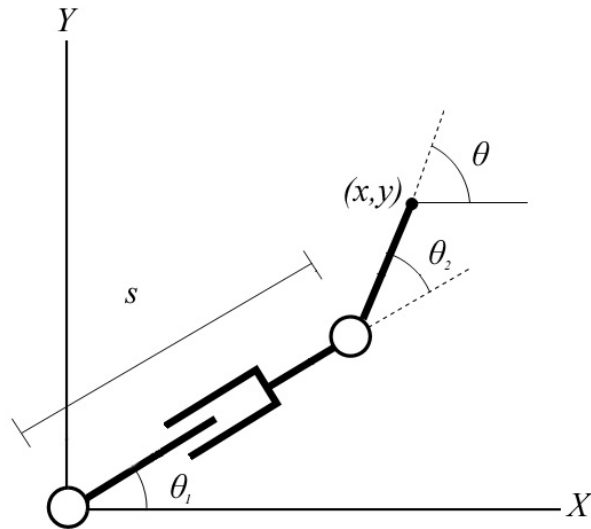


# Final Exam: Introduction to Robotics 16-311 Fall 2005

## Problem #1 [20 pts]

Given  $x$ ,  $y$ , and  $\theta$ ; perform the inverse kinematics of the revolute-prismatic-revolute robot shown below:

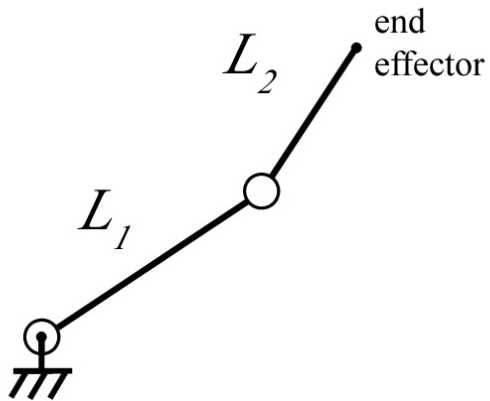


**Problem #2 [20 pts]**

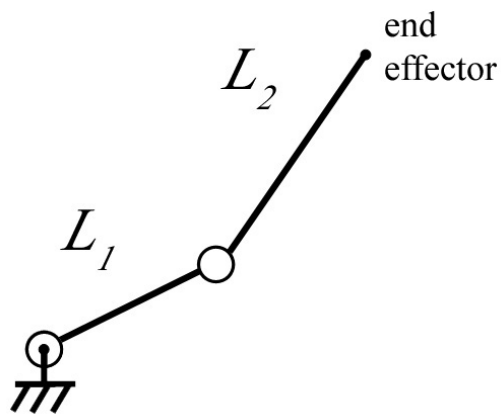
Draw or concisely describe the following:

Note: A circle is different from a disk; remember to shade in if needed. If there is no answer, write NONE. If joint limits are not specified, assume no joint limits.

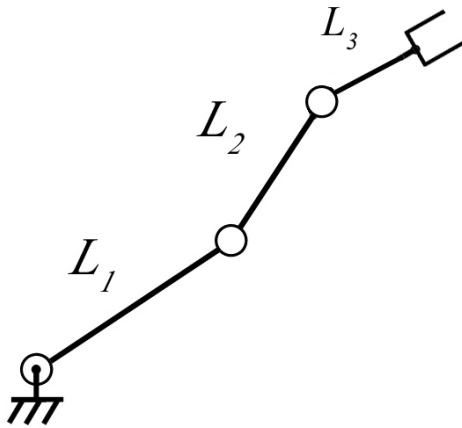
A. the set of points that the end effector can reach for the two-link robot where  $L_1 > L_2$



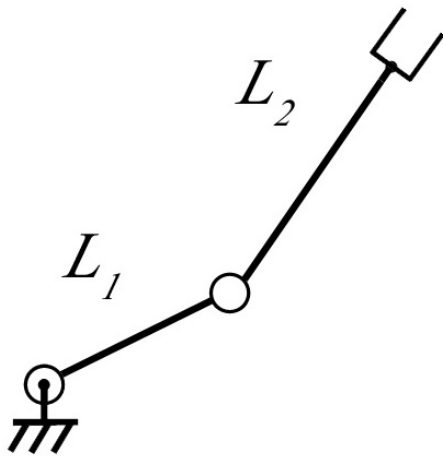
B. the set of points that the end effector can reach for the two-link robot where  $L_1 < L_2$



C. the set of points that the end effector can position and orient the last link for a three link manipulator where  $L_1 > L_2 > L_3$  and  $L_2 + L_3 > L_1$ .



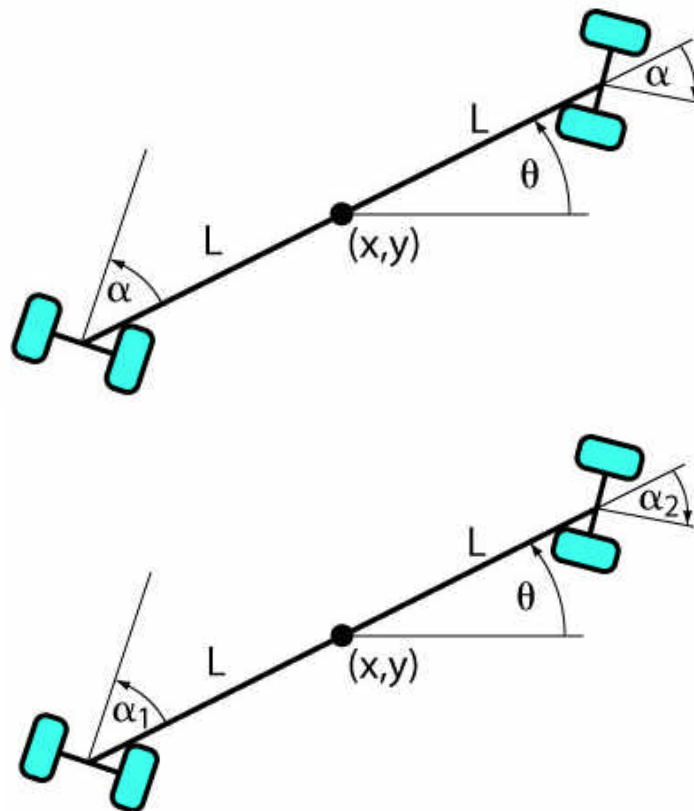
D. the set of points that the end effector can position and orient the last link for a two-link manipulator where  $L_1 < L_2$



**Problem #3 [30 pts]**

Consider the snakeboard-like mechanism described in figure below. This robot can position and orient itself in the plane and has two internal steering wheels, but they are coupled so that they steer in opposite directions.

It has a four dimensional state space:  $x$ ,  $y$ ,  $\theta$ , and  $\alpha$ . Note that  $\alpha$  steers the front and back wheels, but in opposite directions because the front and back wheels are coupled.



A. What is the state space, i.e., what is the  $q$  vector? [2pts]

B. Write out the non-holonomic constraints for this system. [10pts]

C. Verify whether or not the following 5 vectors are in the initial allowable motions. [10pts]

$$\begin{array}{l}
 \text{a) } \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
 \text{b) } \begin{bmatrix} -L \cos(\mathbf{q}) \cot(\mathbf{a}) \\ -L \cot(\mathbf{a}) \sin(\mathbf{q}) \\ 1 \\ 0 \end{bmatrix} \\
 \text{c) } \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}
 \end{array}$$

$$\text{d) } \begin{bmatrix} \cos(\mathbf{q}) \\ \sin(\mathbf{q}) \\ \frac{1}{L} \tan(\mathbf{a}) \\ 0 \end{bmatrix} \quad \text{e) } \begin{bmatrix} -L \cos(\mathbf{q}) \cot(\mathbf{a}) \\ -L \cot(\mathbf{a}) \sin(\mathbf{q}) \\ 0 \\ 1 \end{bmatrix}$$

D. Perform the lie bracket on two of the initial allowable motions [8pts].

**Problem #4 [25 pts]**

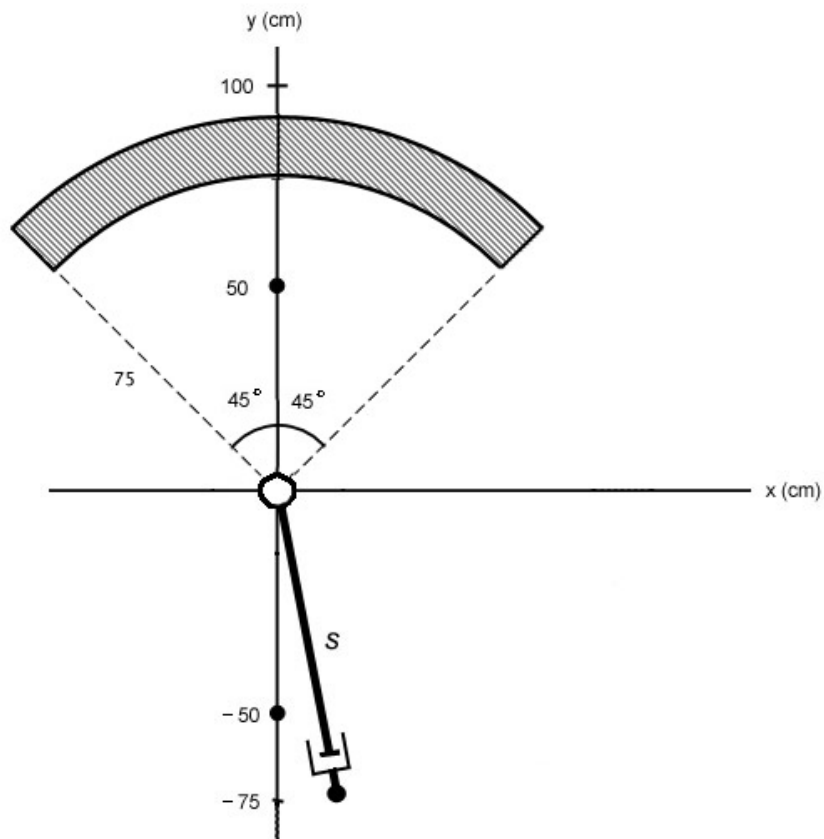
The robot below has a revolute joint with angle  $\theta$  at the base which rotates a prismatic joint with length  $s$  whose range of motion is 0 to 100 cm. The base joint has no limits. The robot is shown in its initial position.

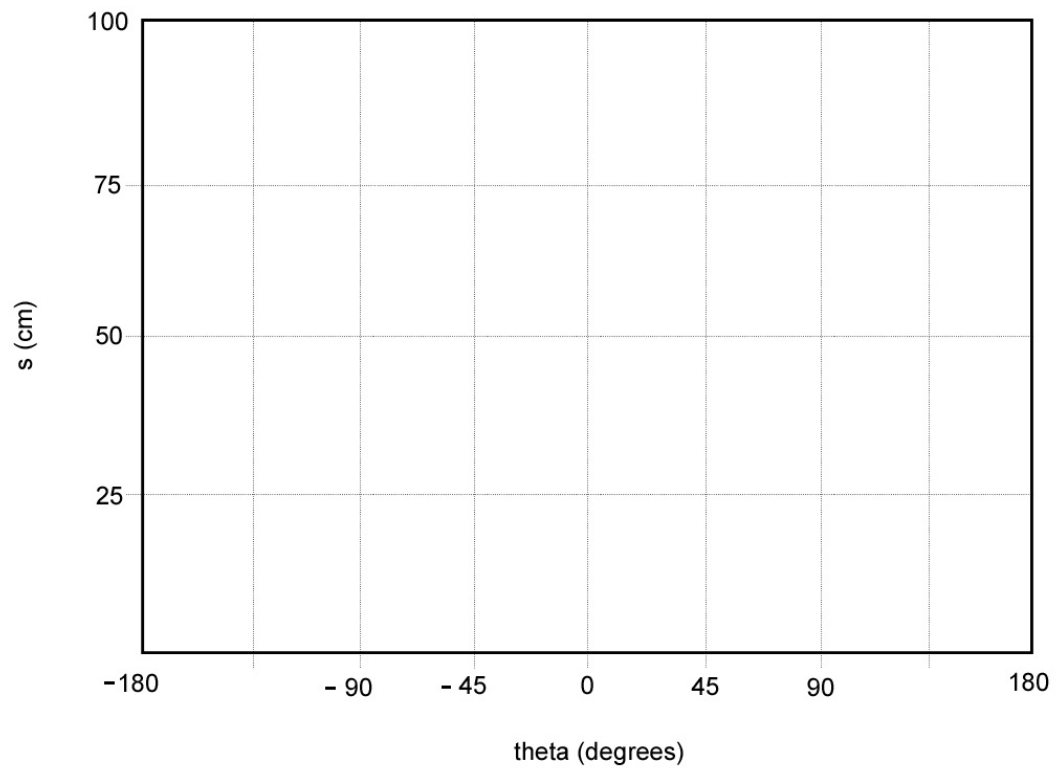
There are 3 obstacles: two point obstacles at  $(0,50)$  and  $(0,-50)$ , and a curved quarter circular wall with radius 75 cm.

Given the following workspace, draw the configuration space [10pts]:

Chose a metric and draw the shortest path in configuration space when the end-effector starts at  $(45,60)$  and arrives at  $(-45,60)$  [10pts]

Draw the path of the end-effector in the work space for the above shortest path [5pts].





**Problem #5 [5 pts]**

Define a Robot (limited to 30 words):