Lecture 22

Locality Analysis and Prefetching

I. Locality Analysis
   A. Temporal
   B. Spatial
   C. Group
   D. Localized Iteration Space

II. Prefetching Pointer-Based Structures

[ALSU 11.5]

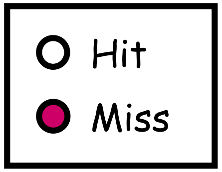
I. Recall: Steps in Locality Analysis

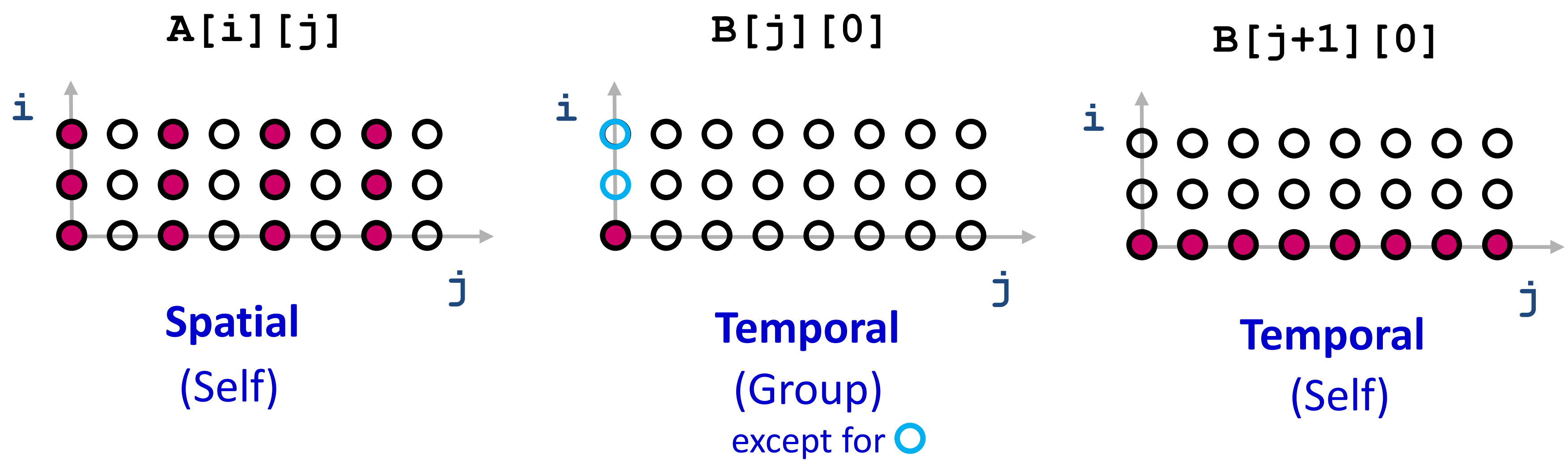
1. Find data reuse ("reuse analysis")
   - if caches were infinitely large, we would be finished
2. Determine "localized iteration space"
   - set of inner loops where the data accessed by an iteration is expected to fit within the cache
3. Find data locality:
   - reuse $\cap$ localized iteration space $\Rightarrow$ locality

Recall: Types of Data Reuse/Locality

```
double A[3][N], B[N][3];

for i = 0 to 2
  for j = 0 to N-2
    A[i][j] = B[j][0] + B[j+1][0];
```





(assume row-major, 2 elements per cache line, N small)

Recall: Reuse Analysis Representation

```
for i = 0 to 2
 for j = 0 to N-2
  A[i][j] = B[j][0] + B[j+1][0];
```

- Map *n* loop indices into *d* array indices via array indexing function:

$$\vec{f}(\vec{i}) = H\vec{i} + \vec{c}$$

$$\texttt{A[i][j]} = \texttt{A}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$

$$\texttt{B[j][0]} = \texttt{B}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$

$$\texttt{B[j+1][0]} = \texttt{B}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

A. Finding Temporal Reuse

- Temporal reuse occurs between iterations $\vec{\imath}_1$ and $\vec{\imath}_2$ whenever:

$$H\vec{\imath}_1 + \vec{c} = H\vec{\imath}_2 + \vec{c}$$
$$H(\vec{\imath}_1 - \vec{\imath}_2) = \vec{0}$$

- For **`B[j+1][0]`** reuse between iterations (i_1,j_1) and (i_2,j_2) whenever:

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ j_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ j_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 - i_2 \\ j_1 - j_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

  - i.e., whenever $j_1 = j_2$, and regardless of the difference between i_1 and i_2

Nullspace and Basis Vectors

- Temporal reuse occurs between iterations $\vec{\imath}_1$ and $\vec{\imath}_2$ whenever:

$$H\vec{\imath}_1 + \vec{c} = H\vec{\imath}_2 + \vec{c}$$
$$H(\vec{\imath}_1 - \vec{\imath}_2) = \vec{0}$$

- There is a well-known concept from linear algebra that characterizes when $\vec{\imath}_1$ and $\vec{\imath}_2$ satisfy the above equation:
  - Set of all solutions to *H v = 0* is called the *nullspace* of *H*
  - Two iterations refer to the same array element iff the difference of their loop-index vectors is in the nullspace of *H*

- A nullspace can be summarized by its basis vectors
  - Any vector in the nullspace is a linear combination of the basis vectors

Nullspace & Basis Vector Example

```
for i = 0 to 2
 for j = 0 to 100
  A[i][j] = B[j][0] + B[j+1][0];
```

- For **B[j+1][0]** reuse between iterations (i_1,j_1) and (i_2,j_2) whenever:

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ j_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i_2 \\ j_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i_1 - i_2 \\ j_1 - j_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

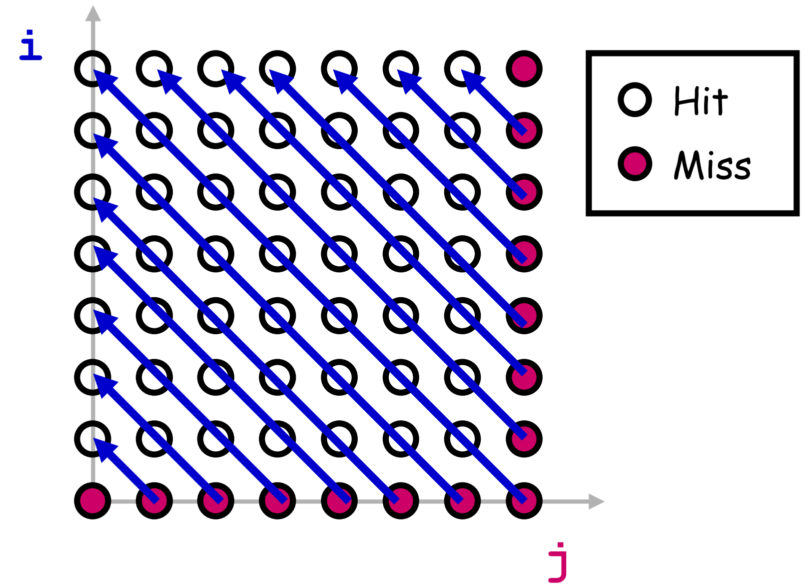
- The nullspace of $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is summarized by the basis vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ because **c** $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ represents all the vectors **v** such that $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ **v** = $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- So reuse occurs whenever $\begin{bmatrix} i_1 - i_2 \\ j_1 - j_2 \end{bmatrix}$ = **c** $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
  - i.e., whenever $j_1 = j_2$, and regardless of the difference between i_1 and i_2

inner or outer loop?
outer

More Complicated Example

```
for i = 0 to N-1
   for j = 0 to N-1
      A[i+j][0] = i*j;
```

$$\texttt{A[i+j][0]} = \texttt{A}\left(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)$$



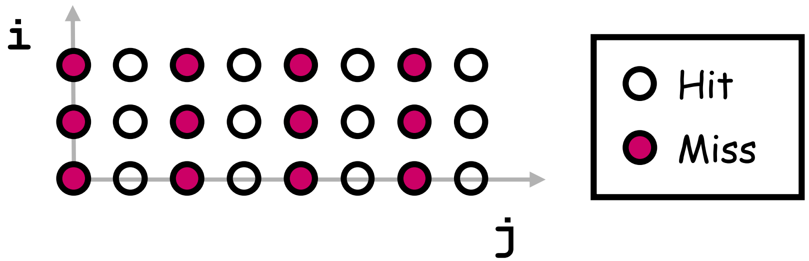
- Nullspace of $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is summarized by the basis vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- So reuse occurs whenever $\begin{bmatrix} i_1 - i_2 \\ j_1 - j_2 \end{bmatrix} = \mathbf{c}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
  - i.e., when $\Delta i = -\Delta j$

B. Computing Spatial Reuse

- We assume two array elements share the same cache line iff they differ only in the last dimension
  - E.g., share the same row in a 2-dimensional array
  - Why is this a reasonable approximation? row major order
  - What are its limitations? A row is made up of many cache lines
    Large row could be larger than the cache
- Replace last row of H with zeros, creating H_s
- Find the nullspace of H_s
- Result: vector along which we access the same row

Computing Spatial Reuse: Example

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```



$$\texttt{A[i][j]} = \texttt{A}\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}i\\ j\end{bmatrix}+\begin{bmatrix}0\\ 0\end{bmatrix}\right)$$

- $H = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $H_s = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$
- Nullspace of H_s is summarized by the basis vector $\begin{bmatrix}0\\ 1\end{bmatrix}$
- So spatial reuse occurs whenever $\begin{bmatrix}i_1 - i_2\\ j_1 - j_2\end{bmatrix} = c\begin{bmatrix}0\\ 1\end{bmatrix}$
  - i.e., whenever $i_1 = i_2$, and regardless of the difference between j_1 and j_2

inner or outer loop?
inner

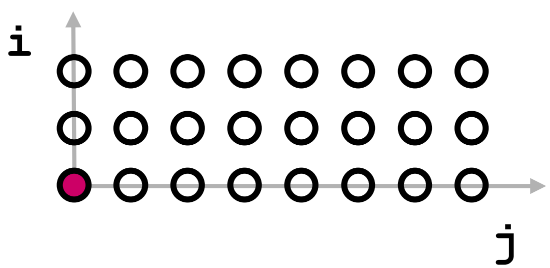
C. Group Reuse (reuse from different static accesses)

```
for i = 0 to 2
  for j = 0 to 100
    A[i][j] = B[j][0] + B[j+1][0];
```

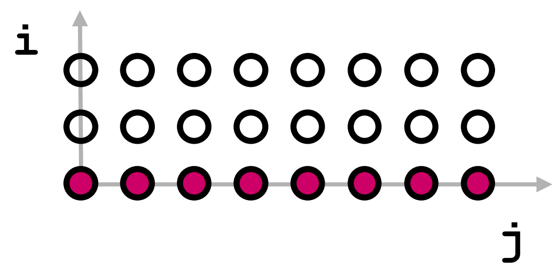
$$H = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

- Limit the analysis to consider only accesses with same H
  - i.e., index expressions that differ only in their constant terms
- Determine when access same location (temporal) or same row (spatial)
- Only the "leading reference" suffers the bulk of the cache misses

B[j][0]



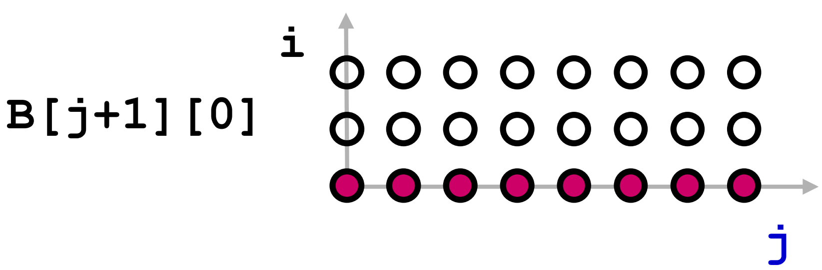
B[j+1][0]



D. Localized Iteration Space

- Given finite cache, when does reuse result in locality?
- Localized if accesses less data than *effective cache size*

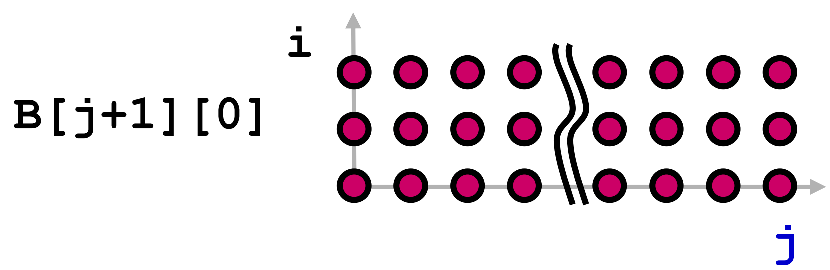
```
for i = 0 to 2
  for j = 0 to 7
    A[i][j] = B[j][0] + B[j+1][0];
```



Localized: both i and j loops

Basis = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

```
for i = 0 to 2
  for j = 0 to 1000000
    A[i][j] = B[j][0] + B[j+1][0];
```



Localized: j loop only

Basis = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Computing Locality

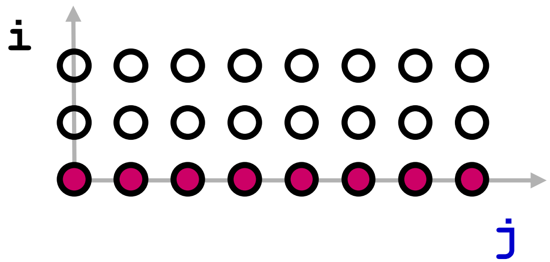
Reuse Vector Space ∩ Localized Vector Space ⇒ Locality Vector Space

- Example:

```
for i = 0 to 2
 for j = 0 to N-2
  A[i][j] = B[j][0] + B[j+1][0];
```

- If N is small, then both loops are localized:
  - $\text{span}\{\begin{bmatrix}1\\0\end{bmatrix}\} \cap \text{span}\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\} \Rightarrow \text{span}\{\begin{bmatrix}1\\0\end{bmatrix}\}$
  - i.e., temporal reuse does result in temporal locality

B[j+1][0]



Computing Locality

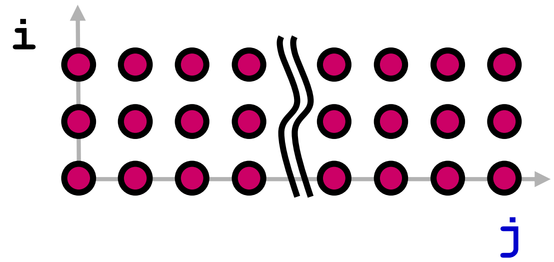
Reuse Vector Space ∩ Localized Vector Space ⇒ Locality Vector Space

- Example:

```
for i = 0 to 2
  for j = 0 to N-2
    A[i][j] = B[j][0] + B[j+1][0];
```

- If N is large, then only the innermost loop is localized:
  - $\text{span}\{\begin{bmatrix}1\\0\end{bmatrix}\} \cap \text{span}\{\begin{bmatrix}0\\1\end{bmatrix}\} \Rightarrow \text{span}\{\}$
  - i.e., no temporal locality

B[j+1][0]



Locality Analysis Summary

1. Find data reuse
   - Temporal reuse: Compute the nullspace of H
   - Spatial reuse: Compute the nullspace of H_s, which is H with last row zeroed out
   - If caches were infinitely large, we would be finished

2. Determine "localized iteration space"
   - set of inner loops where the data accessed by an iteration is expected to fit within the cache

3. Find data locality:
   - reuse $\cap$ localized iteration space $\Rightarrow$ locality

II. Prefetching

Recall: Compiler Algorithm

Analysis: what to prefetch

- Locality Analysis

Scheduling: when/how to issue prefetches

- Loop Splitting
- Software Pipelining

Recall: Prefetch Predicate

Locality Type	Miss Instance	Predicate on Iteration Space
None	Every Iteration	True
Temporal	First Iteration	**i** = 0
Spatial	Every L iterations (L elements/cache line)	(**i** mod L) = 0

Example:

```
for i = 0 to 2
 for j = 0 to N-2
  A[i][j] = B[j][0] + B[j+1][0];
```

Reference	Locality	Predicate on Iteration Space
A[i][j]	$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} \text{none} \\ \text{spatial} \end{bmatrix}$	(**j** mod L) = 0
B[j+1][0]	$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} \text{temporal} \\ \text{none} \end{bmatrix}$	**i** = 0

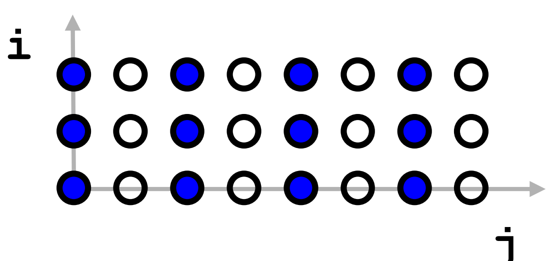
Recall: Loop Splitting for Prefetching Arrays

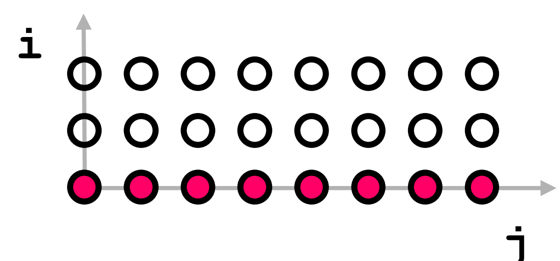
- Decompose loops to isolate cache miss instances
  - cheaper than inserting IF(Prefetch Predicate) statements

Locality Type	**Predicate**	**Loop Transformation**
None	True	None
Temporal	**i** = 0	**Peel** loop **i**
Spatial	(**i** mod L) = 0	**Unroll** loop **i** by L

(L elements/cache line)

Loop peeling: split any problematic first (or last) few iterations from the loop & perform them outside of the loop body





Recall: Example Code with Prefetching

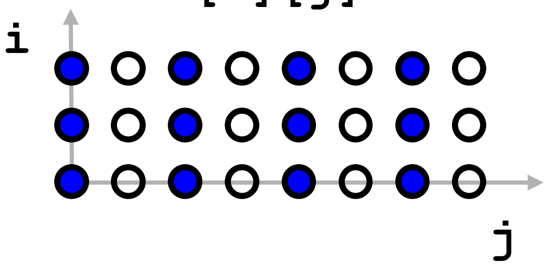
Original Code

```
for (i = 0; i < 3; i++)
  for (j = 0; j < 100; j++)
    A[i][j] = B[j][0] + B[j+1][0];
```

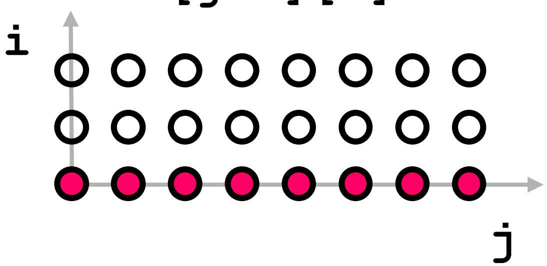
○ Cache Hit

● ● Cache Miss

A[i][j]



B[j+1][0]



i = 0

```
prefetch(&B[0][0]);
for (j = 0; j < 6; j += 2) {
  prefetch(&B[j+1][0]);
  prefetch(&B[j+2][0]);
  prefetch(&A[0][j]);
}
for (j = 0; j < 94; j += 2) {
  prefetch(&B[j+7][0]);
  prefetch(&B[j+8][0]);
  prefetch(&A[0][j+6]);
  A[0][j] = B[j][0]+B[j+1][0];
  A[0][j+1] = B[j+1][0]+B[j+2][0];
}
for (j = 94; j < 100; j += 2) {
  A[0][j] = B[j][0]+B[j+1][0];
  A[0][j+1] = B[j+1][0]+B[j+2][0];
}
```

i > 0

```
for (i = 1; i < 3; i++) {
  for (j = 0; j < 6; j += 2)
    prefetch(&A[i][j]);
  for (j = 0; j < 94; j += 2) {
    prefetch(&A[i][j+6]);
    A[i][j] = B[j][0] + B[j+1][0];
    A[i][j+1] = B[j+1][0] + B[j+2][0];
  }
  for (j = 94; j < 100; j += 2) {
    A[i][j] = B[j][0] + B[j+1][0];
    A[i][j+1] = B[j+1][0] + B[j+2][0];
  }
}
```

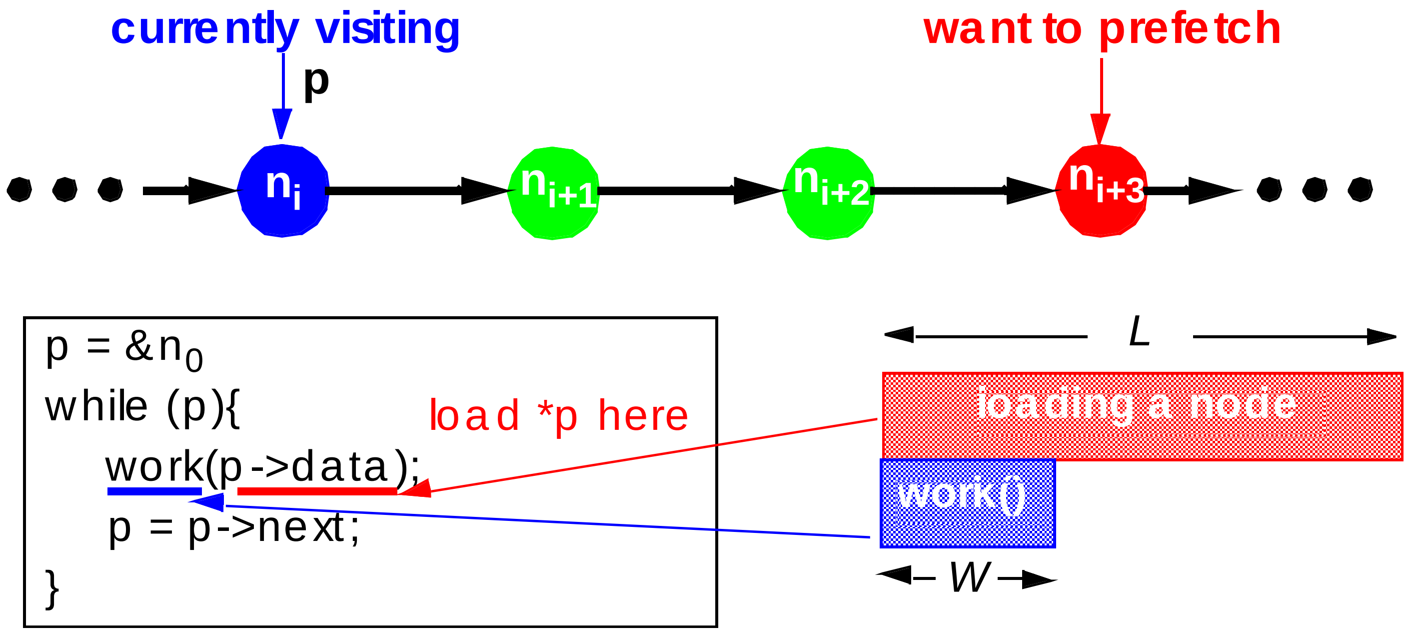
Today: Prefetching for Pointer-Based Structures

- Examples:
  - linked lists, trees, graphs, ...
- A common method of building large data structures
  - especially in non-numeric programs
- Cache miss behavior is a concern because:
  - large data set with respect to the cache size
  - temporal locality may be poor
  - little spatial locality among consecutively-accessed nodes

Goal:

- Automatic compiler-based prefetching for pointer-based data structures

Scheduling Prefetches for Pointer-Based Data Structures

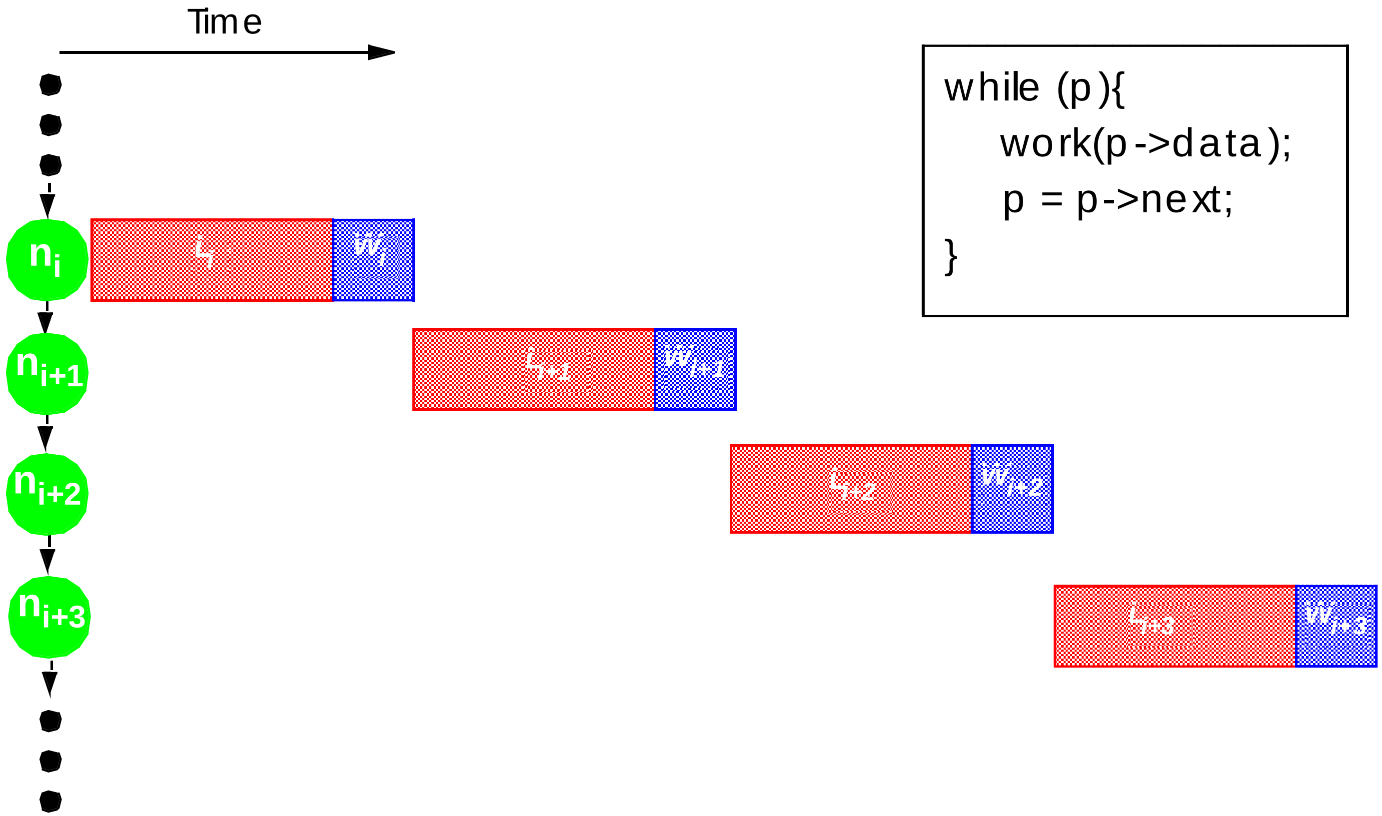


Our Goal: *fully hide latency*

– thus achieving fastest possible computation rate of 1/W

- e.g., if L = 3W, we must prefetch 3 nodes ahead to achieve this

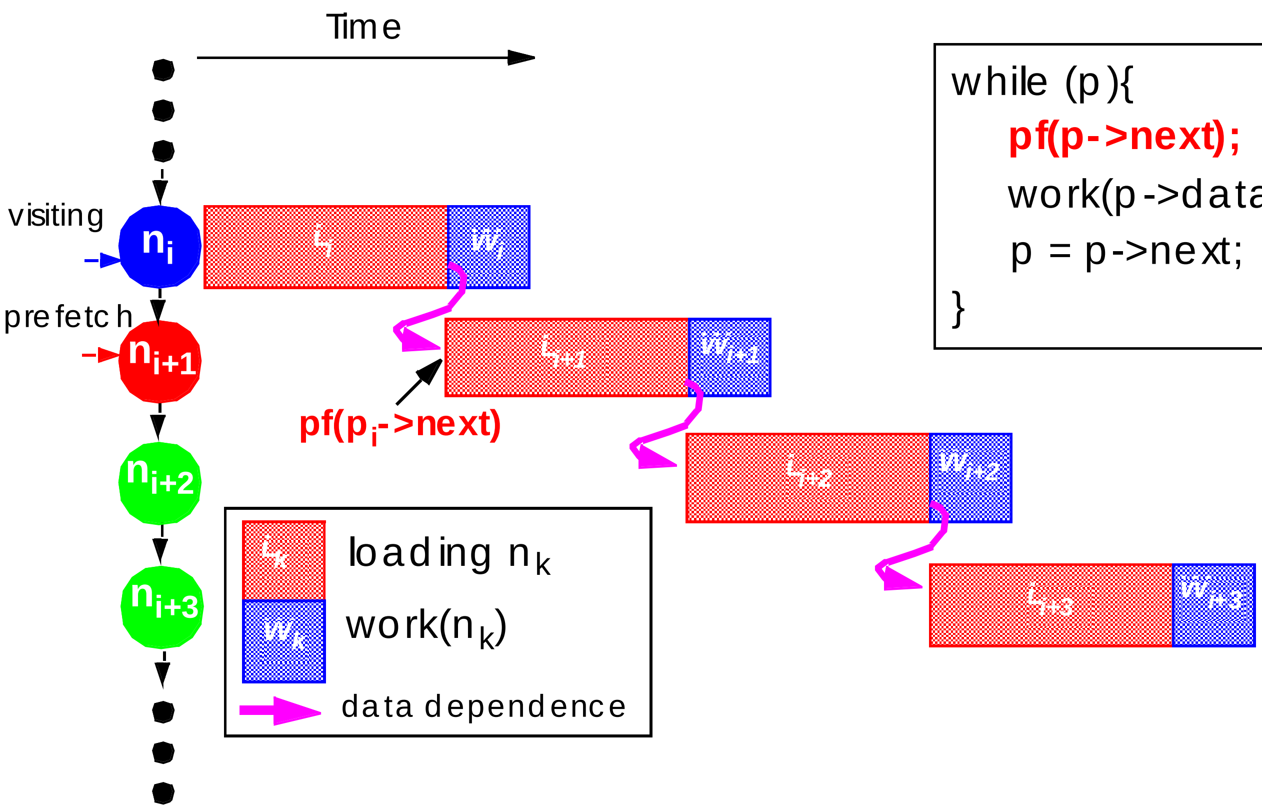
Performance without Prefetching




```
while (p){
    work(p->data);
    p = p->next;
}
```

computation rate = 1 / (L+W)

Prefetching One Node Ahead



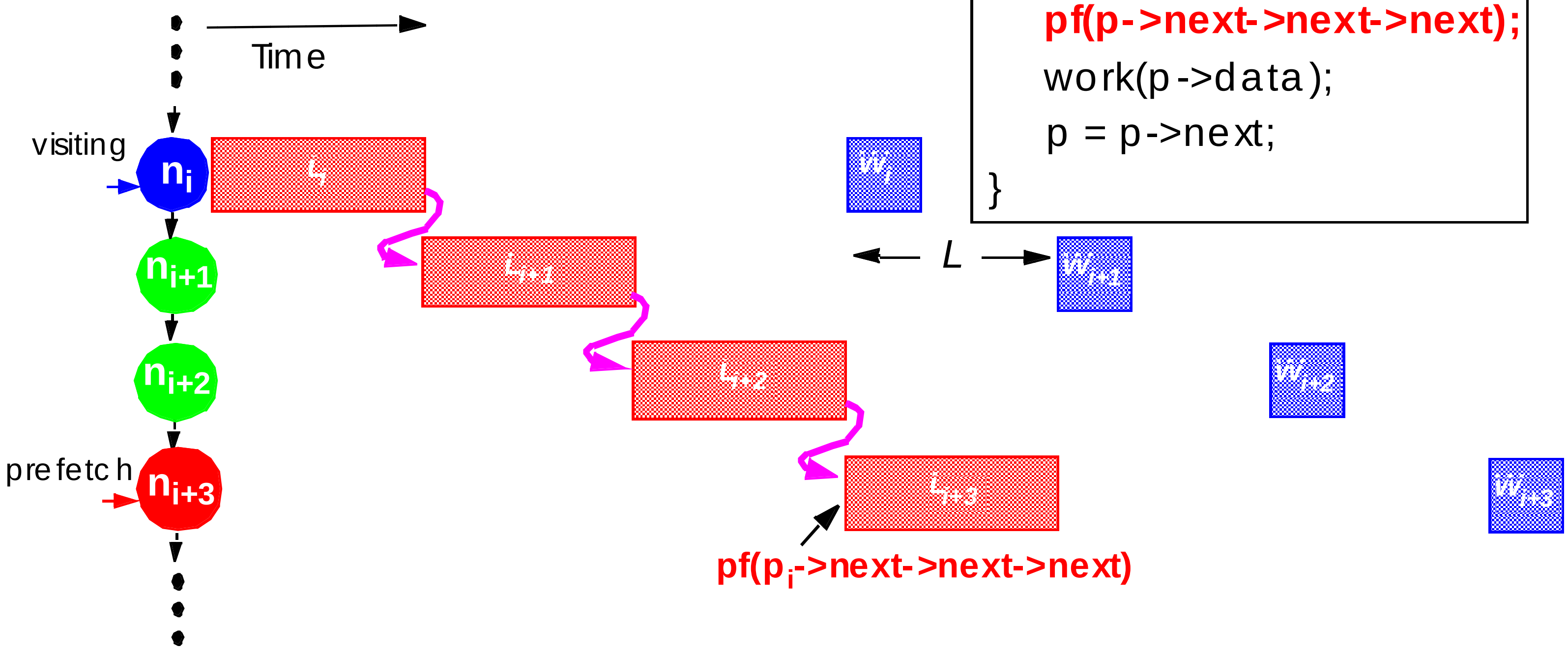

```
while (p){
    pf(p->next);
    work(p->data);
    p = p->next;
}
```

- Computation is overlapped with memory accesses

computation rate = 1/L

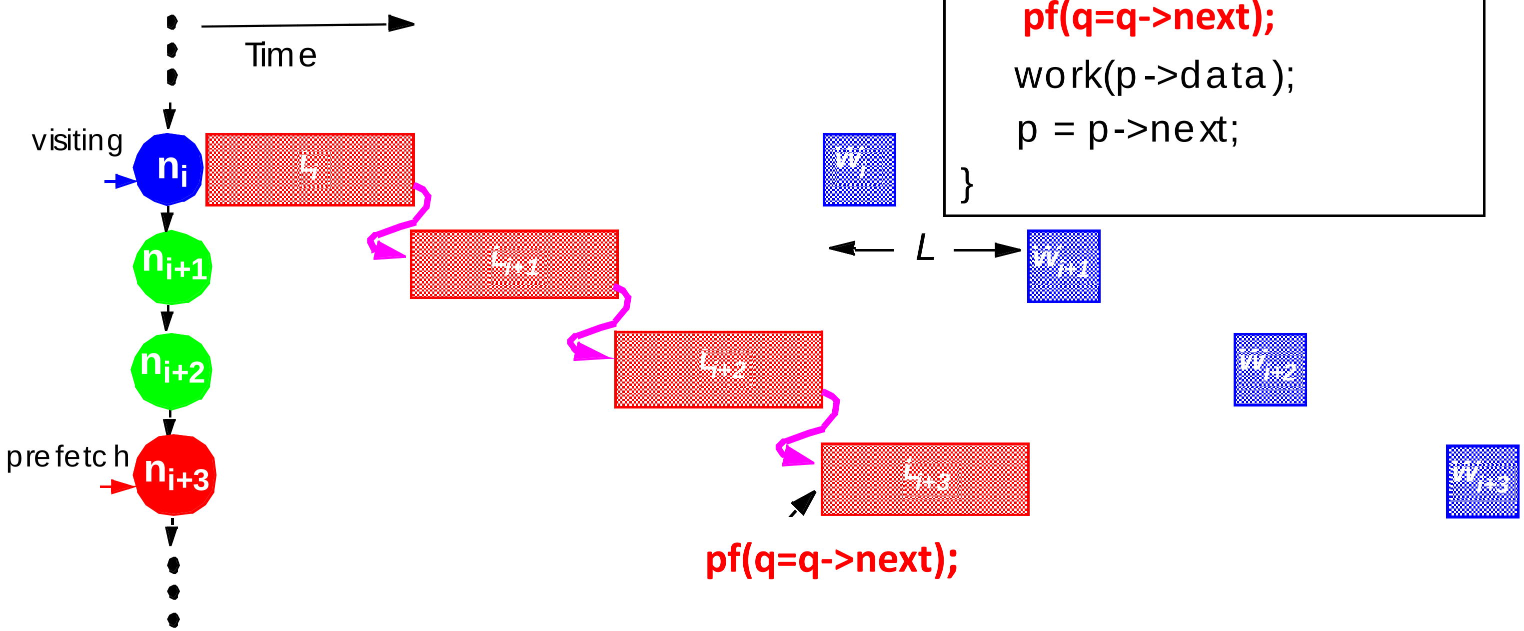
Prefetching Three Nodes Ahead

```
while (p){
    pf(p->next->next->next);
    work(p->data);
    p = p->next;
}
```



Prefetching Three Nodes Ahead

```
q=p->next->next;
while(q) {
    pf(q=q->next);
   work(p->data);
   p = p->next;
}
```

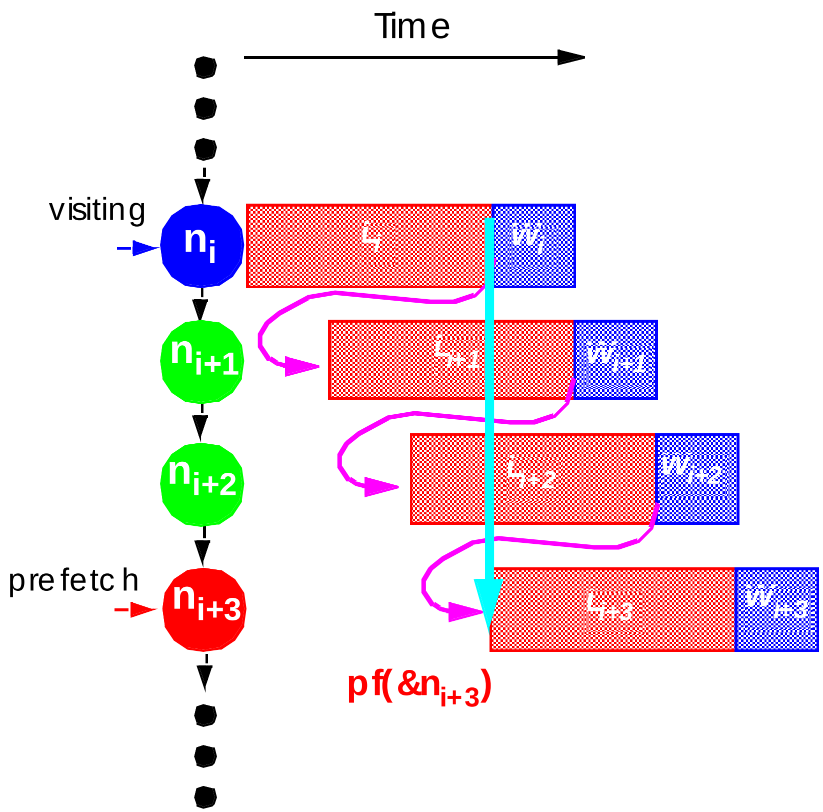


computation rate does not improve (still = 1/L)!

Pointer-Chasing Problem:

- any scheme which follows the pointer chain is limited to a rate of 1/L

Our Goal: Fully Hide Latency




```
while (p){
    pf(&n_{i+3});
    work(p->data);
    p = p->next;
}
```

- achieves the fastest possible computation rate of 1/W

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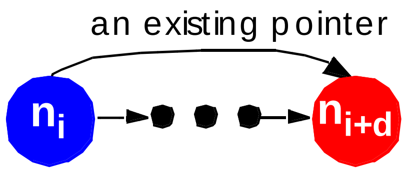
Overcoming the Pointer-Chasing Problem

Key:

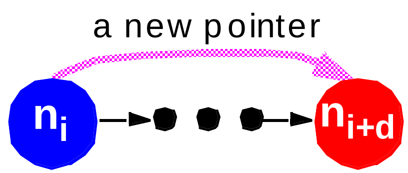
- n_i needs to know $\&n_{i+d}$ without referencing the d-1 intermediate nodes

Three Algorithms:

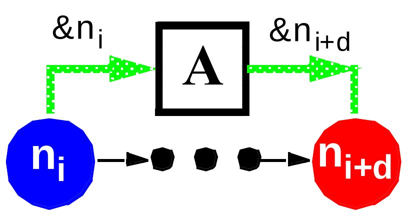
- use *existing* pointer(s) in n_i to approximate $\&n_{i+d}$
  - Greedy Prefetching



- add *new* pointer(s) to n_i to approximate $\&n_{i+d}$
  - History-Pointer Prefetching



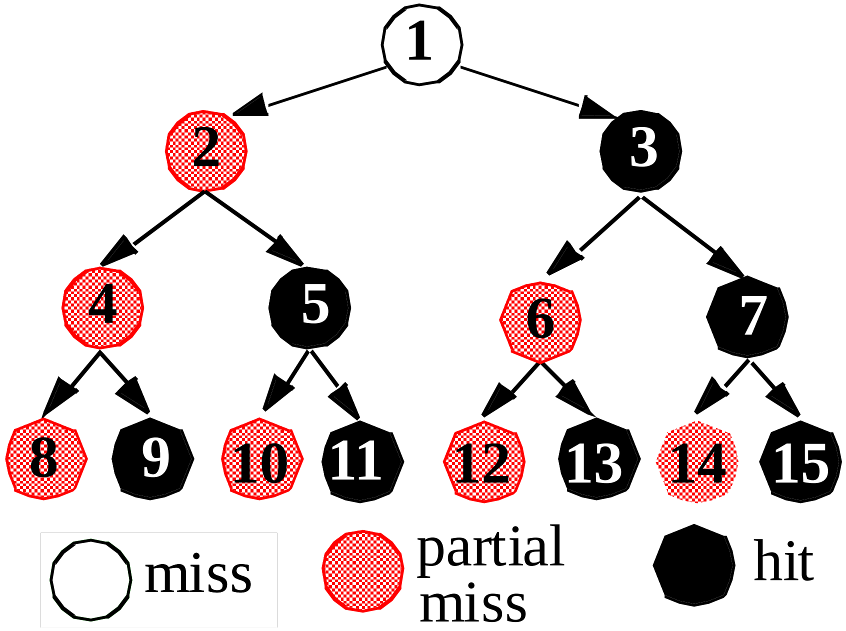
- compute $\&n_{i+d}$ *directly* from $\&n_i$ (no ptr deref)
  - Data-Linearization Prefetching



Greedy Prefetching

- Prefetch all neighboring nodes (simplified definition)
  - only one will be followed by the immediate control flow
  - hopefully, we will visit other neighbors later

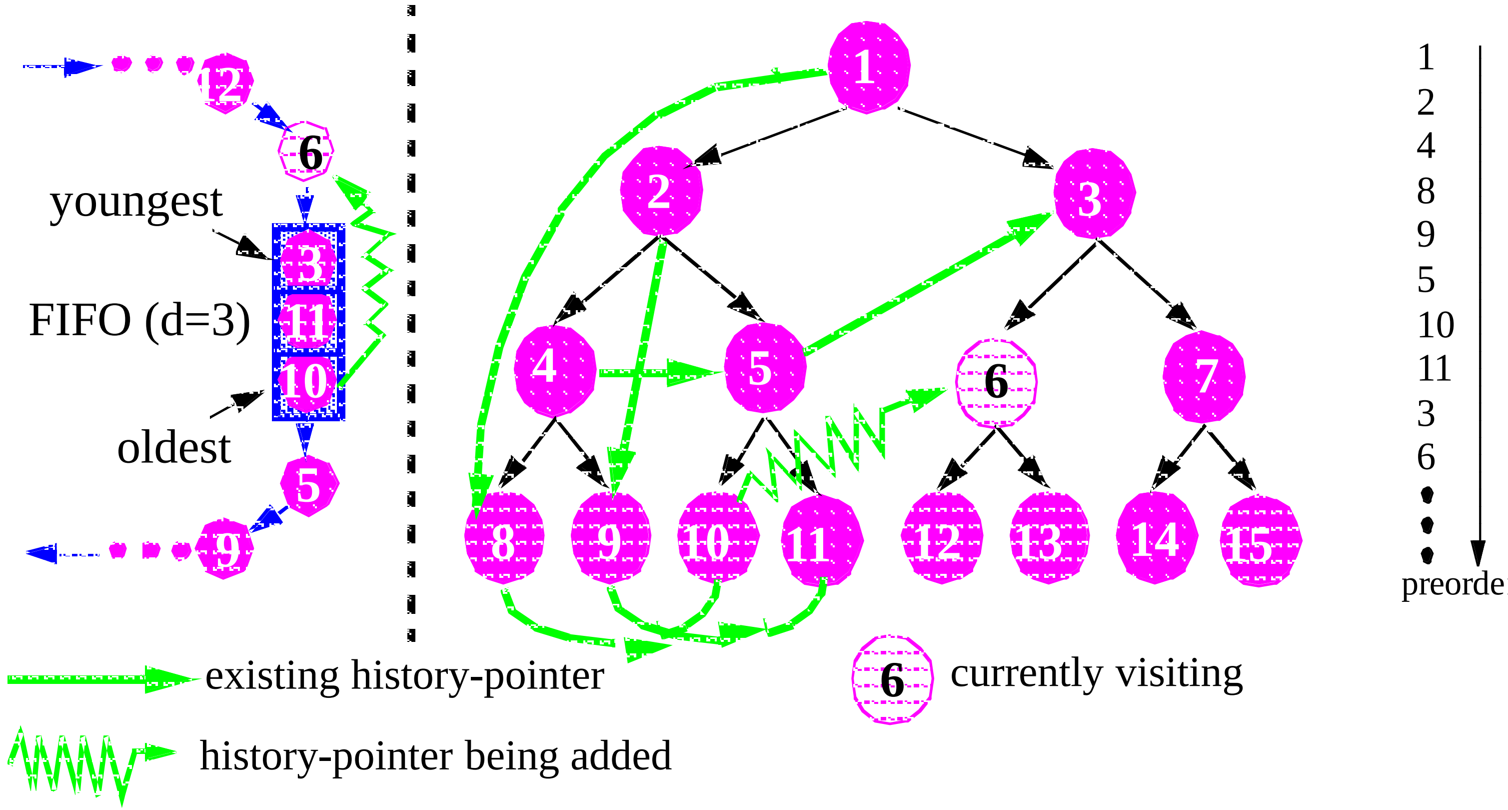
```
preorder(treeNode * t){
  if (t != NULL){
    pf(t->left);
    pf(t->right);
    process(t->data);
    preorder(t->left);
    preorder(t->right);
  }
}
```



- Reasonably effective in practice
- However, little control over the prefetching distance

History-Pointer Prefetching

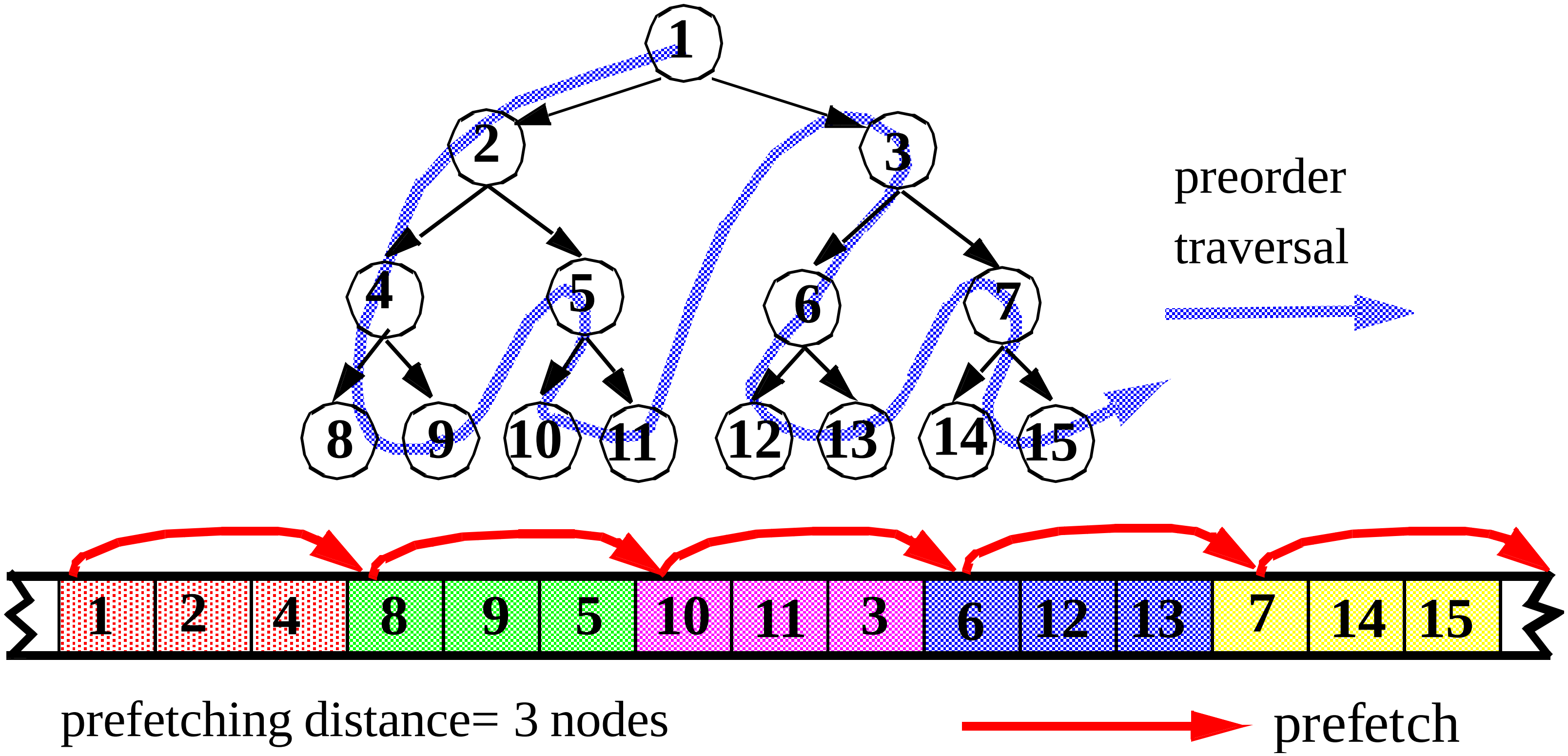
- Add new pointer(s) to each node
  - history-pointers are obtained from some recent traversal



- Trade space & time for better control over prefetching distances

Data-Linearization Prefetching

- No pointer dereferences are required
- Map nodes close in the traversal to contiguous memory



Summary of Prefetching Algorithms for Pointer Structures

	Greedy	History-Pointer	Data-Linearization
Control over Prefetching Distance			
Applicability to Pointer-Based Data Structures			
Overhead in Preparing Prefetch Addresses			
Ease of Implementation			

Summary of Prefetching Algorithms for Pointer Structures

	Greedy	History-Pointer	Data-Linearization
Control over Prefetching Distance	little	more precise	more precise
Applicability to Pointer-Based Data Structures	any	revisited; changes only slowly	must have a major traversal order; changes only slowly
Overhead in Preparing Prefetch Addresses	none	space + time	space if done as shadow structure
Ease of Implementation	relatively straightforward	more difficult	more difficulty

- Greedy prefetching is the most widely applicable algorithm

Today's Class: Locality Analysis and Prefetching

I. Locality Analysis
   A. Temporal
   B. Spatial
   C. Group
   D. Localized Iteration Space

II. Prefetching Pointer-Based Structures

Friday's Class

- Register Allocation: Coalescing