# Lecture 15

# Register Allocation

I. Introduction
II. Abstraction and the Problem
III. Algorithm

Reading: ALSU 8.8.4

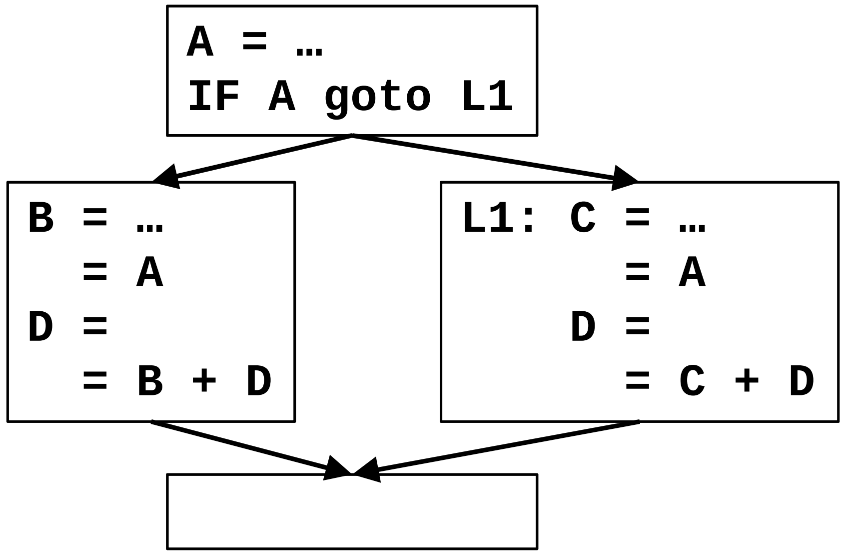
# I. Motivation

- **Problem**
  - Allocation of variables (pseudo-registers) to hardware registers in a procedure
- **Perhaps the most important optimization**
  - Directly reduces running time
    - (memory access → register access)
  - Useful for other optimizations
    - e.g. CSE assumes old values are kept in registers.

# Goals

- Find an allocation for all pseudo-registers, if possible.
- If there are not enough registers in the machine, choose registers to spill to memory

# Example



# II. An Abstraction for Allocation & Assignment

- **Intuitively**
  - Two pseudo-registers **interfere** if at some point in the program they cannot both occupy the same register.
- **Interference graph**: an undirected graph, where
  - nodes = pseudo-registers
  - there is an edge between two nodes if their corresponding pseudo-registers interfere
- **What is not represented**
  - Extent of the interference between uses of different variables
  - Where in the program is the interference

# Register Allocation and Coloring

- A graph is **n-colorable** if:
  - every node in the graph can be colored with one of the n colors such that two adjacent nodes do not have the same color.
- Assigning n register (without spilling) = Coloring with n colors
  - assign a node to a register (color) such that no two adjacent nodes are assigned same registers(colors)
- Is spilling necessary? = Is the graph n-colorable?
- To determine if a graph is n-colorable is NP-complete, for n>2
  - Too expensive
  - Heuristics

# III. Algorithm

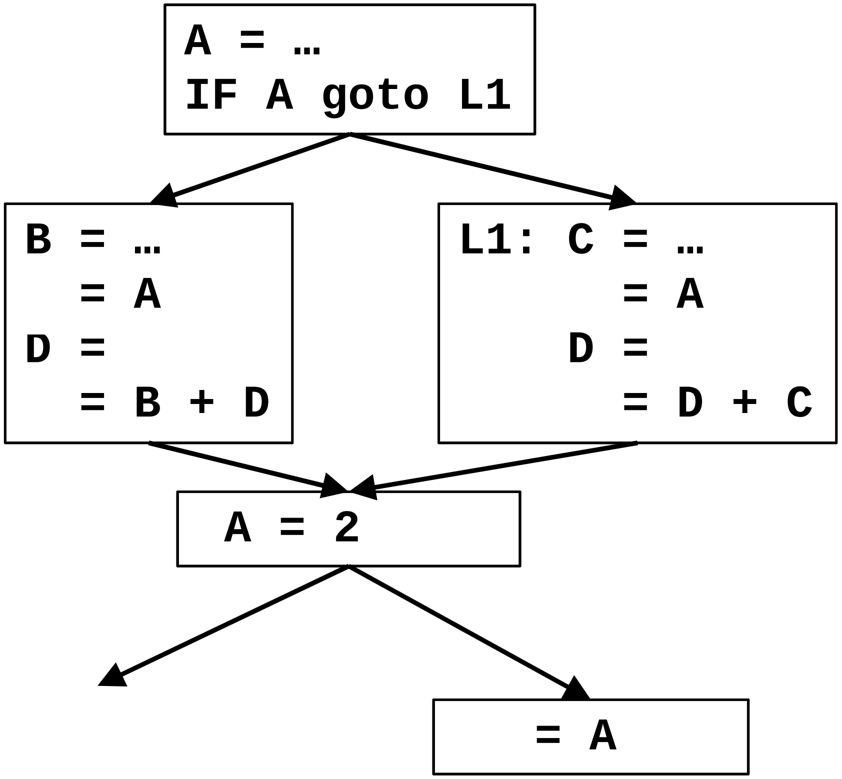
**Step 1. Build an interference graph**

a. refining notion of a node
b. finding the edges

**Step 2. Coloring**

- use heuristics to try to find an n-coloring
  - Success:
    - colorable and we have an assignment
  - Failure:
    - graph not colorable, or
    - graph is colorable, but it is too expensive to color

# Step 1a. Nodes in an Interference Graph

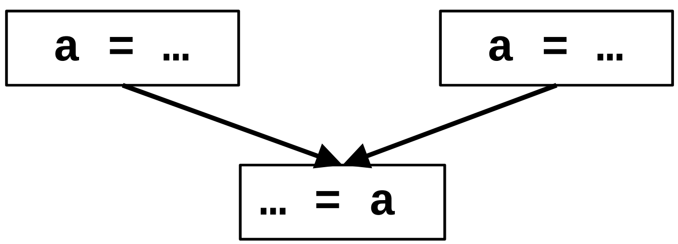


# Live Ranges and Merged Live Ranges

- **Motivation: to create an interference graph that is easier to color**
  - Eliminate interference in a variable's "dead" zones.
  - Increase flexibility in allocation:
    - can allocate same variable to different registers

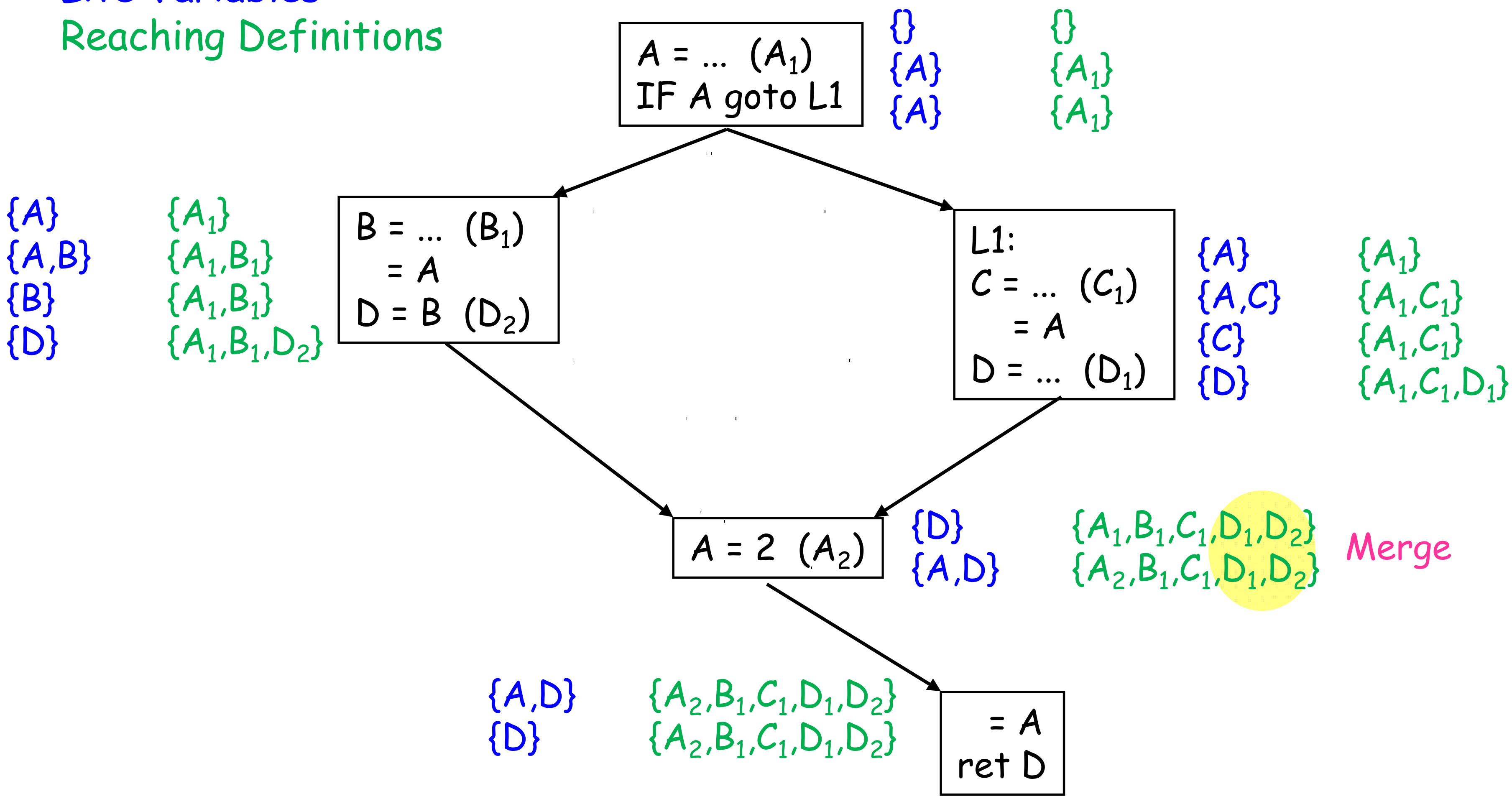
- A **live range** consists of a definition and all the points in a program (e.g. end of an instruction) in which that definition is live.
  - How to compute a live range?

- Two overlapping live ranges for the **same** variable must be merged



# Example (Revisited)

Live Variables
Reaching Definitions



# Merging Live Ranges

- **Merging definitions into equivalence classes**
  - Start by putting each definition in a different equivalence class
  - For each point in a program:
    - if (i) variable is live, and (ii) there are multiple reaching definitions for the variable, then:
      - merge the equivalence classes of all such definitions into one equivalence class

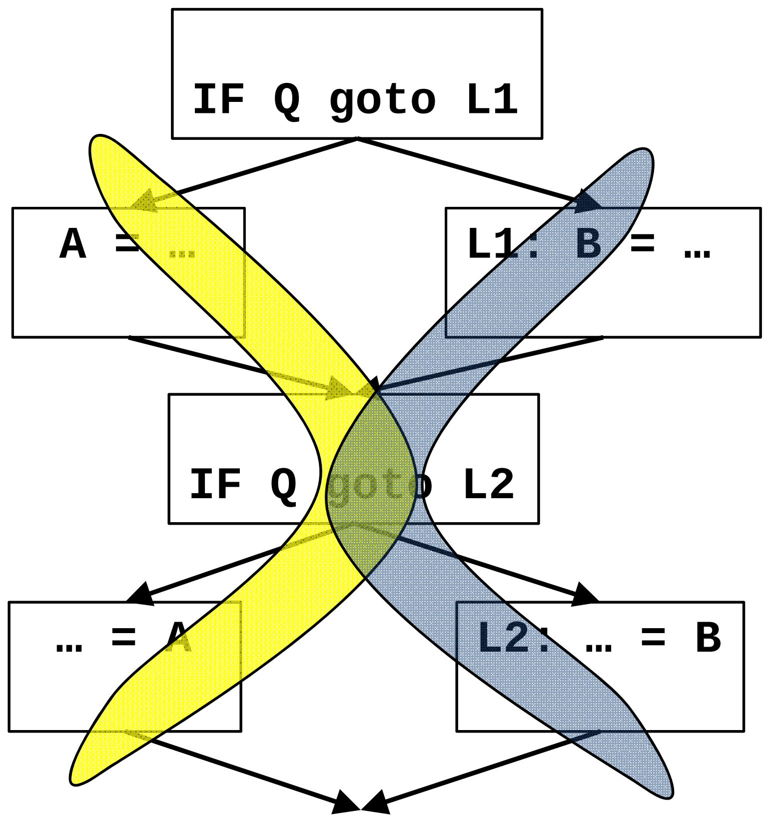
- **From now on, refer to merged live ranges simply as live ranges**
  - merged live ranges are also known as "webs"

# Step 1b. Edges of Interference Graph

- **Intuitively**:
  - Two live ranges (necessarily of different variables) may interfere if they overlap at some point in the program.
  - Algorithm:
    - At each point in the program:
      - enter an edge for every pair of live ranges at that point.

- **An optimized definition & algorithm for edges:**
  - Algorithm:
    - check for interference only at the start of each live range
  - Faster
  - Better quality

# Example 2



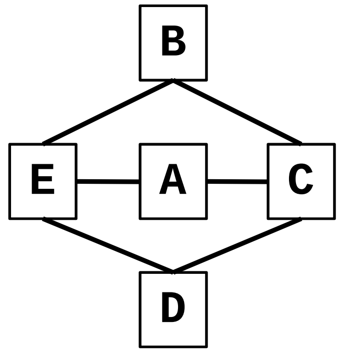
# Step 2. Coloring

- **Reminder:** **coloring for n > 2 is NP-complete**

- **Observations:**
  - a node with degree < n $\Rightarrow$
    - can always color it successfully, given its neighbors' colors
  - a node with degree = n $\Rightarrow$
  - a node with degree > n $\Rightarrow$

# Coloring Algorithm

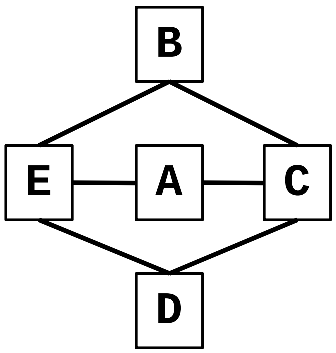
- Algorithm:
  - Iterate until stuck or done
    - Pick any node with degree < n
    - Remove the node and its edges from the graph
  - If done (no nodes left)
    - reverse process and add colors
- Example (n = 3):




- Note: degree of a node may drop in iteration
- Avoids making arbitrary decisions that make coloring fail

# What Does Coloring Accomplish?

- **Done**:
  - colorable, also obtained an assignment
- **Stuck**:
  - colorable or not?



# What if Coloring Fails?

- Use heuristics to improve its chance of success and to spill code

```
Build interference graph

Iterative until there are no nodes left
    If there exists a node v with less than n neighbor
        place v on stack to register allocate
    else
        v = node chosen by heuristics
          (least frequently executed, has many neighbors)
        place v on stack to register allocate (mark as spilled)
    remove v and its edges from graph

While stack is not empty
     Remove v from stack
     Reinsert v and its edges into the graph
     Assign v a color that differs from all its neighbors
     (guaranteed to be possible for nodes not marked as spilled)
```

# Summary

- **Problems:**
  - Given n registers in a machine, is spilling avoided?
  - Find an assignment for all pseudo-registers, whenever possible.
- **Solution:**
  - Abstraction: an **interference graph**
    - nodes: live ranges
    - edges: presence of live range at time of definition
  - Register Allocation and Assignment problems
    - equivalent to **n-colorability** of interference graph
      - ➔ NP-complete
  - Heuristics to find an assignment for n colors
    - successful: colorable, and finds assignment
    - not successful: colorability unknown & no assignment