

15-780 Homework 5

Deadline: 10:30 am on April 30 (Thursday)

There are 100 total points: point values are listed with each question.

1. **Split or Steal (45 pts)**

This question will refer to the following video clip:

<http://www.youtube.com/watch?v=p3Uos2fzIJ0>

- (a) Draw the matrix for the game the contestants are playing. (5 pts)

Now consider the game defined by the following matrix, which is similar to the above game:

	<i>Split</i>	<i>Steal</i>
<i>Split</i>	(60, 60)	(0, 100)
<i>Steal</i>	(100, 0)	(5, 5)

The following questions will refer to this game.

- (b) List all the strictly-dominated and weakly-dominated strategies for each player. (5 pts)
- (c) What are all of the Nash equilibria of this game? (5 pts)
- (d) Now suppose that the players are repeating this game  $k$  times. What are all of the subgame-perfect equilibria of the repeated game? Hint: try solving the last repetition first (i.e., set  $k = 1$ ). With a simultaneous-moves game like this one, this backward-induction-like strategy doesn't always work, but when it does it is often the best way to solve a game. (10 pts)
- (e) Now suppose that at each time step, we will repeat the game with probability 0.9, and end the game with probability 0.1. Consider the following strategy, which is known as *tit for tat*: split in the first round, and at each future round play the strategy selected by the other player in the previous round. Prove that the strategy profile in which both players play tit for tat is a Nash equilibrium of this game. (10 pts)
- (f) Now let  $p \geq \frac{8}{19}$  be arbitrary and suppose that at each time step, we will repeat the game with probability  $p$  and end the game with probability  $1 - p$ . Please find two different Nash equilibria of this game, and justify your answer. Note that you must find two equilibria for each  $p$ , and that your equilibria could be different for different values of  $p$ . (10 pts)

Hint: to construct an equilibrium of the repeated game, consider the idea of *punishment*: suppose the two players agree to play a particular way, and then one (say the row player) decides to do something different from the agreement. What could the column player do on future rounds to make the row player regret her choice?

2. **Three-card Poker (55 pts)**

In three-card poker, two players are both dealt a card from a deck consisting of a king (K), a queen (Q) and a jack (J) (the final card is not dealt). Both players ante \$1, and player 1 (P1) is first to act. He has two choices: he can bet \$1 (B) or check (C). If P1 bets, player 2 (P2) can either call or fold. If P1 bets and P2 calls, then whoever has the higher card ( $K > Q > J$ ) wins the entire pot of \$4. If P1 bets and P2 folds, then P1 wins the pot of \$2. If P1

checks, then P2 can either bet or check. If P1 checks and P2 checks, then whoever has the higher card wins \$2. If P1 checks and P2 bets, then P1 can either call or fold. If P1 calls, then whoever has the higher card wins \$4. If P1 folds, then P2 wins \$2.

- (a) Draw the extensive-form game tree of this game. (10 pts)
- (b) How large would the equivalent normal-form representation of this game be? Note that you do not need to explicitly write down the normal-form representation of this game. (5 pts)
- (c) Say that an action  $a$  for player  $i$  at information set  $s$  in an extensive-form game is weakly dominated if there exists another action  $b$  at the same information set, such that for all leaf nodes  $n_x$  reachable by playing action  $x$  at  $s$ , we have  $u_i(n_b) \geq u_i(n_a)$ . Based on your extensive-form representation, perform iterated elimination of weakly dominated actions and list all actions that end up being eliminated for each player. (10 pts)
- (d) Now reconstruct the extensive-form representations of the game from part a with the dominated actions eliminated. (5 pts)
- (e) Write down the equivalent normal-form representation of the game you just constructed in part d. (5 pts)
- (f) What is one equilibrium of this game? What is the value of the game to player 1? You can use any method you want (we recommend solving it manually from the extensive-form representation), but must show your work and justify your answer. (10 pts)
- (g) What are all of the equilibria of this game? You can use any method you want (again we recommend solving it manually from the extensive-form representation), but must show your work and justify your answer. (10 pts)

Hint: There are infinitely many equilibria, so you should provide as detailed a description as possible of the set of equilibria. For example, you could give a parameterized strategy for each player, and say what sets the parameters can range over. Of course, your parameterized strategies should include your answer to 2(f) as a special case.