Lecture 10:

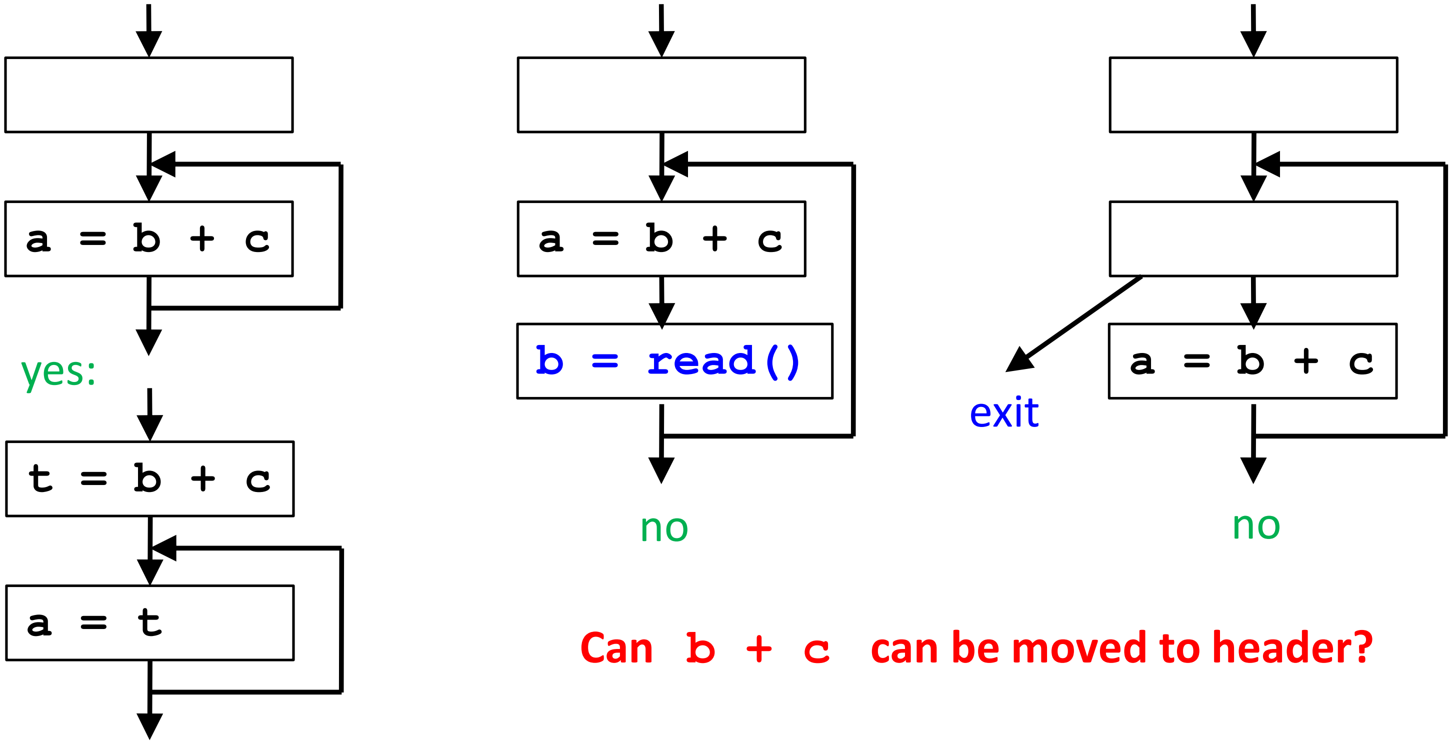
Lazy Code Motion

I. Mathematical concept: a cut set

II. Lazy Code Motion Algorithm
  - Pass 1: Anticipated Expressions
  - Pass 2: (Will be) Available Expressions
  - Pass 3: Postponable Expressions
  - Pass 4: Used Expressions

ALSU 9.5.3-9.5.5

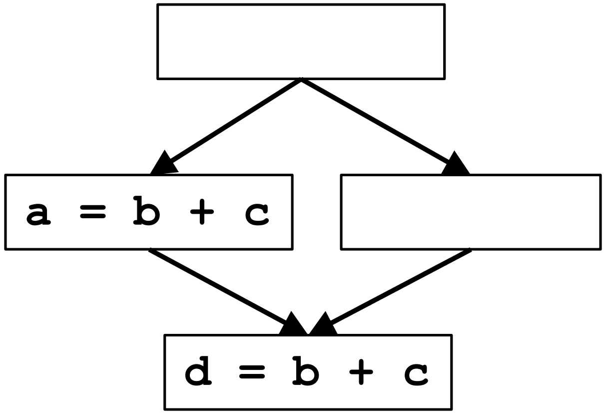
Review: Loop Invariant Code Motion

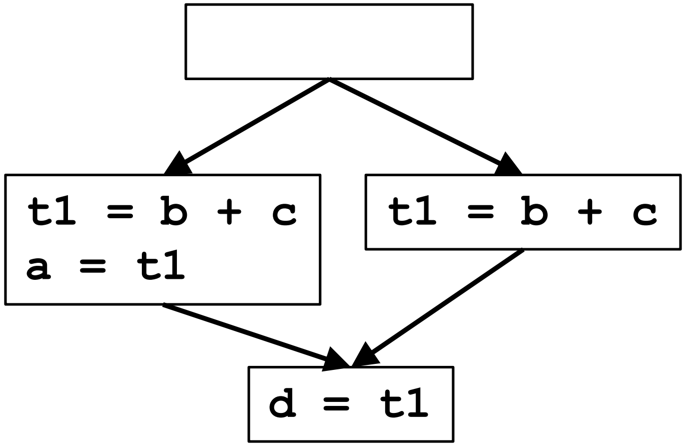


Can `b + c` can be moved to header?

- Given an expression (b+c) inside a loop,
  - does the value of b+c change inside the loop?
  - is the code executed at least once?

Review: Partial Redundancy Elimination



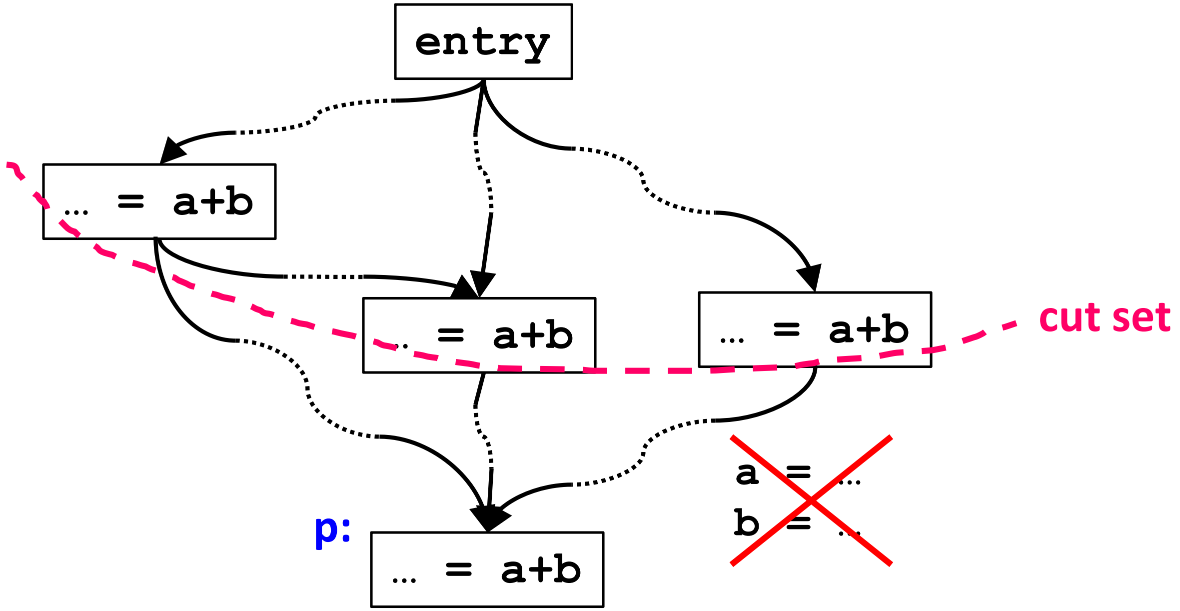


- Can we place calculations of b+c
  such that no path re-executes the same expression?

- Partial Redundancy Elimination (PRE)
  - subsumes:
    - global common subexpression (full redundancy)
    - loop invariant code motion (partial redundancy for loops)

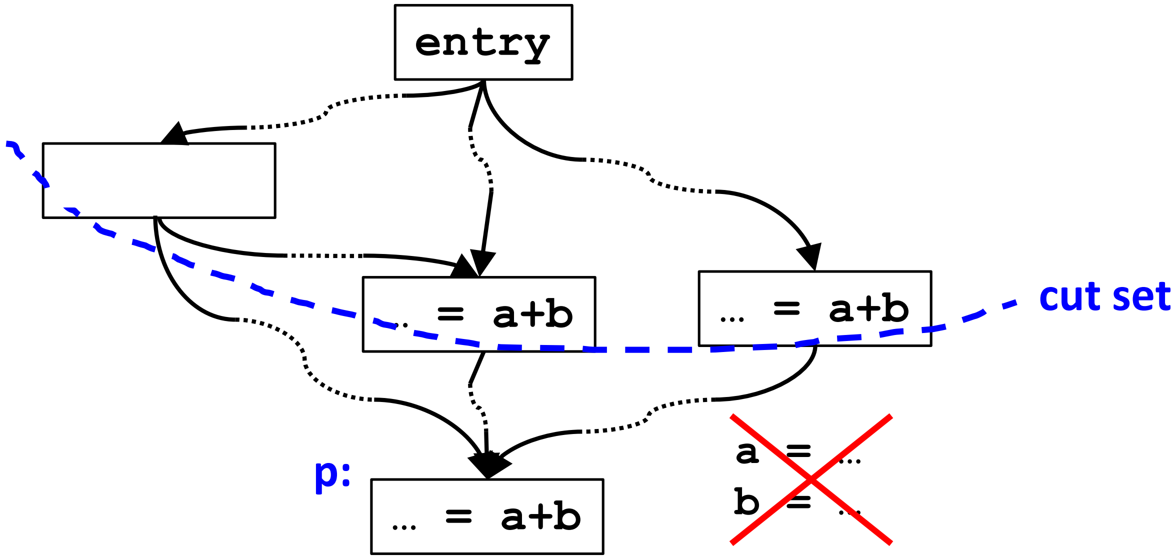
I. Full Redundancy: A Cut Set in a Graph

Key mathematical concept



- **Full redundancy at p: expression a+b redundant on all paths**
  - a cut set: nodes that separate entry from p (there can be many cut sets)
  - each node in a cut set contains a calculation of a+b
  - a, b not redefined

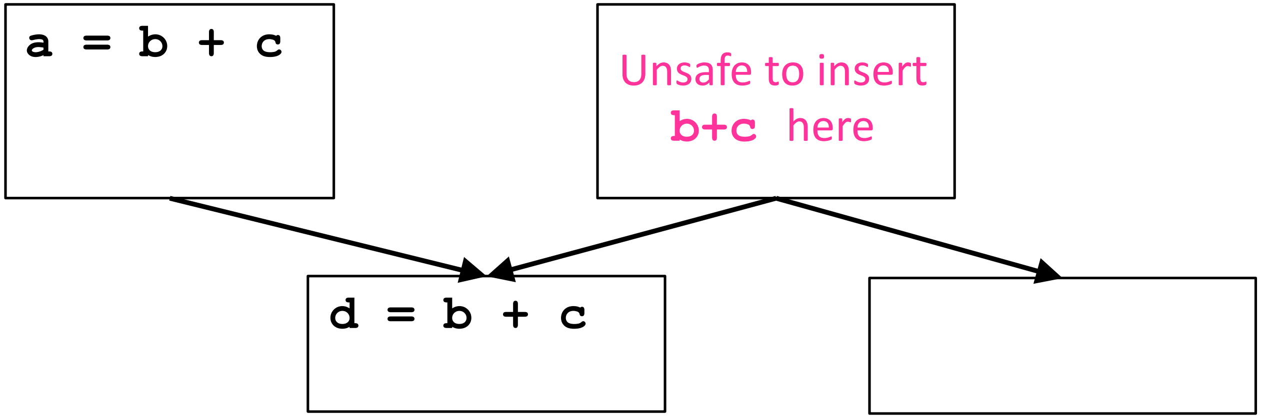
Partial Redundancy: Completing a Cut Set



- **Partial redundancy at p: redundant on some but not all paths**
  - Add operations to create a cut set containing a+b
  - Note: Moving operations up can eliminate redundancy
- **Constraint on placement: no wasted operation**
  - a+b is "anticipated" at B if its value computed at B will be used along ALL subsequent paths
  - a, b not redefined, no branches that lead to exit without use
- **Range where a+b is anticipated → Choice**

Review: Where Can We Insert Computations?

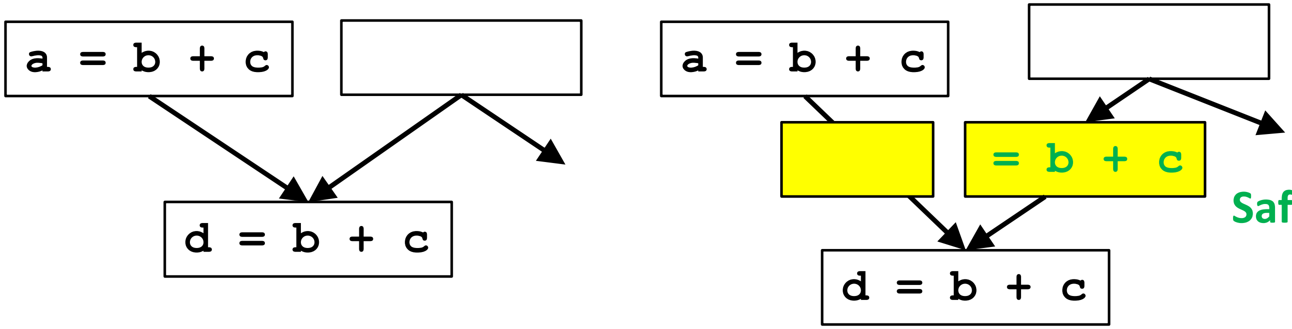
- **Safety: never introduce a new expression along any path.**



  - Insertion could introduce exception, change program behavior.
  - Solution: insert expression only where it is **anticipated**, i.e., its value computed at point p will be used along ALL subsequent paths

- **Performance: never increase the # of computations on any path.**
  - Under simple model, guarantees program won't get worse.
  - Reality: might increase register lifetimes, add copies, lose.

Preparing the Flow Graph



- **Definition: Critical edges**
  - source basic block has multiple successors
  - destination basic block has multiple predecessors
- **Modify the flow graph:**
  - Add a basic block for every edge that leads to a basic block with multiple predecessors (not just on critical edges)
    - How does this help the example?
  - To keep algorithm simple: consider each statement as its own basic block and restrict placement of instructions to the beginning of a basic block

II. Lazy Code Motion Algorithm

- Pass 1: Anticipated Expressions
- Pass 2: (Will be) Available Expressions
- Pass 3: Postponable Expressions
- Pass 4: Used Expressions

Big picture:

- First calculates the “earliest” set of blocks for insertion
  - this maximizes redundancy elimination
  - but may also result in long register lifetimes
- Then it calculates the “latest” set of blocks for insertion
  - achieving the same amount of redundancy elimination as “earliest”
  - but hopefully reducing the lifetime of the register holding the value of the expression

Pass 1: Anticipated Expressions

This pass does most of the heavy lifting in eliminating redundancy

- **Backward pass: Anticipated expressions**
  Anticipated[b].in: Set of expressions anticipated at the entry of b
  - An expression is anticipated if its value computed at point p will be used along ALL subsequent paths

	Anticipated Expressions
Domain	Sets of expressions
Direction	
Transfer Function	
$\wedge$	
Boundary	in[exit] =
Initialization	in[b] =

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	Anticipated Expressions
Domain	Sets of expressions
Direction	backward
Transfer Function	$f_b(x) = EUse_b \cup (x - EKill_b)$ EUse: used exp, EKill: exp killed
$\wedge$	$\cap$
Boundary	in[exit] = $\emptyset$
Initialization	in[b] = {all expressions}

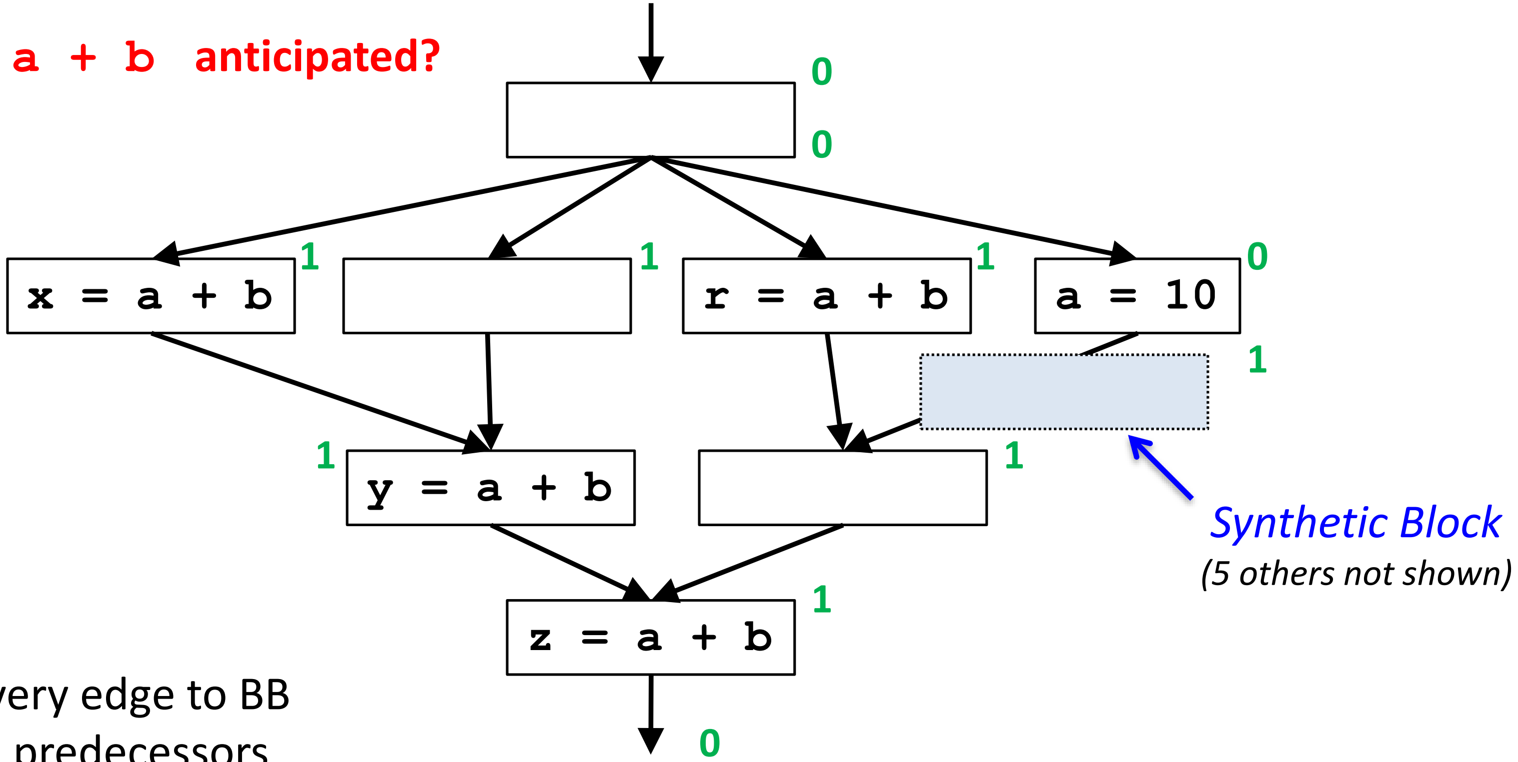
- **First approximation:**
  - place operations at the frontier of anticipation
    (boundary between not anticipated and anticipated)

Example 1

See the algorithm in action

IN[i] = EUse[i] ∪ (OUT[i] - EKill[i])
Meet = ∩

Where is `a + b` anticipated?



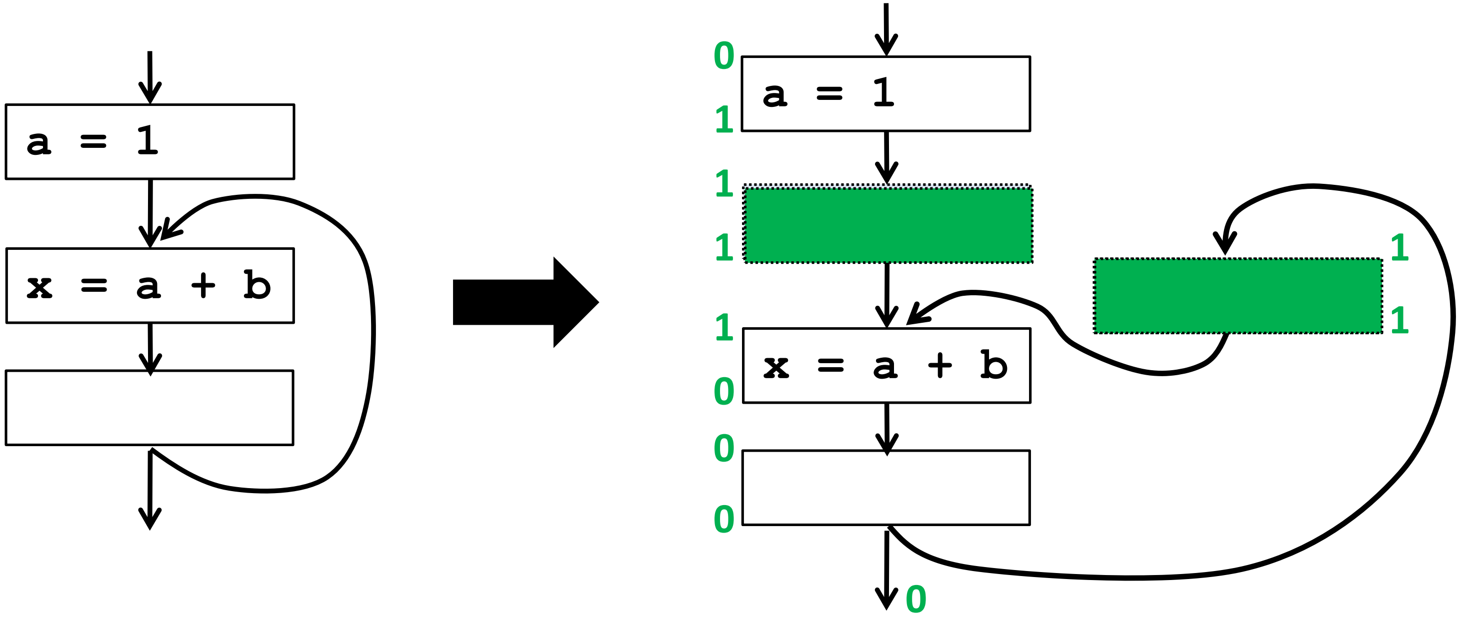
Add BB for every edge to BB with multiple predecessors

- What is the result if we insert `t = a + b` at the frontier of anticipation?

Example 2

(Loop Invariant Code)

IN[i] = EUse[i] ∪ (OUT[i] - EKill[i])
Meet = ∩



Add BB for every edge to BB
with multiple predecessors

Where is `a + b` anticipated?

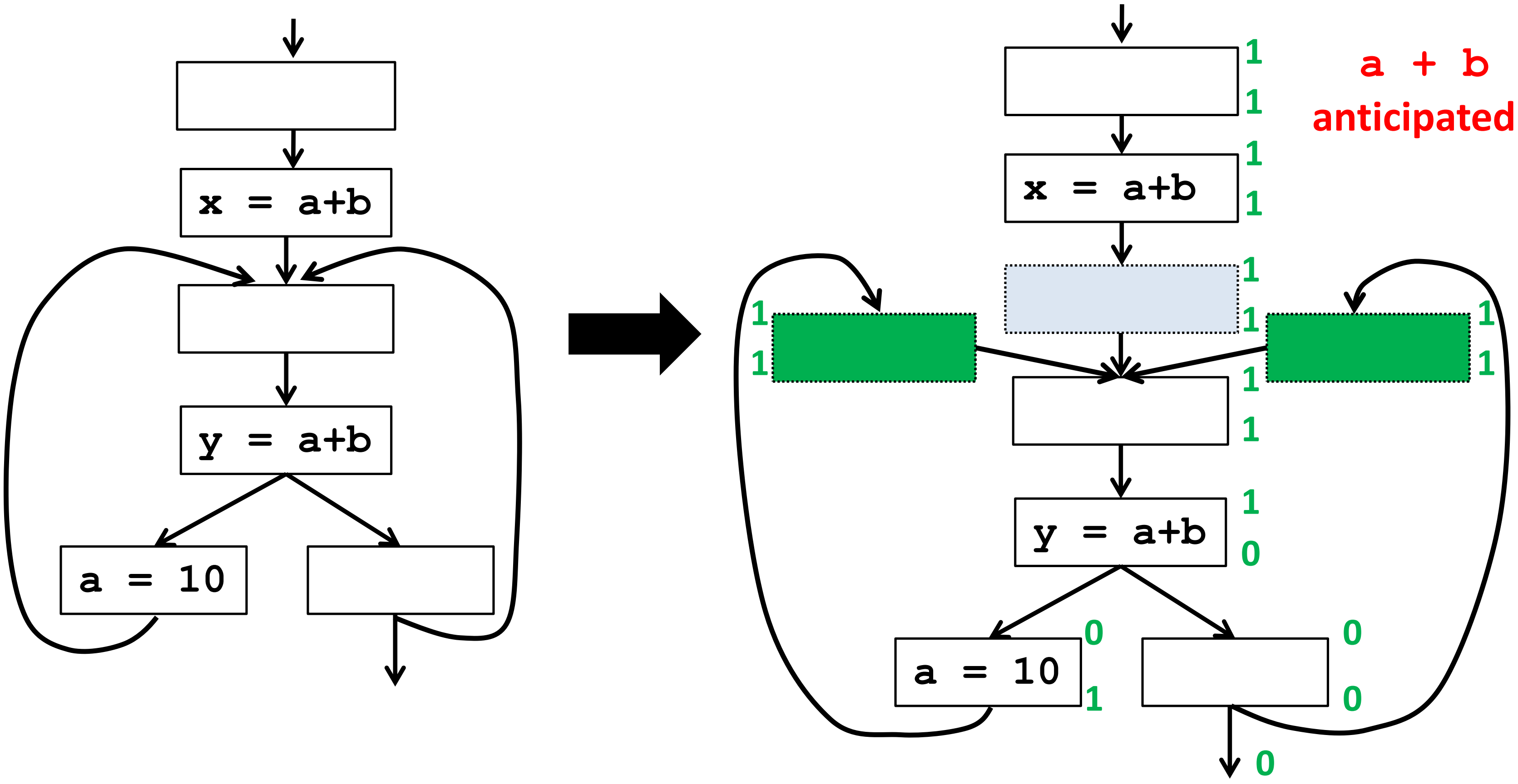
- Which blocks comprise the frontier of anticipation?
- Was inserting `a + b` at the frontier of anticipation the right thing to do in this case?
  - **doesn't eliminate redundancy within loop (why not?)**

Example 3

(More Complex Loop)

$IN[i] = EUse[i] \cup (OUT[i] - EKill[i])$

Meet = $\cap$



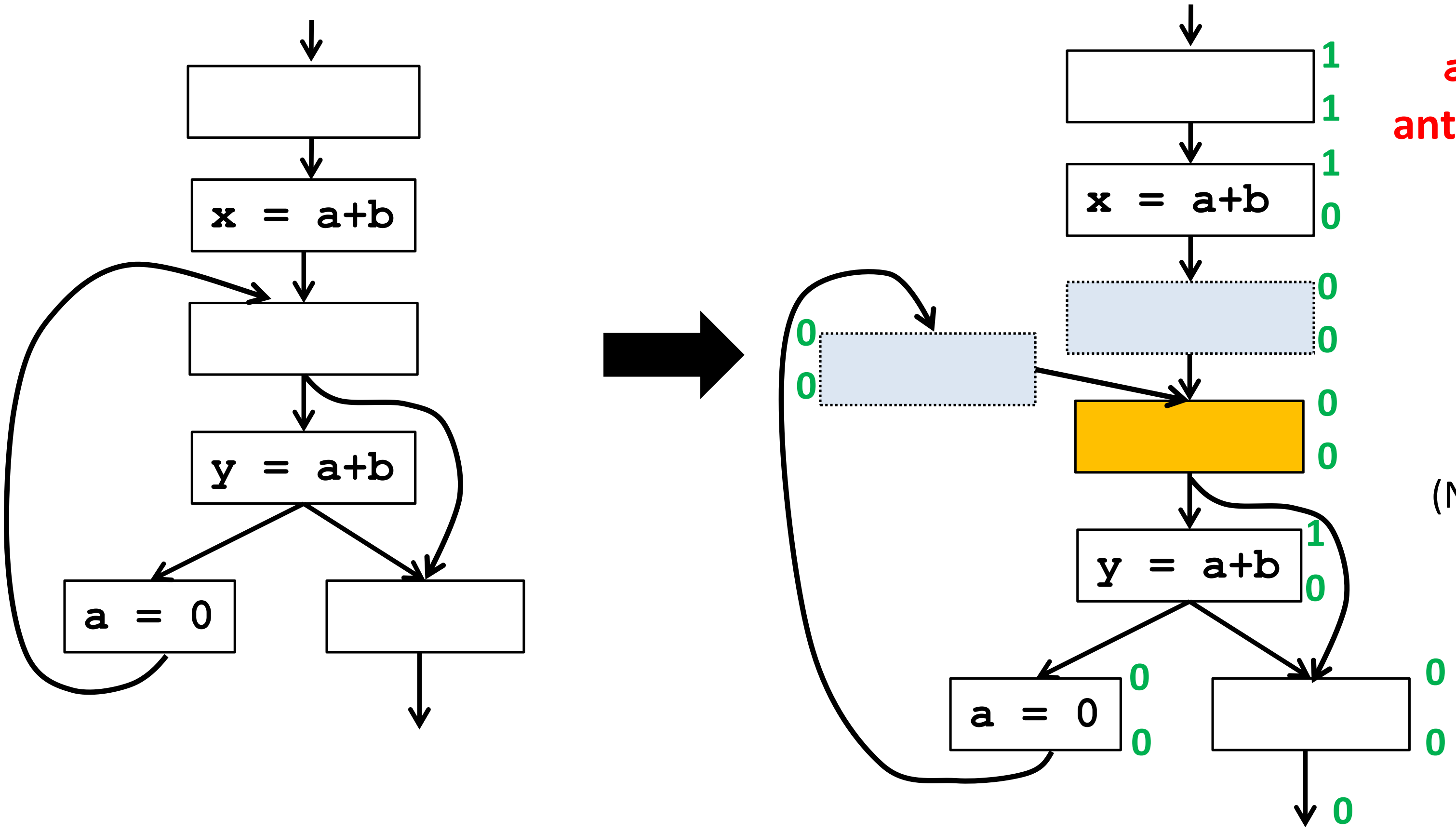
a + b anticipated?

- Where would we ideally like to insert “**a+b**” in this case? only in added block on left
- What happens if we insert at the frontier of anticipation? insert in both green blocks

Example 4

(Variation on Previous Loop)

IN[i] = EUse[i] ∪ (OUT[i] - EKill[i])
Meet = ∩



a + b anticipated?

(Not shown: 2 more synthetic blocks)

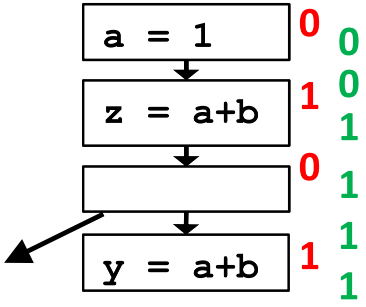
- Is there any opportunity to eliminate redundancy here?

  no: unsafe to insert in left added block (a+b not anticipated there)
  (e.g. “a+b” could be “b/a” & orange block could be “if a > 0”)

Pass 2: Place As Early As Possible

There is still some redundancy left!

- **First approximation: frontier between "not anticipated" & "anticipated"**
- **Complication: anticipation may oscillate**



- Pretend we calculate expression e whenever it is anticipated
- e **will be available at p** if e has been "anticipated but not subsequently killed" on all paths reaching p

	(will be) Available Expressions
Domain	Sets of expressions
Direction	forward
Transfer Function	$f_b(x) = (\text{Anticipated}[b].\text{in} \cup x) - \text{EKill}_b$
$\wedge$	$\cap$
Boundary	out[entry] = $\varnothing$
Initialization	out[b] = {all expressions}

Early Placement

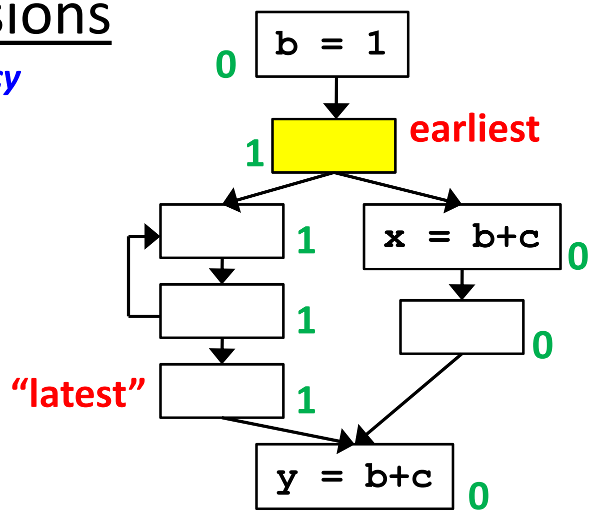
- **earliest(b)**
  - set of expressions added to block b under early placement
  - calculated from results of first 2 passes
- **Place expression at the earliest point anticipated and not already available**
  - earliest(b) = anticipated[b].in – available[b].in
- **Algorithm**
  - For all basic block b, if x+y ∊ earliest[b]
    - at beginning of b:
      create a new variable t
      t = x+y,
      replace every original x+y by t

Result:

- Maximized redundancy elimination
- Placed as early as possible
- But: register lifetimes?

Pass 3: Postponable Expressions

Let's be lazy without introducing redundancy



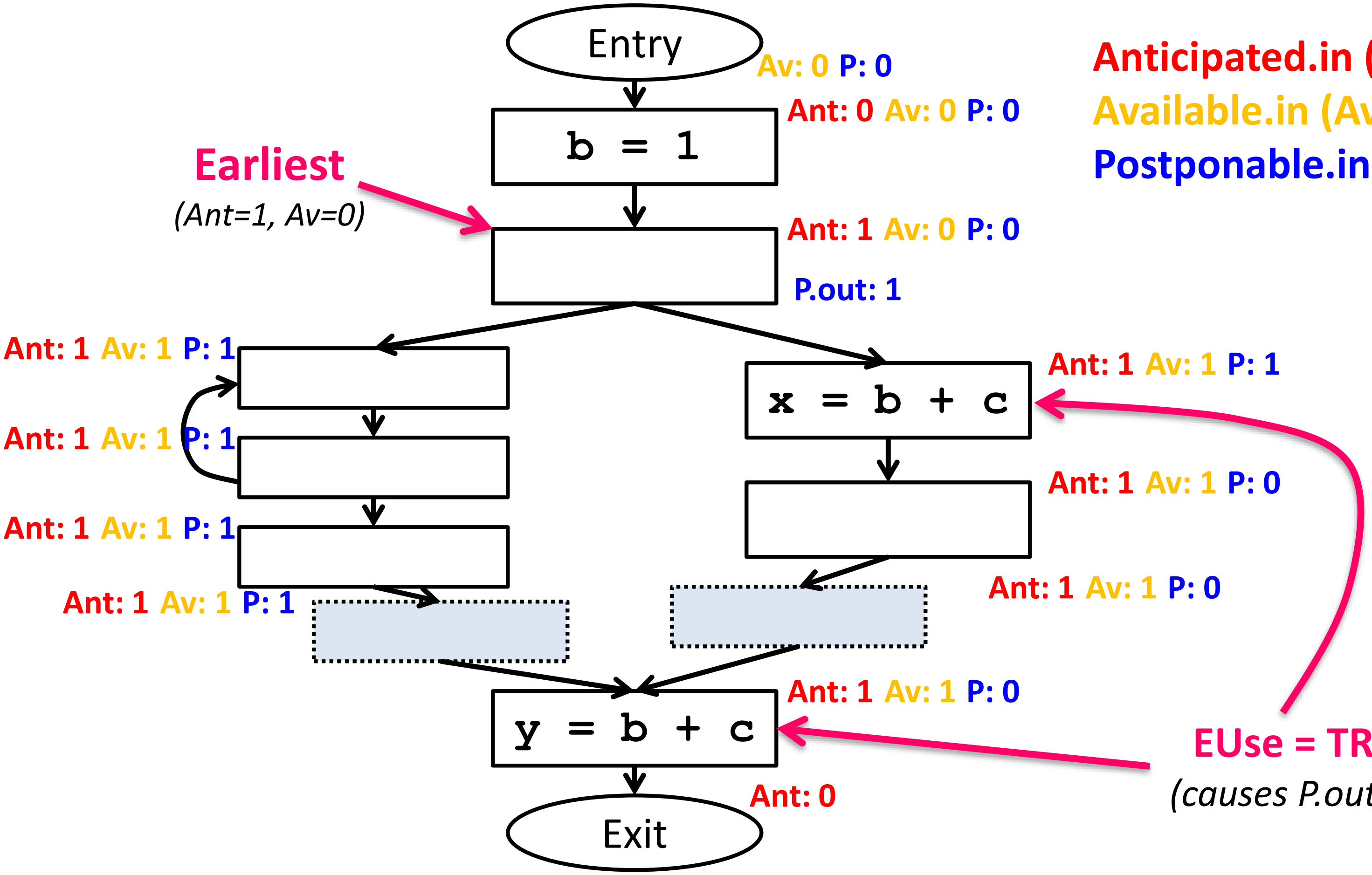
- **Delay creating redundancy to reduce register pressure**

- **An expression e is postponable at a program point p if**
  - all paths leading to p have seen earliest placement of e but not a subsequent use

	Postponable Expressions
Domain	Sets of expressions
Direction	forward
Transfer Function	$f_b(x) = (\text{earliest}[b] \cup x) - \text{EUse}_b$
$\wedge$	$\cap$
Boundary	out[entry] = $\emptyset$
Initialization	out[b] = {all expressions}

Example Illustrating “Postponable”

Anticipated.in (Ant)
Available.in (Av)
Postponable.in (P)



Entry

Av: 0 P: 0

`b = 1`

Ant: 0 Av: 0 P: 0

Earliest
(Ant=1, Av=0)

Ant: 1 Av: 0 P: 0

P.out: 1

Ant: 1 Av: 1 P: 1

Ant: 1 Av: 1 P: 1

Ant: 1 Av: 1 P: 1

Ant: 1 Av: 1 P: 1

`x = b + c`

Ant: 1 Av: 1 P: 1

Ant: 1 Av: 1 P: 0

Ant: 1 Av: 1 P: 0

`y = b + c`

Ant: 1 Av: 1 P: 0

Ant: 0

Exit

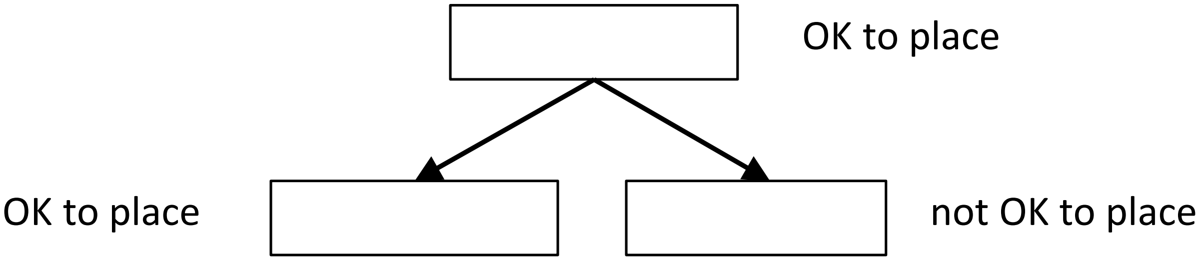
EUse = TRUE
(causes P.out = 0)

Ant.IN[i] = EUse[i] ∪ (Ant.OUT[i]-EKill[i])
Avail.OUT[i] = (Ant.IN[i] ∪ Avail.IN[i])-EKill[i]

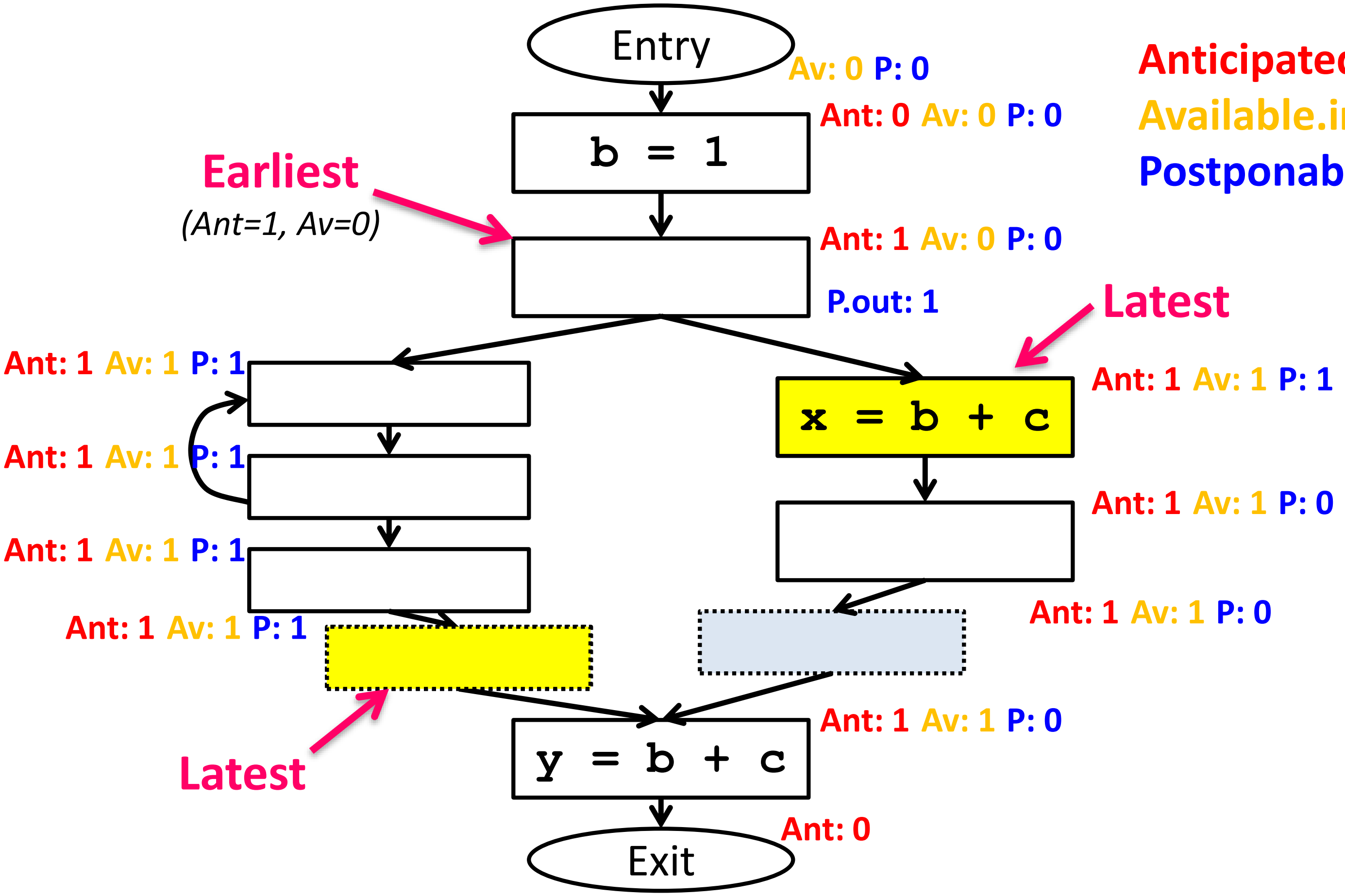
Post.OUT[i] = (Earliest[i] ∪ Post.IN[i])-EUse[i]

Latest: frontier at the end of "postponable" cut set

- $\text{latest}[b] = (\text{earliest}[b] \cup \text{postponable.in}[b]) \cap$
  $(\text{EUse}_b \cup \neg(\bigcap_{s \in \text{succ}[b]}(\text{earliest}[s] \cup \text{postponable.in}[s])))$
  - OK to place expression: earliest or postponable
  - Need to place at b if either
    - used in b, or
    - not OK to place in one of its successors
- Works because of pre-processing step (an empty block was introduced to an edge if the destination has multiple predecessors)
  - if b has a successor that cannot accept postponement, b has only one successor
  - The following does not exist:



Example Illustrating "Latest"



Entry

Av: 0 P: 0

b = 1

Ant: 0 Av: 0 P: 0

Earliest

(Ant=1, Av=0)

Ant: 1 Av: 0 P: 0

P.out: 1

Latest

Ant: 1 Av: 1 P: 1

x = b + c

Ant: 1 Av: 1 P: 1

Ant: 1 Av: 1 P: 0

Ant: 1 Av: 1 P: 1

Ant: 1 Av: 1 P: 1

Ant: 1 Av: 1 P: 1

Ant: 1 Av: 1 P: 0

Ant: 1 Av: 1 P: 0

y = b + c

Latest

Ant: 0

Exit

Anticipated.in (Ant)

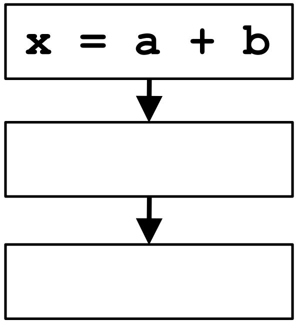
Available.in (Av)

Postponable.in (P)

- latest[b] = (earliest[b] $\cup$ postponable.in[b]) $\cap$ (EUse$_b$ $\cup \neg(\bigcap_{s \in succ[b]}$(earliest[s] $\cup$ postponable.in[s])))

Pass 4: Used Expressions

Finally… this is easy, it is like liveness (for expressions)



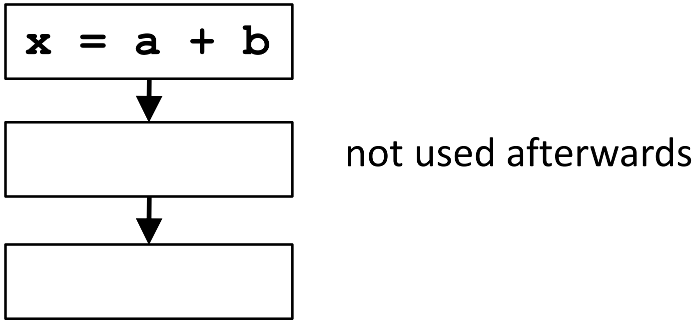
not used afterwards

- **Eliminate temporary variable assignments unused beyond current block**
- **Compute: Used.out[b]: sets of used (live) expressions at exit of b.**

	Used Expressions
Domain	Sets of expressions
Direction	
Transfer Function	
∧	
Boundary	in[exit] =
Initialization	in[b] =

Pass 4: Used Expressions

Finally… this is easy, it is like liveness (for expressions)



- **Eliminate temporary variable assignments unused beyond current block**
- **Compute: Used.out[b]: sets of used (live) expressions at exit of b.**

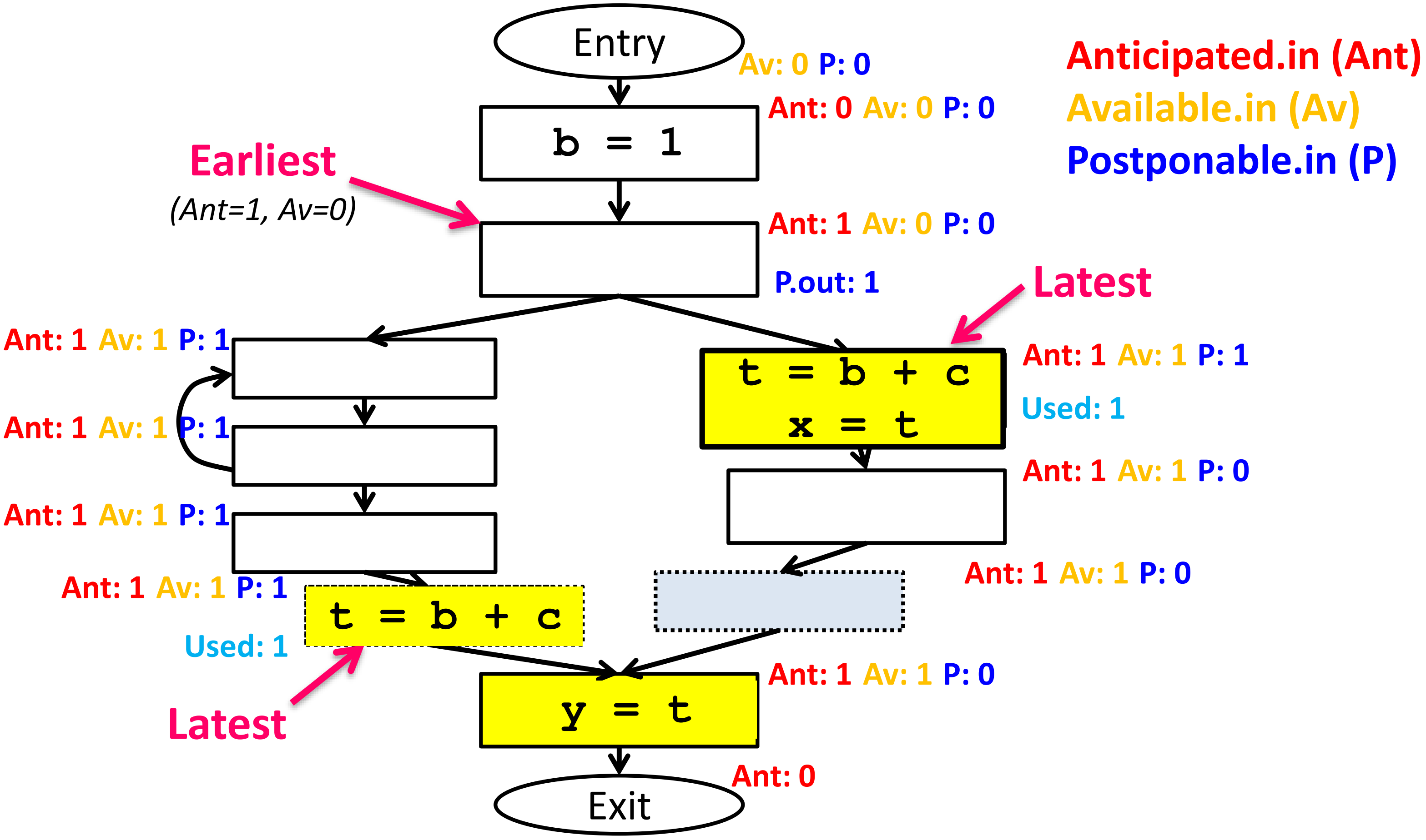
	Used Expressions
Domain	Sets of expressions
Direction	backward
Transfer Function	$f_b(x) = (EUse[b] \cup x) - latest[b]$
$\wedge$	$\cup$
Boundary	$in[exit] = \emptyset$
Initialization	$in[b] = \emptyset$

Code Transformation

- For all basic blocks b,

```
if (x+y) ∈ (latest[b] ∩ used.out[b])
    at beginning of b:
    add new t = x+y
    replace every original x+y by t
```

Transformed Code



Anticipated.in (Ant)
Available.in (Av)
Postponable.in (P)

If (x+y) ∈ (latest[b] ∩ used.out[b]) then add t = x+y. Replace every original x+y by t

4 Passes for Partial Redundancy Elimination

1. ***Safety*: Cannot introduce operations not executed originally**
   - Pass 1 (backward): Anticipation: range of code motion
   - Placing operations at the frontier of anticipation gets most of the redundancy
2. ***Squeezing the last drop of redundancy*:**
   An anticipation frontier may cover a subsequent frontier
   - Pass 2 (forward): Availability
   - Earliest: anticipated, but not yet available
3. ***Push the cut set out -- as late as possible***
   To minimize register lifetimes
   - Pass 3 (forward): Postponability: move it down provided it does not create redundancy
   - Latest: where it is used or the frontier of postponability
4. ***Cleaning up***
   - Pass 4 (backward): **Remove unneeded temporary assignments**

Remarks

- **Powerful algorithm**
  - Finds many forms of redundancy in one unified framework
- **Illustrates the power of data flow**
  - Multiple data flow problems

Today's Class

1. Mathematical concept: a cut set
2. Lazy Code Motion Algorithm
    - Pass 1: Anticipated Expressions
    - Pass 2: (Will be) Available Expressions
    - Pass 3: Postponable Expressions
    - Pass 4: Used Expressions

Monday's Class

- Static Single Assignment
    - ALSU 6.2.4