Lecture 15:

Memory Hierarchy Optimizations

I. Caches: A Quick Review
II. Iteration Space & Loop Transformations
III. Intro to Locality Analysis

ALSU 7.4.2-7.4.3, 11.2-11.5.1

I. Caches: A Quick Review

- How do they work?
- Why do we care about them?
- What are typical configurations today?
- What are some important cache parameters that will affect performance?

Optimizing Cache Performance

- Things to enhance:
  - temporal locality
  - spatial locality
- Things to minimize:
  - conflicts (i.e. bad replacement decisions)

What can the *compiler* do to help?

Two Things We Can Manipulate

- Time:
  - When is an object accessed?
- Space:
  - Where does an object exist in the address space?

How do we exploit these two levers?

Time: Reordering Computation

- What makes it difficult to know *when* an object is accessed?
- How can we predict a better time to access it?
  - What information is needed?
- How do we know that this would be safe?

Space: Changing Data Layout

- What do we know about an object's location?
  - scalars, structures, pointer-based data structures, arrays, code, etc.
- How can we tell what a better layout would be?
  - how many can we create?
- To what extent can we safely alter the layout?

Types of Objects to Consider

- Scalars
- Structures & Pointers
- Arrays

Scalars

- Locals
- Globals
- Procedure arguments

```
int x;
double y;
foo(int a){
  int i;
  …
  x = a*i;
  …
}
```

- Is cache performance a concern here?
- If so, what can be done?

Structures and Pointers

- What can we do here?
  - within a node
  - across nodes

```
struct {
   int count;
   double velocity;
   double inertia;
   struct node *neighbors[N];
} node;
```

Example: Can rearrange field order to improve cache performance

- What limits the compiler’s ability to optimize here?

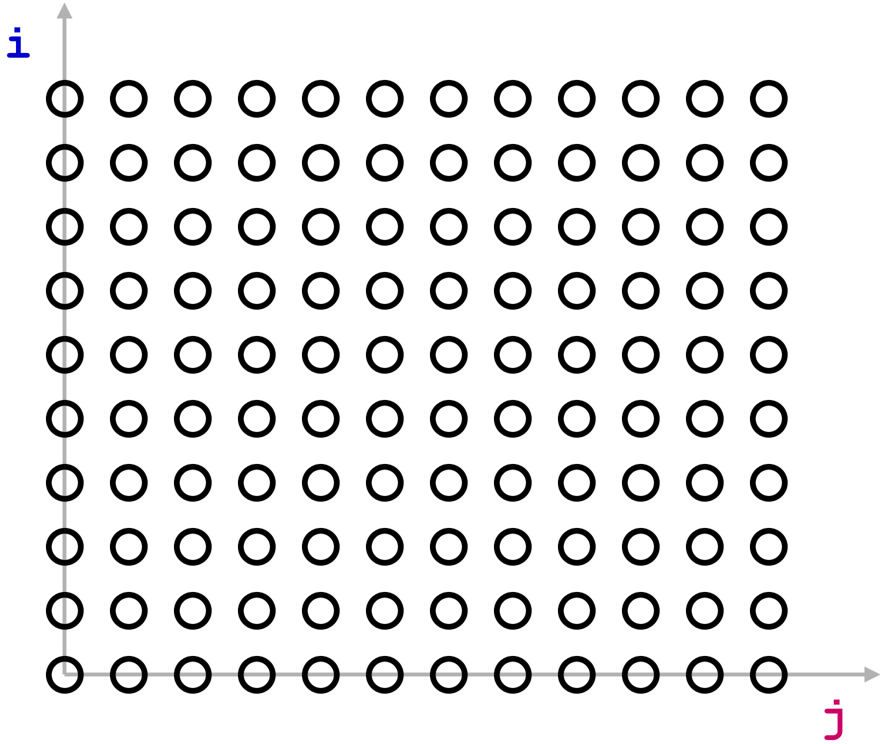
Arrays / Matrices

```
double A[N][N], B[N][N];
…
for i = 0 to N-1
   for j = 0 to N-1
      A[i][j] = B[j][i];
```

- usually accessed within loops nests
  - makes it easy to understand "time"
- what we know about array element addresses:
  - start of array?
  - relative position within array

II. Iteration Space and Loop Transformations

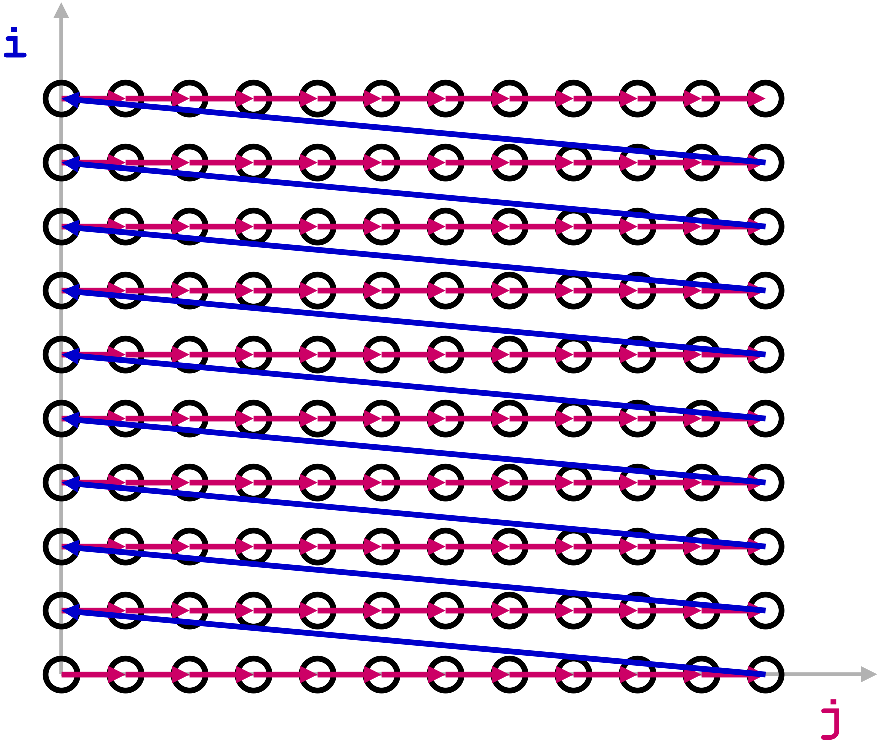
```
for i = 0 to N-1
   for j = 0 to N-1
      A[i][j] = B[j][i];
```



- each position represents an iteration (not an array element)

Visitation Order in Iteration Space

```
for i = 0 to N-1
  for j = 0 to N-1
    A[i][j] = B[j][i];
```



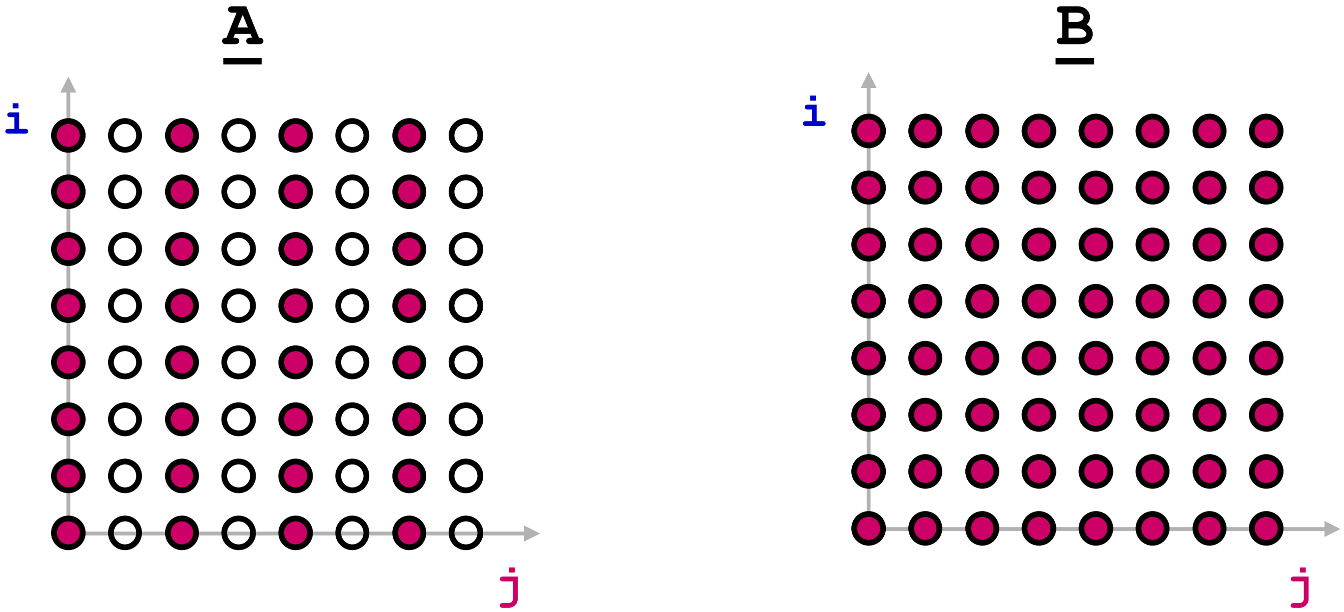
- Note: iteration space ≠ data space

When Do Cache Misses Occur?

```
for i = 0 to N-1
  for j = 0 to N-1
    A[i][j] = B[j][i];
```

○ Hit

● Miss



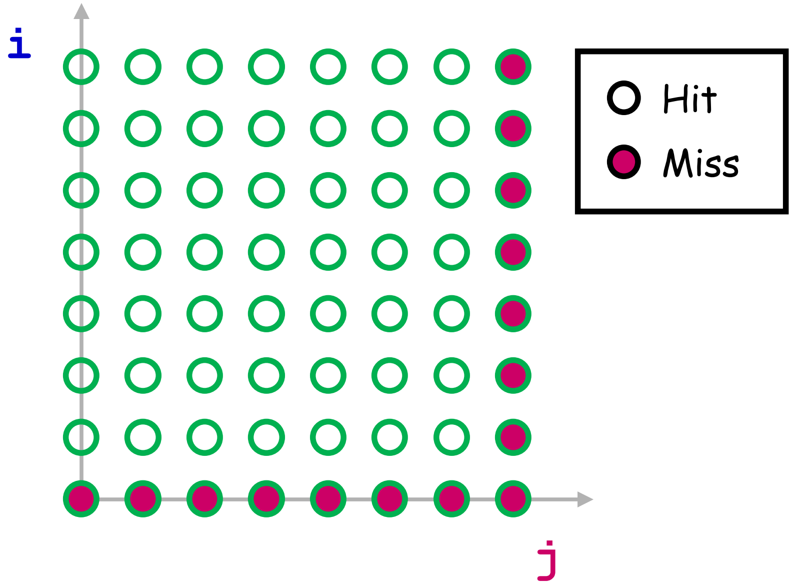
Assume row major order, N large, 2 elements per cache line

Row major layout: A[0][0] A[0][1]…A[0][N-1] A[1][0] A[1][1]…A[1][N-1] A[2][0]…

When Do Cache Misses Occur?

```
double A[2N-1][N];

for i = 0 to N-1
   for j = 0 to N-1
      A[i+j][0] = i*j;
```



Assume row major order, 2 elements per cache line

Row major layout of A:

A[0][0] A[0][1]…A[0][N-1] A[1][0]…A[1][N-1]…A[2N-2][0]…A[2N-2][N-1]

If N large then all misses. What if N is small? see above

Types of Data Reuse/Locality

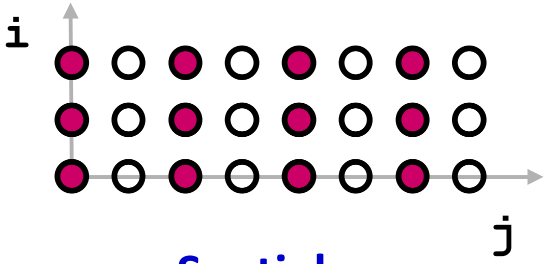
```
double A[3][N], B[N][3];

for i = 0 to 2
 for j = 0 to N-2
   A[i][j] = B[j][0] + B[j+1][0];
```

○ Hit

● Miss

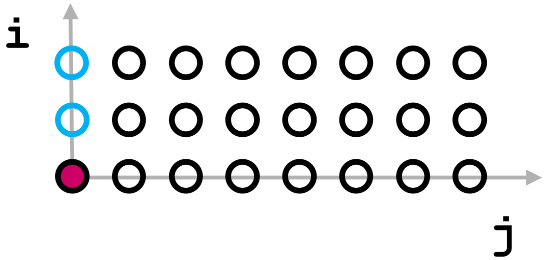
`A[i][j]`



Spatial

(Self)

`B[j][0]`

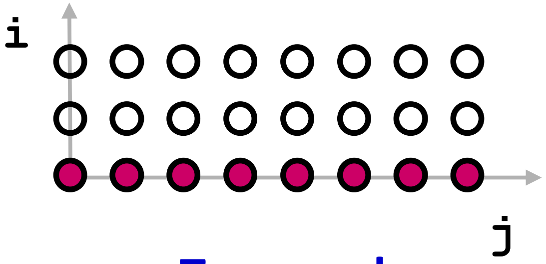


Temporal

(Group)

except for ○

`B[j+1][0]`



Temporal

(Self)

(assume row-major, 2 elements per cache line, N small)

Optimizing the Cache Behavior of Array Accesses

- We need to answer the following questions:
  - when do cache misses occur?
    - use “locality analysis”
  - can we change the order of the iterations (or possibly data layout) to produce better behavior?
    - evaluate the cost of various alternatives
  - does the new ordering/layout still produce correct results?
    - use “dependence analysis”

Examples of Loop Transformations

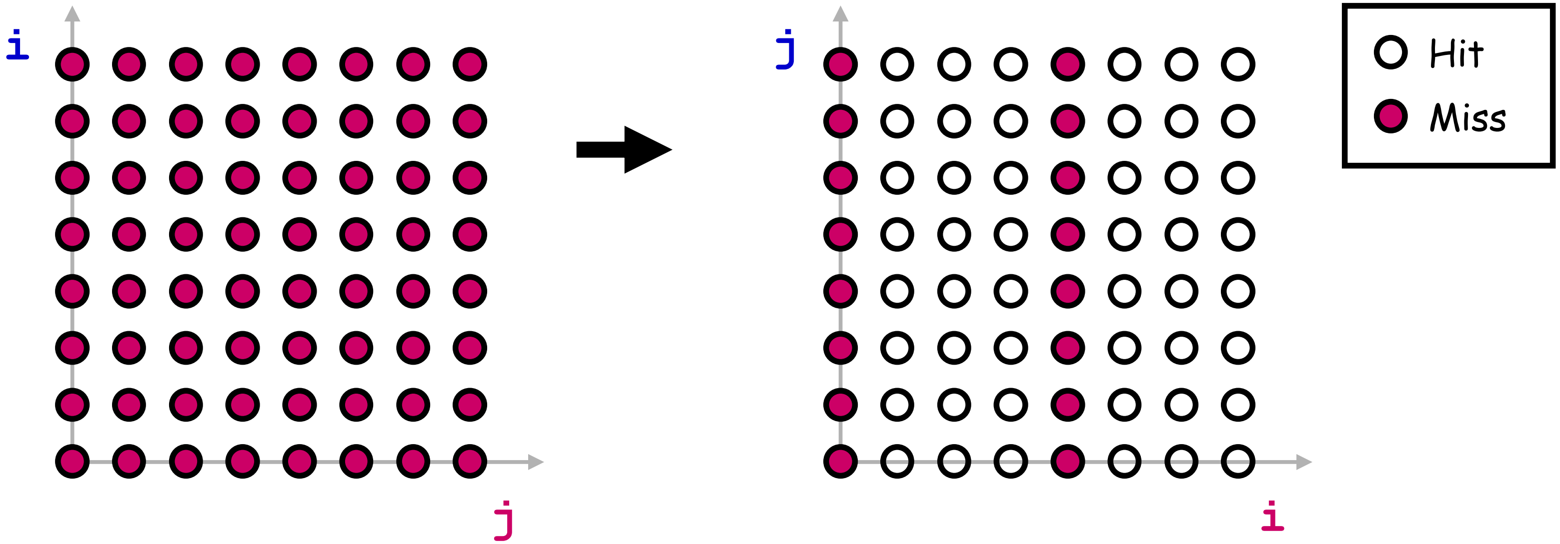
- Loop Interchange
- Cache Blocking
- Skewing: iterate through iteration space in the loops at an angle
- Loop Reversal: execute iterations in a loop in reverse order
- …

(we will briefly discuss the first two;
see ALSU 11.7.8 for others)

Loop Interchange

```
for i = 0 to N-1
  for j = 0 to N-1
    A[j][i] = i*j;
```

```
for j = 0 to N-1
  for i = 0 to N-1
    A[j][i] = i*j;
```



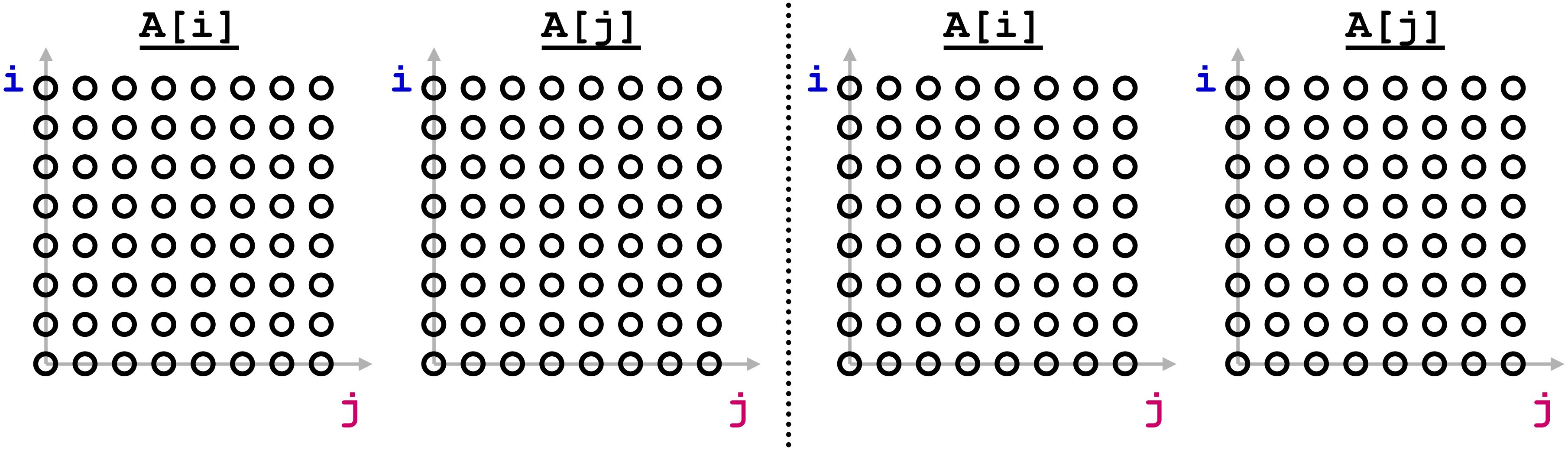
Assume row major order, N large, 4 elements per cache line

Cache Blocking (aka "Tiling")

L elements per cache line

```
for i = 0 to N-1
    for j = 0 to N-1
        f(A[i],A[j]);
```

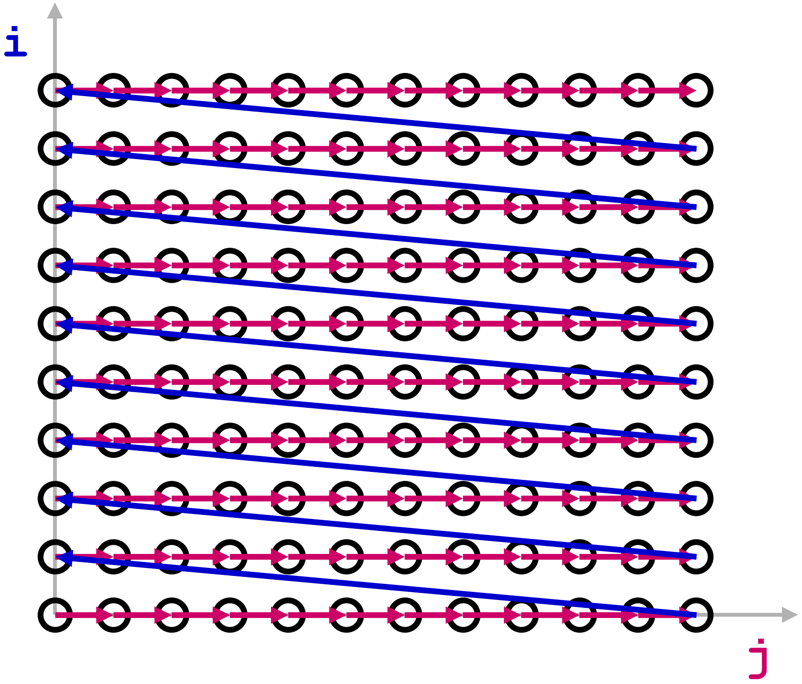
```
for JJ = 0 to N-1 by L
    for i = 0 to N-1
        for j = JJ to min(N-1,JJ+L-1)
            f(A[i],A[j]);
```



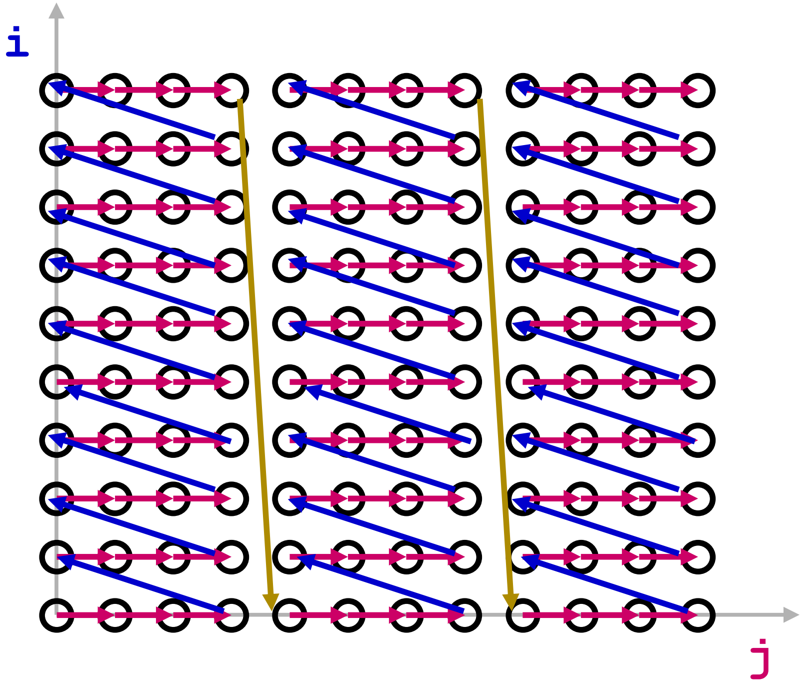
now we can exploit temporal locality

Impact on Visitation Order in Iteration Space

```
for i = 0 to N-1
    for j = 0 to N-1
        f(A[i],A[j]);
```



```
for JJ = 0 to N-1 by L
    for i = 0 to N-1
        for j = JJ to min(N-1,JJ+L-1)
            f(A[i],A[j]);
```



Cache Blocking in Two Dimensions

```
for i = 0 to N-1
  for j = 0 to N-1
    for k = 0 to N-1
      c[i,k] += a[i,j]*b[j,k];
```

```
for JJ = 0 to N-1 by B
  for KK = 0 to N-1 by B
    for i = 0 to N-1
      for j = JJ to min(N-1,JJ+B-1)
        for k = KK to min(N-1,KK+B-1)
          c[i,k] += a[i,j]*b[j,k];
```

- brings square sub-blocks of matrix "**b**" into the cache
- completely uses them up before moving on
- reduces the number of misses from $\frac{N^3}{L}$ or N^3 to only $\frac{2N^3}{L\,C}$ (C=cache size, L=line size)

III. Intro to Locality Analysis

- Definitions:
  - Reuse:
    - accessing a location that has been accessed in the past
  - Locality:
    - accessing a location that is now found in the cache

- Key Insights
  - Locality only occurs when there is reuse!
  - BUT, reuse does not necessarily result in locality.
    - why not?

Steps in Locality Analysis

1. Find data reuse ("reuse analysis")
    - if caches were infinitely large, we would be finished
2. Determine "localized iteration space"
    - set of inner loops where the data accessed by an iteration is expected to fit within the cache
3. Find data locality:
    - reuse $\cap$ localized iteration space $\Rightarrow$ locality

Reuse Analysis: Representation

```
for i = 0 to 2
 for j = 0 to N-2
  A[i][j] = B[j][0] + B[j+1][0];
```

- Map *n* loop indices into *d* array indices via array indexing function:

$$\vec{f}(\vec{\imath}) = H\vec{\imath} + \vec{c}$$

$$\mathtt{A[i][j]} = \mathtt{A}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$

$$\mathtt{B[j][0]} = \mathtt{B}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$

$$\mathtt{B[j+1][0]} = \mathtt{B}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

More Complicated Example

```
for i = ...
 for j = 0 to m
  A[2i+2][m-j][i+3j+1] = ...;
```

$$\texttt{A[2i+2][m-j][i+3j+1]} = \texttt{A}\left(\begin{bmatrix} 2 & 0 \\ 0 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 2 \\ m \\ 1 \end{bmatrix} \right)$$

Note: Representation is for Affine Array Indexes, i.e.
the index for each dimension of the array is an affine expression of surrounding loop variables and symbolic constants

An expression of one or more variables $x_1, x_2, \ldots, x_n$ is affine if it can be expressed as
$c_0 + c_1x_1 + c_2x_2 + \cdots + c_nx_n$ for constants $c_0, c_1, \ldots, c_n$

Temporal Reuse

- Temporal reuse occurs between iterations $\vec{\imath}_1$ and $\vec{\imath}_2$ whenever:

$$H\vec{\imath}_1 + \vec{c} = H\vec{\imath}_2 + \vec{c}$$

$$H(\vec{\imath}_1 - \vec{\imath}_2) = \vec{0}$$

- For **`B[j+1][0]`** reuse between iterations (i_1,j_1) and (i_2,j_2) whenever:

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ j_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ j_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 - i_2 \\ j_1 - j_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

  - i.e., whenever $j_1 = j_2$, and regardless of the difference between i_1 and i_2

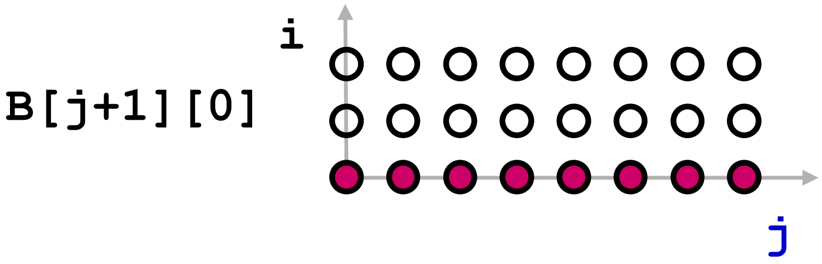
Steps in Locality Analysis

1. Find data reuse (“reuse analysis”)
   - if caches were infinitely large, we would be finished
2. Determine “localized iteration space”
   - set of inner loops where the data accessed by an iteration is expected to fit within the cache
3. Find data locality:
   - reuse $\cap$ localized iteration space $\Rightarrow$ locality

Localized Iteration Space

- Given finite cache, when does reuse result in locality?
- Localized if accesses less data than *effective cache size*

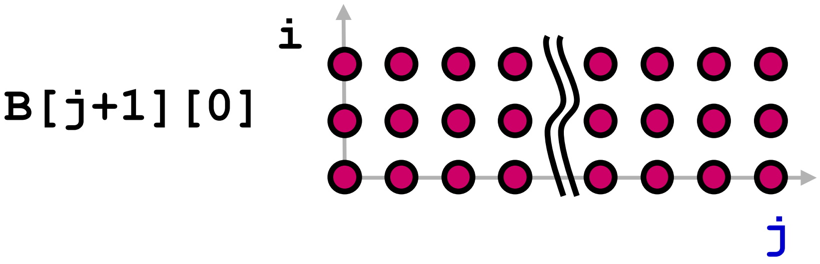
```
for i = 0 to 2
  for j = 0 to 7
    A[i][j] = B[j][0] + B[j+1][0];
```



Localized: both i and j loops

reuse implies locality

```
for i = 0 to 2
  for j = 0 to 1000000
    A[i][j] = B[j][0] + B[j+1][0];
```



Localized: j loop only

reuse but no locality

Steps in Locality Analysis

1. Find data reuse ("reuse analysis")
   - if caches were infinitely large, we would be finished
2. Determine "localized iteration space"
   - set of inner loops where the data accessed by an iteration is expected to fit within the cache
3. Find data locality:
   - reuse ∩ localized iteration space ⇒ locality

Big picture, but more to come in a future lecture...

Today's Class: Memory Hierarchy Optimizations

I. Caches: A Quick Review
II. Iteration Space & Loop Transformations
III. Intro to Locality Analysis

Friday's Class

- Abhilasha leads discussion of Assignments 1 & 2 (Phil out of town)
- Discussion Lead sign up sheet goes live at 1:30 pm

Monday's Class

- Array Dependence Analysis; Parallelization
  - ALSU 11.6, 11.7.8