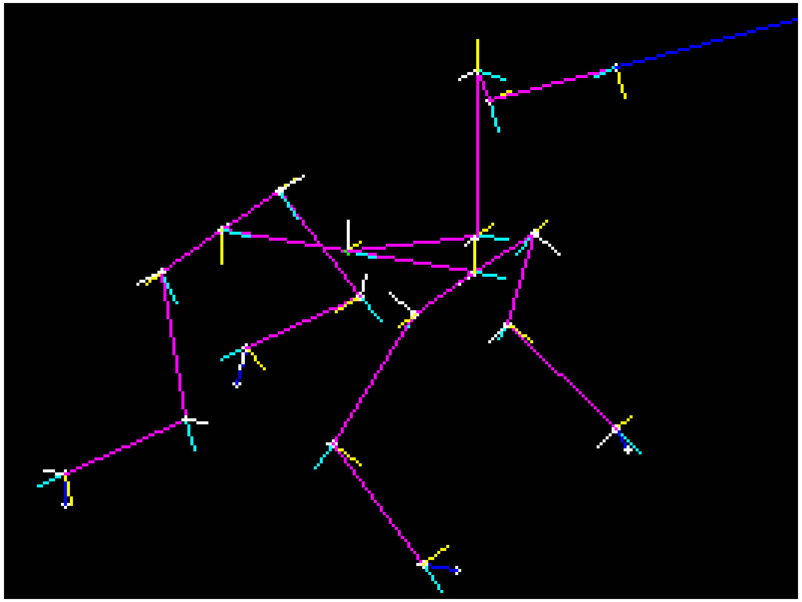


Image from http://www.futuristgerd.com/2015/09/10

Outline

- Kinematics is the study of how things move.
- Kinematic chains
- Reference frames
- Homogeneous coordinates
- Forward kinematics: calculating limb positions from joint angles. (Easy.)
- Inverse kinematics: calculating joint angles to achieve desired limb positions/trajectories. (Hard.)

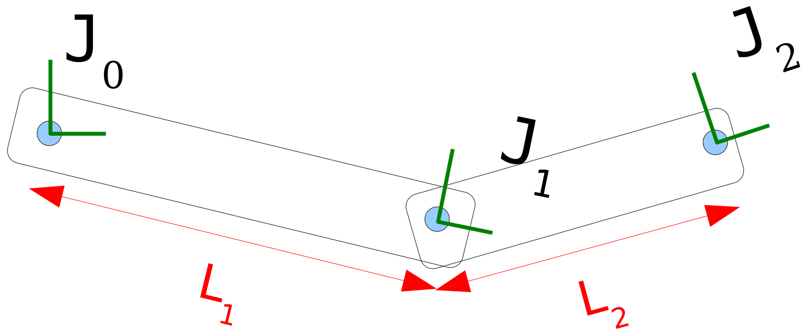
Robots As Kinematic Chains or Trees



- The root is called the *Base Frame*.
- Typically at the center of the robot's body..

Chains = Joints + Links

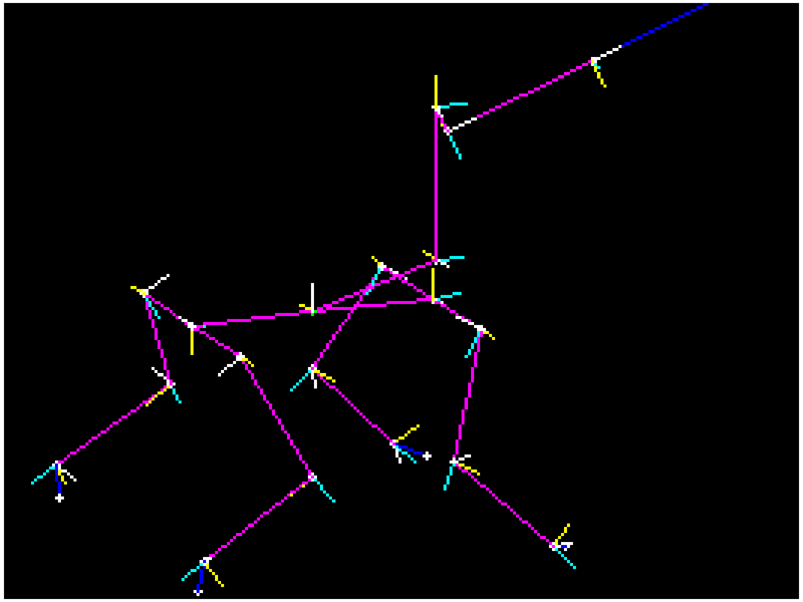
- A chain is a sequence of alternating joints and links.

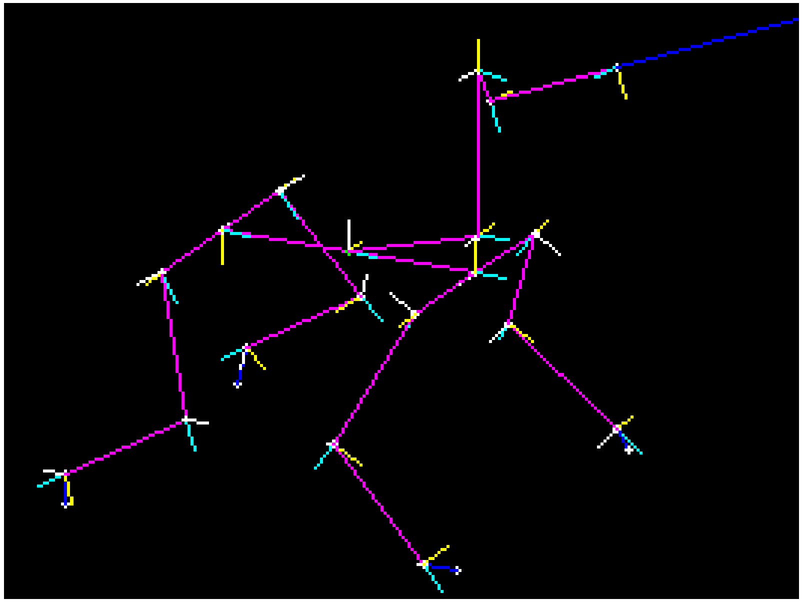


- We can use transformation matrices to calculate the position of the tip of the chain (joint J_2) from the joint angles θ_0, θ_1 and the link lengths L_1, L_2.
- Each rotational joint has a rotation transform; each link has a translation transform.
- The math for this will be shown later in this lecture.

AIBO Kinematic Chains

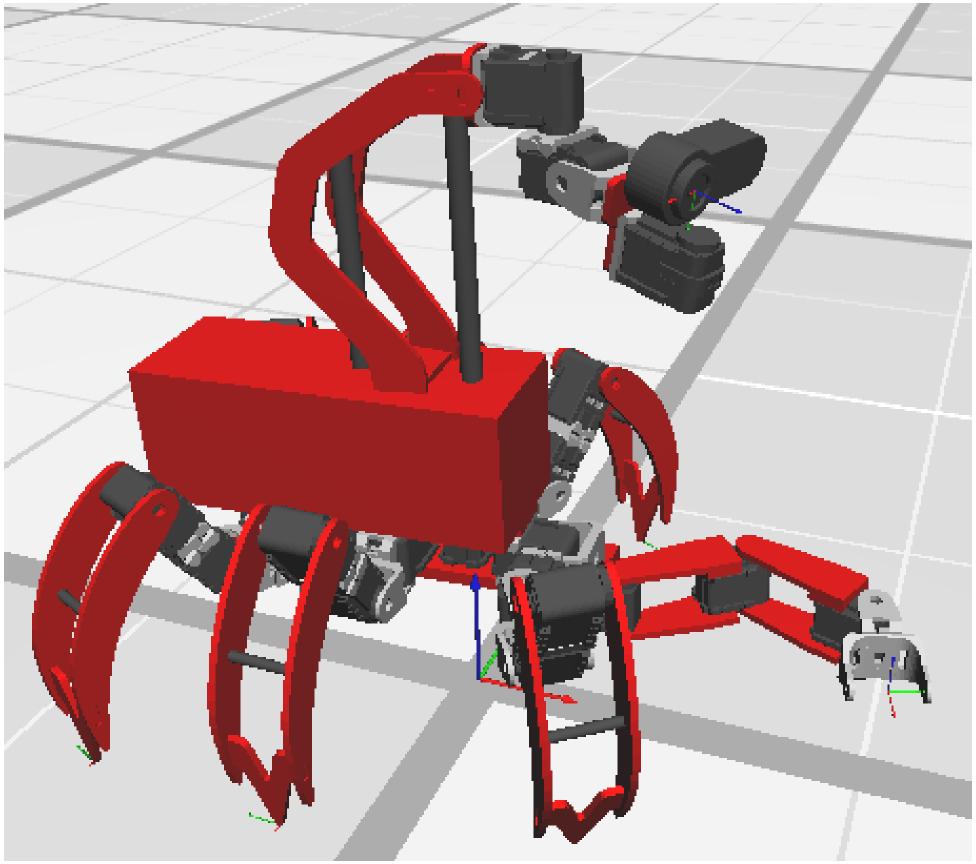
- The AIBO had 9 kinematic chains.
  - 4 for the legs
  - 1 for the head (the camera), 1 for the mouth
  - 3 for the IR range sensors
- All chains began at the center of the body (base frame).





Chiara Kinematic Chains

- The Chiara had 8 major kinematic chains:
  - Head / camera / IR
  - Arm
  - Left front leg
  - Right front leg (4-dof)
  - Left middle leg
  - Right middle leg
  - Left back leg
  - Right back leg



Calliope Kinematic Chains

BaseFrame

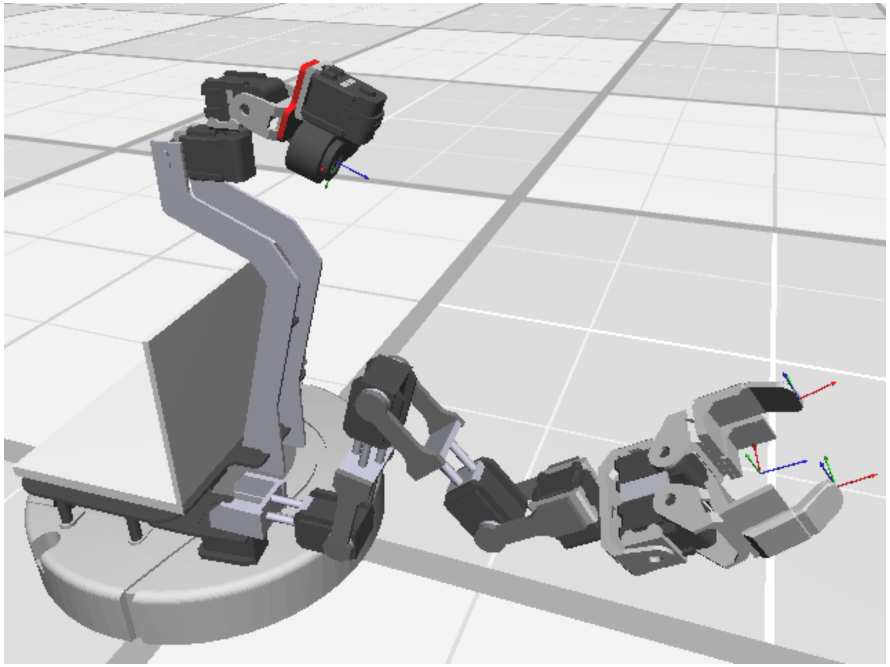
center of axle
  WHEEL:L, WHEEL:R

NECK:PAN
  NECK:TILT
    CameraFrame

ARM:base
  ARM:shoulder
    ARM:elbow
      ARM:wrist
        ARM:wristrot
          GripperFrame
          ARM:gripperleft
            LeftFingerFrame
          ARM:gripperright
            RightFingerFrame

In Tekkotsu you can use the DisplayKinTree demo to show the kinematic tree of the robot.

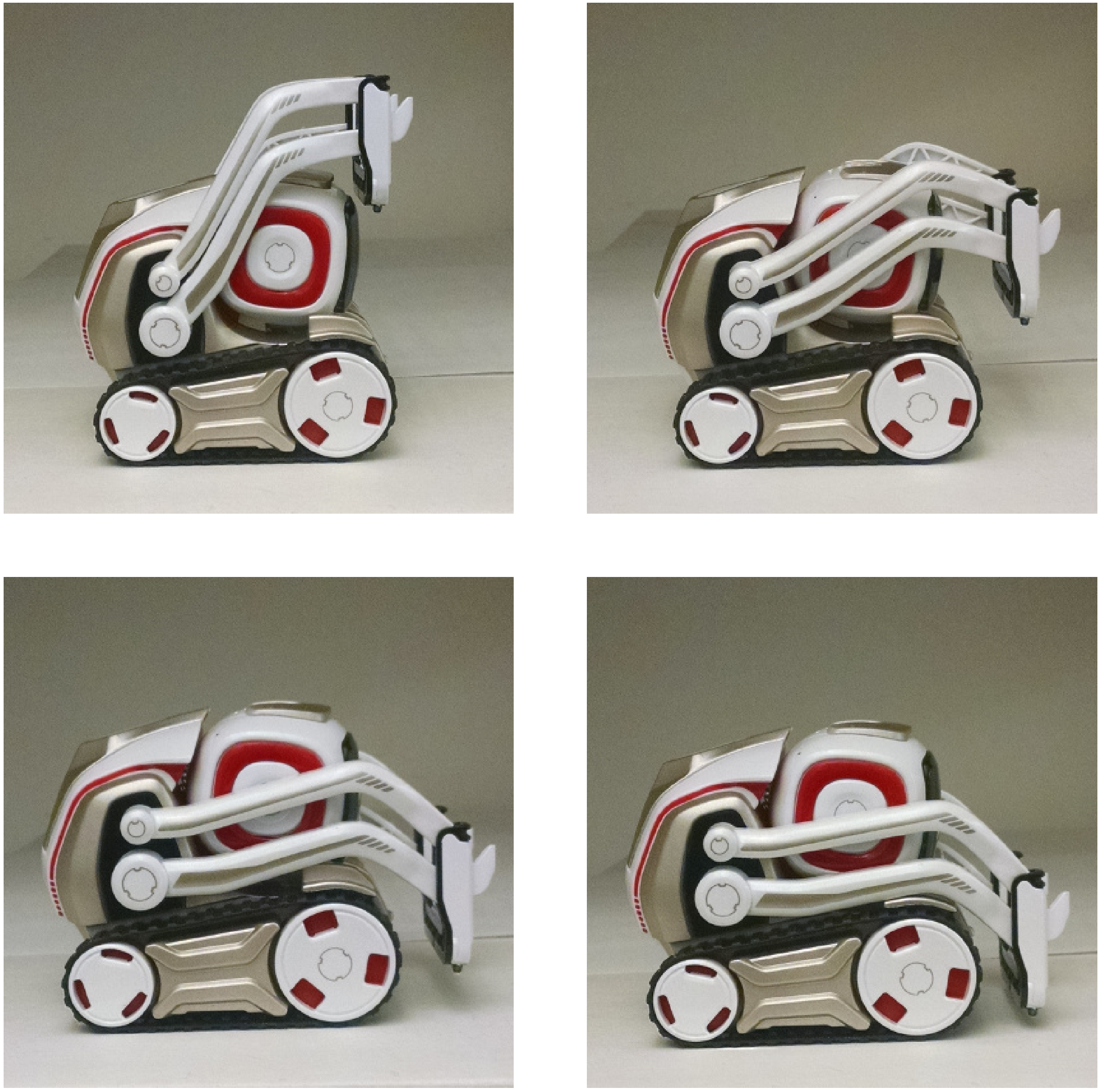
Root Control
  > Framework Demos
    > Kinematics Demos
      > DisplayKinTree



Cozmo Kinematic Chains

- Base frame is on the floor at the center of the front axle. Only two joints!
- Reference frames of interest:
  - Base frame
  - Head joint → Camera
  - Shoulder joint → Lift
  - Center of rotation
  - All four wheels
  - Cliff detector
  - IR Headlight

Cozmo's Lift: Four-Bar Linkage

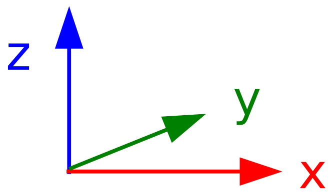


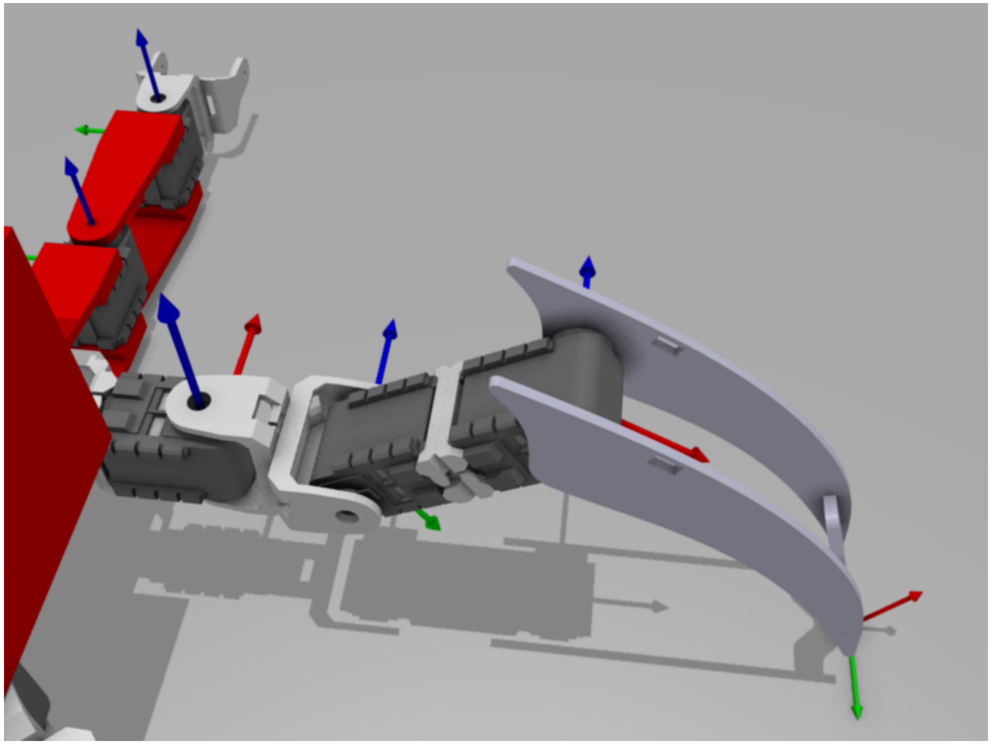
VEX AIM Kinematic Chains

- Very simple kinematic structure:
  - Base frame (center of robot’s body)
  - Camera frame
  - Kicker frame
  - World frame

Reference Frames

- Every joint has an associated reference frame.
- Additional reference frames for camera, toes, etc.





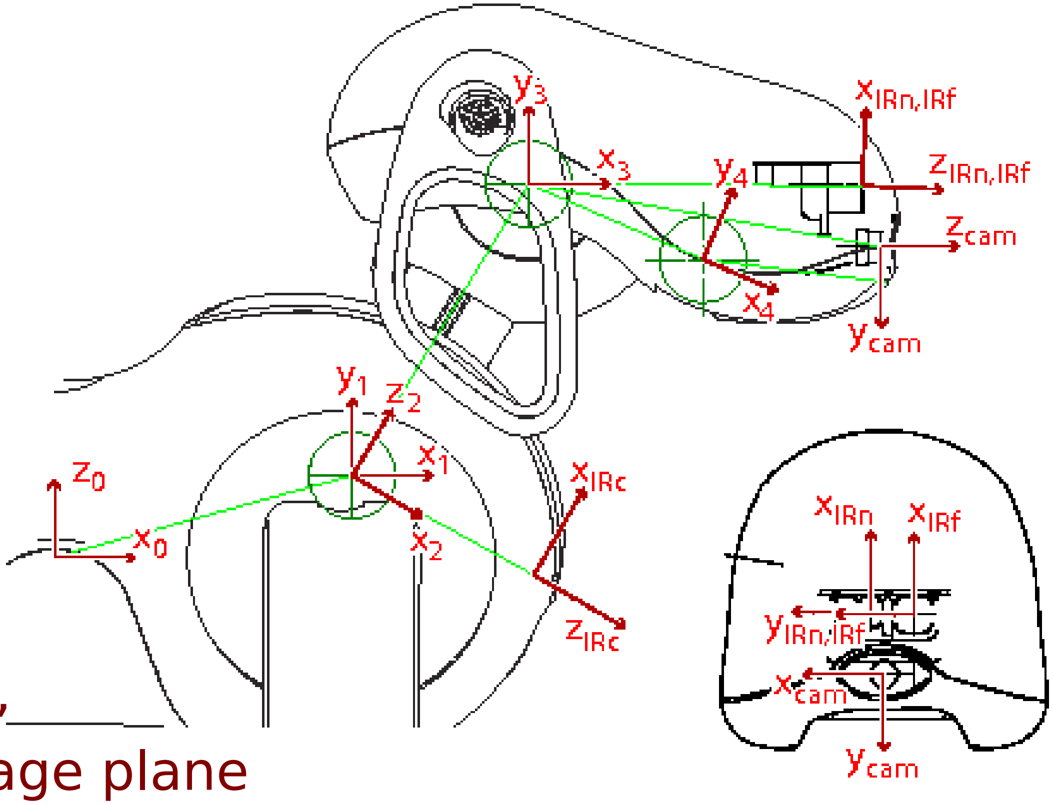
- Denavit-Hartenberg conventions: joints rotate about their z-axes.
- The x and y axes follow the *right hand rule*.

Chains of Reference Frames

- BaseFrame: z is up, x is forward, y is left.
  - This convention is also used for world coordinates.

- Axis of rotation determines z for a joint.

- The head chain:
  - Base frame 0 z_0 = "up"
  - Tilt joint 1 y_1 = "up"
  - Pan joint 2
  - Nod joint 3
  - Camera 4 z_4 = "out", x_4,y_4 = image plane



Moving Along A Chain

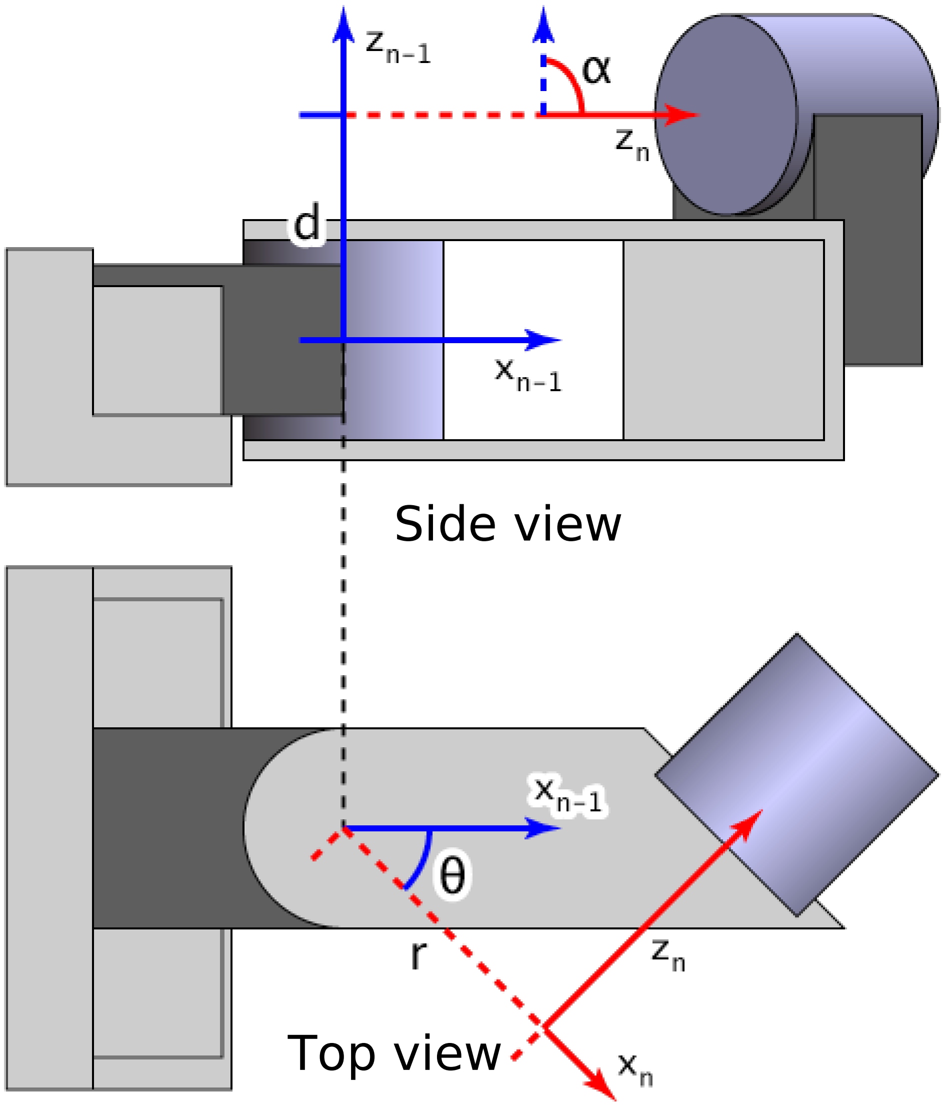
- Denavit-Hartenberg conventions specify how to express the relationship between one reference frame and the next.
- We use a modified version, to allow for kinematic trees instead of simple chains.

    - d: translation along previous z axis
    - θ: rotation around previous z axis
    - r: translation along new x axis
    - α: rotation around new x axis

Denavit-Hartenberg Video

http://www.youtube.com/watch?v=rA9tm0gTln8

Summary of D-H Conventions

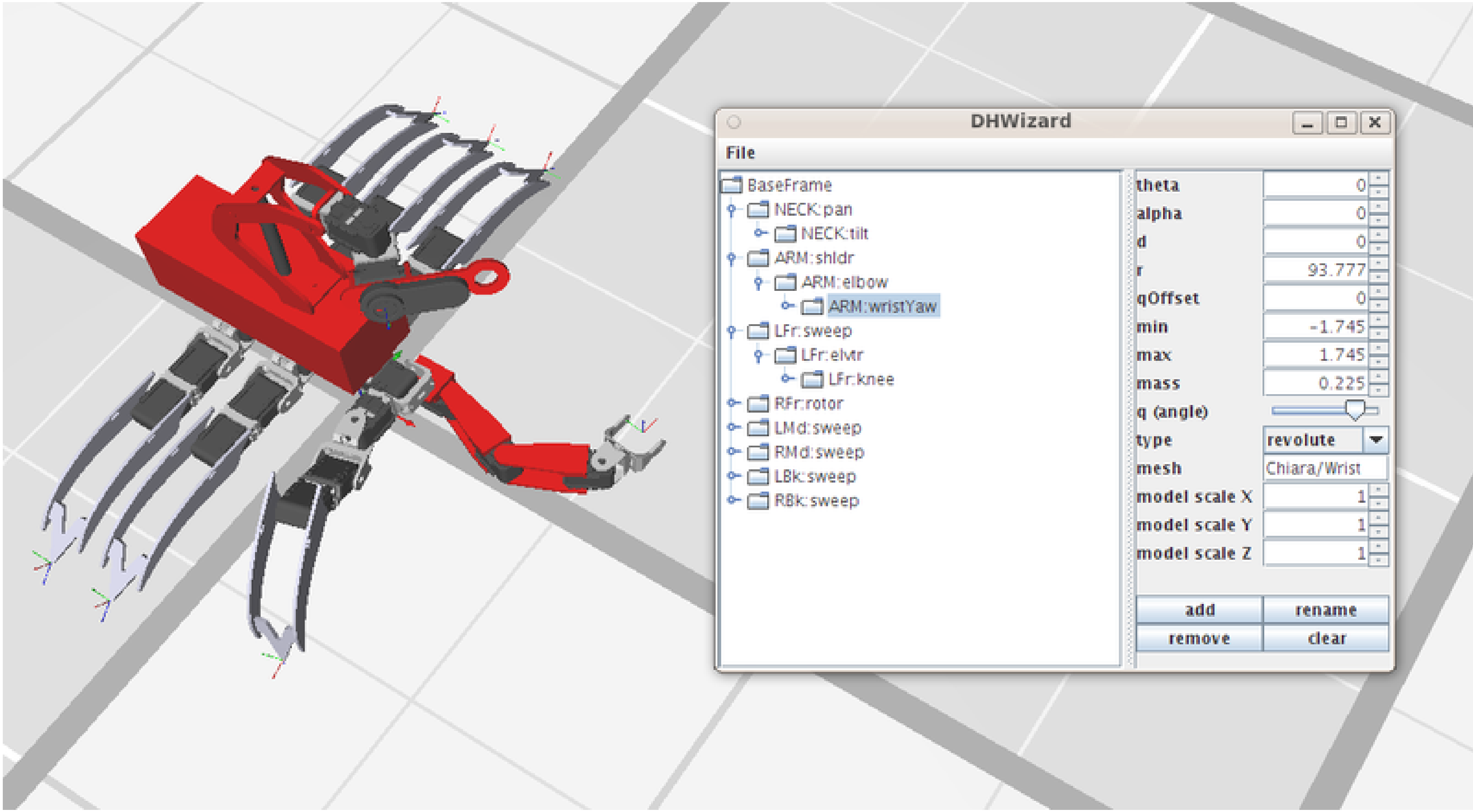


1) Move by d along z_{n-1}
2) Rotate by θ around z_{n-1}
3) Move by r along x_n, which is the common normal of z_{n-1} and z_n
4) Rotate by α along x_n

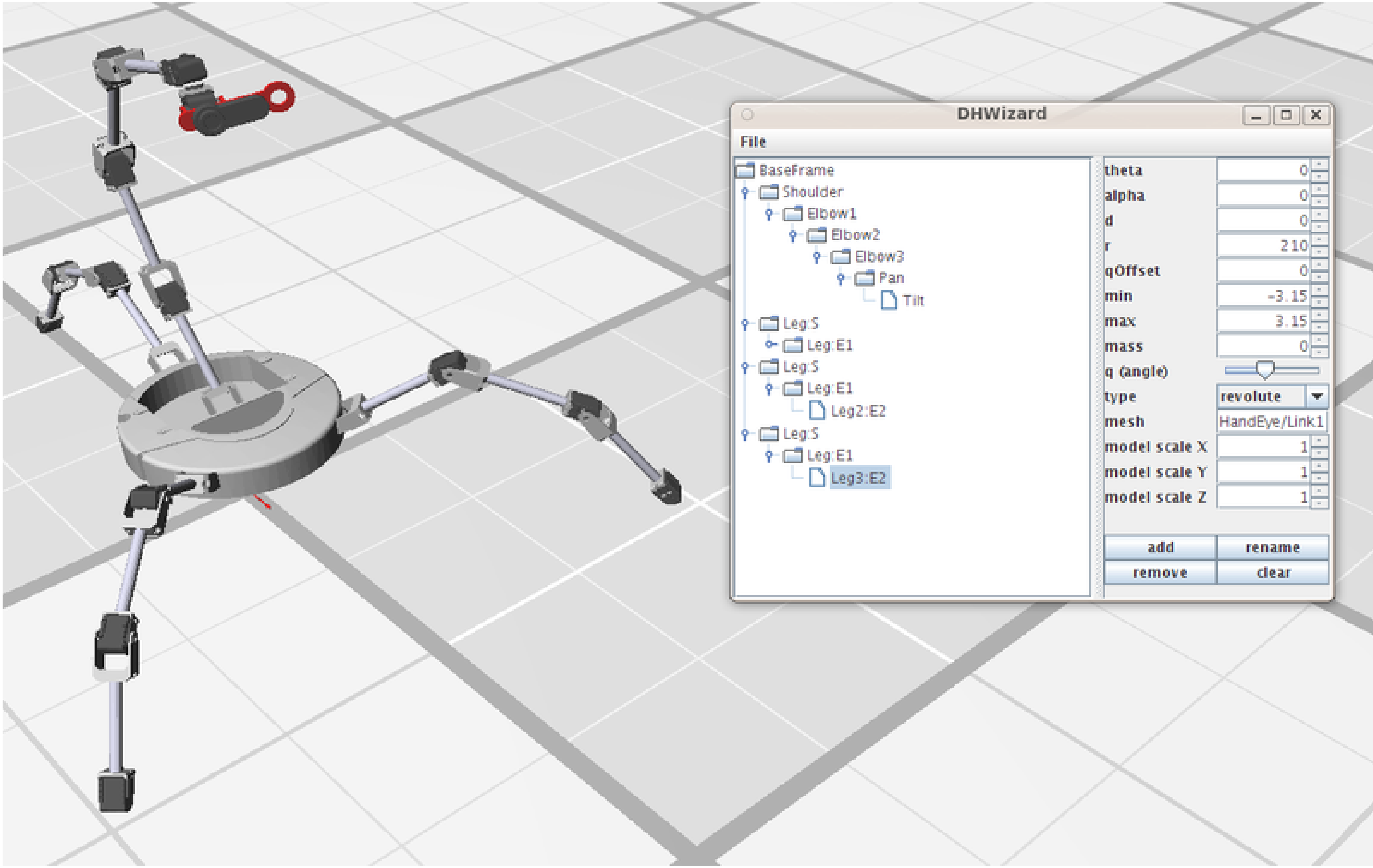
When z_{n-1} and z_n are parallel:

- d is arbitrary
- α is 0

Tekkotsu's DH Wizard Tool



DH Wizard


DHWizard
File
BaseFrame
Shoulder
Elbow1
Elbow2
Elbow3
Pan
Tilt
Leg:S
Leg:E1
Leg:S
Leg:E1
Leg2:E2
Leg:S
Leg:E1
Leg3:E2
theta 0
alpha 0
d 0
r 210
qOffset 0
min -3.15
max 3.15
mass 0
q (angle)
type revolute
mesh HandEye/Link1
model scale X 1
model scale Y 1
model scale Z 1
add
rename
remove
clear

Now, The Math...

- How do we represent transformations from one reference frame to the next in a kinematic chain?
  - Homogeneous coordinates
  - Transformation matrices
- How do we perform these calculations in Python?
  - The numpy package
- How do I get the computer to do the work for me?
  - Forward kinematics solver

Homogeneous Coordinates

- Represent a point in 3-space by an (3+1)-dimensional vector. (Extra component is an inverse scale factor.)
  - In "normal" form, last component is always 1.

$$\vec{v} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  - For points at infinite distance: last component is 0.
- Allows us to perform a variety of transformations using matrix multiplication:

  Translation, Rotation, Scaling

- vex-aim-tools uses 3D coordinates (so 4-dimensional vectors) for everything.

Translation Matrix

$$Translate(dx, dy, dz) = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$Translate(dx, dy, dz) \cdot \vec{v} = \begin{bmatrix} x + dx \\ y + dy \\ z + dz \\ 1 \end{bmatrix}$$

Rotation About Z (In X-Y Plane)

$$RotZ(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$RotZ(\theta) \cdot \vec{\mathrm{v}} = \begin{bmatrix} x\cos\theta + y\sin\theta \\ -x\sin\theta + y\cos\theta \\ z \\ 1 \end{bmatrix}$$

General X-Y Transformation

- Let θ be rotation angle in the x-y plane.
  Let dx, dy, dz be translation amounts.
  Let 1/s be a scale factor.

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & dx \\ -\sin\theta & \cos\theta & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & s \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$T\,\vec{\mathrm{v}} = \begin{bmatrix} x\cos\theta + y\sin\theta + dx \\ -x\sin\theta + y\cos\theta + dy \\ z + dz \\ s \end{bmatrix} = \begin{bmatrix} (x\cos\theta + y\sin\theta + dx)/s \\ (-x\sin\theta + y\cos\theta + dy)/s \\ (z + dz)/s \\ 1 \end{bmatrix}$$

Transformations Are Composable

- To rotate in the x-y plane about point p: translate p to the origin, rotate, then translate back.

$$Translate(p) = \begin{bmatrix} 1 & 0 & 0 & p.x \\ 0 & 1 & 0 & p.y \\ 0 & 0 & 1 & p.z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RotZ(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RotateAbout(p, \theta) = Translate(p) \cdot RotZ(\theta) \cdot Translate(-p)$$

Most General Form of a Transformation Matrix

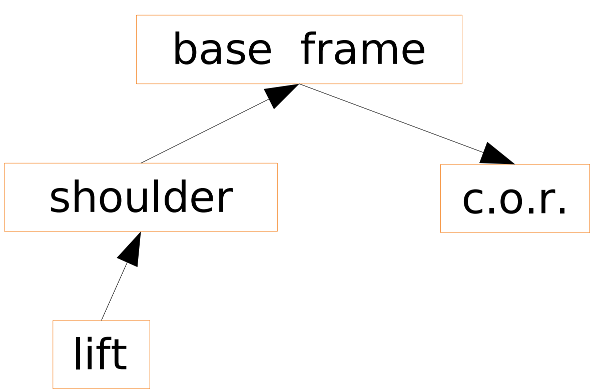
<table>
<tr><td colspan="3" rowspan="3">Full 3D Rotation Matrix</td><td>dx</td></tr>
<tr><td>dy</td></tr>
<tr><td>dz</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>scale</td></tr>
</table>

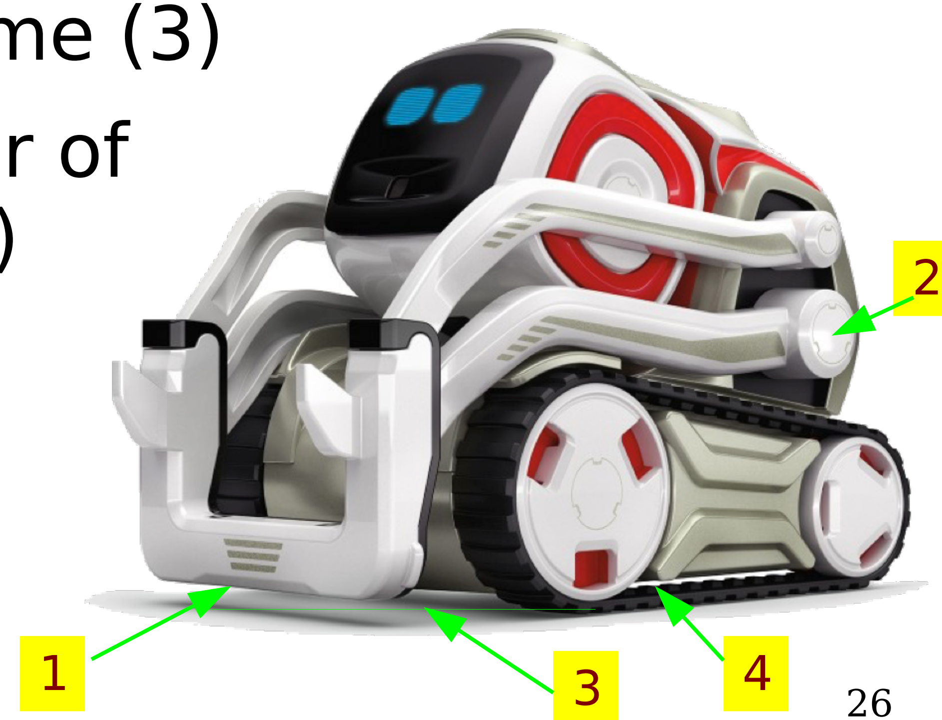
Forward Kinematics

- Given a set of joint angles, calculate the position of an end-effector.
- Example: suppose the lift joint is at +30 degrees.
- What is the position of the bottom edge of the lift relative to the robot's center of rotation?

Solution to FK Problem

- Convert between reference frames in the kinematic tree:
  - **Start** at the lift edge reference frame (1)
  - **Up** to the shoulder reference frame (2)
  - **Up** to the base frame (3)
  - **Down** to the center of rotation frame (4)



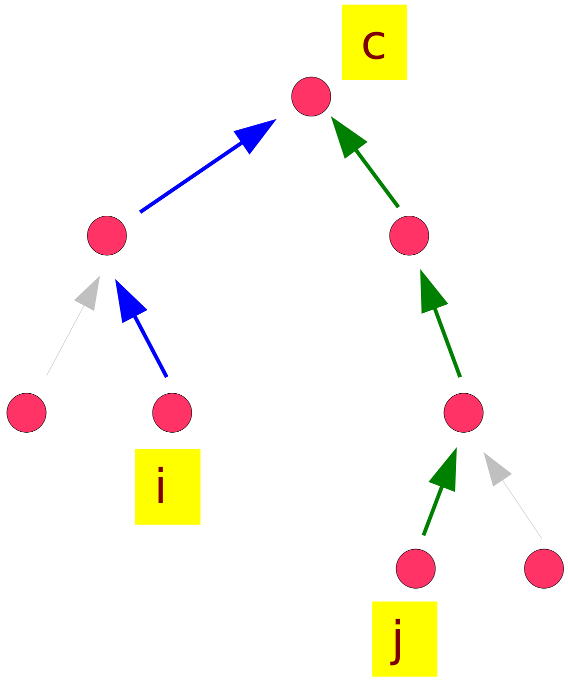


Converting Between Reference Frames

- Common conversions are between the base frame (body coordinates) and a limb or camera frame.
- Each step requires a transformation matrix.
- Where do these matrices come from?
  - The Denavit-Hartenberg parameters:

$$\text{RotX}(\alpha) \cdot \text{Translate}(r,0,d) \cdot \text{RotZ}(\theta)$$

From Frame i to Frame j



Search upward from i to common frame c, forming T_{ci}.

Search upward from j to common frame c, forming T_{cj}.

Compute inverse $T_{jc} = (T_{cj})^{-1}$

Desired transformation is:

$$T_{ji} = T_{jc} \cdot T_{ci}$$

The numpy Package

- We will use numpy to represent coordinates and transformation matrices.
- Represent points as column vectors, which are $n \times 1$ matrices.

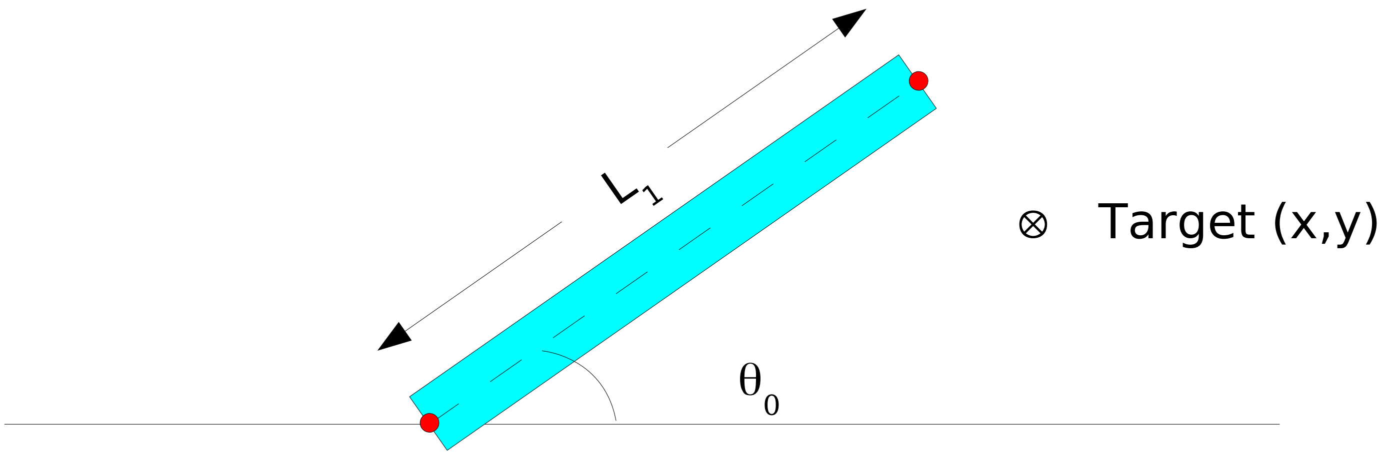
```
import numpy as np
v = np.array([ [5.75], [30], [115], [1] ])
w = np.array([ [17], [-4.2], [100], [1] ])

innerprod = v.T.dot(w)      a 1x1 matrix
outerprod = v.dot(w.T)      a 4x4 matrix
t = np.random.rand(4,4)     random matrix
tinv = np.linalg.inv(t)     matrix inverse
```

Inverse Kinematics

- Inverse kinematics finds the joint angles to put an effector at a particular point in space.
- Hard problem:
  - Solution space can be discontinuous
  - Can be highly nonlinear
  - Multiple solutions may be possible
  - Maybe no solution (so find closest approximation)
- Example: lookAtPoint(x,y,z)
  - point described in base frame coordinates
  - calculate head (and body?) angles

Solving the 1-Link Arm

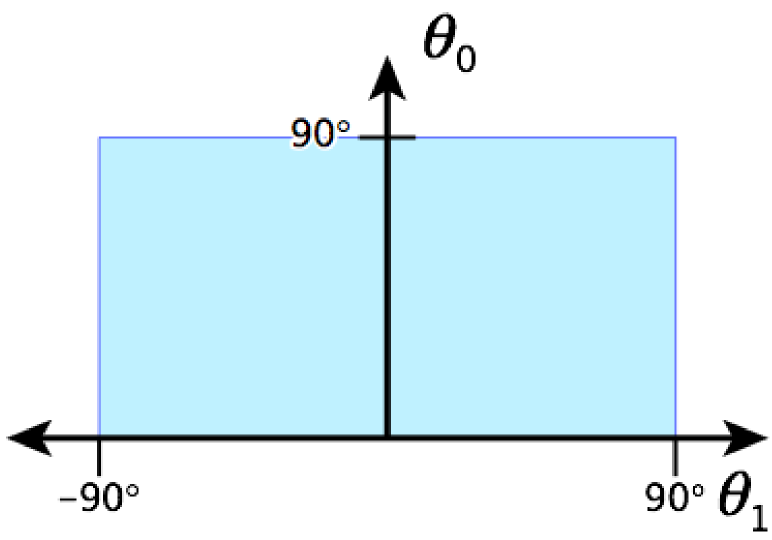


Reachable if: $L_1 = \sqrt{x^2 + y^2}$

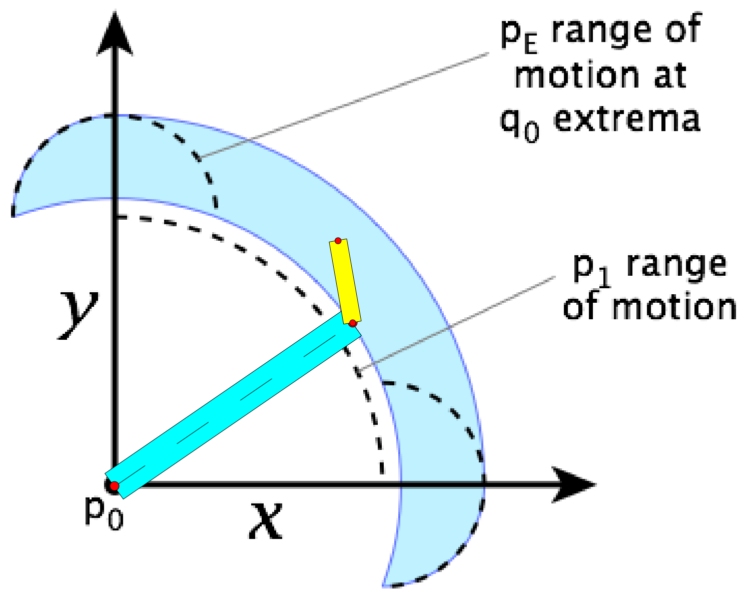
Solution: $\theta_0 = \operatorname{atan2}(y, x)$

Configuration Space vs. Work Space

Consider a 2-link arm, with joint constraints
$0° < \theta_0 < 90°$, $-90° < \theta_1 < 90°$

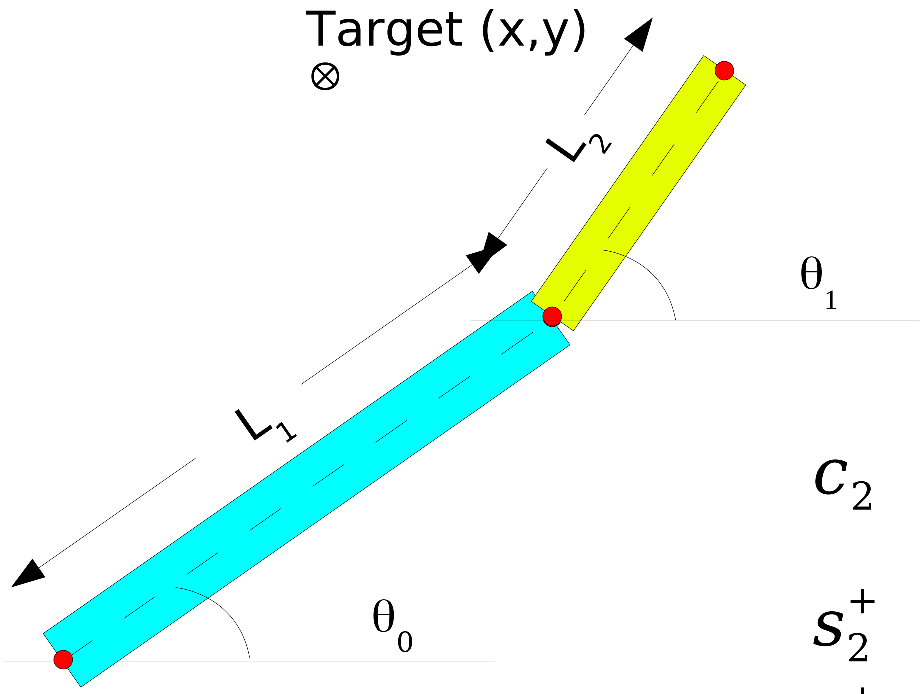


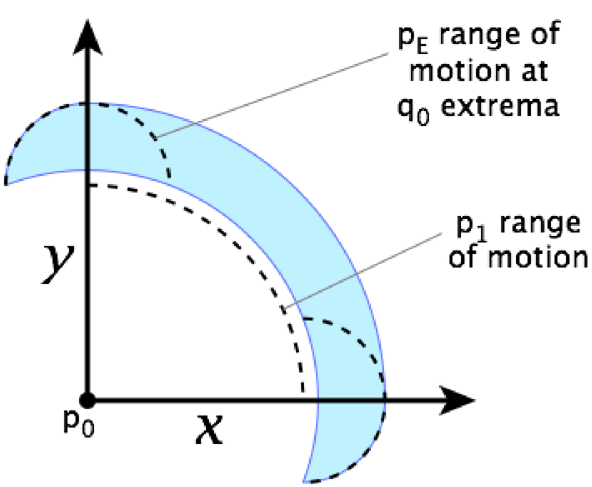
Configuration Space: robot's internal state space (e.g. joint angles)



Work Space: set of all possible end-effector positions

Solving the 2-Link Planar Arm

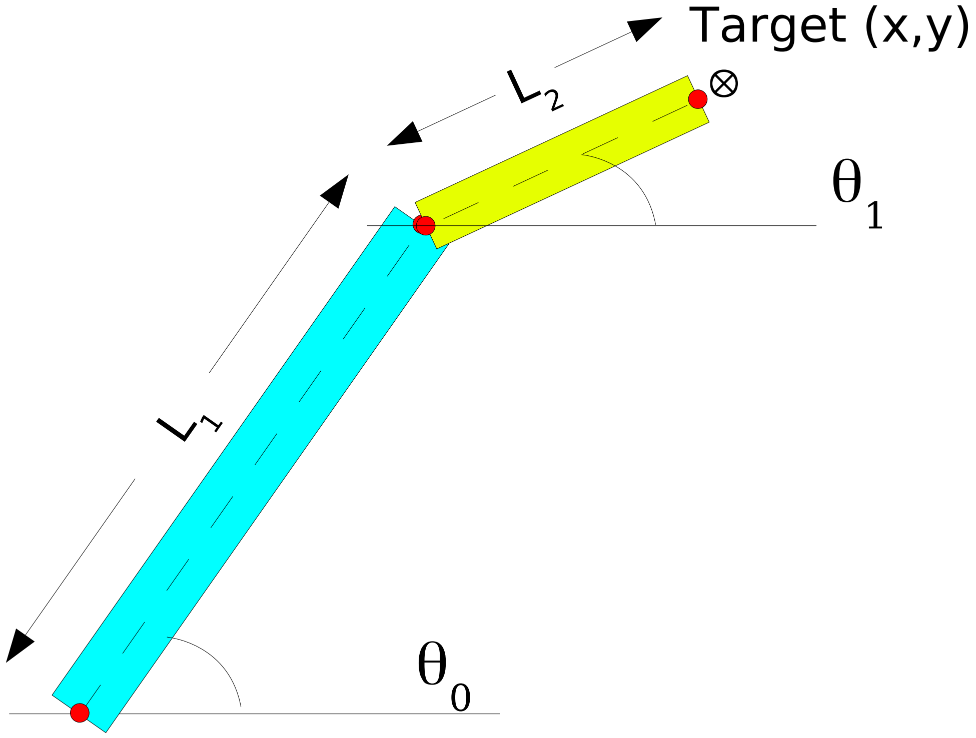


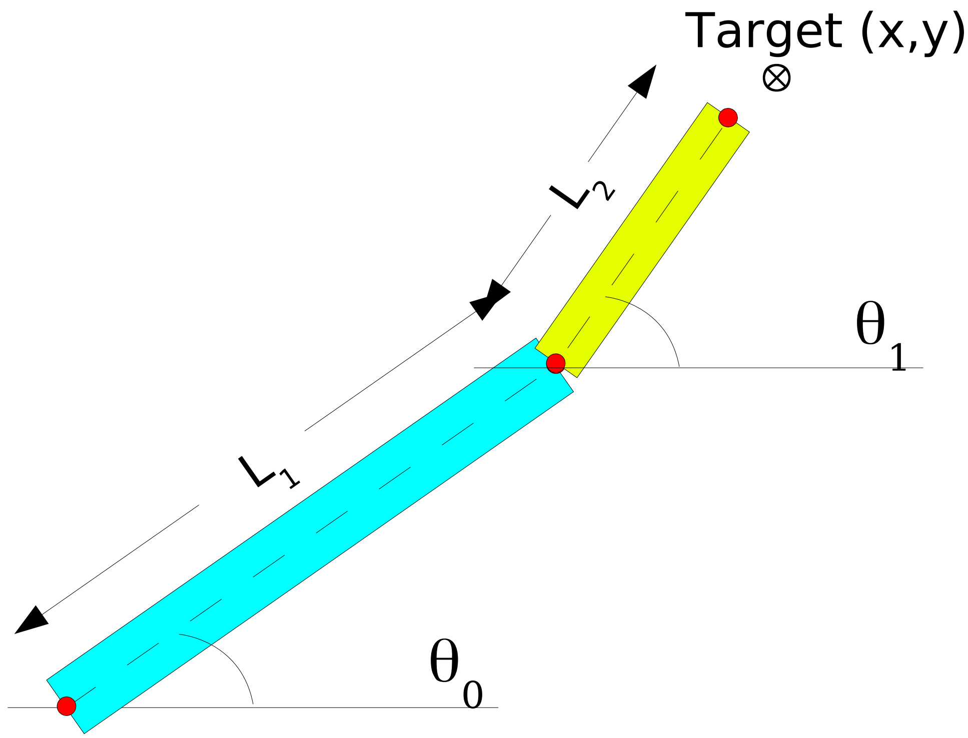


$$
\begin{aligned}
c_2 &= \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2} \\
s_2^+ &= \sqrt{1-c_2^2} \\
\theta_1^+ &= \operatorname{atan2}(s_2^+, c_2) \\
K_1 &= L_1 + c_2 L_2 \\
K_2 &= s_2^+ L_2 \\
\theta_0 &= \operatorname{atan2}(y, x) - \operatorname{atan2}(K2, K1)
\end{aligned}
$$

Reachable if: $c_2^2 \le 1$

Two Possible Solutions





$$s_2^- = -\sqrt{1-c_2^2}$$
$$\theta_1^- = \operatorname{atan2}(s_2^-, c_2)$$

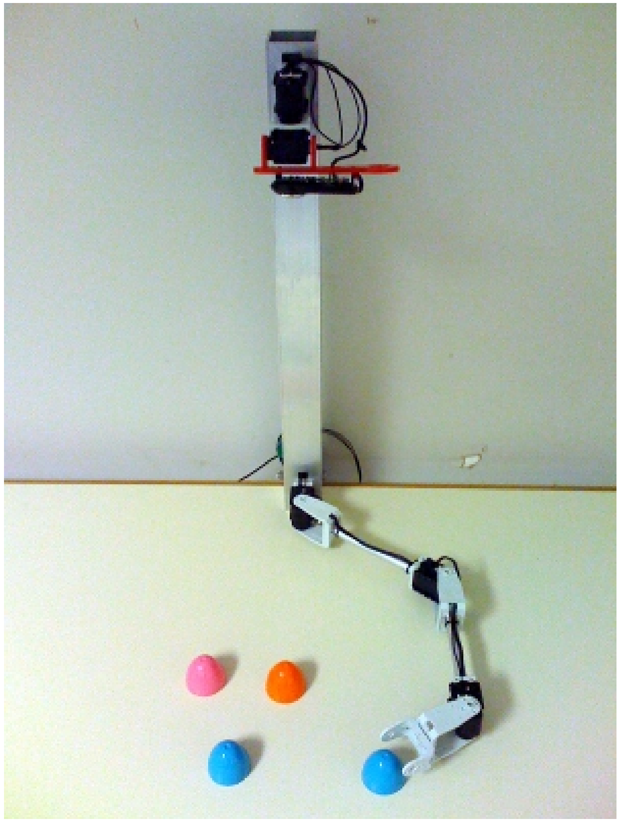
"Elbow up"

$$s_2^+ = \sqrt{1-c_2^2}$$
$$\theta_1^+ = \operatorname{atan2}(s_2^+, c_2)$$

"Elbow down"

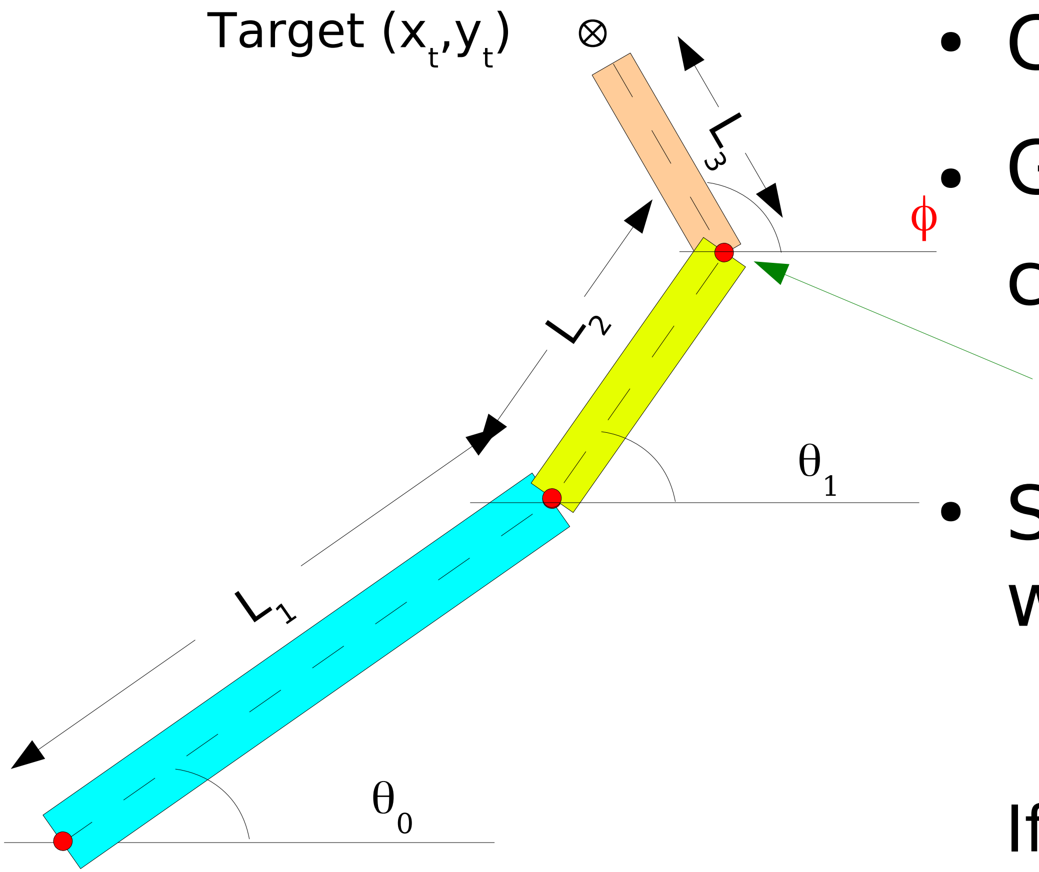
How Many Degrees of Freedom Are Enough?

- With 2 dof you can put the end effector at any point in the workspace.
- But you can't control end-effector orientation.
  - What if the arm is holding a screwdriver?



- With 3 dof in the same plane you can control both position and orientation.

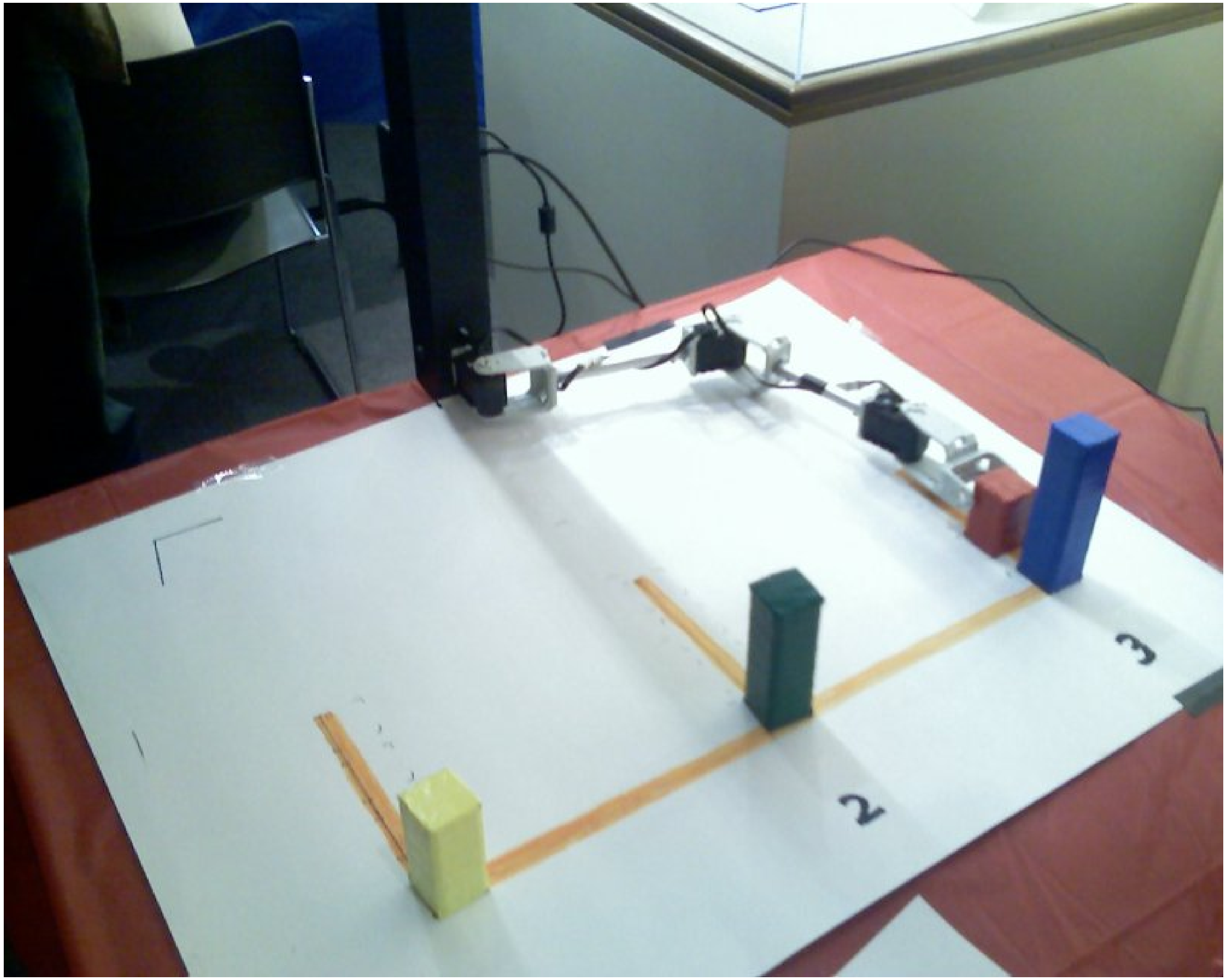
Solving the 3-Link Planar Arm



- Choose tool angle ϕ
- Given target position x_t, y_t, calculate wrist position: x_w and y_w
- Solve 2-link problem to put wrist at x_w, y_w.

If you don't know ϕ, pick an arbitrary starting value and search from there until you find a solution that works.

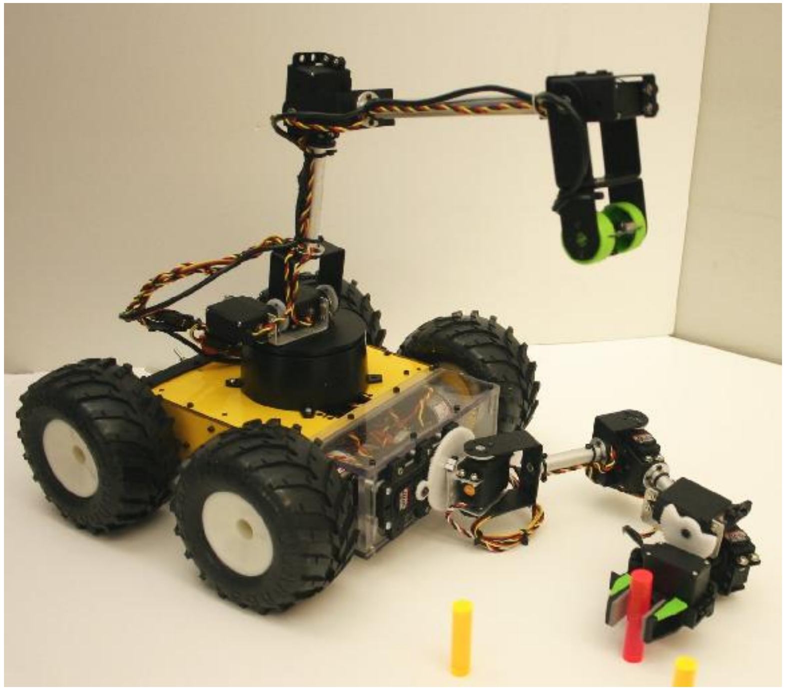
Towers of Hanoi in the Plane

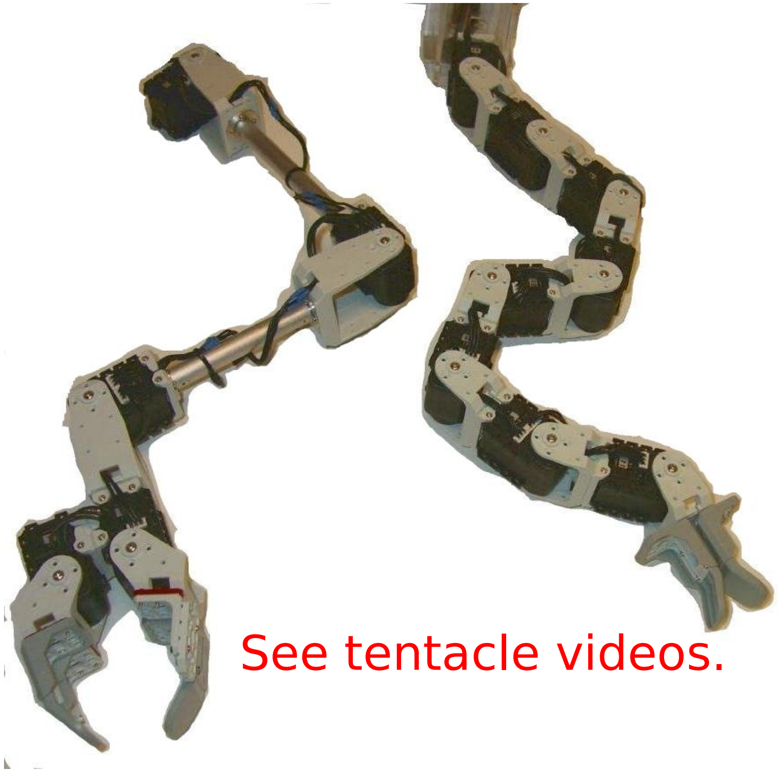


Video by Michel Brudzinski and Evan Patton at RPI.
https://www.youtube.com/watch?v=QahSf4fbi0g
Poses crafted by hand: IK solver wasn't written yet!

Customized Kinematics Solvers

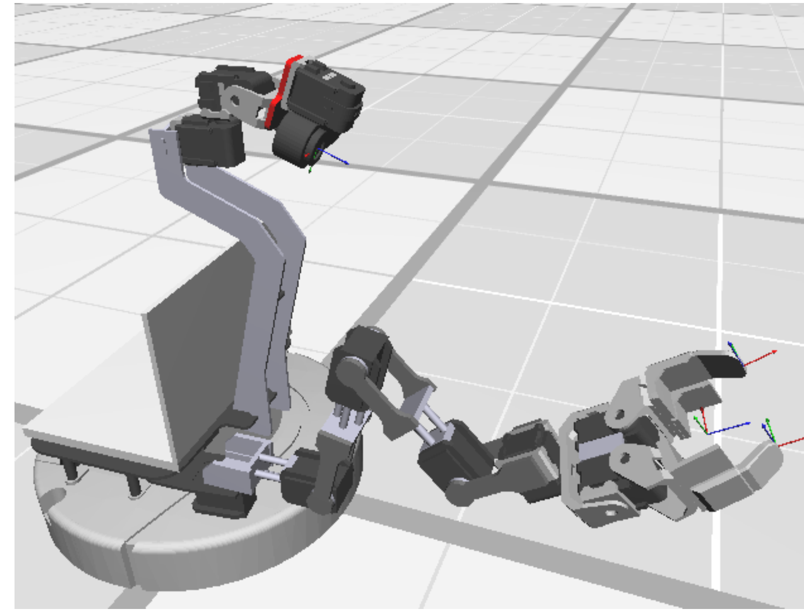
- For some simple kinematic chains, such as a pan/tilt, we can write analytic solutions to the IK problem.
- For the general case, must use gradient descent search.





See tentacle videos.

Calliope's 5-dof ARM



- Only one degree of freedom in the horizontal plane:
  - ARM:base
- Three degrees of freedom in a vertical plane:
  - ARM:shoulder, ARM:elbow, ARM:wrist
- An additional degree of freedom in an orthogonal plane:
  - ARM:wristrot
- Conclusion: can only partially control the 3D pose of the end-effector.
  - What kinds of motions can this arm not make?

Why VEX AIM Needs Kinematics

- Forward kinematics:
  - Calculate robot bounding box based on body plus carried objects, for collision avoidance during path planning.

- Inverse kinematics:
  - Put the kicker in the right place for object manipulation tasks.
  - Calculate required heading and base frame location given desired relationship between the kicker and an object.

Kinematics in vex-aim-tools

- Kinematics engine is in:

  `aim_fsm/kine.py`

- Robot's kinematic description is in:

  `aim_fsm/aim_kin.py`

- You can display kinematic info in simple_cli using the commands:

  show kine

  show kine *joint_name*