Lecture 18

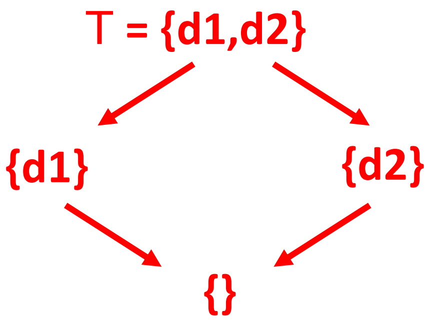
Region-Based Analysis

I. Basic Idea
II. Algorithm
III. Optimization and Complexity
IV. Comparing region-based analysis with iterative algorithms

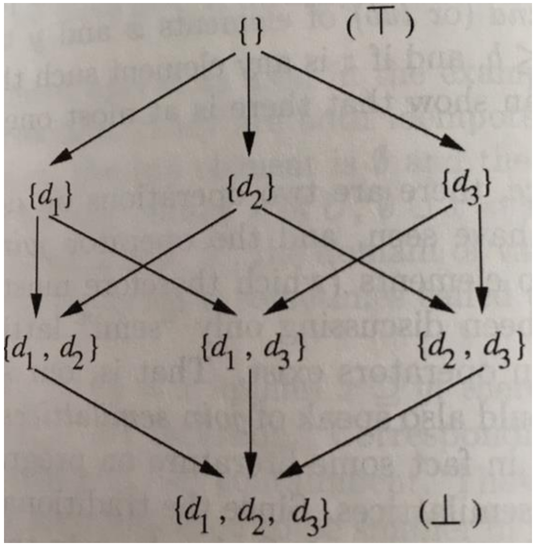
[ALSU 9.7]

Review: Iterative Data Flow Analysis

- **Semi-lattice**
  - set of values **V**
  - meet operator $\wedge$
  - Top T
  - finite descending chain?



Meet Operator:
Intersection



Meet Operator:
Union

Review: Iterative Data Flow Analysis

- **Semi-lattice**
  - set of values **V**
  - meet operator $\wedge$
  - Top T
  - finite descending chain?
- **Transfer functions**
  - function of a basic block **f: V $\rightarrow$ V**
  - closed under composition
  - meet-over-paths **MOP**
  - monotone
  - distributive?

For each node *n*: MOP(*n*) = $\wedge f_{p_i}$(T), for all paths *p* in data-flow graph reaching *n*.

If data flow framework is monotone (i.e., x ≤ y implies f(x) ≤ f(y)) then if the algorithm converges, IN[b] ≤ MOP[b] *, so analysis is ? **safe.**

Data flow framework (monotone) converges if its lattice has ?
a finite descending chain.

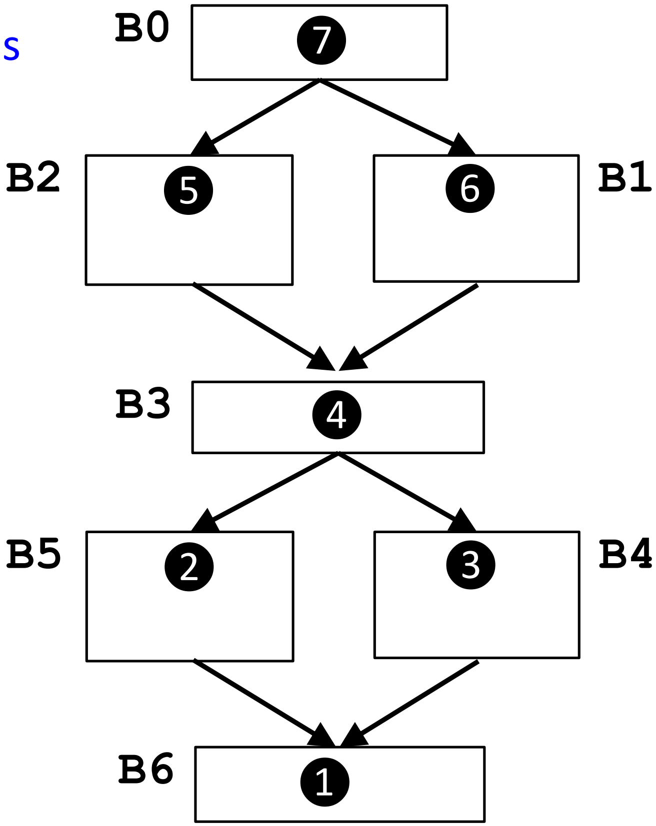
If data flow framework is distributive (i.e., f(x $\wedge$ y) = f(x) $\wedge$ f(y)) then if the algorithm converges, IN[b] = MOP[b] *, so ? **precision is high.**

* for backward analysis OUT[b]

Review: Iterative Data Flow Analysis

- **Semi-lattice**
  - set of values **V**
  - meet operator $\wedge$
  - Top T
  - finite descending chain?
- **Transfer functions**
  - function of a basic block **f: V $\rightarrow$ V**
  - closed under composition
  - meet-over-paths **MOP**
  - monotone
  - distributive?
- **Algorithm**
  - initialization step (entry/exit, other nodes)
  - repeated passes until find fixedpoint solution
  - visit order of each pass: **rPostOrder**

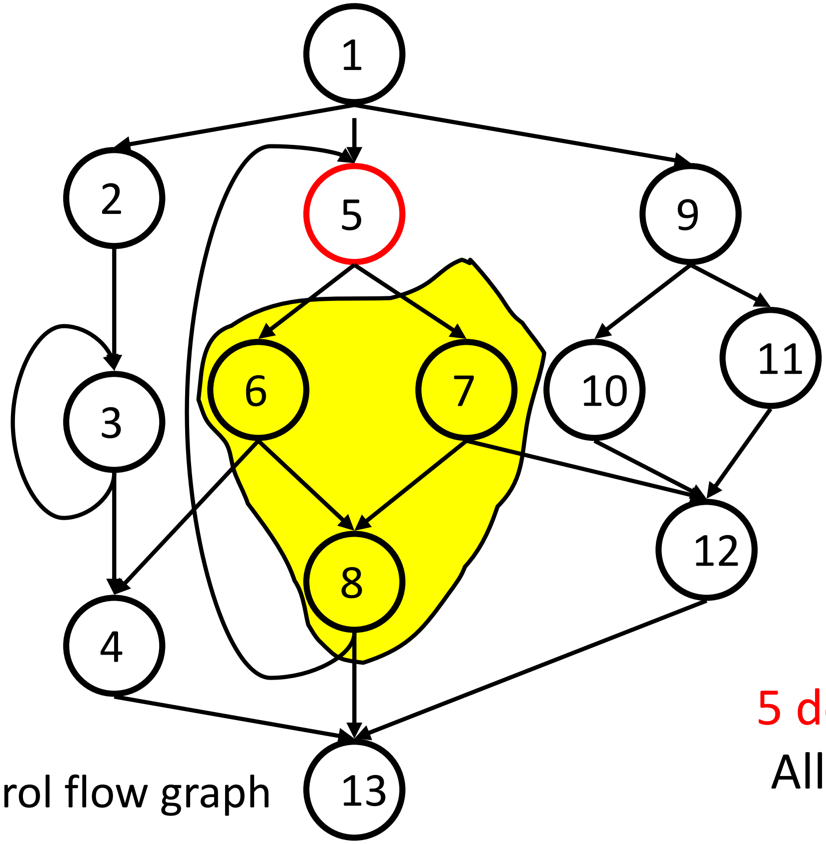
B0,B1,…,B6 is rPostOrder



Region-Based Analysis: Motivation

- **Exploit the structure of block-structured programs in data flow**
- **Tie in several concepts studied:**
  - Use of structure in induction variables, loop invariant
    - motivated by nature of the problem
    - *This lecture: can we use structure for speed?*
  - Iterative algorithm for data flow
    - *This lecture: an alternative algorithm*
  - Reducibility
    - all retreating edges of DFS Tree are back edges (t->h, h dominates t)
    - reducible graphs converge quickly
    - *This lecture: algorithm exploits & requires reducibility*
- **Usefulness in practice**
  - Faster for "harder" analyses
  - Useful for analyses related to structure, e.g., global scheduling (Lecture 20)
- **Theoretically interesting: better understanding of data flow**

Review: Dominance



Control flow graph

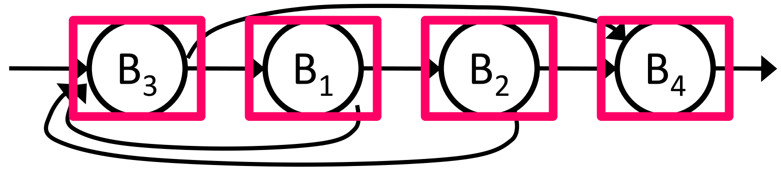
5 dominates {5,6,7,8}:
All paths to 6, 7, or 8 must visit 5 first

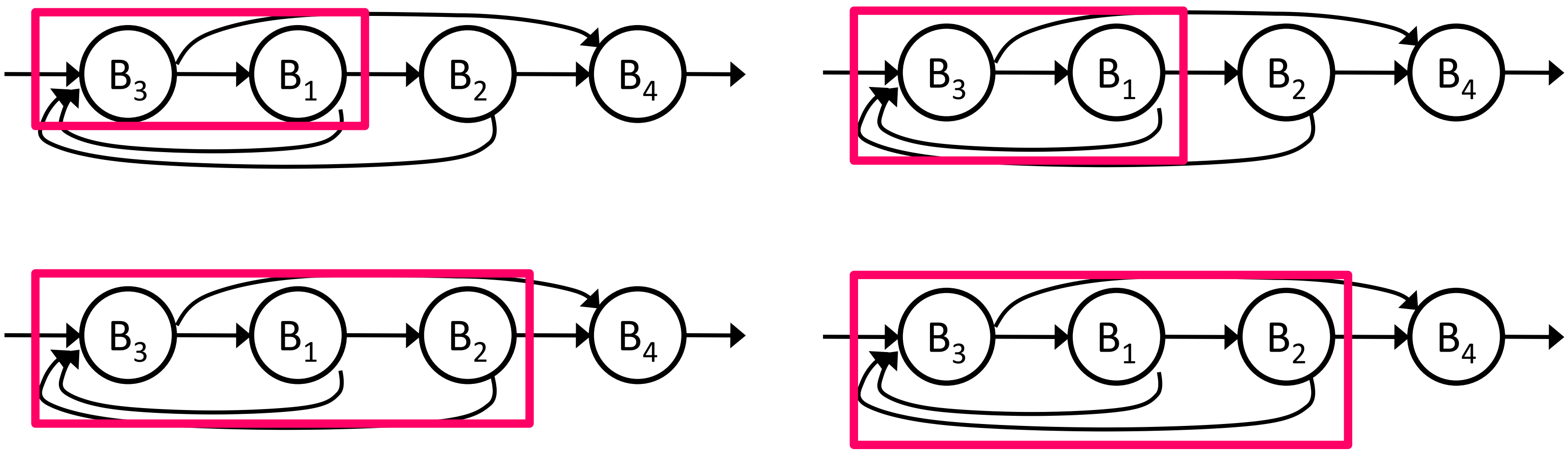
x strictly dominates w (x sdom w) iff impossible to reach w without passing through x first

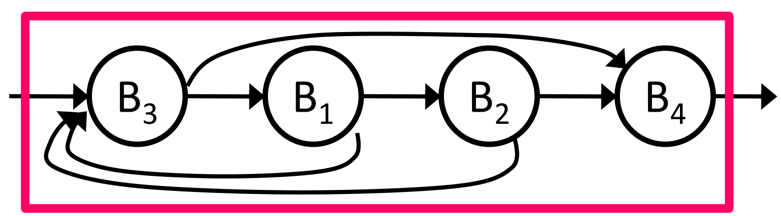
x dominates w (x dom w) iff x sdom w OR x = w

I. Big Picture

A **region** in a flow graph is a set of nodes with a **header** that dominates all other nodes in a region



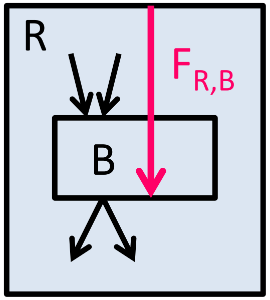




Basic Idea

- **In Iterative Analysis:**
  - DEFINITION: Transfer function F_B:
    summarize effect from beginning to end of basic block B

- **In Region-Based Analysis:**
  - DEFINITION: Transfer function $F_{R,B}$:
    summarize effect from beginning of region R to end of basic block B



  - Recursively
    construct a larger region R from smaller regions
    construct $F_{R,B}$ from transfer functions for smaller regions
    until the program is one region

  - Let P be the region for the entire program,
    and v be initial value at entry node
    – out[B] = $F_{P,B}$ (v)
    – in [B] = $\wedge_{B'}$ out[B'], where B' is a predecessor of B

II. Algorithm

A. Operations on transfer functions

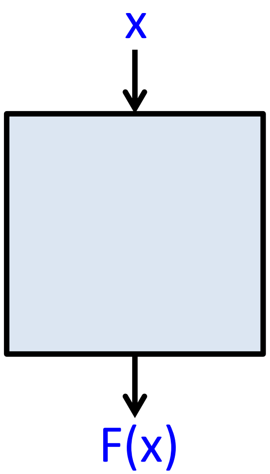
B. How to build nested regions?

C. How to construct transfer functions that correspond to the larger regions?

A. Operations on Transfer Functions

Example: **Reaching Definitions**

- Transfer function over a block:



$$F(x) = \mathbf{Gen} \cup (x - \mathbf{Kill})$$

Input parameters

- Resulting transfer functions (after operations) must be consistent with this form:
  - same equation
  - updated values for **Gen** and **Kill** set parameters

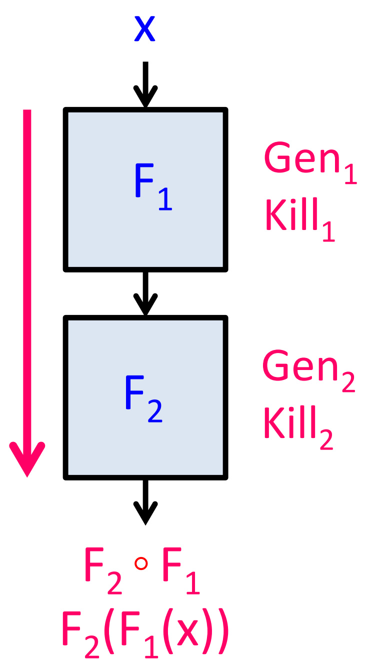
Operations on Transfer Functions: Composition

$F_2 \circ F_1$

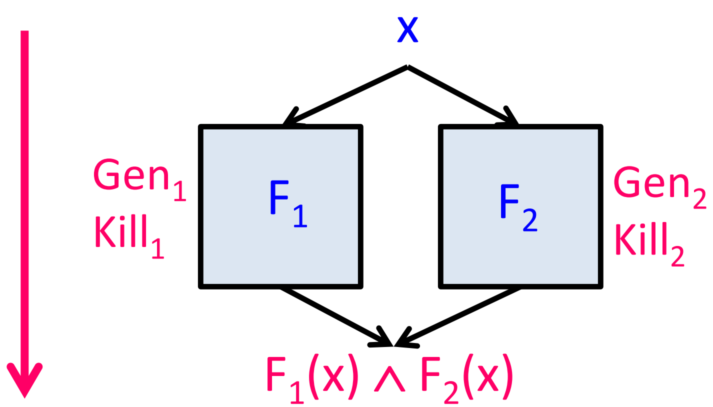
$$
\begin{aligned}
F_2(F_1(x)) &= Gen_2 \cup (F_1(x) - Kill_2) \\
&= Gen_2 \cup (Gen_1 \cup (x - Kill_1)) - Kill_2) \\
&= Gen_2 \cup (Gen_1 - Kill_2) \cup (x - (Kill_1 \cup Kill_2))
\end{aligned}
$$

Gen set
after composition

Kill set
after composition



Operations on Transfer Functions: Meet



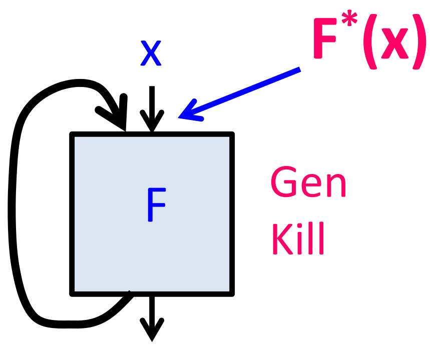
(Recall that for Reaching Definitions, $\wedge = \cup$.)

$$F_1(x) \wedge F_2(x) = Gen_1 \cup (x - Kill_1) \cup Gen_2 \cup (x - Kill_2)$$

$$= (Gen_1 \cup Gen_2) \cup (x - (Kill_1 \cap Kill_2))$$

Gen set after $\wedge$ **Kill** set after $\wedge$

Operations on Transfer Functions: Closure



New Feature!
(We don't have this in iterative data flow analysis.)

What is the value at the input of the block?

- *including* the possible effects of the back edge
  → it may iterate 0, 1, 2, ..., ∞ number of times

$$
\begin{aligned}
F^*(x) &= \wedge_{\{n\geq 0\}} F^n(x) \\
&= x \wedge F(x) \wedge F(F(x)) \wedge \ldots \\
&= x \cup (Gen \cup (x - Kill)) \cup (Gen \cup ((Gen \cup (x - Kill)) - Kill)) \cup \ldots \\
&= \mathbf{Gen} \cup (x - \varnothing)
\end{aligned}
$$

For Reaching Definitions

Gen set **Kill** set (after closure)

Recap of Operations on Transfer Functions

For **Reaching Definitions:**

- Transfer Function (**F(x)**):

$$F(x) = \mathbf{Gen} \cup (x - \mathbf{Kill})$$

- Composition ($\mathbf{F_2(F_1(x))}$):

$$\mathbf{Gen} = Gen_2 \cup (Gen_1 - Kill_2)$$
$$\mathbf{Kill} = Kill_1 \cup Kill_2$$

- Meet: ($\mathbf{F_1(x) \wedge F_2(x)}$):

$$\mathbf{Gen} = Gen_1 \cup Gen_2$$
$$\mathbf{Kill} = Kill_1 \cap Kill_2$$

- Closure: ($\mathbf{F^*(x)}$):

$$\mathbf{Gen} = Gen$$
$$\mathbf{Kill} = \varnothing$$

B. Structure of Nested Regions

- A **region** in a flow graph is a set of nodes that
  - includes a **header**, which dominates all other nodes in a region

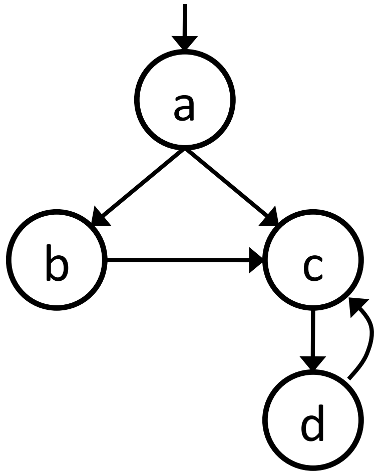
- **T1-T2 rule (Hecht & Ullman) for Flow Graphs:**

  - T1: Remove a loop
    If n is a node with a loop, i.e. an edge n->n, delete that edge (all such edges for n)

  - T2: Remove a vertex
    If there is a node n that has a unique predecessor, m,
    then m may consume n by
    deleting n and making all successors of n be successors of m.

Reducible Flow Graph

T1: Remove n->n loops
T2: Remove a vertex
w/unique predecessor

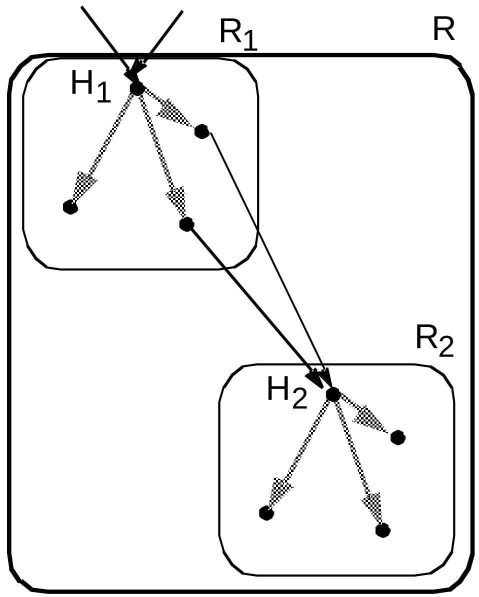


- In reduced graph:
  - each vertex represents a subgraph of original graph (a **region**).
  - each edge represents an edge in original graph
- **Limit flow graph**: result of exhaustive application of T1 and T2
  - independent of order of application
  - reducible flow graph: limit flow graph has a single vertex

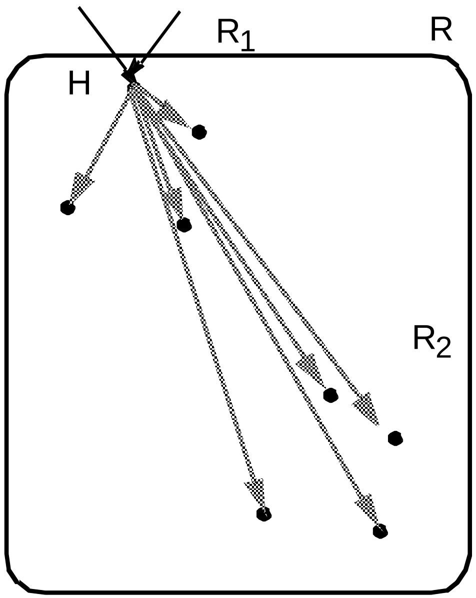
T2: Remove a vertex w/unique predecessor

C. Transfer Functions for T2 Rule

Before



After



- **Transfer function**

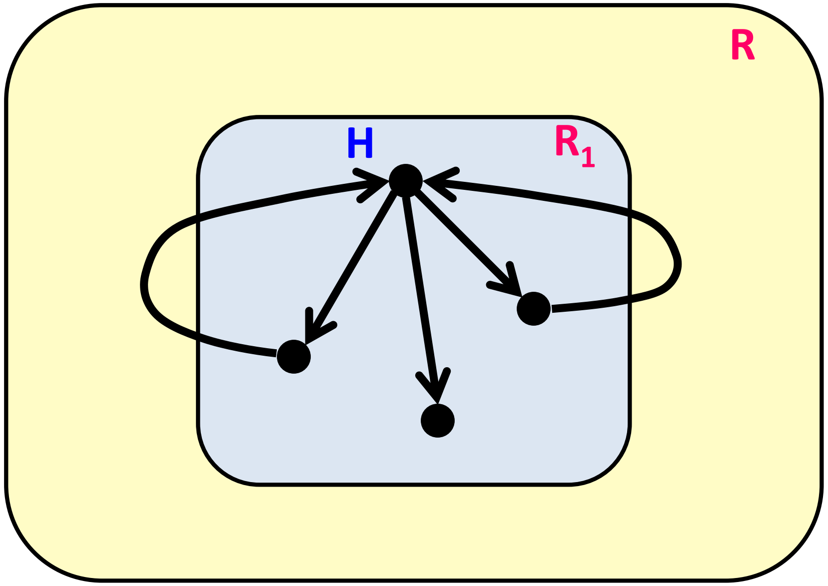
  $F_{R,B}$: summarizes the effect from beginning of R to end of B

  $F_{R,in(H2)}$: summarizes the effect from beginning of R to beginning of H2

  - Unchanged for blocks B in region R_1 ($F_{R,B} = F_{R1,B}$)
  - $F_{R,in(H2)} = \wedge_P F_{R,P}$, where p is a predecessor block of H_2
  - For blocks B in region R_2: $F_{R,B} = F_{R2,B} \circ F_{R,in(H2)}$

T1: Remove n->n loops

Transfer Functions for T1 Rule



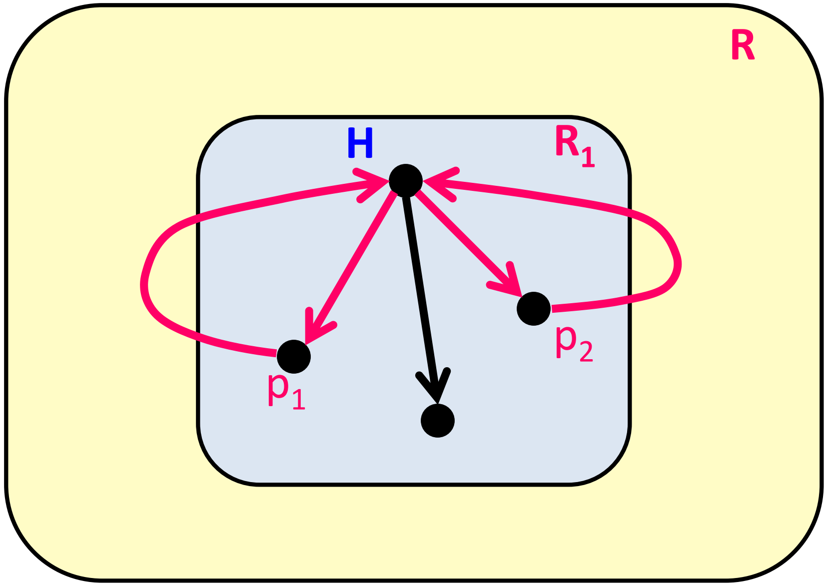
R: new region
(subsumes back edges from $R_1 \rightarrow R_1$)

Observations:

- the header of $\mathbf{R_1}$ (i.e. **H**) is also the header of **R**
- we already know how to get from **H** to **B** for every block B in $\mathbf{R_1}$: i.e. $\mathbf{F_{R1,B}}$
  - this will be the *last step* in getting from the new **R** to **B** (**composition**)
- what's new: we need to get from **R** to the input of **H**, *including back edges!*
  - this involves both **meet** ($\wedge$) and **closure** (*****) operations

T1: Remove n->n loops

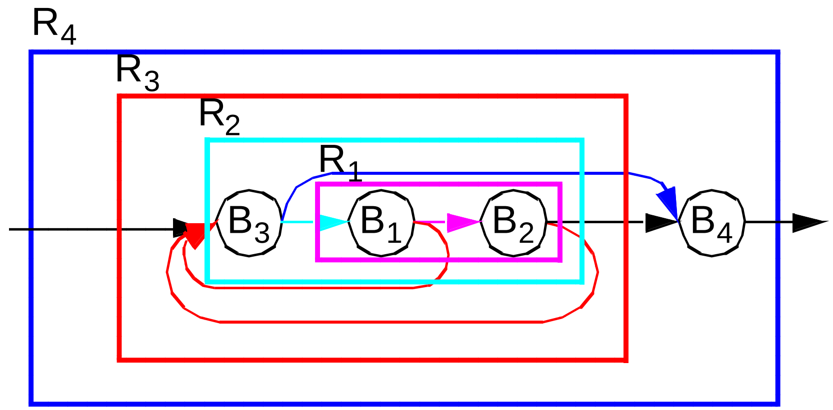
Transfer Functions for T1 Rule

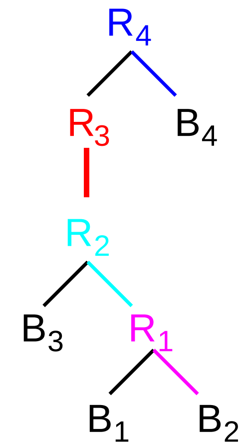


R: new region
(subsumes back edges from $R_1 \rightarrow R_1$)

- **Transfer Function $F_{R,B}$**
  - $F_{R,in(H)} = (\wedge_P F_{R1,P})^*$, where p is a predecessor block of H in R
  - $F_{R,B} = F_{R1,B} \circ F_{R,in(H)}$

Example





R	Rule	R’	$F_{R,in(R')}$	$F_{R,B1}$	$F_{R,B2}$	$F_{R,B3}$	$F_{R,B4}$
R_1	T_2	B_2				--	--
R_2	T_2	R_1					--
R_3	T_1	R_2					--
R_4	T_2	B_4					

- R: region name; R’: region w/subsumed header; R”: region w/header that remains
- T_2: $F_{R,B} = F_{R'',B}$ for B ϵ R”; $F_{R,B} = F_{R',B} \circ F_{R,in(R')}$ for B ϵ R’; $F_{R,in(R')} = \wedge_P F_{R,P}$, p ϵ pred(HR’)
- T_1: $F_{R,B} = F_{R',B} \circ F_{R,in(R')}$; $F_{R,in(R')} = (\wedge_P F_{R',P})^*$, p ϵ pred(HR’)

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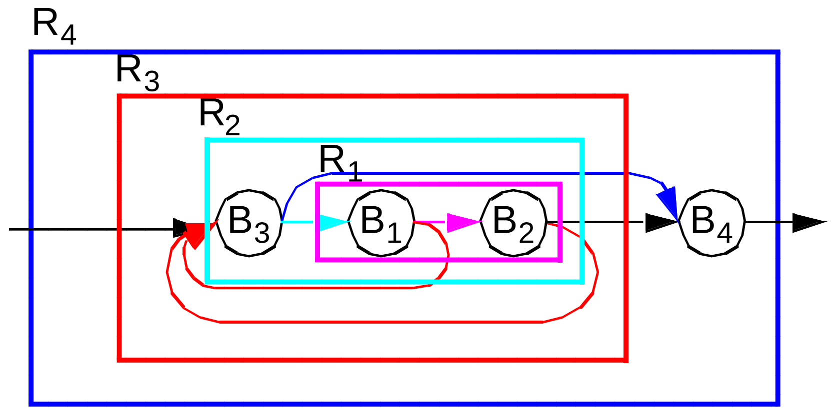
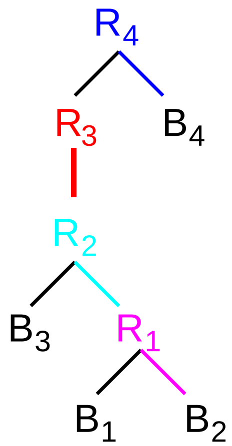
Example





R	Rule	R'	$F_{R,in(R')}$	$F_{R,B1}$	$F_{R,B2}$	$F_{R,B3}$	$F_{R,B4}$
R_1	T_2	B_2	F_{B1}	F_{B1}	$F_{B2} \circ F_{R1,in(B2)}$	--	--
R_2	T_2	R_1	F_{B3}	$F_{R1,B1} \circ F_{R2,in(R1)}$	$F_{R1,B2} \circ F_{R2,in(R1)}$	F_{B3}	--
R_3	T_1	R_2	$(F_{R2,B1} \wedge F_{R2,B2})^*$	$F_{R2,B1} \circ F_{R3,in(R2)}$	$F_{R2,B2} \circ F_{R3,in(R2)}$	$F_{R2,B3} \circ F_{R3,in(R2)}$	--
R_4	T_2	B_4	$F_{R3,B3} \wedge F_{R3,B2}$	$F_{R3,B1}$	$F_{R3,B2}$	$F_{R3,B3}$	$F_{B4} \circ F_{R4,in(B4)}$

- R: region name; R': region w/subsumed header; R'': region w/header that remains
- T_2: $F_{R,B} = F_{R'',B}$ for $B \in R''$; $F_{R,B} = F_{R',B} \circ F_{R,in(R')}$ for $B \in R'$; $F_{R,in(R')} = \wedge_P F_{R,P}$, $p \in pred(HR')$
- T_1: $F_{R,B} = F_{R',B} \circ F_{R,in(R')}$; $F_{R,in(R')} = (\wedge_P F_{R',P})^*$, $p \in pred(HR')$

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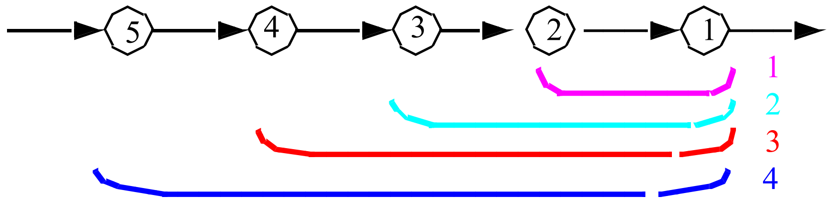
III. Complexity of Algorithm



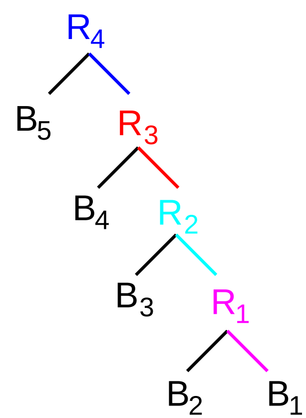
R	Rule	R'	$F_{R,in(R')}$	$F_{R,B1}$	$F_{R,B2}$	$F_{R,B3}$	$F_{R,B4}$	$F_{R,B5}$
R_1	T_2	B_1	F_{B2}	$F_{B1} \circ F_{B2}$	F_{B2}			
R_2	T_2	R_1	F_{B3}	$F_{R1,B1} \circ F_{B3}$	$F_{R1,B2} \circ F_{B3}$	F_{B3}		
R_3	T_2	R_2	F_{B4}	$F_{R2,B1} \circ F_{B4}$	$F_{R2,B2} \circ F_{B4}$	$F_{R2,B3} \circ F_{B4}$	F_{B4}	
R_4	T_2	R_3	F_{B5}	$F_{R3,B1} \circ F_{B5}$	$F_{R3,B2} \circ F_{B5}$	$F_{R3,B3} \circ F_{B5}$	$F_{B4} \circ F_{B5}$	F_{B5}

$O(n^2)$ entries

$O(n)$ entries

R	$F_{R4,in(R)}$
R_4	I
R_3	$F_{B5} \circ F_{R4,in(R4)}$
R_2	$F_{B4} \circ F_{R4,in(R3)}$
R_1	$F_{B3} \circ F_{R4,in(R2)}$
B_1	$F_{B2} \circ F_{R4,in(R1)}$

B	$F_{R4,B}$
B_5	$F_{B5} \circ F_{R4,in(R4)}$
B_4	$F_{B4} \circ F_{R4,in(R3)}$
B_3	$F_{B3} \circ F_{R4,in(R2)}$
B_2	$F_{B2} \circ F_{R4,in(R1)}$
B_1	$F_{B1} \circ F_{R4,in(B1)}$

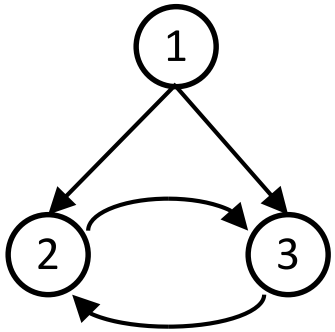


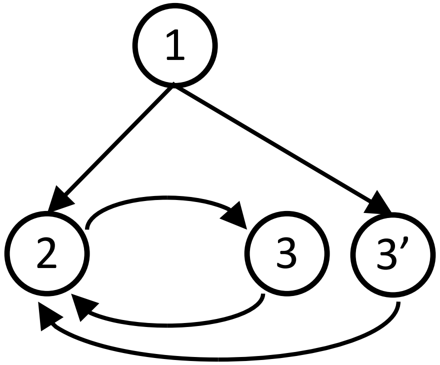
Optimization

- **Let m = number of edges, n = number of nodes**
- **Ideas for optimization**
  - If we compute $F_{R,B}$ for every region B is in, then it is very expensive
  - We are ultimately only interested in the entire region (E);
    we need to compute only $F_{E,B}$ for every B.
    - There are many common subexpressions between $F_{E,B1}$, $F_{E,B2}$, ...
    - Number of $F_{E,B}$ calculated = m
  - Also, we need to compute $F_{R,in(R')}$, where R' represents the region whose header is subsumed.
    - Number of $F_{R,B}$ calculated, where R is not final = n
- Total number of $F_{R,B}$ calculated: (m + n)
  - Data structure keeps "header" relationship
    - Practical algorithm: O(m log n)
    - Complexity: $O(m\alpha(m,n))$, α is inverse Ackermann function

Reducibility

T1: Remove n->n loops
T2: Remove a vertex
w/unique predecessor





- If no T1, T2 is applicable before graph is reduced to single node, then **split node** (make k copies of node, one per predecessor) and continue
- Worst case: exponential
- Most graphs (including GOTO programs) are reducible

IV. Comparison with Iterative Data Flow Analysis

- **Applicability**
  - Definitions of F* can make technique more powerful than iterative algorithms
  - Backward flow: reverse graph is not typically reducible.
    - Requires more effort to adapt to backward flow than iterative algorithm
  - More important for interprocedural optimization, optimizations related to loop nesting structure
- **Speed**
  - Irreducible graphs
    - Iterative algorithm can process irreducible parts uniformly
    - Serious "irreducibility" can be slow with region-based analysis
  - Reducible graphs?

Review: Speed of Convergence of Iterative Data Flow

- **If cycles do not add information***
  - information can flow in one pass down nodes of increasing order number:
    - e.g., $\underbrace{\text{1 -> 4 -> 5 -> 7}}_{\text{first pass}}$ -> 2 -> 6 ...
  - passes determined by number of back edges in the path
    - essentially the nesting depth of the graph
  - Number of iterations = number of back edges in any acyclic path + 2

- **What is the depth?**
  – corresponds to depth of intervals for "reducible" graphs

* E.g., Reaching Definitions: if a defn *d* in node n_1 reaches a node n_k along a path that contains a cycle (i.e., a repeated node), then the cycle can be removed to form a shorter path from n_1 to n_k such that *d* reaches n_k.

Comparison with Iterative Data Flow Analysis

- **Applicability**
  - Definitions of F* can make technique more powerful than iterative algorithms
  - Backward flow: reverse graph is not typically reducible.
    - Requires more effort to adapt to backward flow than iterative algorithm
  - More important for interprocedural optimization, optimizations related to loop nesting structure
- **Speed**
  - Irreducible graphs
    - Iterative algorithm can process irreducible parts uniformly
    - Serious "irreducibility" can be slow with region-based analysis
  - Reducible graph & Cycles do not add information (common)
    - Iterative: (depth + 2) passes, O(m*depth) steps
      depth is 2.75 average, independent of code length
    - Region-based analysis: Theoretically almost linear, typically O(m log n) steps
  - Reducible graph & Cycles add information*
    - Iterative takes longer to converge
    - Region-based analysis remains the same

*E.g., Constant Propagation

```
L: a = b
   b = c
   c = 1
   goto L
```

Today's Class: Region-Based Analysis

I. Basic Idea
II. Algorithm
III. Optimization and Complexity
IV. Comparing region-based analysis with iterative algorithms

Wednesday

- Day of discussions on Project Proposal ideas

Friday's Class

- Instruction Scheduling [ALSU 10.1-10.3]
- Discussion of Assignment #3