Name: **Andrew/CS ID:**

15-780 Midterm, Fall 2007

- Place your name and your andrew/cs email address on the front page.
- The exam is open-book, open-notes, no electronics other than calculators.
- The maximum possible score on this exam is 100. You have 90 minutes.
- We have given you plenty of space to do problems (do not be alarmed by the number of pages). There are 13 pages. Please check that you have them all.
- Dont spend too much time on any one problem. If you get stuck on any of the problems, move on to another one and come back to that problem if you have time. Good luck!

1. **Search (14 pts)**

   (a) Please circle the correct multiple choice answer for the following questions. (2 pts each)

      i. On a **4-connected** grid, A* with a zero heuristic reduces to:
      A) Breadth First Search B) Depth First Search
      C) Iterative Deepening A* D) Depth First Iterative Deepening
      Answer: A

      ii. On a **4-connected** grid, Iterative Deepening A* with a zero heuristic reduces to:
      A) Breadth First Search B) Depth First Search
      C) A* D) Depth First Iterative Deepening
      Answer: D

   (b) Now consider an **8-connected** grid with obstacles. Which of the following heuristics is admissible for A* (circle YES or NO below heuristic)? (1 pt each)

      i. Euclidean distance to goal
      YES NO
      Answer: YES. Generally on a grid, diagonal moves cost $\sqrt{2}$, which makes Euclidean distance admissible. Since we didn't say this, if you said that diagonal moves were worth one (or drew a picture with this), we accepted NO here.

      ii. Euclidean distance to goal times 2
      YES NO
      Answer: NO

      iii. Euclidean distance to goal divided by 2
      YES NO
      Answer: YES

      iv. Manhattan distance to goal
      YES NO
      Answer: NO

      v. Distance to goal along optimal path
      YES NO
      Answer: YES

   (c) For each heuristic that you deemed admissible in Part (b), order them from least informed to most informed. (5 pts)
   Answer: Euclidean divided by 2 then Euclidean then Distance along optimal

2. **Propositional and First-Order Logic (10 pts)**

   For this problem, define your own vocabulary, but be consistent throughout.

   (a) Translate the following sentences into first-order logic. (3 pts each)

      i. Every person who buys insurance is smart.
         Answer: P(x) means x is a person, I(x) means x buys insurance, S(x) means x is smart
         $(\forall x)(P(x) \wedge I(x)) \Rightarrow S(x)$

      ii. No one goes to baseball games.
         Answer: G(x,y) means x goes to y. B(x) means x is a baseball game.
         $(\forall x)(\forall y)(P(x) \wedge B(y)) \Rightarrow \neg G(x, y)$

   (b) Suppose the universe consists of two people, Ted and Lindsey, and one baseball game, Yankees vs. Red Sox. Using this universe, write out a satisfying model for your translation from problem 2(a)ii. Your model should include a table of values of the functions and predicates used. Please also state whether this is the only satisfying model using this universe (circle ONLY or NOT ONLY)? (4 pts)

      i. No one goes to baseball games.
         Answer: P(T) = true, P(L) = true, P(YvR) = false, B(YvR) = true, B(T) = false, B(L) = false, G(T,YvR) = false, G(L,YvR) = false
         This is the ONLY satisfying model in this universe with this translation.
         ONLY   NOT ONLY

3. **Planning**

   (a) Consider the problem of planning to pass the Graduate AI class. Suppose we simplify this problem so that its state consists of the three predicates Awake (indicating whether the student is awake enough to function), PassedMidterm (indicating whether the student has passed the midterm), and ProjectDone (indicating whether the student has finished the final project). The initial state is

      Awake $\wedge$ ¬PassedMidterm $\wedge$ ¬ProjectDone

      There are three operators:

      - DrinkCoffee (no preconditions, postcondition Awake)
      - TakeMidterm (precondition Awake, postcondition PassedMidterm $\wedge$ ¬Awake)
      - DoProject (precondition Awake, postcondition ProjectDone $\wedge$ ¬Awake)

The goal description is

PassedMidterm $\wedge$ ProjectDone

Please write out the plan graph for this problem, up to and including the third state level (counting the initial conditions as the first state level), **omitting** all mutexes.

Solution: In the hand drawn plan graph attached, we've used the abbreviations A=Awake, PM=PassedMidterm, and PD=ProjectDone for literals, and DC=DrinkCoffee, TM=TakeMidterm, and DP=DoProject for actions.

The most common error on this part was to add additional arcs into action nodes corresponding to extra preconditions. E.g., people often put an arc from ¬Awake to DrinkCoffee, which would correspond to a precondition that you have to be un-awake to drink coffee. This precondition is not listed in the formal action definition above: the definition above says that it is legal to drink coffee if you're already awake, and doing so doesn't change your awakeness status.

Another common error was to duplicate state nodes. E.g., one can achieve ¬Awake either using DP or TM, so it was tempting to put one copy of ¬Awake connected to each of these action nodes.

(b) In the plan graph from problem 3a, which of the following pairs are mutex? Why or why not?

i. the actions TakeMidterm and DoProject at the first action level
Solution: These are mutex, since they delete each other's precondition Awake.

ii. the predicates PassedMidterm and ProjectDone at the second state level
Solution: These are mutex, since the only way to achieve both is to do both TakeMidterm and DoProject at the first action level, and TM and DP are mutex at the first action level (as shown immediately above).

iii. the predicates PassedMidterm and Awake at the second state level
Solution: To achieve PM $\wedge$ A at state level 2, at the first action level we must do TM and either DC or the maintenance action for A. But, TM is mutex with both DC and maintain(A), since TM deletes A. So, PM and A are mutex at state level 2.

iv. the predicates PassedMidterm and Awake at the third state level
Solution: PM and A are *not* mutex at the third state level, since the plan TakeMidterm, DrinkCoffee achieves both.

4. **Bayesian Network**

This question focus on the Bayesian network (BN) in Figure 1.

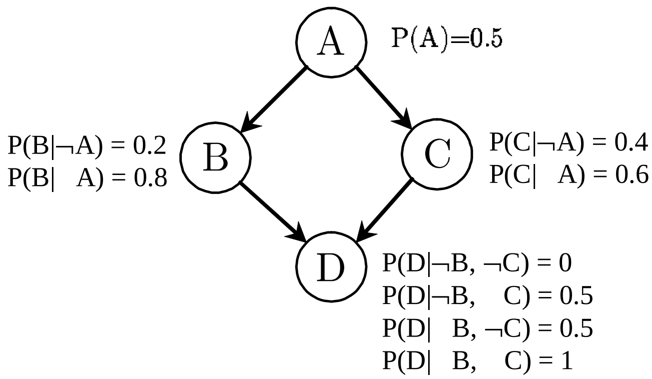


Figure 1: A simple BN and the conditional probability table (CPT).

(a) Specify TRUE or FALSE for each of the statements below (no need to explain).

i. B is conditionally independent of C given A.
TRUE FALSE

Answer: TRUE.

ii. The Markov blanket of C is $\{A, D\}$.
TRUE FALSE

Answer: FALSE. The Markov blanket of C is $\{A, D, B\}$.

iii. If a new BN is constructed by adding an edge from B to C and removing the edge from B to D, the new BN cannot represent the original distribution, no matter how you adjust the CPT.
TRUE FALSE

Answer: TRUE. The inexistence of an edge is more important in a BN. In the new BN, B is always conditionally independent of D given C, which is not true in the original distribution.

iv. $P(A, B, D) < 0.4$.
TRUE FALSE

Answer: TRUE. You can use $P(A,B,D) < P(A,B) = 0.5 \times 0.8 = 0.4$, or

$$\begin{aligned} P(A,B,C,D) &= P(A)P(B|A)P(C|A)P(D|B,C) \\ &= 0.5 \times 0.8 \times 0.6 \times 1 = 0.24 \\ P(A,B,\neg C,D) &= P(A)P(B|A)P(\neg C|A)P(D|B,\neg C) \\ &= 0.5 \times 0.8 \times 0.4 \times 0.5 = 0.08 \\ P(A,B,D) &= P(A,B,C,D) + P(A,B,\neg C,D) = 0.32 < 0.4 \end{aligned}$$

(b) During MCMC (Gibbs) sampling, what is the probability of sampling C as true, if all variables are currently assigned as true?

Answer: $MB(C) = \{A, B, D\}$, so we have

$$\begin{aligned} P(A,B,C,D) &= P(A)P(B|A)P(C|A)P(D|B,C) \\ &= 0.5 \times 0.8 \times 0.6 \times 1 = 0.24 \\ P(A,B,\neg C,D) &= P(A)P(B|A)P(\neg C|A)P(D|B,\neg C) \\ &= 0.5 \times 0.8 \times 0.4 \times 0.5 = 0.08 \\ P(C|A,B,D) &= \frac{P(A,B,C,D)}{P(A,B,C,D) + P(A,B,\neg C,D)} \\ &= 0.24/(0.24 + 0.08) = 3/4 \end{aligned}$$

5. **Decision Tree**

The best-selling computer game *Black and White* lets a player train a Tiger using the decision tree technique. The player can *stroke* (reward) or *slap* (punish) the Tiger, for example based on what it eats. The tiger tries to learn the player's preferences based on four attributes: *Type*, *Size*, *Time*, and *EnergyLevel*, which is real-valued. In this question, you need to draw the decision tree learned from the following data set. You can use $\log_2(3) = 1.5850, \log_2(5) = 2.3219$. Hint: for the root attribute, $H(F) = 0.8813$, $H(F|T) = 0.6755$, $H(F|S) = 0.7900$, $H(F|W) = 0.7900$.

(a) To handle the real-valued attribute E, we search for one threshold θ that splits the data into the two groups, $E \geq \theta$ and $E < \theta$. The best threshold is the one that results in highest information gain,

$$IG(F|E:\theta) = H(F) - H(F|E \geq \theta)P(E \geq \theta) - H(F|E < \theta)P(E < \theta)$$

No	Type	Size	When	EnergyLevel	Feedback?
1	deer	small	night	91	slap
2	sheep	small	day	85	slap
3	deer	big	night	79	slap
4	sheep	small	night	75	slap
5	deer	big	day	65	stroke
6	human	big	night	58	slap
7	human	small	night	40	slap
8	human	small	day	29	slap
9	sheep	small	day	20	stroke
10	sheep	big	night	15	stroke

Figure 2: Tiger food training data set, ordered by *EnergyLevel*.

How many possible thresholds should we consider in order to compute $IG(F|E : \theta)$? (2 pts)

Answer: 3. There is no need to compute all 8 possible thresholds. You only need to compute the thresholds between two adjacent values that have different labels, i.e. 70 (between 75 and 65), 62 (between 65 and 58), and 25 (between 20 and 29).

(b) What is the best split, and the corresponding IG, on EnergyLevel at the root? (2 pts)

Answer:

$$
\begin{aligned}
IG(F|E:70) &= .8813 + .6 \times (.5\log_2(.5) + .5\log_2(.5)) = .2813 \\
IG(F|E:62) &= .8813 + .5 \times (.2\log_2(.2) + .8\log_2(.8)) \\
&\quad + .5 \times (.4\log_2(.4) + .6\log_2(.6)) = .0349 \\
IG(F|E:25) &= .8813 + .8 \times (1/8\log_2(1/8) + 7/8\log_2(7/8)) = .4464
\end{aligned}
$$

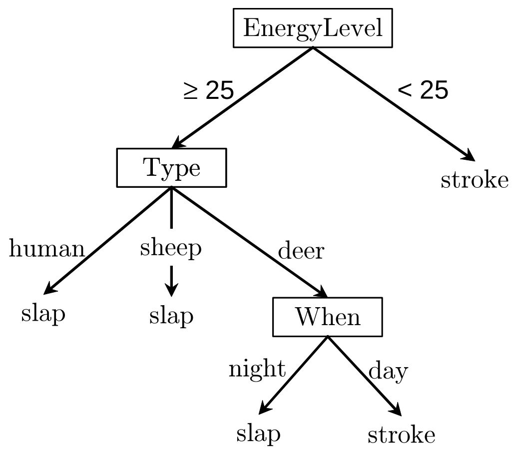
The best split is 25 (between 20 and 29), and the IG would be 0.4464.

(c) A discrete attribute in the decision tree will not appear again under itself. Can a real-valued attribute re-appear under itself? Why or why not? (3 pts)

Answer: Yes, real-valued attribute can re-appear under itself. After a discrete attribute is split upon, all branch under the split will have the same value for this attribute, so IG will be zero. However for a real-valued attribute, the values will still be different, and may benefit from a further split. Hence you need to search real-valued attributes again even if it had been split upon.

(d) Now draw the decision tree learned from this data set. (5 pts)

Answer:



Root level: Using the hints, $IG(F|T) = H(F) - H(F|T) = 0.2058$, $IG(F|S) = IG(F|W) = 0.0913$. Hence *EnergyLevel* has highest IG with $IG(F|E : 25) = 0.4464$, see above.

Second level: Under $E \geq 25$, $H(F) = .5436$, and we have a tie since

$$H(F|T) = H(F|S) = H(F|W) = -3/8 \times (1/3 \log_2(1/3) + 2/3 \log_2(2/3)) = .3444$$

This is lower than the conditional entropies of splitting *EnergyLevel*,

$$H(F|E : 70) = -.5 \times (.25 \log_2(.25) + .75 \log_2(.75)) = .4056$$

$$H(F|E : 62) = -5/8 \times (.2 \log_2(.2) + .8 \log_2(.8)) = .4512$$

Only the tree based on the first attribute *Type* is shown above, but any tree created by choosing *Type*, *Size*, and *When* is considered correct (all with IG 0.1992).

Third level: Under $E \geq 25$ and Type being deer, again we have a tie since *When* or *EnergyLevel* with threshold 70 can separate *Feedback* perfectly. Either attribute will be considered correct, but only the tree splitting on *When* is shown.

6. **Hidden Markov Models (HMMs) (15 pts)**

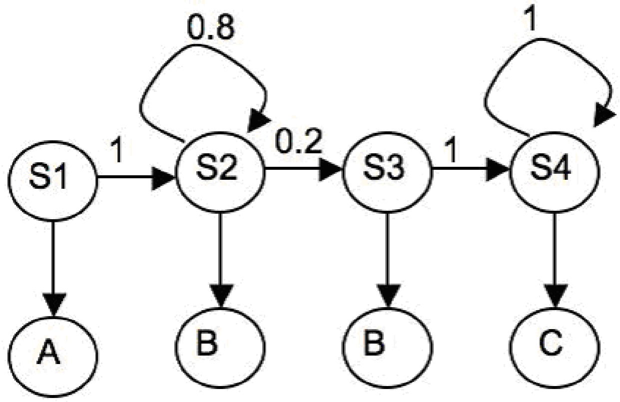


Figure 3: HMM for part 6a

(a) Inference in HMMs

Consider the HMM in Figure 3. S1 through S4 are the states in the HMM. Transition probabilities are listed above the transition edges. Emissions are deterministic, that is each state can only output one symbol which is listed below that state. We always start at state S1 (that is $\pi_{S1} = 1$). (3 pts each)

i. What is $p(O_4 = B)$?

There are two ways to output B as the fourth observation: Either we are at state S2 or at state S3. To be in S2 after four steps we need to stay there between the 2nd and 4th steps which will happen with probability $1 * 0.8^2$. To reach state S3 after 4 steps we need to stay in state S2 for one step and then transition. This will happen with probability 1*0.8*0.2. Thus the total probability is: $0.8 *^2 + 0.8 * 0.2 = 0.8$

ii. What is $p(O_4 = C)$?

We can only output B or C in the fourth step and since p(O4=B) = 0.8 we get p(O4=C) = 0.2

iii. Circle the sign that describes the relationship between the following two values. Use 'cannot be determined' for cases where either there is not enough data to determine the relationship.

$p(O_{100} = B)$? $p(O_{100} = C)$ $<$ $>$ $=$ cannot be determined

Circle $<$. To reach a state emitting B after 100 steps we need to state in state S2 for 97 or 98 steps. This will happen with probability $0.8^{97} * 0.2 + 0.8^{98}$. This number is clearly less then the probability that we will stay in state S2 for only one step (0.2), which is a lower bound on the probability of emitting C in the 100th step.

(b) Learning in HMMs

We are trying to determine the optimal number of states for our HMM. We are considering HMMs with 4 or 5 states. Assume that the optimal number is 4 states (that is, an HMM with 4 states accurately captures the underlying joint distribution). Also assume that we have *unlimited training data*. Answer yes or no (no explanation needed). (6 pts total)

i. A HMM with 5 states will likely lead to a better likelihood for the training data.
NO. With unlimited training data both models will perform the same for the training data. With such data we will learn the correct model for the 4 states HMM. The 5 states HMM will also be perfect for the data (you cannot reduce the training data likelihood by adding more states).

ii. A HMM with 5 states will likely lead to a better likelihood for the test data.
NO. With unlimited training data we will recover the correct HMM for the 4 states model and it will perform as well as the 5 states HMM.

7. **MDPs, LPs, Duality**

(a) Consider the following stochastic planning problem: we have two biased coins, labeled A and B. Coin A turns up heads with probability $P_A = 0.7$, while coin B turns up heads with probability $P_B = 0.4$. The coins start in state HT, that is, coin A shows heads and coin B shows tails. On each turn, if both coins show the same side, we receive a reward of \$1; otherwise we receive \$0. Then, we pick one of the two coins to flip, and leave the other one in its current state.

Draw a transition diagram for this Markov decision process. Label the states and indicate the reward for each state, and write the transition probabilities on each arc.

Solution: See attached hand drawn MDP.

(b) What is the optimal action in state HH for the MDP of problem 7a? Hint: there's a reason that we haven't specified the discount factor.

Solution: The optimal action is to flip coin A, since it has a higher probability of keeping us in state HH. Note that HH is the highest-value state in this MDP, so we want to stay in state HH no matter what the discount factor is. (In HT or TH there is no immediate reward. TT has an immediate reward, but we will leave it faster than we leave HH: the highest probability of staying in TT comes from flipping coin B, which only yields a 60% chance of remaining in TT (as compared to flipping coin A in state HH, which yields a 70% chance of remaining).)

(c) Now consider the smaller MDP with states 1 and 2 and actions A and B, whose transitions are given by the following tables:

$P(s_{t+1} \mid s_t, A) =$

	$s_{t+1} = 1$	$s_{t+1} = 2$
$s_t = 1$	$\frac{2}{3}$	$\frac{1}{3}$
$s_t = 2$	$\frac{1}{2}$	$\frac{1}{2}$

$P(s_{t+1} \mid s_t, B) =$

	$s_{t+1} = 1$	$s_{t+1} = 2$
$s_t = 1$	$\frac{1}{3}$	$\frac{2}{3}$
$s_t = 2$	$\frac{1}{3}$	$\frac{2}{3}$

and whose rewards are

$$r(1) = 0 \qquad r(2) = 1$$

Assuming a discount factor of $\gamma = \frac{1}{2}$, and assuming that we start in state 1, we can find the optimal value function for this MDP using the following linear program:

$$\text{minimize } x \text{ subject to}$$
$$\begin{aligned}
x &\geq 0 + \frac{1}{2}\left(\frac{2}{3}x + \frac{1}{3}y\right) \\
y &\geq 1 + \frac{1}{2}\left(\frac{1}{2}x + \frac{1}{2}y\right) \\
x &\geq 0 + \frac{1}{2}\left(\frac{1}{3}x + \frac{2}{3}y\right) \\
y &\geq 1 + \frac{1}{2}\left(\frac{1}{3}x + \frac{2}{3}y\right)
\end{aligned}$$

In this LP, x refers to the value of state 1, while y refers to the value of state 2. For example, the first constraint tells us that the value of state 1 must be at least as good as the value we get by starting in state 1 and choosing action A.

Please sketch the feasible region for this LP. Label each corner of the feasible region with its coordinates.

Solution: See attached drawing of feasible region.

(d) In your sketch from problem 7c (or a copy of it if you prefer), please draw the objective function as a vector with its head pointing in the direction of better solutions, and label the optimal solution. What is the value of the optimal solution in this LP?

Solution: See previous plot. The optimal solution is $(2/3, 5/3)$, with value $2/3$.

(e) Now derive the dual of the LP from problem 7c. What is the value of the optimal solution to the dual? Hint: you should not need to solve the dual LP explicitly.

Solution: The optimal solution to the dual has value $2/3$, the same value as the optimal solution to the original LP. (Note that we didn't need to construct the dual at all in order to get its optimal value.)

To compute the dual, we can first collect terms, then use dual variables (Lagrange multipliers) to combine the inequalities, and finally use the result to bound the original objective. (Note: many people also scaled the equations before taking the dual, e.g., by dividing by the coefficient of y to get an inequality of the form $y \geq 2 + x/2$. This is technically not permitted, since it changes the scaling of the dual variables; however, it still leads to a valid bound on the original objective, and so we did not deduct any points for doing so.)

So, to begin, collect terms:

$$\text{minimize } x \text{ subject to}$$
$$\begin{aligned}
\frac{2}{3}x - \frac{1}{6}y &\geq 0 \\
-\frac{1}{4}x + \frac{3}{4}y &\geq 1 \\
\frac{5}{6}x - \frac{1}{3}y &\geq 0 \\
-\frac{1}{6}x + \frac{2}{3}y &\geq 1
\end{aligned}$$

Now add dual variables $a, b, c, d \geq 0$ which multiply the constraints:

$$a\left(\frac{2}{3}x - \frac{1}{6}y\right) + b\left(-\frac{1}{4}x + \frac{3}{4}y\right) + c\left(\frac{5}{6}x - \frac{1}{3}y\right) + d\left(-\frac{1}{6}x + \frac{2}{3}y\right)$$
$$\geq 0a + b + 0c + d$$

In this combined constraint, the coefficient of x is

$$\frac{2}{3}a - \frac{1}{4}b + \frac{5}{6}c - \frac{1}{6}d$$

And, the coefficient of y is

$$-\frac{1}{6}a + \frac{3}{4}b - \frac{1}{3}c + \frac{2}{3}d$$

Comparing these to the coefficients in the original LP's objective $(x + 0y)$, and noting that we want a lower bound on this objective, we get the constraints

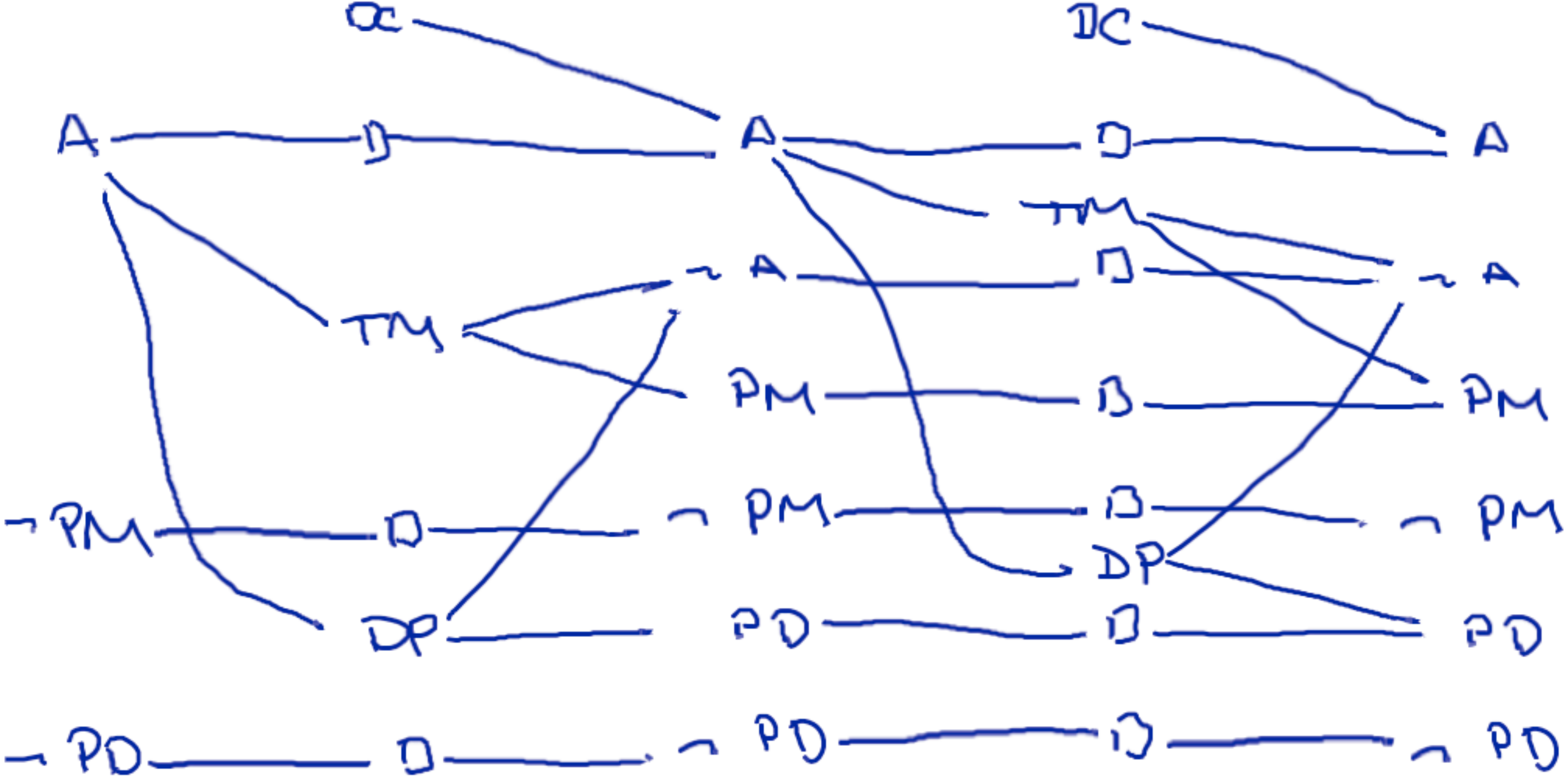
$$\frac{2}{3}a - \frac{1}{4}b + \frac{5}{6}c - \frac{1}{6}d \leq 1 \tag{1}$$

$$-\frac{1}{6}a + \frac{3}{4}b - \frac{1}{3}c + \frac{2}{3}d \leq 0 \tag{2}$$

If these two constraints hold, we get that

$$x + 0y \geq 0a + b + 0c + d \tag{3}$$

To find the tightest lower bound, we can maximize the right-hand side of 3. So, the dual LP is to maximize $b+d$ subject to the constraints 1–2 using the variables $a, b, c, d \geq 0$.



OC
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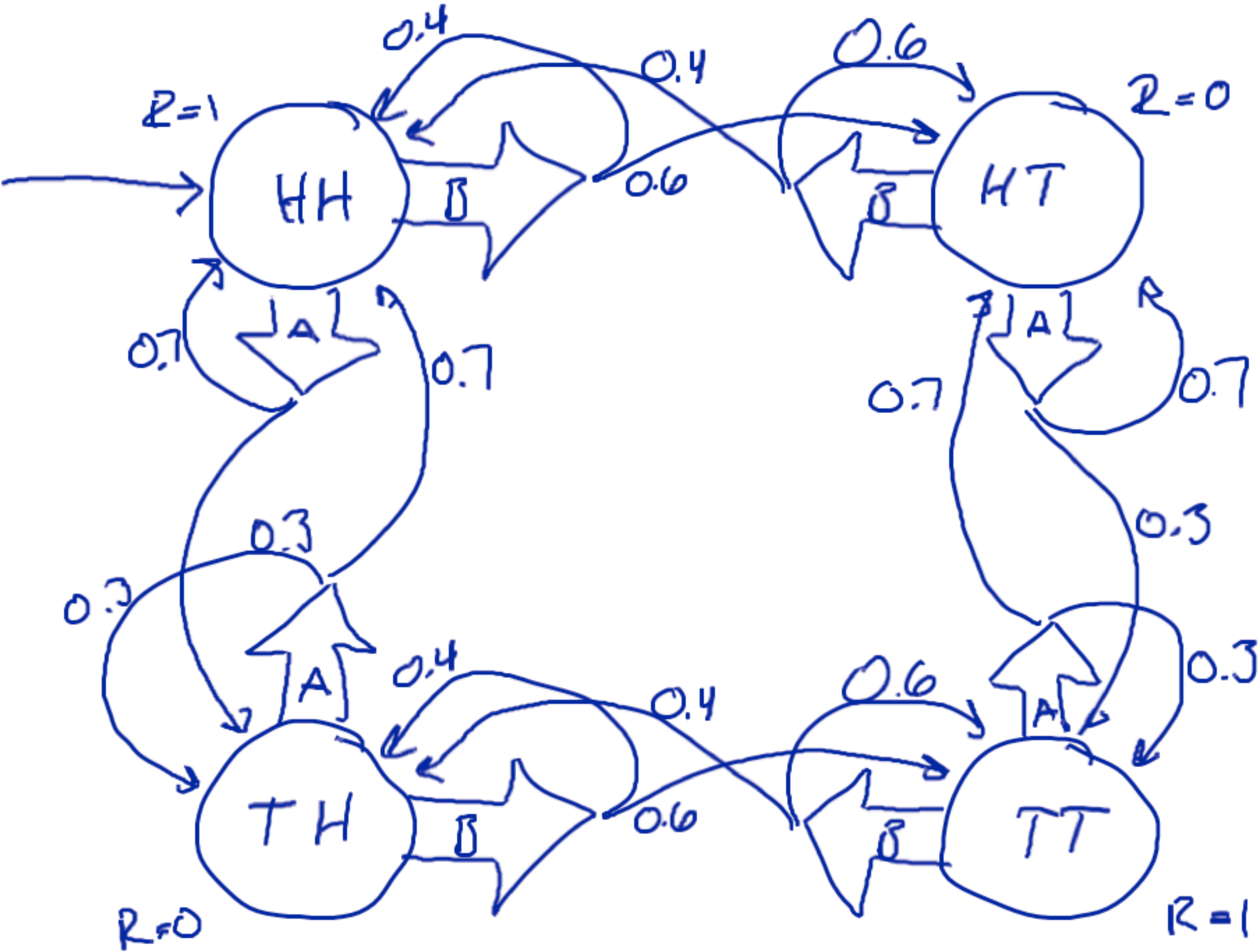

R=1
HH
R=0
HT
R=0
TH
R=1
TT
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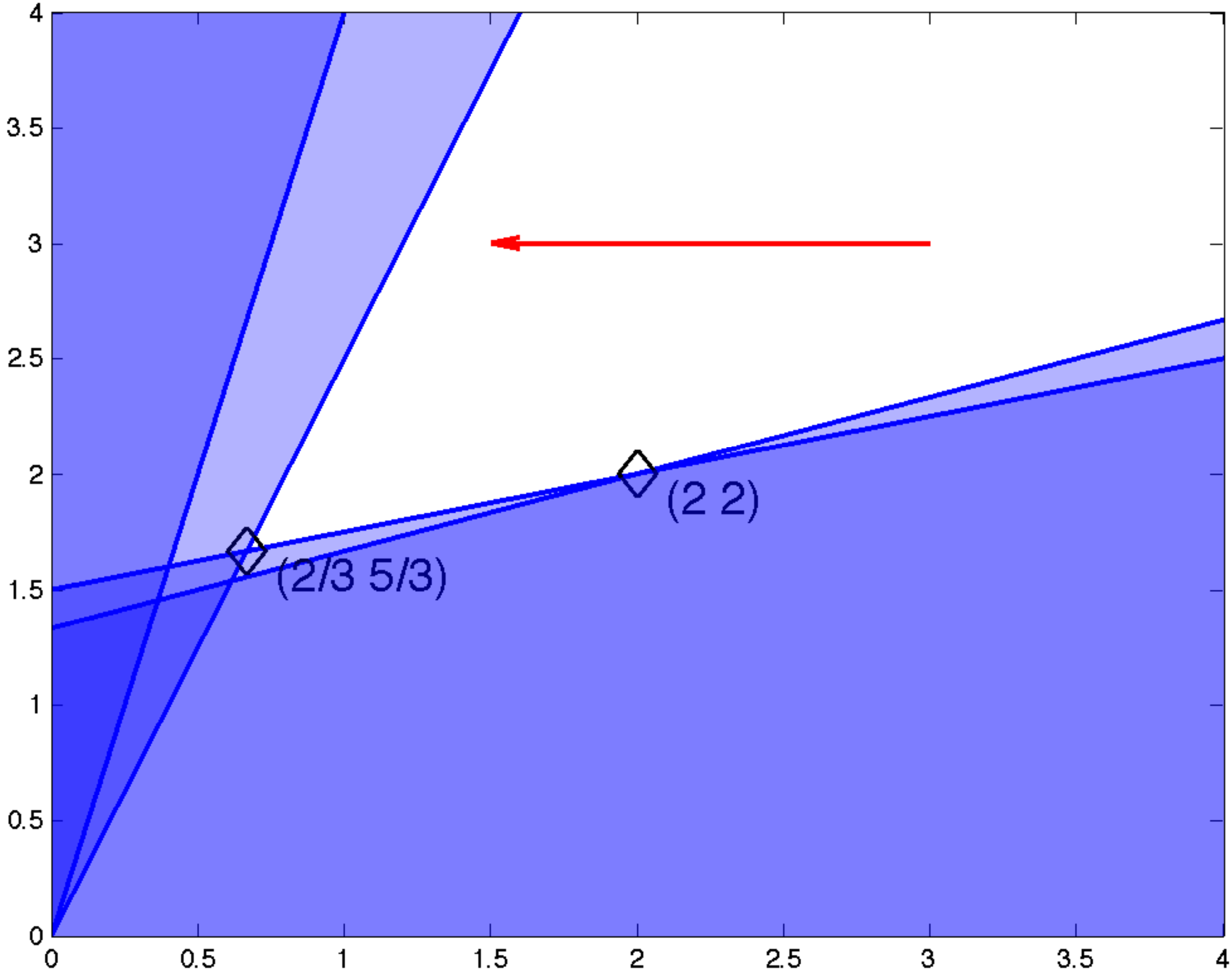
0.6
0.7
0.3



(2 2)
(2/3 5/3)
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0.5
1
1.5
2
2.5
3
3.5
4