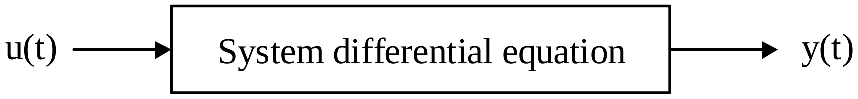
Dynamic systems that are described by linear, constant-coefficient, differential equations are called *linear time-invariant* (LTI) *systems*.

2. Transfer Function of Linear Time-Invariant (LTI) Systems

The transfer function of a linear, time-invariant system is defined as the ratio of the Laplace transform of the output (response function), $Y(s) = \mathcal{L}\{y(t)\}$, to the Laplace transform of the input (driving function) $U(s) = \mathcal{L}\{u(t)\}$, under the assumption that all initial conditions are zero.



Taking the Laplace transform with zero initial conditions,

Transfer function: $G(s) = \dfrac{Y(s)}{U(s)}$

A dynamic system can be described by the following time-invariant differential equation:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t)$$
$$= b_m \frac{d^m u(t)}{dt^m} + b_{m-1} \frac{d^{m-1} u(t)}{dt^{m-1}} + \cdots + b_1 \frac{du(t)}{dt} + b_0 u(t)$$

Taking the Laplace transform and considering zero initial conditions we have:

$$\left(a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0\right)Y(s) = \left(b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0\right)U(s)$$

The transfer function between u(t) and y(t) is given by:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} = \frac{M(s)}{N(s)}$$

where G(s) = M(s)/N(s) is the transfer function of the system; the roots of N(s) are called *poles* of the system and the roots of M(s) are called *zeros* of the system. By setting the denominator function to zero, we obtain what is referred to as the *characteristic equation*:

$$a_n s_n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = 0$$

We shall see later that the stability of linear, SISO systems is completely governed by the roots of the characteristic equation.