Grad AI. 15-780 Fall, 2007

Homework 2

- *Homework deadline: 10:30am on October 16*
- *Please hand in a hard copy of this homework. You need not submit any code by e-mail.*

1. **First-Order Logic (40 pts)**

   (a) Represent the following sentences in first-order logic, using a consistent vocabulary, which you must define. (Some from Russell and Norvig). (3 pts each)

   i. Every student who takes graduate AI passes it.

   ii. If any gas-turbine cars are inexpensive then some gas-turbine cars are successful.

   iii. If all propositions are true or false and no commands are true or false then no commands are propositions.

   iv. Anyone can give advice, but a person accepts advice only if he/she is wise.

   (b) For each pair of atomic sentences, give the most general unifier if it exists. (From Russell and Norvig). (2 pts each)

   i. Older(Father(y),y), Older(Father(x),John)

   ii. Q(y,G(A,B)), Q(G(x,x),y)

   (c) Prove the following using the inference rules of first-order logic. (4 pts each)

   i. Given:
   $(\forall x)(P(x) \Rightarrow S(x))$
   $(\forall x)(P(x) \vee E(x))$
   $\neg(\forall x)S(x)$
   Prove:
   $(\exists x)E(x)$

   ii. Given:
   $(\forall x)(P(x) \Rightarrow W(x))$
   $(\forall x)(I(x) \Rightarrow S(x))$
   Prove:
   $(\forall x)(W(x) \Rightarrow I(x)) \Rightarrow (\forall y)(P(y) \Rightarrow S(y))$

   (d) Translate the following arguments into first-order logic and prove them using the inference rules of first-order logic. (6 pts each)

   i. All computer scientists are wise people. Some computer scientists are bearded. Therefore, some wise people are bearded.

   ii. When some politician is dishonest the country suffers. Senator Gordon is dishonest. Therefore, the country is suffering.

(e) Take the assumptions of Problem 1(d)ii (i.e., "When some politician is dishonest the country suffers. Senator Gordon is dishonest.") as a knowledge base. Write a satisfying model M of this knowledge base. M should contain a list of objects and tables of the values of each function and predicate mentioned in the knowledge base. Demonstrate that your model is a satisfying model. Is the conclusion of 1(d)ii (i.e., "The country is suffering.") true in M? Does this constitute a proof that the country must be suffering given our assumptions? Why or why not? (4 pts)

2. **Planning (25 pts)**

In this problem, you will be writing a planning problem in a STRIPS-like language and then solving it by drawing out the plan graph by hand. The Rocket and Aliens planning domain is as follows.

- A science rocket must travel to Mars and Saturn to study alien specimens on each planet. It must study both specimens before completing its mission.
- The rocket starts at Earth.
- The rocket can only travel from Earth to Mars and from Mars to Saturn (you may assume that it never makes a return trip).

(a) Write the planning domain above using a STRIPS-like language. States should be of the form "$At(x, y)$" and "$\neg At(x, y)$," where x and y are strings that describe objects in the domain (e.g., rockets, planets, little green men, or whatever other objects you feel are relevant). Actions should be of the form "$Move(x, y, z)$." You should (at least) have actions for studying specimens and moving between planets. Be sure to describe the pre-conditions and post-conditions of all of the actions you listed, as well as the initial state and goal predicates. (8 pts)

(b) Draw the plan graph that represents the task of studying alien specimens as described above (**you need only draw enough levels to reach the first appearance of all of the goal predicates**). Draw "mutually exclusive" (mutex) edges in another color between actions with contradictory post-conditions, actions that delete pre-conditions of each other, and actions with mutex pre-conditions. Also include mutex edges between literals that cannot both be achieved without executing mutex actions. You need not include mutex edges between contradictory literals, but these mutex edges should be considered when drawing mutexes at the next level. Note: if your graph gets too cluttered with mutexes, you may write them out separately. (10 pts)

(c) Use your plan graph to find a plan that achieves the goal predicates. Write the plan as a partially-ordered list of actions. How many operators did it take to solve this problem? How many levels did you need in the plan graph? Did you need to extend your plan graph farther than required by 2b? (Recall that the algorithm for finding plans from a plan graph dictates that, if we fail to find a solution using a graph with a given number of levels, we should add another level and try again.) (7 pts)

3. **Linear Programming (35 pts)**

WARNING: Problem 3 (particularly parts b–d) will take a significant amount of time. Do not leave this problem for the last minute!

(a) Part 3a deals with the following linear program:

maximize:

$$x + y$$

subject to:

$$\begin{aligned} \frac{x}{2} + y &\leq 2 \\ 3x + 2y &\leq 8 \\ x, y &\geq 0 \end{aligned}$$

i. What is the optimal solution of this LP? Please find by hand by sketching the feasible region and determining the intersection of the active constraints. You may use a linear programming toolbox (like Matlab's *linprog* function) to check. (5 pts)

ii. Derive the dual of this linear program by hand. (5 pts)

(b) Using the techniques described in class (and in RN on p. 402), take the plan graph of Problem 2b and write it as a SAT problem in CNF. You should only write out the SAT formula corresponding to the minimum number of levels required so that all goal predicates appear in the last level of the graph (as requested in Problem 2b); if you extended the graph further to find a plan in part 2c, you do not need to translate the extra levels. To save space, you should use only a single SAT variable to represent both a literal and its negation (even though the plan graph translation would still be valid if you added two separate variables connected by a mutex). Note: if we deducted points for your plan-graph in Problem 2b, you will not be further penalized in this problem as long as you correctly translate the graph you showed. (Within limits. Correctly translating an empty plan graph does not get full credit. . .) (8 pts)

(c) Consider the version of the Rocket and Aliens problem below.

- There are specimens on Mars and Saturn. The science rocket starts at Earth and can travel from Earth to Mars and from Mars to Saturn.
- The rocket does not need to study both specimens. Instead, it must maximize profit until a maximum number of actions have been completed.
- It costs 10 units to fly from Earth to Mars, and it costs 15 units to fly from Mars to Saturn. Studying the specimen on Mars yields 20 units of utility, and studying the specimen on Saturn yields 40 units of utility. The science rocket does not gain further utility by studying a specimen more than once.

Write the problem of finding a maximum-profit plan for this problem as an integer linear program (ILP) using the CNF constraints you wrote in Problem 3b.

Show your constraints, and provide the cost of one possible solution with positive profit. Note: you should formulate the ILP for a known plan length (the length corresponding to the termination of the graph in Problem 2b). (7 pts)

Hint 1: the clause $a \vee b \vee \neg c$ is equivalent to the ILP constraints

$$\begin{aligned} 1 &\le a + b + (1 - c) \\ 0 &\le a \le 1 \\ 0 &\le b \le 1 \\ 0 &\le c \le 1 \end{aligned}$$

for integer variables a, b, c.

Hint 2: Our encoding of the ILP has about 50 variables and about 150 constraints. The size of your encoding may be significantly larger or smaller depending on how you wrote out the operators, but if your encoding gets too large (say, more than twice the above figures) you may want to look for a simplification.

(d) Recall that the LP relaxation of an ILP uses the same equality and inequality constraints and the same objective, but drops the integrality requirements on the variables. Using Matlab's *linprog* function (or a similar tool), solve the LP relaxation of your Rocket and Aliens ILP. If you wish to use a linear program toolbox other than Matlab's, please check with the TAs first.

i. Provide a printout of the output of your LP solver. What is the cost of the resulting solution? Does it correspond to a feasible plan? What can you conclude about the cost of the optimal plan (using any information from this part or any previous part)? Note: depending on your formulation of the ILP and LP relaxation, your answers may differ from those of your classmates. (5 pts)

ii. Suppose we were to drop the mutex constraints from the ILP. Would it change the set of feasible solutions of the ILP? What about the set of feasible solutions of the LP relaxation? Try solving your LP relaxation again without the mutex constraints and report the results. (5 pts)