15-494/694: Cognitive Robotics

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Lecture 5:

Particle Filters and Localization

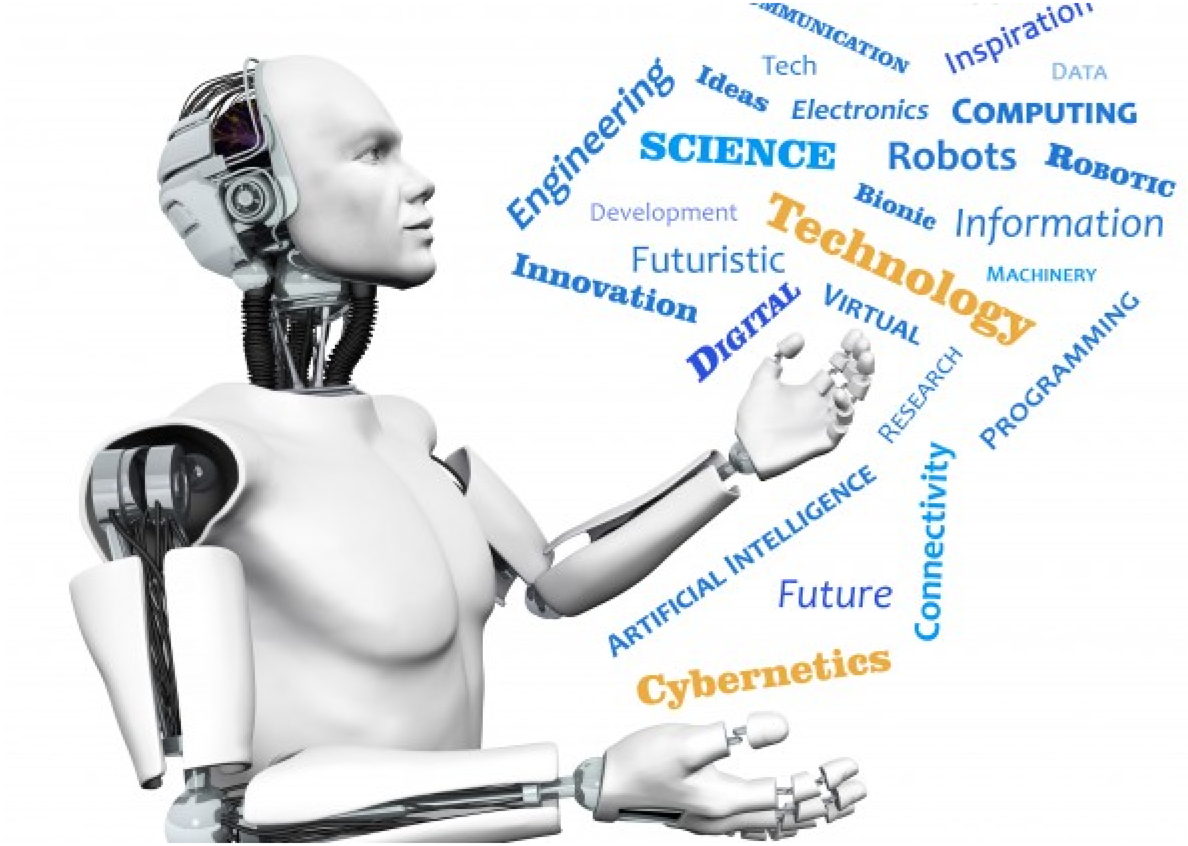


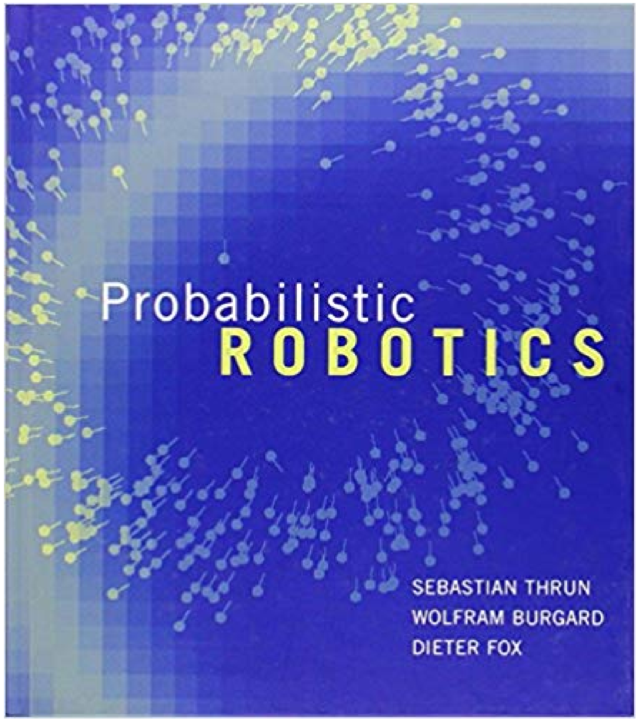
Image from http://www.futuristgerd.com/2015/09/10

Outline

- Probabilistic Robotics
- Belief States
- Parametric and non-parametric representations
- Motion model
- Sensor model
- Evaluation and resampling
- Demos

Probabilistic Robotics

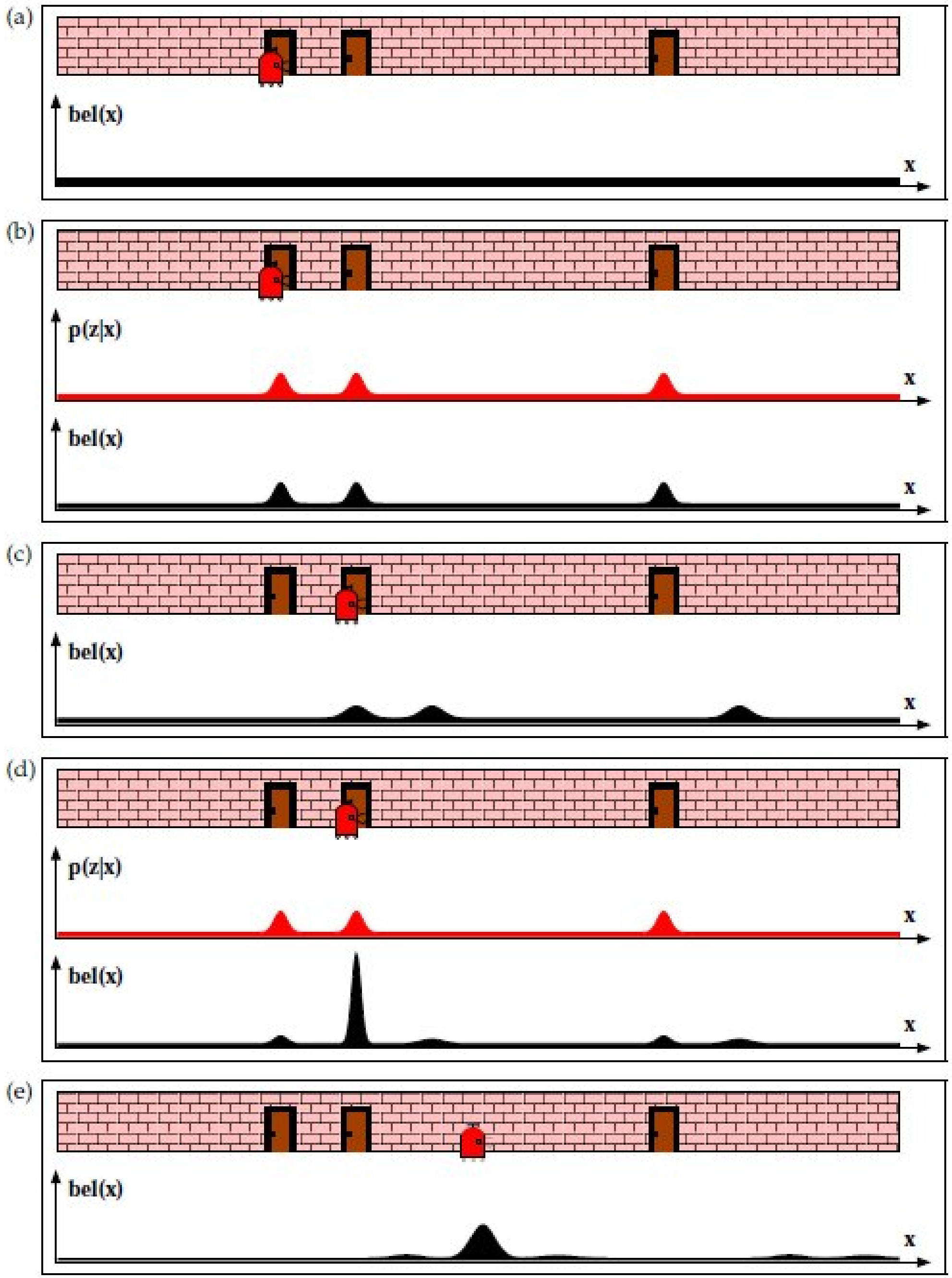
- The world is uncertain:
  - Sensors are noisy and inaccurate.
  - Actuators are unreliable.
  - Other actors can affect the world.

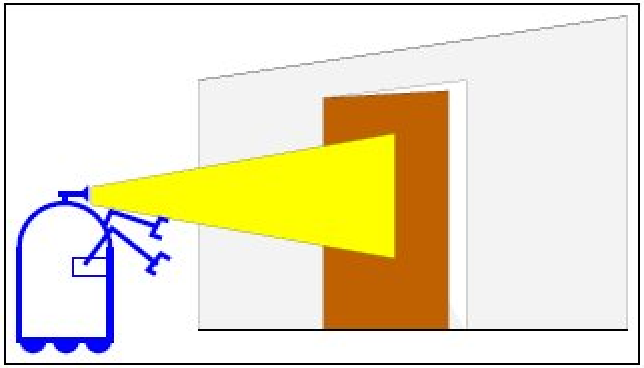


- Embrace the uncertainty!
- How?
  - Explicitly *model* our uncertainty about sensors and actions.
  - Replace discrete states with beliefs: *probability distributions* over states.
  - Use Bayesian filtering to update our beliefs.

Beliefs

are probability distributions





Figures from Thrun, Burgard, and Fox (2005) *Probabilistic Robotics*

Some Notation

- x_t = state at time *t*
- u_t = *control signal at time t*
- z_t = *sensor input at time* t
- We don't know x_t with certainty; we have an *a priori* (before measurement) belief $\overline{bel}(x_t)$ about it:

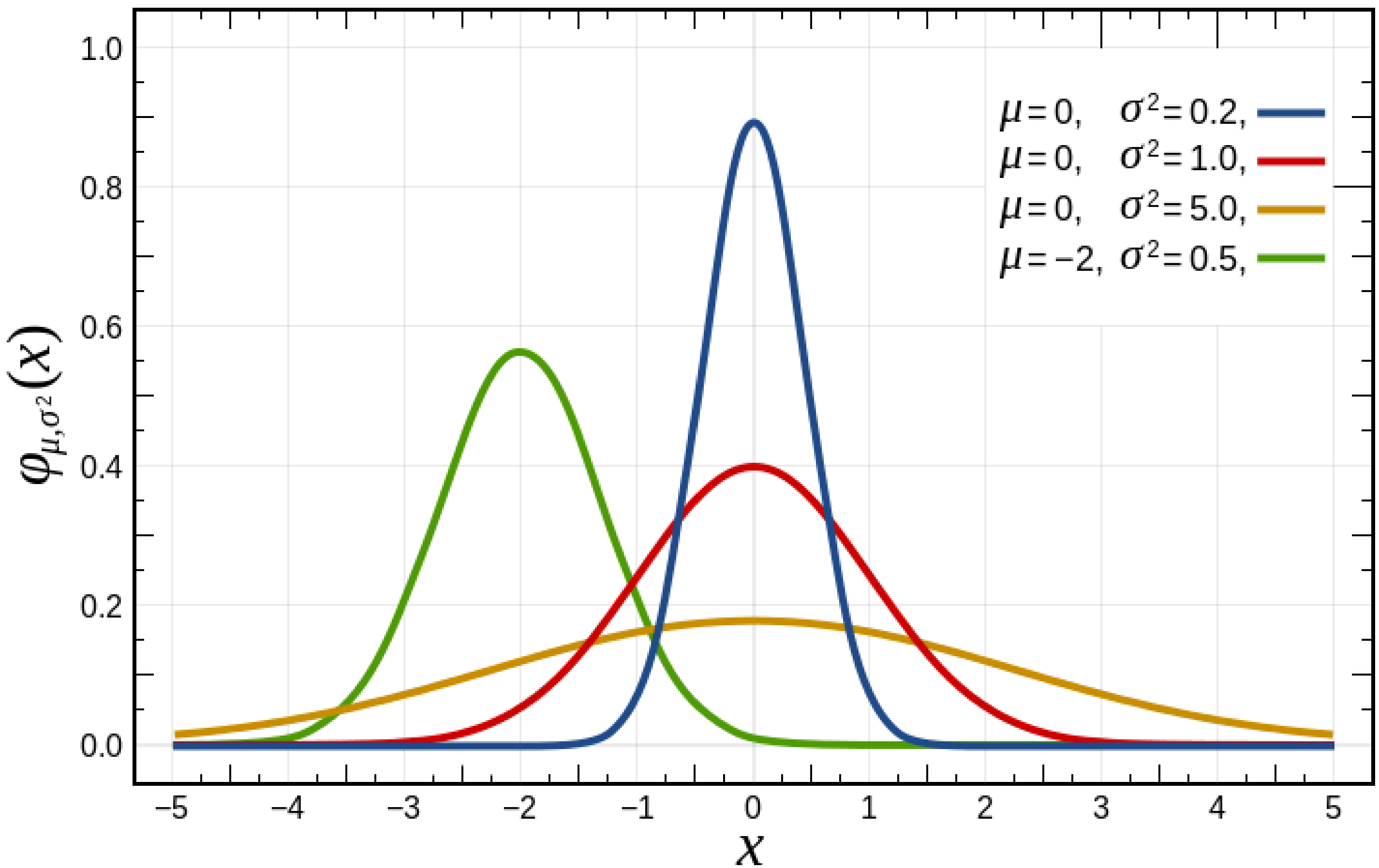
$$\overline{bel}(x_t) = p(x_t \mid z_{1:t-1}, u_{1:t})$$

- New sensor data z_t updates our belief, giving an *a posterior* belief $bel(x_t)$:

$$bel(x_t) = \eta \; p(z_t \mid x_t) \cdot \overline{bel}(x_t)$$

Parametric Representations (1)

- Represent a probability distribution using an analytic function described by a small number of parameters.
- Most common example: Gaussian



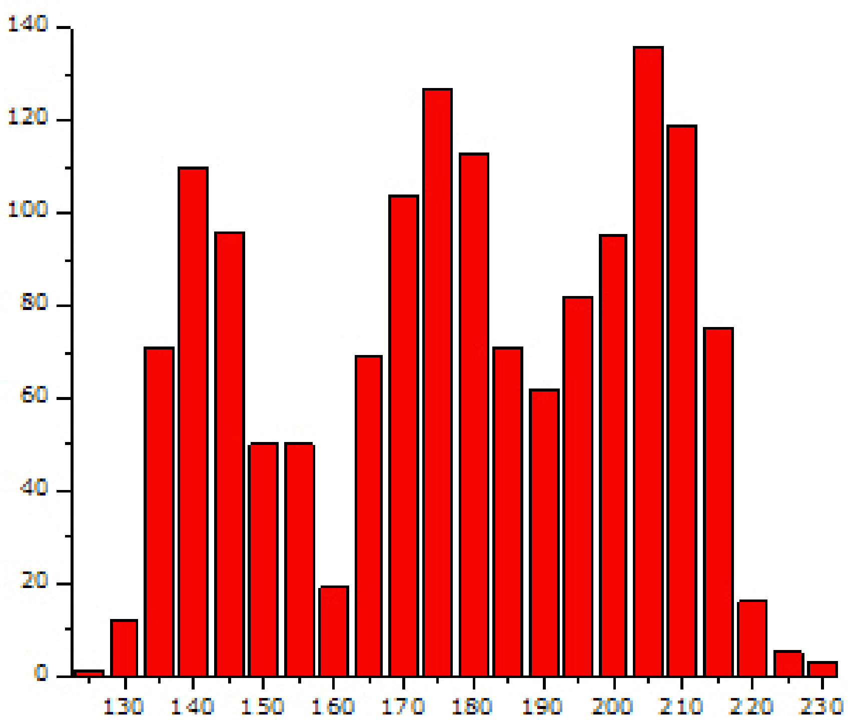
Parametric Representations (2)

- Good points:
  - Compact representation: just a few numbers
    - For a Gaussian: mean μ and variance σ^2
  - Fast to compute
  - Nice mathematical properties
  - **Easy to sample from**

- Drawbacks:
  - May not match the data very well
  - Can give bad results if the fit is poor

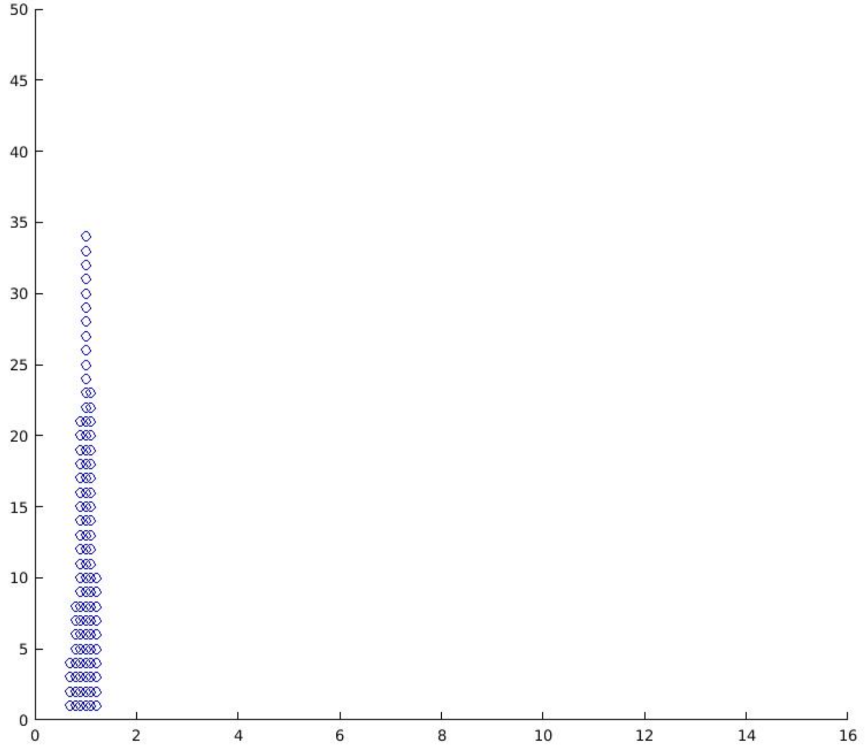
Nonparametric Representations

- No preconceived formula for the distribution.
- Instead, maintain a representation of the actual distribution, via *sampling*.
- Example: histogram
- Good points:
  - Can represent completely arbitrary distributions
- Drawbacks:
  - Requires more storage
  - Expensive to update



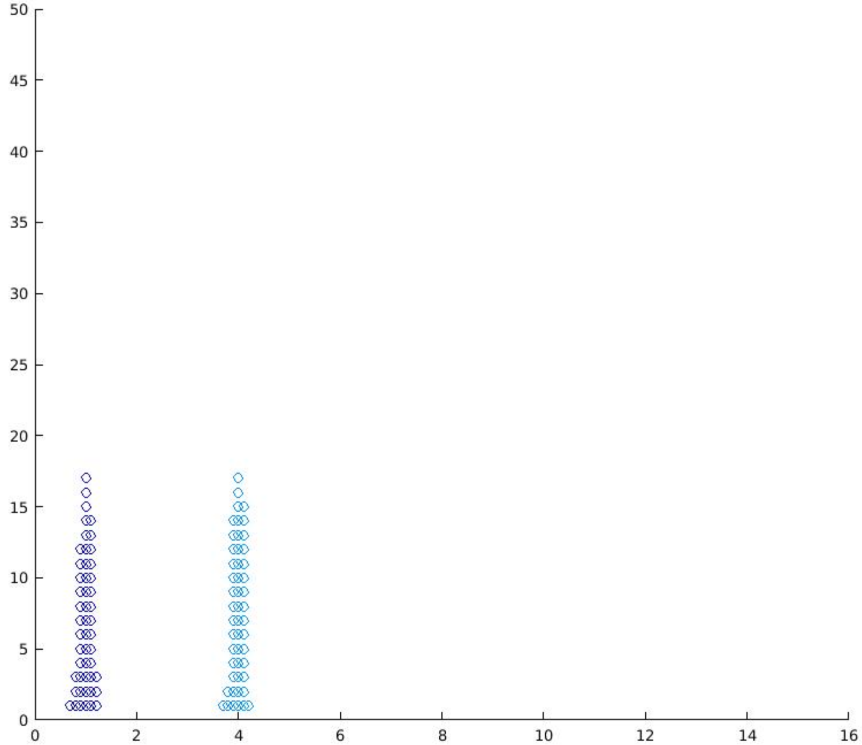
Where Is The Robot?

- Parametric: the robot is at x=1 with $\sigma^2 = 0.2$
- Non-parametric: 100 samples indicating robot position.



Where Is The Robot?

- Parametric: fail (or put robot at the mean: x=2.5)
- Non-parametric: 100 samples.



Particle Filters

- A particle filter is an efficient non-parametric representation of a distribution.
- Each particle represents a sample drawn from the distribution.
- As the distribution changes, we update the particles.
- Three kinds of updating:
  - Change the *value* the particle encodes (motion model).
  - Change the *weight* assigned to the particle (sensor model).
  - *Resample* the distribution, getting a fresh set of particles with initially equal weights.

Bayesian Filter, part 1

- Our belief about the robot's position x at time *t*-1 is a probability distribution $p(x_{t-1})$, which we represent as a set of *samples*.

- At time *t* the robot moves, following some control signal u_t, producing a new distribution $p(x_t)$.

- A *motion model* defines how our new prediction $\overline{bel}(x_t)$ arises from applying u_t.

$$\overline{bel}(x_t) = \int p(x_t | x_{t-1}, u_t) \cdot bel(x_{t-1}) \; d x_{t-1}$$

Why Are We Integrating?

$$\overline{bel}(x_t) = \int_{x_{t-1}} p(x_t | x_{t-1}, u_t) \cdot bel(x_{t-1}) \, dx_{t-1}$$

Probability of arriving at x_t given that we were previously at x_{t-1} and got control signal u_t.

Belief that we were previously at location x_{t-1}

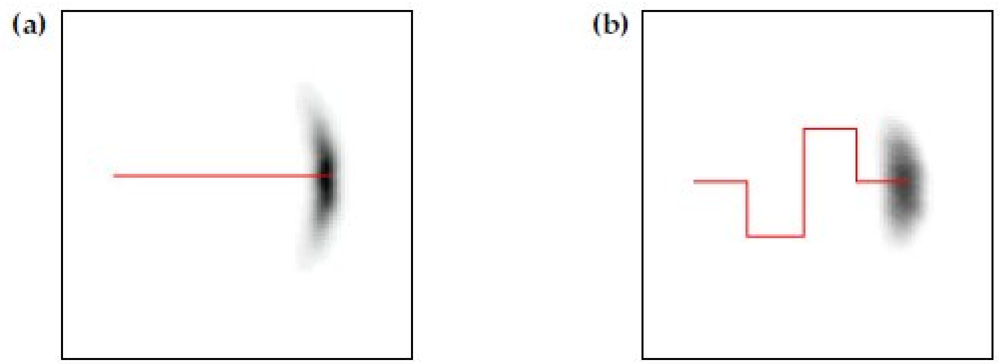
All possible previous locations x_{t-1}

Integrated over all possible starting locations x_{t-1}.

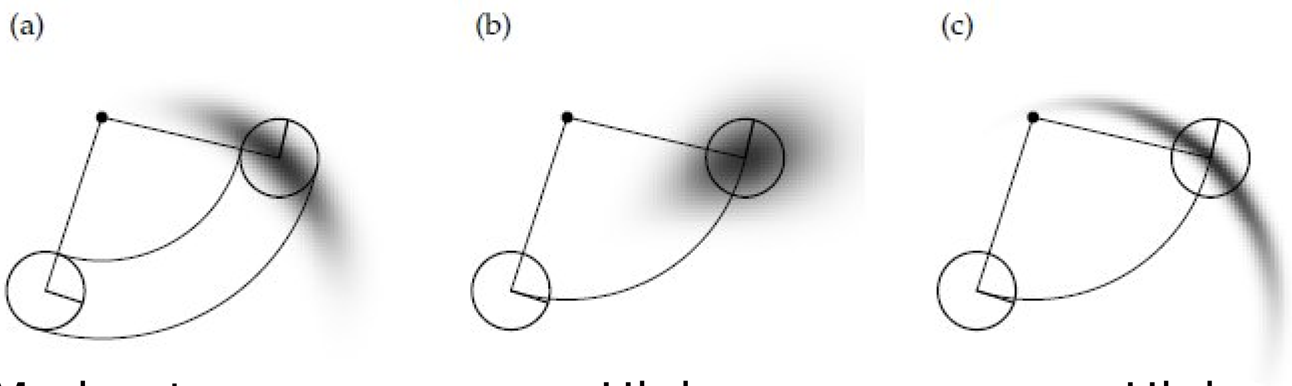
Motion Models

- Motion models express the noisiness of motion u_t.
- Typically use a simple parametric distribution.
  - Easy to sample.
- We represented the distribution $p(x_{t-1})$ as a set of *a posteriori* samples $bel(x_{t-1})$. Motion gives us $\overline{bel}(x_t)$.
- How do we sample $\overline{bel}(x_t)$?
- Solution: for each sample in $bel(x_{t-1})$, draw a value from the motion model's distribution and add it to the sample value.

Motion Model $p(x_t|x_{t-1},u_t)$



Figures from Thrun, Burgard, and Fox (2005) *Probabilistic Robotics*

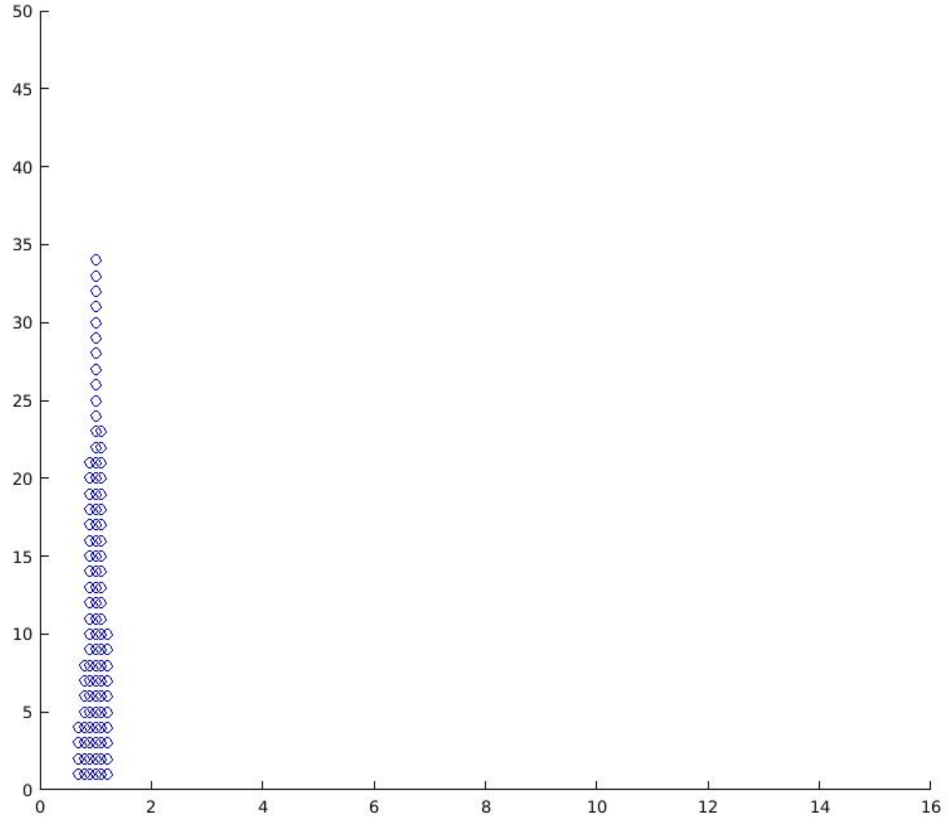


Moderate Noise Values

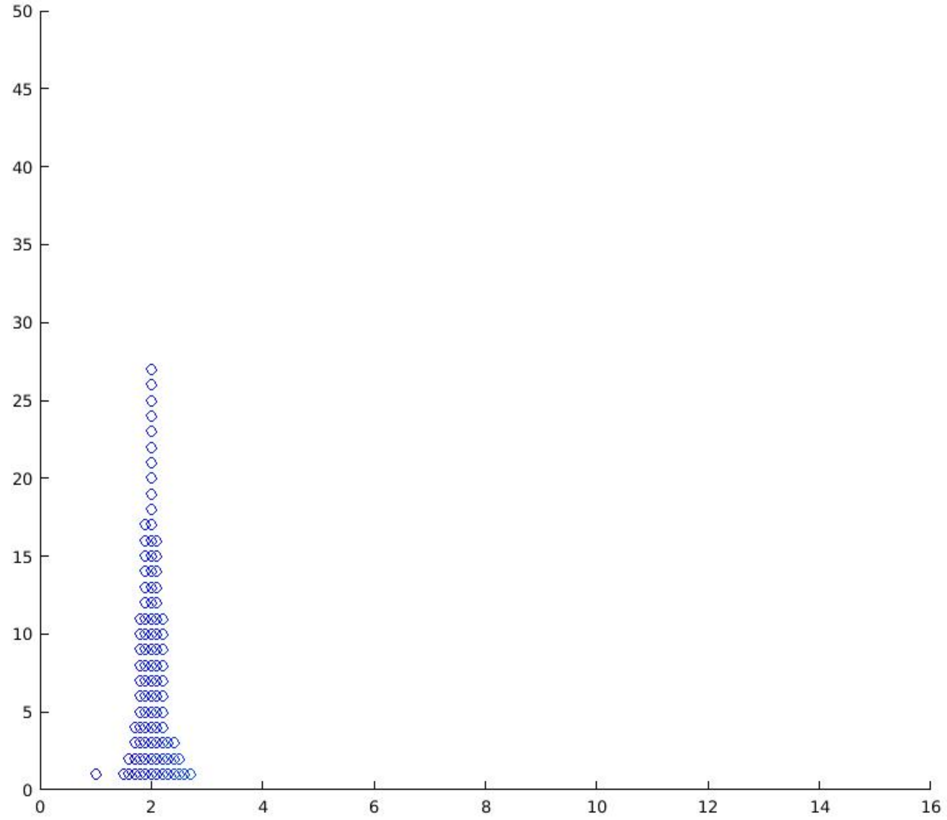
High Translational Uncertainty

High Rotational Uncertainty

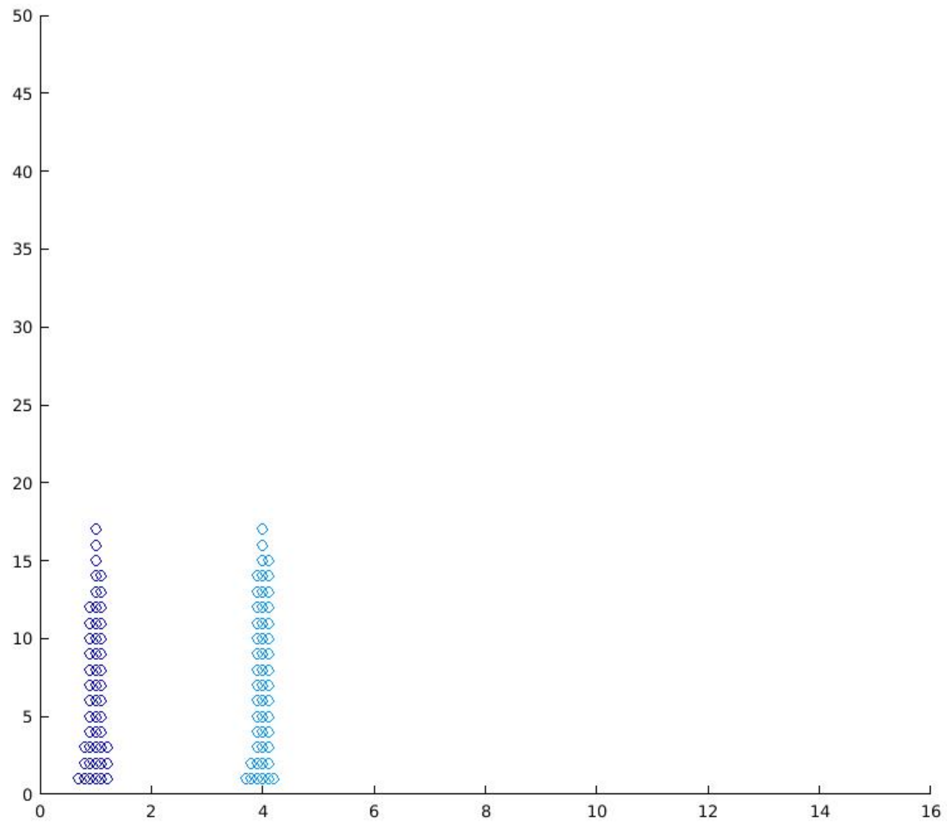
Robot at t=0: bel(x_0)



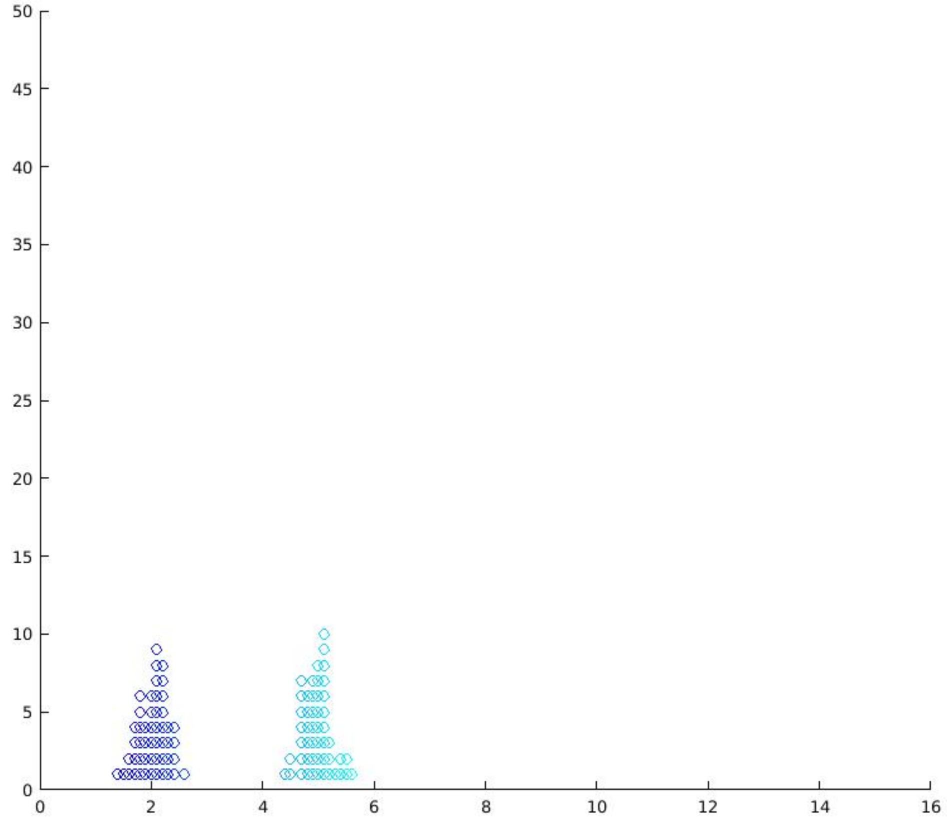
Prediction at t=1: $\overline{bel}(x_1)$



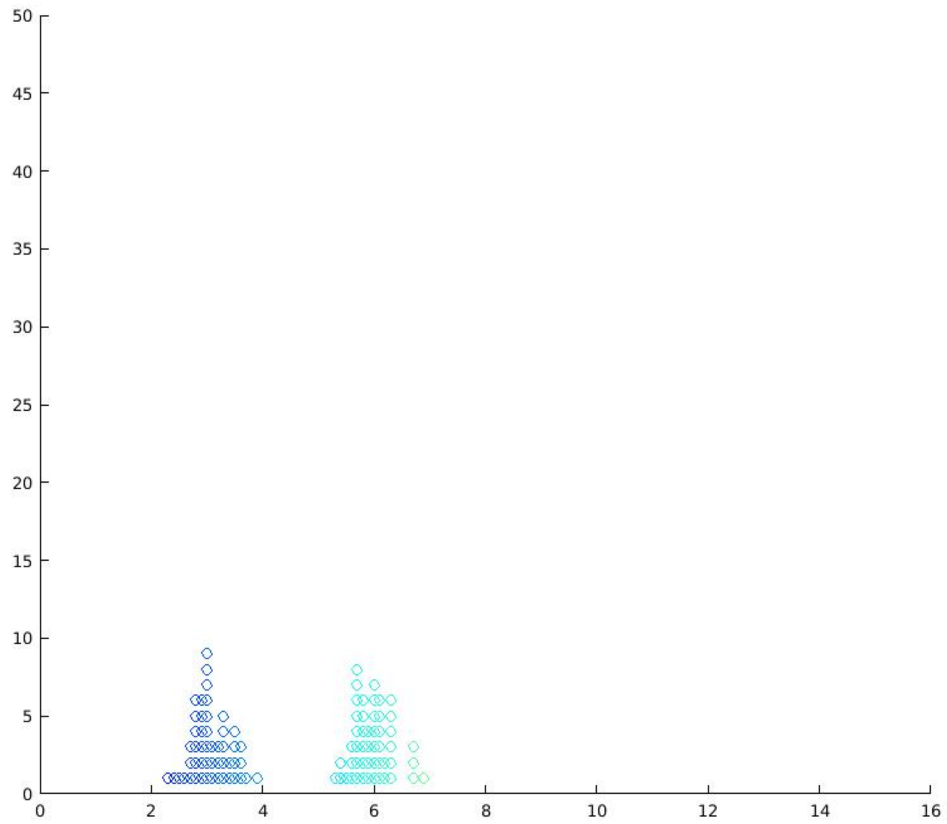
Robot at t=0: bel(x_0)



Prediction at t=1: $\overline{bel}(x_1)$



Prediction at t=2: $\overline{bel}(x_2)$



Correcting Our Prediction

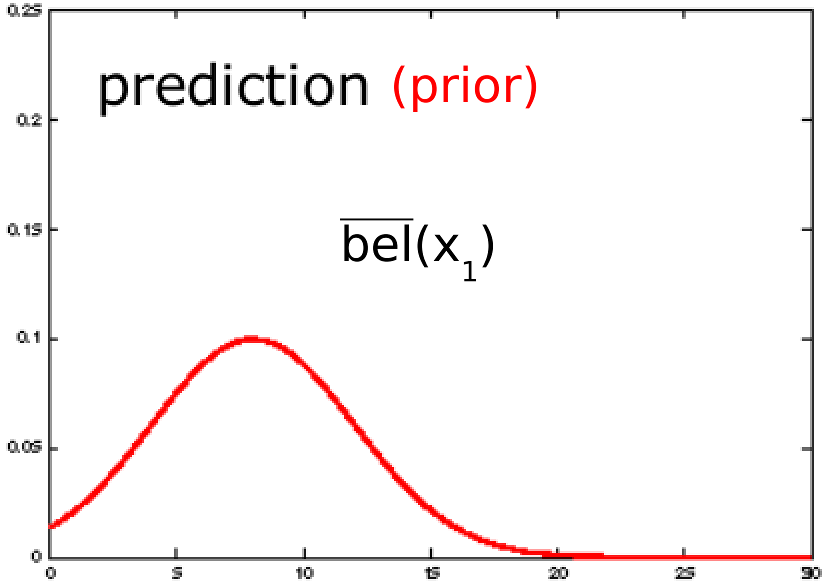
- To mitigate the noisiness of our motion model, we use sensor readings z_t to correct our belief distribution.
- Our sensors give us a probability distribution $p(x_t|z_t)$.
- Can't our sensors just tell us where we are?
- **NO!**
  - They're noisy.
  - An individual reading may not be that informative because the world can be ambiguous (e.g., doors look alike).
  - Need to combine information.

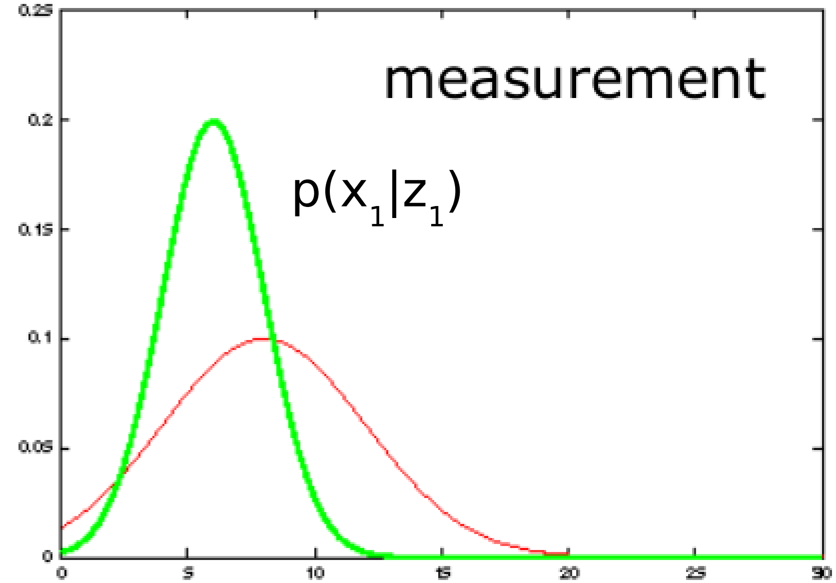
Sensor Model

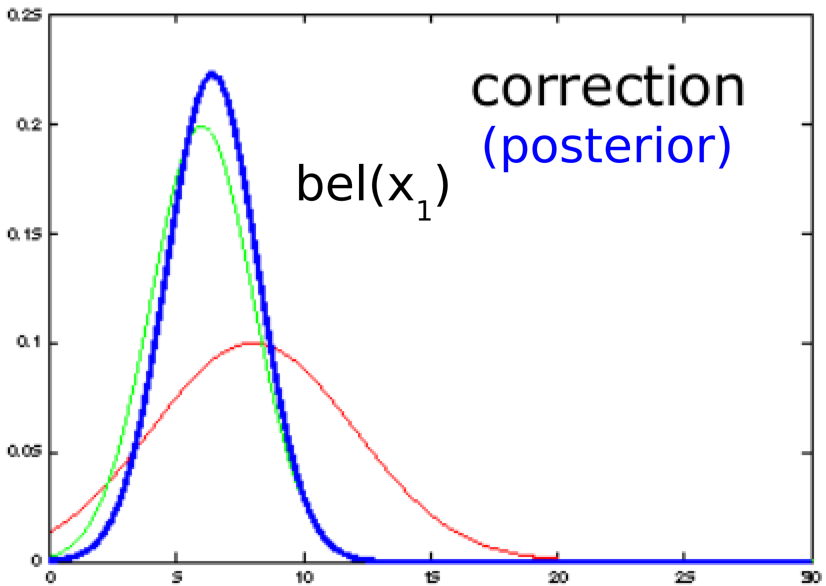
- We should try to model uncertainty in our sensor data.
- Lots of work on sonar and laser rangefinder noise models (e.g., effects of reflections, viewing angle, etc.)
- For visual landmarks:
  - Effects of camera resolution.
  - Distance estimates might have variance proportional to the distance value (larger distances have higher variance).
  - Bearing estimates might have variance inversely proportional to distance.

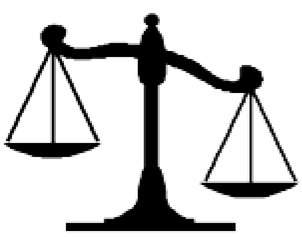
Interlude: The Kalman Filter

If distributions are gaussians, we can combine them using a **Kalman filter**. Weighting is inversely proportional to variance.





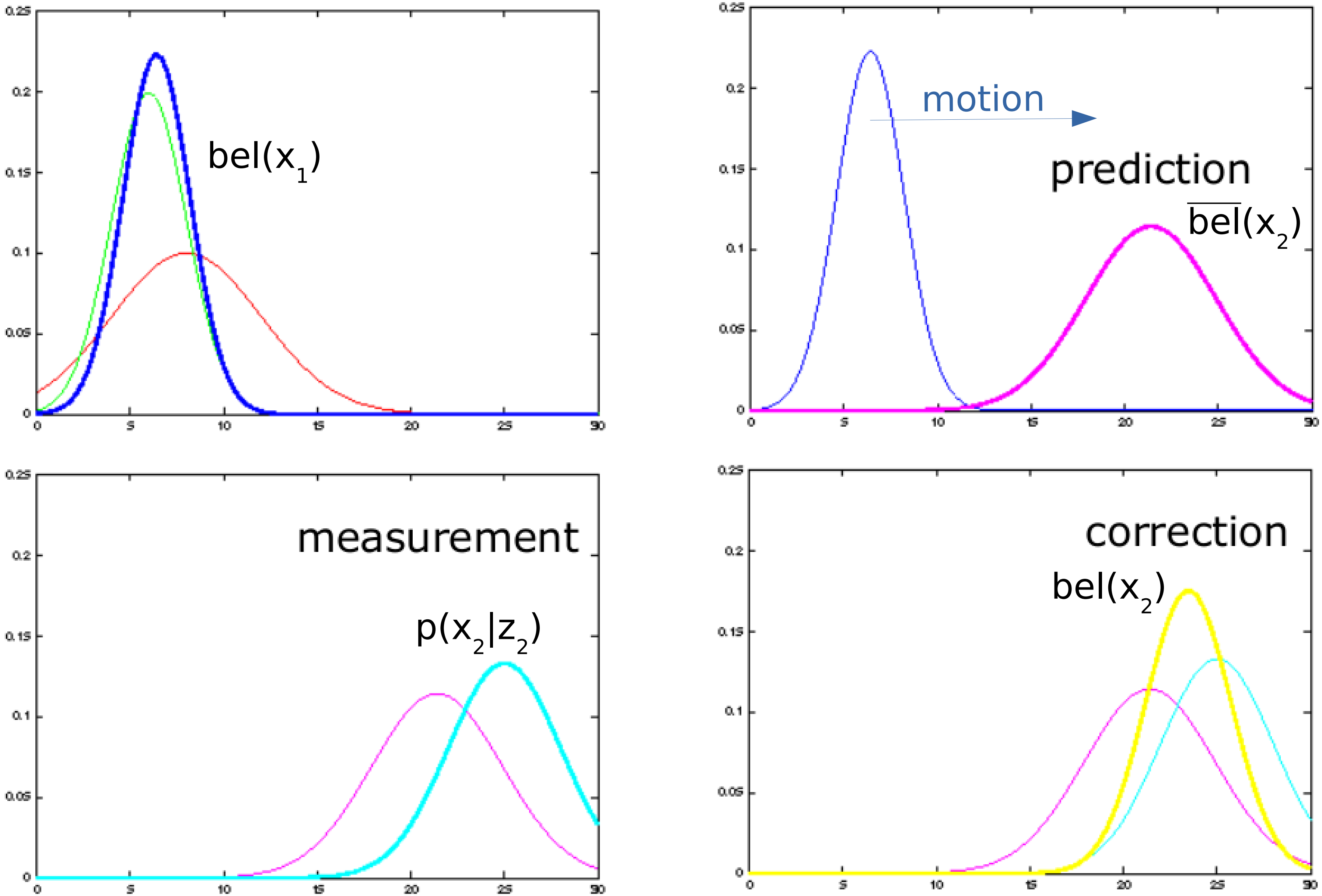




It's a weighted mean!

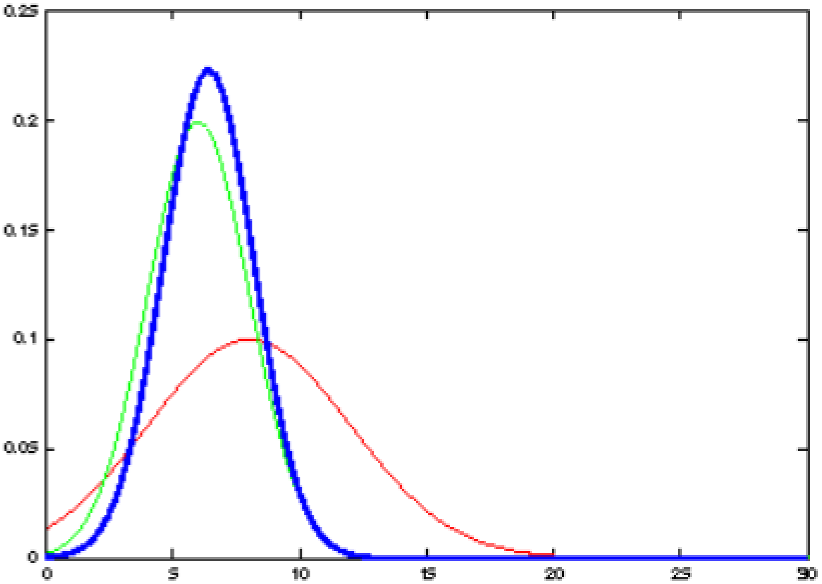
Slide modified from Burgard et al., "Introduction to Mobile Robotics", 2014, lecture 9: "Bayes Filter – Kalman Filter".

Second iteration: prior belief → prediction → measurement → correction.



Slide modified from Burgard et al., “Introduction to Mobile Robotics”, 2014, lecture 9: “Bayes Filter – Kalman Filter”.

Product of Two Gaussians



$$\mu_3 = \frac{\mu_1 \sigma_2 + \mu_2 \sigma_1}{\sigma_1 + \sigma_2}$$

$$\sigma_3 = \frac{\sigma_1 \cdot \sigma_2}{\sigma_1 + \sigma_2}$$

Bayesian Filter, part 2

From part 1:

$$\overline{bel}(x_t) = \int_{x_{t-1}} p(x_t | x_{t-1}, u_t) \cdot bel(x_{t-1}) \; dx_{t-1}$$

Sensor reading z_t gives distribution $p(x_t | z_t)$.

Corrected: $bel(x_t) = \eta \, p(z_t | x_t) \cdot \overline{bel}(x_t)$

η is a normalization constant.

But How Do We Correct Our Beliefs If We’re Using Particles to Represent the Distribution?

Corrected Sampling Representation

- Prior distribution $\overline{bel}(x_t)$ is "corrected" by weight $p(z_t|x_t)$ to give posterior $bel(x_t)$.
- The *weighted* particles are a sampling representation of the new distribution $p(x_t)$.
- The robot can move around and we can move the particles and update their weights.
- But is this a <u>good</u> representation?
- Particles whose weights become low aren't representing useful hypotheses. Eventually the representation falls apart because we're sampling the wrong regions.

Resampling

- Things break down when too many particles are representing the wrong regions of bel(x_t), so their weights are low.
- We can fix this by resampling bel(x_t), giving a fresh set of particles distributed correctly.
- But we have no formula for bel(x_t), and no direct representation of it.
- So how do we sample from it?
  - *Importance sampling.*

Sampling a Function y=g(x) From an Arbitrary Distribution x

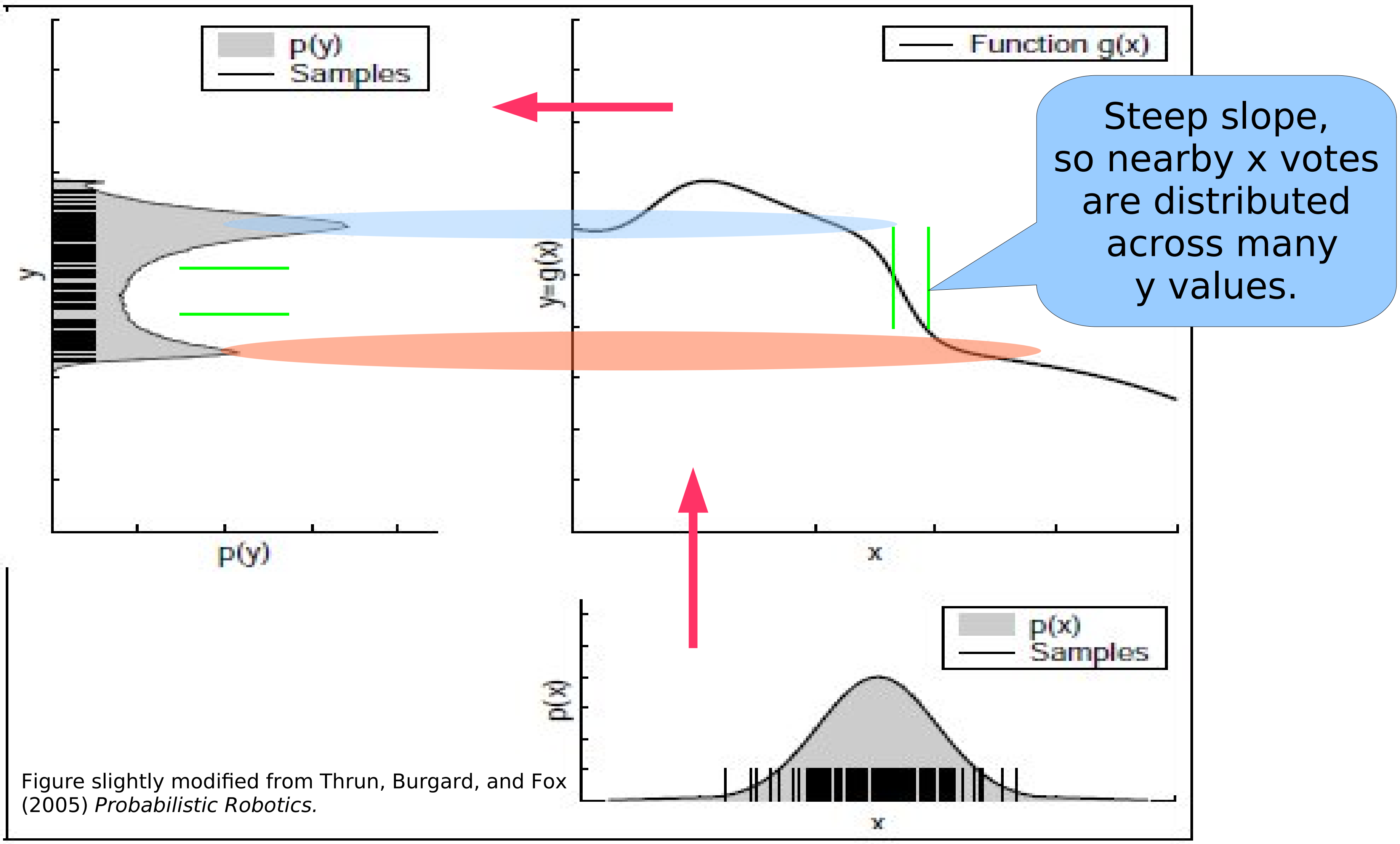
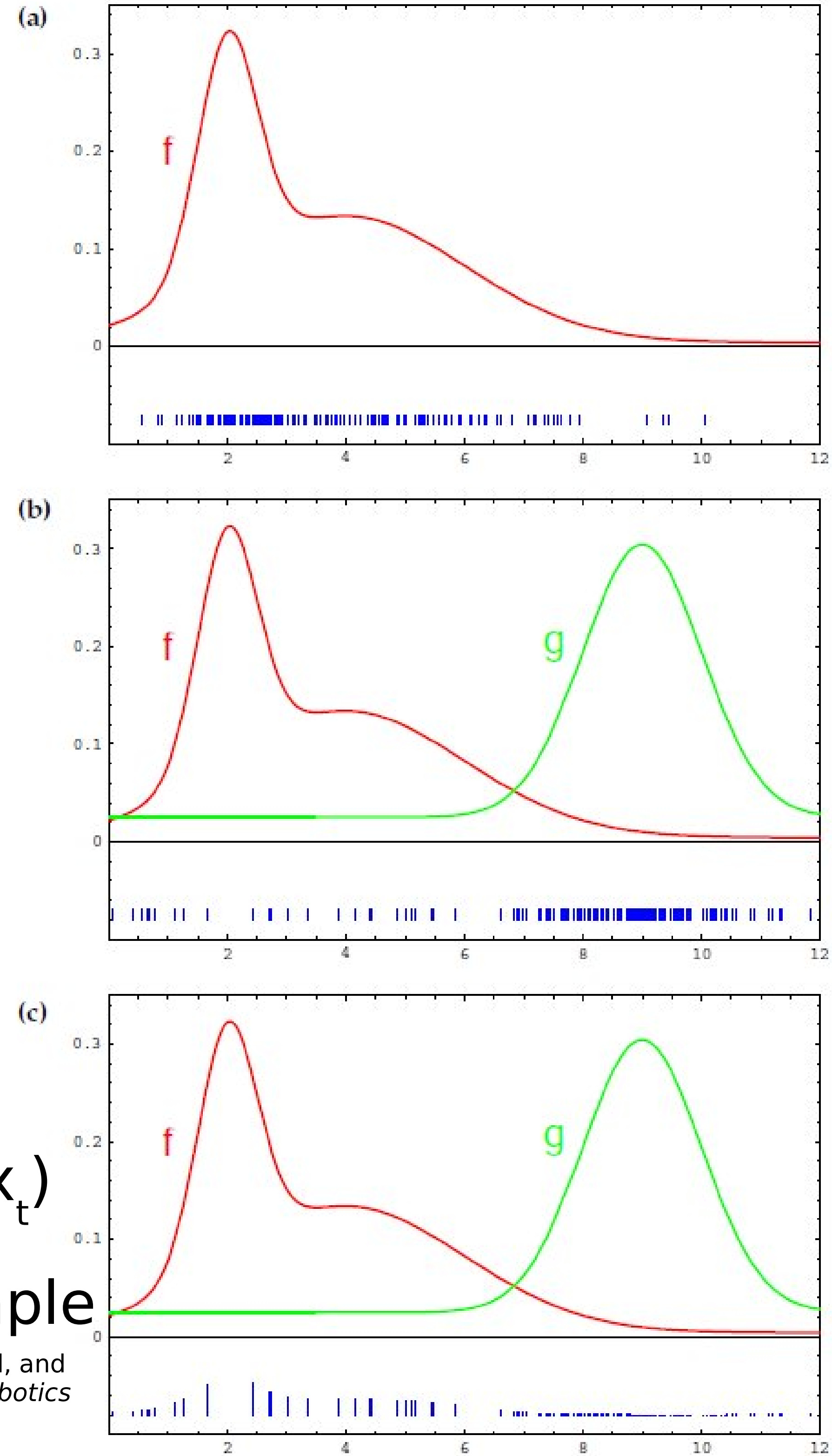


Figure slightly modified from Thrun, Burgard, and Fox (2005) *Probabilistic Robotics.*

Importance Sampling



- Want to sample from f.
- Can only sample from g.
- Weight each sample by f(x) / g(x).
- The weighted samples approximate f.
- g is $\overline{\mathrm{bel}}(x_t)$
- Weighting comes from $p(z_t|x_t)$
- Drawing from weighted sample gives f = $\mathrm{bel}(x_t)$

Figure from Thrun, Burgard, and Fox (2005) *Probabilistic Robotics*

Resampling: Drawing From Weighted Samples

- We don't need to resample on every time step t. We can accumulate sensor data for several time steps, so our weights are more accurate.

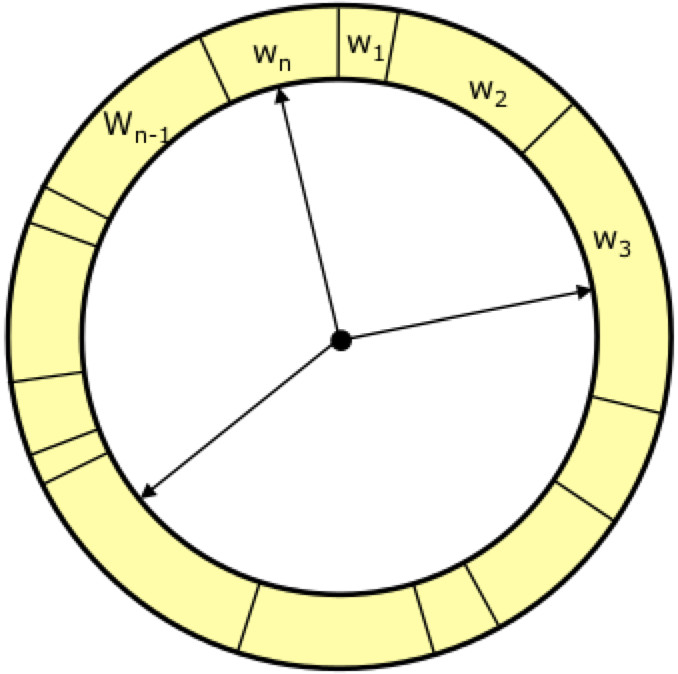
- We can also use the weights to estimate the robot's location (if the distribution is unimodal):

$$\hat{x}(t) = \sum_i w_t^{(i)} \cdot x_t^{(i)}$$

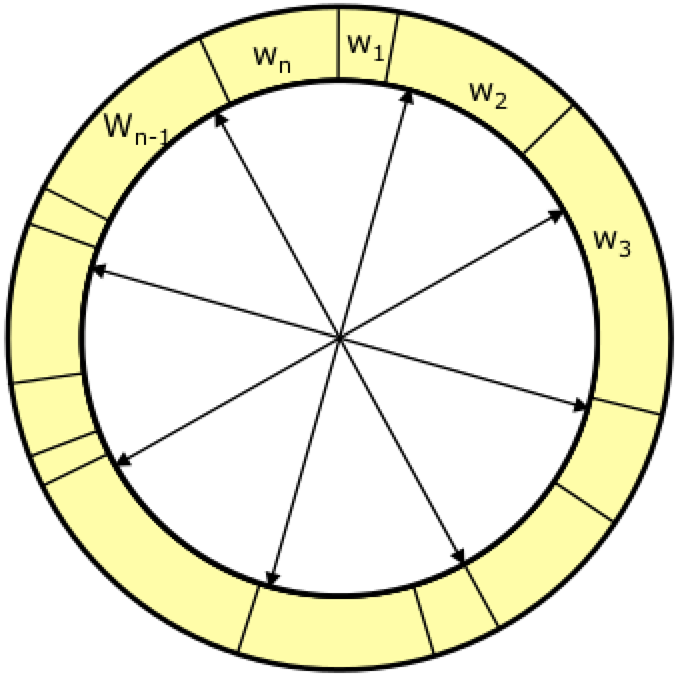
- When to resample?
  - If the variance on the weights is high, then many particles are representing non-useful portions of the space.
  - Resampling redistributes the particles so they are concentrated where the probability density is highest.

How To Resample

- Stochastic universal sampling is a trick for drawing samples from a weighted distribution as fairly as possible (**low variance sampling**).



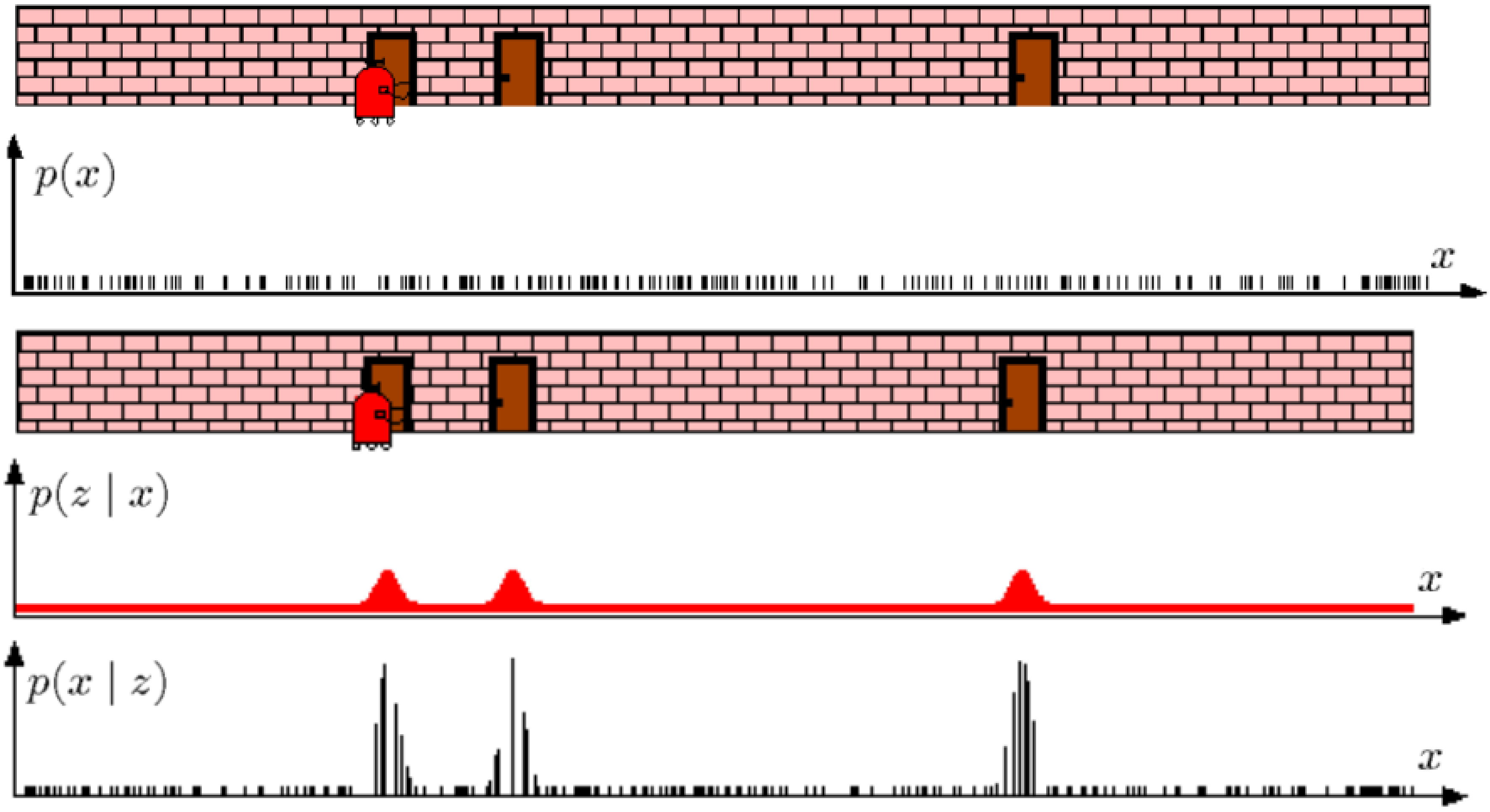
3 samples



8 samples (equal spacing instead of independent sampling lowers the variance)

Image from Burgard et al., "Introduction to Mobile Robotics", 2014, lecture 12: "Bayes Filter – Particle Filter and Monte Carlo Localization".

Weighting in a Corridor



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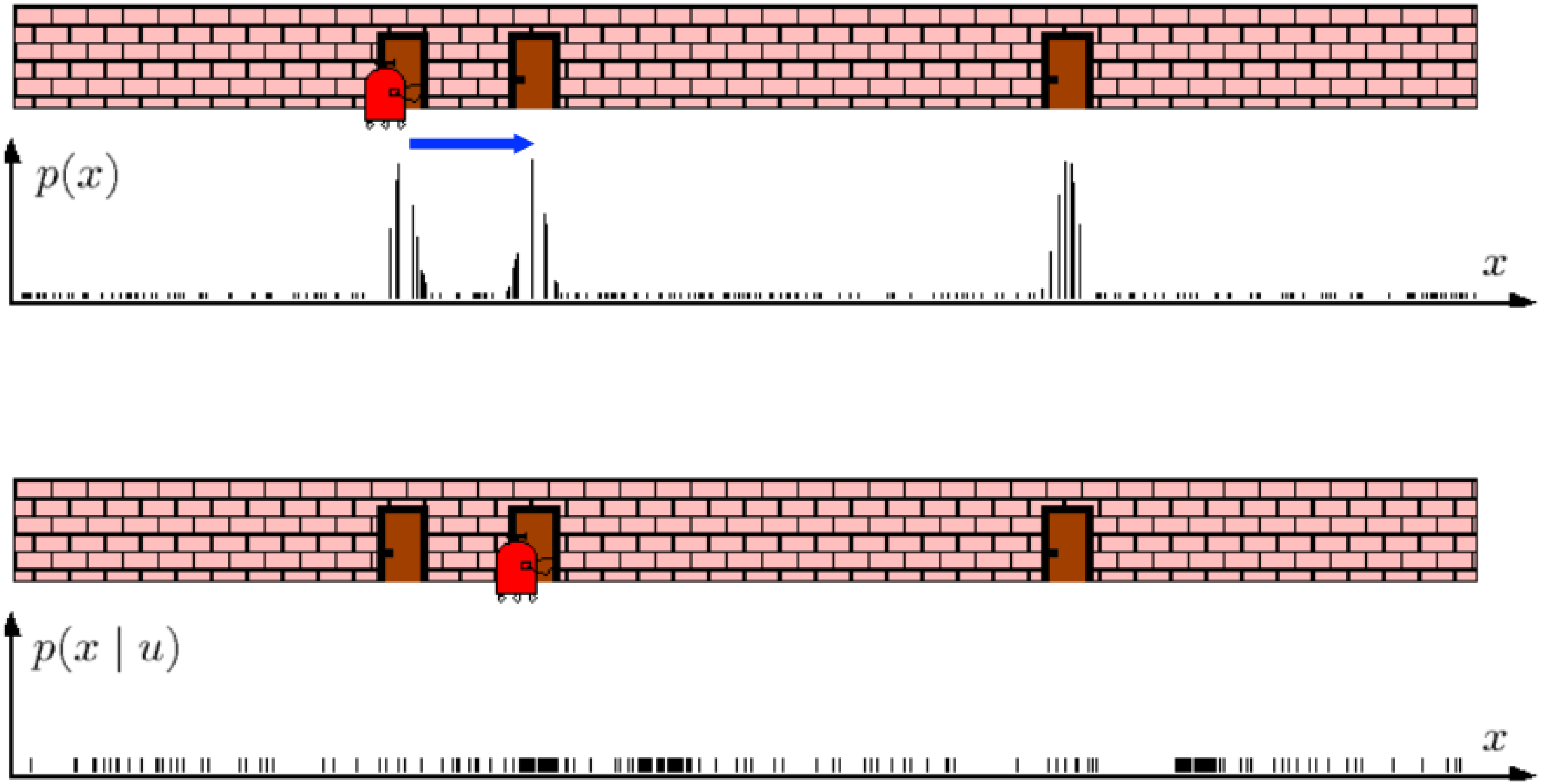
Moving and Then Resampling



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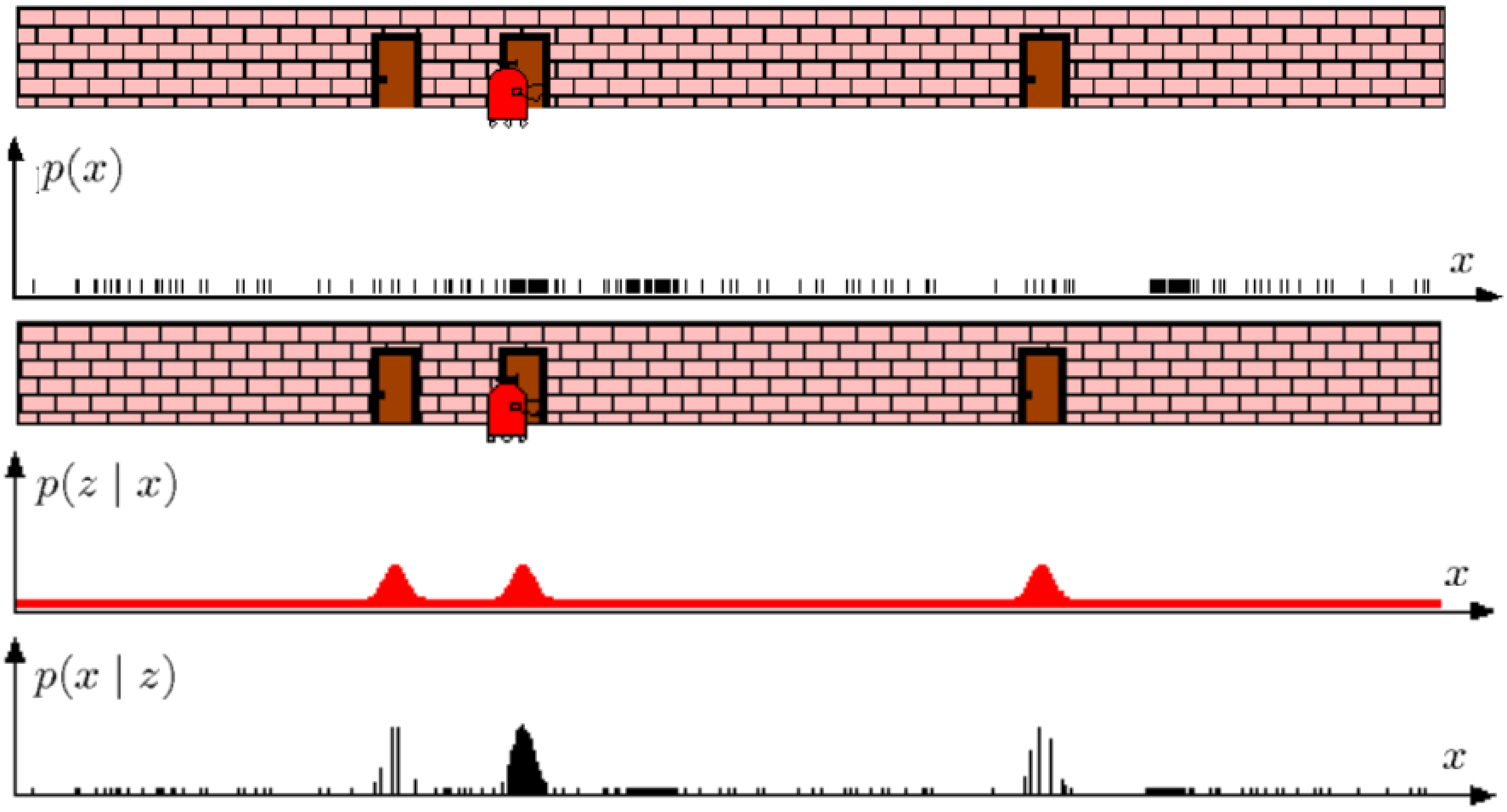
Sensing and Weighting



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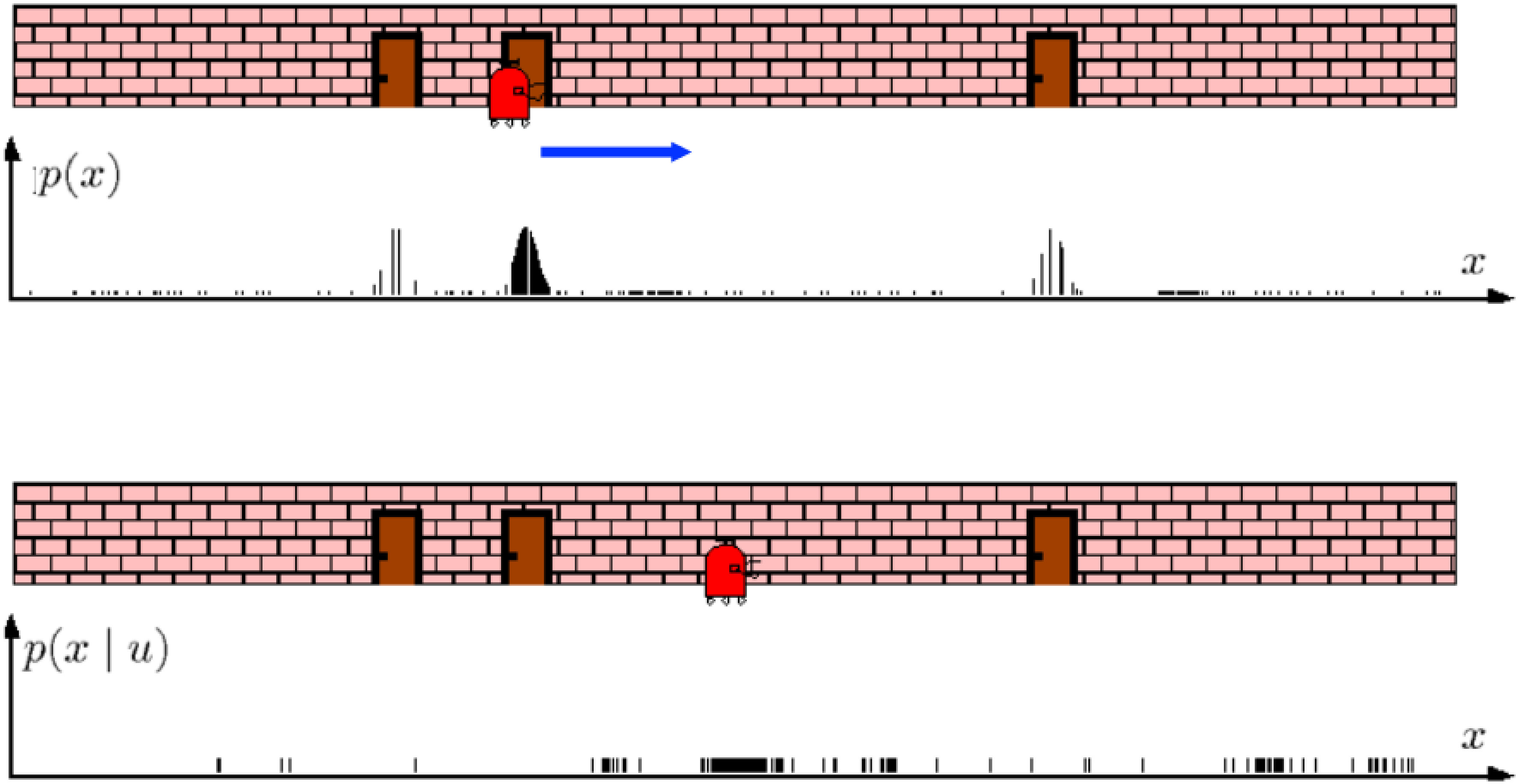
Moving and Then Resampling



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Summary

- Particle filters are the preferred method for robot localization in the real world.
- Robot pose typically encoded as (x,y,θ).
- A map is needed to define how sensor values indicate locations. But what if we don't have a map?
- Particles can be used to represent hypotheses about the map as well as about the robot's location.
  - SLAM: Simultaneous Localization and Mapping.
  - We'll explore this in a later lecture.