Logistics Exercises (from 1st Class)

Exercise 1:

Given a Buy at Bulk problem instance *BB = (G,c,u)* on a graph *G* with *k* cables types having costs $c_i, 1 \le i \le k$ and capacities $u_i, 1 \le i \le k$.

Transform it into a Deep Discount problem instance *DD* by a mapping Φ

$$BB \xrightarrow{\Phi} DD$$

by setting edge costs

$p_i = c_i, \forall i$ (price of installation per unit length),

$r_i = \frac{c_i}{u_i}, \forall i$ (rate of routing per unit flow per unit length)

and call this $\Phi(BB)$.

Conversely, given a *DD* instance *DD = (G,p,r)* with *k* contracts, transform it into a BB instance using a mapping Ψ

$$DD \xrightarrow{\Psi} BB$$

by setting costs and capacities

$c_i = p_i, \forall i$ (cost per unit length),

$u_i = \frac{p_i}{r_i}, \forall i$ (capacity)

and call it $\Psi(DD)$.

For a problem instance X, we abuse notation to let X also denote the cost of a feasible solution to it. Let X^* denote the cost of an optimal solution to X. Show that

$$\Phi^*(BB) \le 2BB^*$$

$$\Psi^*(DD) \le 2DD^*$$

Exercise 2:

The following two statements are true for the Deep Discount problem. Show that they do not hold for the Buy at Bulk problem.

a) In an optimum solution, there is at most contract per edge
b) The support of the optimal solution is acyclic (in undirected graphs), i.e. it is a tree.

Exercise 3:

Show that the following property holds for edge costs.

i.e. show that $f^i(a) + f^j(b) \ge \min[f^i(a+b), f^j(a+b)]$

where *i* and *j* represent contract types, and $f^i(a)$ is the cost at flow *a* using contract *i*. Note that this implies 2 a) for the DD problem.

Exercise 4:

Given a graph G = (V,E) with edge lengths l_e, a single sink $t \in V$, sources $\{v_j\}_1^S = S \subset V$ with demands $\{dem_j\}_1^S$, and

given k cable types, with characteristics (p_i, r_i) (where p_i is the price of installation per unit length, r_i is the rate of routing per unit flow per unit length),

what is the minimum cost solution when the price is very small, i.e. $p_i = \frac{1}{LD}$ and the rate $r_i = 1,\ 1 \le i \le k$

where $L = \sum_{e \in Edges} l_e$ and $D = \sum_{j=1}^{S} dem_j$

with the restriction that all demands are 1? (Hint: It is a specific kind of shortest tree.)