Midterm Exam 16-311 Intro to Robotics

Name:____________________ Team: ____________________

- You will have 1 hour and 15 minutes to complete this exam
- There are 6 questions on 18 pages. Make sure you have all of them.
- When making drawings - be precise. Rounded edges should look rounded, sharp edges should look sharp, sizes should be close to scale. Neatness counts.
- Show your work. Partial credit may apply. Likewise, justify algerbraically your work to ensure full credit, where applicable.
- It should be *very* clear what your final answer is, circle it if necessary.
- You may need to make certain assumptions to answer a problem. State them (e.g. what is optimal).
- You are allowed one *handwritten* two-sided reference sheet for the exam. No cell phones, laptops, neighbors, etc. allowed.
- Good Luck!

1 Vision - 20 pts

A certain video game console camera produces images where each pixel contains red, green, and blue color intensities (RGB), as well as the distance from the camera to the point of incidence for each pixel, referred to as the "depth" (D). We would like to use this RGBD image to detect a player standing in front of the camera.

a) Below is an example depth image from the camera. Threshold it and run the double raster algorithm on it to find the pixels corresponding to the player. Specify your threshold and briefly sketch the thresholded image and double raster steps. You may draw and label rectangular areas instead of writing each value. (7 pts)

$$\begin{bmatrix} 9 & 9 & 11 & 8 & 10 & 7 & 9 & 11 & 8 \\ 8 & 12 & 9 & 4 & 5 & 8 & 9 & 10 & 8 \\ 11 & 8 & 7 & 3 & 5 & 10 & 9 & 11 & 8 \\ 10 & 8 & 9 & 4 & 5 & 11 & 9 & 9 & 8 \\ 11 & 9 & 4 & 3 & 4 & 3 & 8 & 10 & 11 \\ 9 & 10 & 5 & 3 & 5 & 4 & 3 & 11 & 9 \\ 10 & 8 & 4 & 4 & 5 & 3 & 4 & 10 & 9 \\ 8 & 9 & 3 & 4 & 5 & 3 & 5 & 3 & 9 \\ 9 & 8 & 3 & 5 & 3 & 9 & 9 & 4 & 8 \\ 8 & 9 & 4 & 3 & 4 & 11 & 10 & 8 & 10 \\ 10 & 10 & 3 & 3 & 4 & 8 & 8 & 9 & 10 \\ 11 & 7 & 8 & 3 & 3 & 9 & 10 & 11 & 9 \\ 9 & 11 & 11 & 3 & 3 & 10 & 9 & 8 & 7 \\ 8 & 9 & 7 & 3 & 3 & 7 & 12 & 8 & 12 \end{bmatrix}$$

Threshold result:

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Raster result:

b) In reality, the depth values are very noisy. What kind of filter would you apply to the image before finding the player? Briefly describe the effect of the filter width on the filtered image, and explain how you would choose the width. (4 pts)

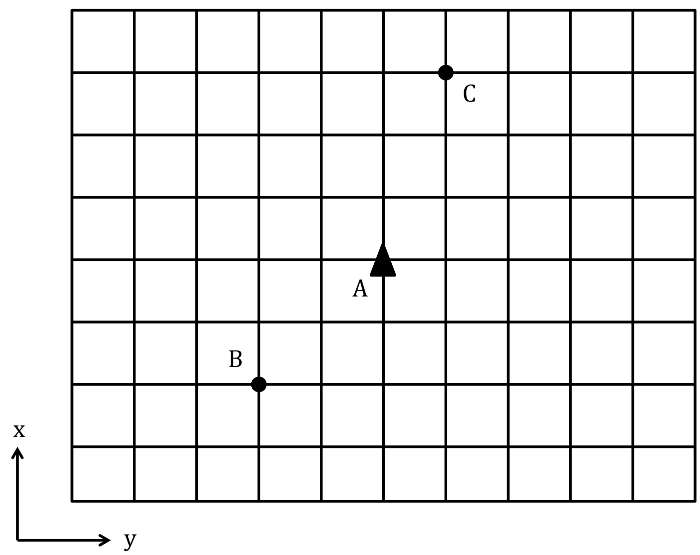
c) Now that we know which pixels correspond to the player and how far away the player is, we would like to estimate the player's height. Let d be the player distance from the camera, f be the camera focal length, and p be the camera pixel density. Distances are in meters, and the pixel density is in pixels/meters. Determine the player's height in meters using these quantities. (3 pts)

d) Finally, we take a photo of the player's face. Describe in **5 sentences or less** how we could use techniques from class with this photo to find the player's face in other images of the player. (6 pts)

2 Path Planning - 20 Points

Many vehicles have constraints that make path planning difficult. For example, bicycles and cars have a limited turn radius and cannot turn in place. In this problem we will explore the effect these constraints have on planning techniques we have learned so far.

Consider the following bicycle start position A. Note its orientation. We are interested in reaching points B or C with any final orientation.

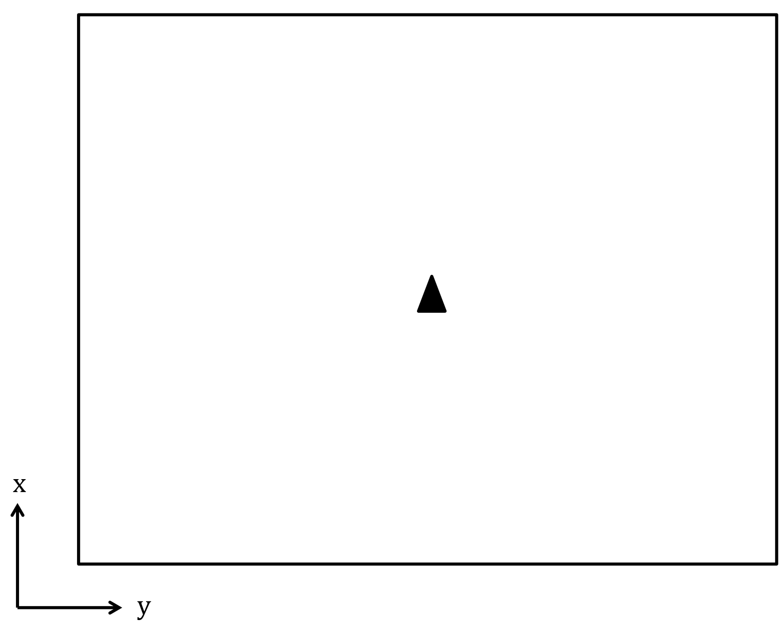


a) Is point B or C closer by the L2 metric? With the L1 metric? (2 pts)

b) Draw in the path the bicycle might take to go from A to B. Does the L2 metric reflect this true path length? What about the L1 metric? (3 pts)

c) Say we try to plan for the bicycle with a wavefront planner using the L2 metric. What problem will the path returned by the planner have? (5 pts)

d) Briefly describe a better distance metric for bicycle motion. How might you utilize your suggested distance metric in a wavefront planner? Sketch some continuous wavefronts with this metric in the space below. (8 pts)

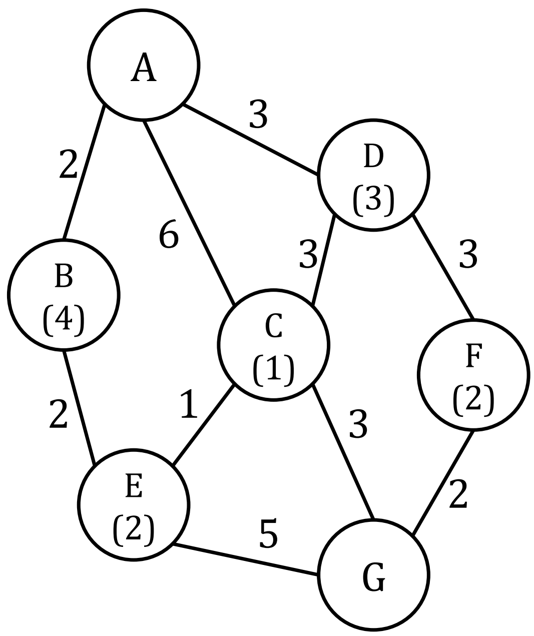


e) Consider now the same planning problem with a Rapidly-exploring Random Tree (RRT). What step of the RRT algorithm would you use your better distance metric in? (2 pts)

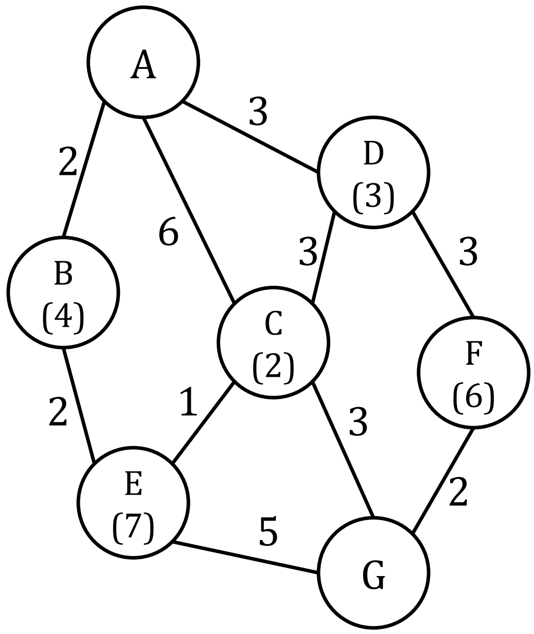
3 Graph Search - 18 Points

A* is a powerful algorithm with many applications in robotics. In class, we have applied A* to a variety of path planning problems with given heuristics. In this problem, we will explore how the choice of heuristic can affect A*'s performance.

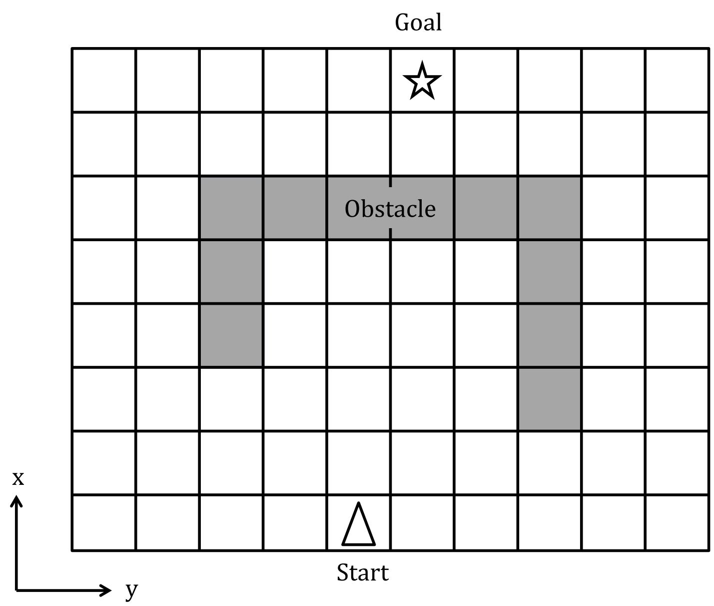
a) Perform A* on the following graph, breaking ties alphabetically. Heuristic values are written in the nodes. Is this path optimal? Why or why not? (6)



b) Perform A* on the following graph again, but with the following heuristic values instead. Is this path optimal? Why or why not? What is different about the heuristic?

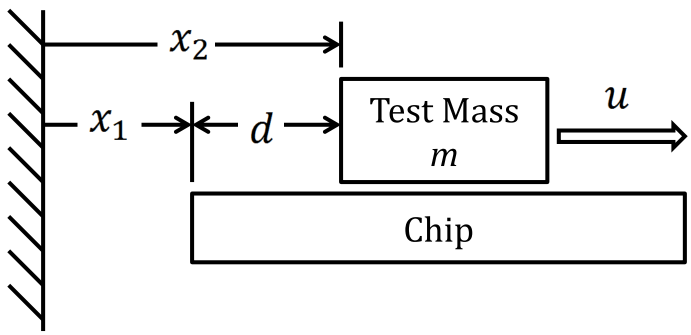


c) Consider the discrete path planning problem depicted below. Assume 4-connectivity. What is an admissible heuristic for a position $h(x, y)$ in this problem?



4 Dynamics and Control - 20 Points

At the heart of every MEMS accelerometer is a small test mass actuated by electromagnetic forces. In this problem, we will analyze the control loop positioning the test mass inside the chip with a 1D example.



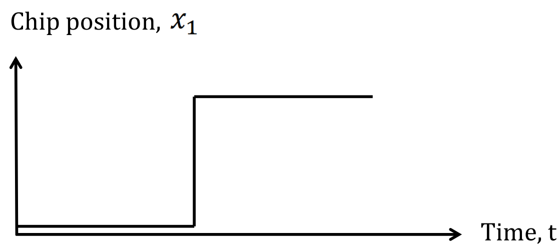
The position of the chip is given by x_1. The position of the test mass is x_2, its mass is m, and the force applied to it by the chip is u. Let $d = x_2 - x_1$ be the position of the test mass relative to the chip. The equations of motion are:

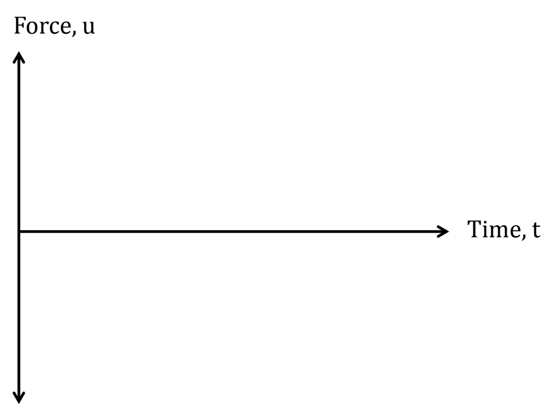
$$\ddot{x_1} = \frac{u}{m} - \ddot{d}$$

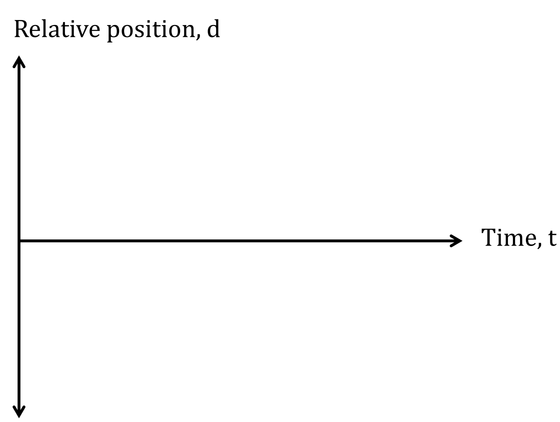
We see directly that we can measure the acceleration of the chip knowing only the test mass, the force, and the second derivative of the relative position.

Note: When sketching responses, please exaggerate small differences for clarity and label the plots.

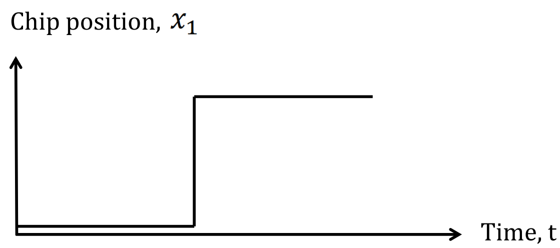
a) Consider a P controller of the form $u = -k_p d$. Sketch the responses u and d over time for a small k_p when the chip is bumped as shown below. On the same plots, sketch the responses for a large k_p for the same bump. (5 pts)

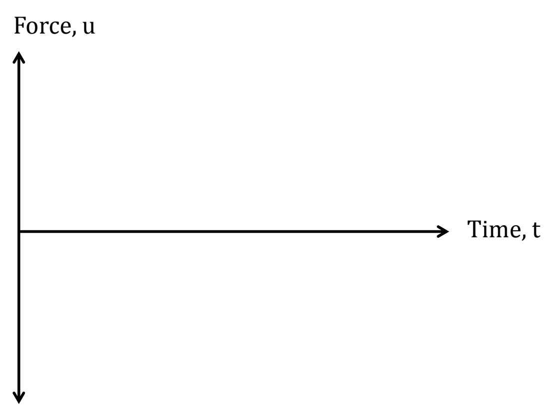


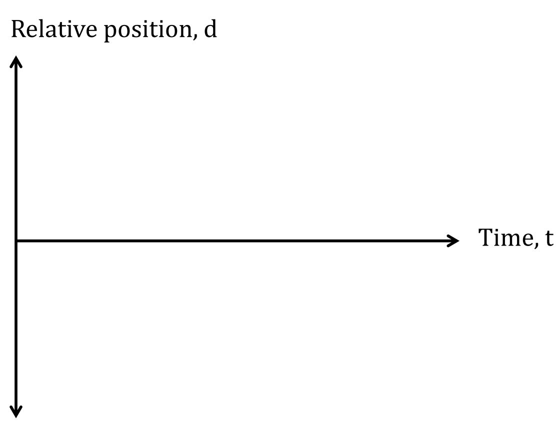




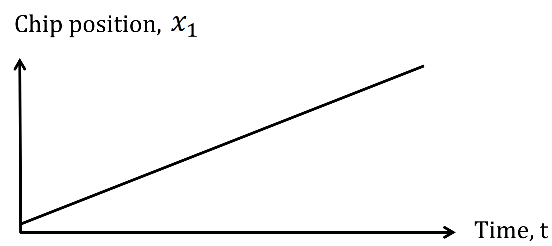
b) We have upgraded to a PD controller of the form $u = -k_p d - k_d \dot{d}$. Sketch the new responses u and d over time for a large k_p and a small k_d when the chip is bumped again. Now sketch the responses for the same large k_p but with a large k_d. (5 pts)

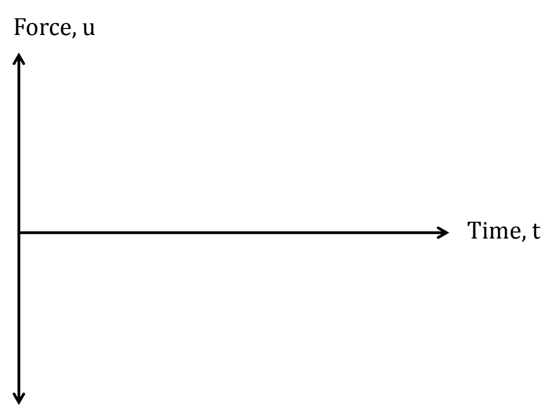


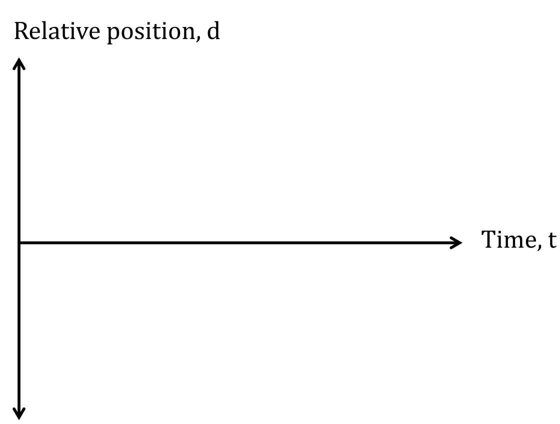




c) Finally, sketch the responses u and d over time for the large k_p and k_d when the chip is moving at constant velocity. (5 pts)







d) In reality it is difficult to estimate $\ddot{d}$. Why? Hint: Think about derivative masks from computer vision. (3 pts)

e) If we assume $\ddot{d} = 0$, what problems will our accelerometer have with a P controller? With a PD controller? (2 pts)

5 Localization - 16 Points

The localization techniques taught in class can be applied to a variety of estimation problems. One common estimation problem is *parameter estimation*, wherein various parameters of a system are estimated by observing its behavior.

Consider again our 1D cart, but with an unknown constant mass. This time, we observe its state x and applied forces u. Let m denote the unknown mass in kg. We also know that its mass is somewhere between 1 kg and 10 kg. In this problem we will formulate the parameter estimation problem using localization techniques from class.

a) Write out the transition function $m_{t+1} = f(m_t)$. (3 pts)

b) Write out the observation function $y_t = g(m_t)$. Specify what the observations y_t are in terms of our known quantities. Hint: Your equations of motion from the previous problem may be useful. (5 pts)

c) We would like to now consider the effect of the mass decreasing over time as the cart expends its fuel. Specifically, we know that the cart's mass decreases by 0.1 kg per unit of u at each time step. Write out the transition function $m_{t+1} = f(m_t, u_t)$. (3 pts)

d) Briefly write the pseudocode for estimating the cart mass using a Bayes Filter. How can you avoid the filter converging to the wrong mass? (5 pts)

6 Guest Speakers - 6 pts

Match the topic to the speaker:

Yaser Sheikh	Robot C
Timothy Friez	Modeling and Control
Nathan Michael	Structure from Motion