Homework 3 — CS 818A4 — Spring 2004

Problem 1

Derive the axiom schemata

$$m \leq j \leq n \Rightarrow \Big(\Big(\bigodot_{i=m}^{n} p(i)\Big) \Rightarrow (p(j) \ * \ \mathbf{true})\Big)$$

from the axiom schematas for iterating separating conjunction given earlier on the first page of Chapter 5 of the class notes.

Problem 2

If τ is an S-expression, then $|\tau|$, called the *flattening* of τ, is the sequence defined by:

$$|a| = [a] \quad \text{when } a \text{ is an atom}$$

$$|(t_1 \cdot t_2)| = |\tau_1| \cdot |\tau_2|.$$

Here $[a]$ denotes the sequence whose only element is a, and the "$\cdot$" on the right of the last equation denotes the concatenation of sequences.

Define and prove correct (by an annotated specification of its body) a recursive procedure $\mathsf{flatten}$ that mutates a tree denoting an S-expression τ into a singly-linked list segment denoting the flattening of τ. This procedure should not do any allocation or disposal of heap storage. However, since a list segment representing $|\tau|$ contains one more two-cell than a tree representing τ, the procedure should be given as input, in addition to the tree representing τ, a single two-cell, which will become the initial cell of the list segment that is constructed.

More precisely, the procedure should satisfy

$$\{\mathsf{tree}\ \tau\ (\mathsf{i}) \ * \ \mathsf{j} \mapsto -, -\}$$
$$\mathsf{flatten}(\mathsf{i}, \mathsf{j}, \mathsf{k})$$
$$\{\mathsf{lseg}\ |\tau|\ (\mathsf{j}, \mathsf{k})\}.$$

(Note that $\mathsf{flatten}$ must not assign to the variables i, j, or k.)

Problem 3

Show, by an informal metaproof, that the following laws hold, whenever p is an arbitrary assertion and i and i' are intuitionistic assertions:

$$i * p \Rightarrow i \qquad i \Rightarrow p \mathrel{-\!\!*} i \qquad i * i' \Rightarrow i \wedge i'.$$