CS 740, Fall 1995
Homework Assignment 5

Assigned: Nov. 14
Due: Tues., Nov. 28, 1:00PM

Policy

You may work in a group of up to 3 people in solving the problems for this assignment. However, you must prepare your own writeup, and you must identify your collaborators in your writeup.

You only need to hand in a hard copy document for this assignment. Formatted text is preferred to hand written. If some question arises during grading regarding the correctness of your code for the programming portion, you may be asked to submit an electronic version.

1. Interconnection Networks

Problem 1

Prove that the bisection width of an N-node hypercube H_N is exactly $N/2$ edges.

To prove an upper bound of $N/2$, describe a set of $N/2$ edges whose removal separates the nodes of the hypercube into two sets of $N/2$ nodes with no edges between the two sets.

For many networks a lower bound on the bisection width can be proven using the following technique. Start by embedding the N-node complete graph K_N in H_N in a uniform fashion. The load of the embedding (i.e., maximum number of nodes of K_N mapped to any node of H_N) should be 1, and the congestion on every edge e of H_N (i.e., maximum number of edges of K_N whose paths cross e) should be the same. Complete the proof by arguing that if the bisection width of the hypercube were less than $N/2$, then the embedding would give you a bisection of K_N with too few edges.

Extra credit: Suppose that instead of removing edges, you are asked to bisect the hypercube by removing some of the nodes. After removing the nodes, the remaining nodes of the hypercube should be partitioned into two sets containing the same number of nodes, to within one. Show that removing $O(N/\sqrt{\log N})$ nodes suffices. Hint: in solving this problem it is helpful to consider the number of bits that are 1 in the log N-bit label of each hypercube node.

Problem 2

Show that the butterfly network can be redrawn so that the pattern of edges between any two consecutive levels is identical. You should be able to do this by permuting the order in which the nodes on each level appear in the drawing. Give a formal explanation for why the network in your drawing (which is called an Ω network) is isomorphic to a butterfly, i.e., show how butterfly nodes are mapped to Ω network nodes, and vice-versa.

2. Parallel Programming in NESL

For this section, you will write code in NESL and use the NESL interpreter to try it out. Refer to the SCANDAL project home WWW page (reachable through the "Research Project Home Pages" link from the SCS home page) for code examples, instructions on running the interpreter, and language documentation.

Problem 3: Set Difference

Implement a procedure `set_diff` in NESL, that when given two integer vectors `a` and `b`, returns a vector consisting of those elements in `a` that do not occur in `b`. You can implement this by a variant of the Quicksort algorithm. You *may not* use the NESL's built-in version of this operation.