Single Point of Contact Manipulation of Unknown Objects

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Introduction

The desire to build systems which can operate reliably in natural environments motivates several areas of modern robotics research. Addressing the inherent uncertainty that exists in a natural environment is a central issue in this research. Often, it is possible for an autonomous system to learn the state of its environment over time by combining the information obtained by several consecutive, noisy, measurements. We wish to design a process analogous to scientific experimentation which will enable a robot to learn aspects of its environment which are initially unknown.

In particular we are interested not only in the usual uncertainties regarding the position of the manipulator, the forces it applies to the object, and the position of the object, but also in the shape, mass, and surface properties of the object itself. We attempt to formulate an optimal learning model that will work in the general case where known forces are applied to a rigid body. Once this model is constructed we consider a specific algorithm for approximating this optimal learning. Once a model of learning exists we can begin to reason about which experiments should be chosen in order to learn the unknown properties of the object in as little time as possible.

Background

This work originated with Dave Hershberger's work on manipulating unknown objects with a single point of contact manipulator. Where he worked with objects constrained to a few classes known a-priori (sphere, box, cone) we address arbitrary shapes. Additionally, the surface used in his work was sand, while this work assumes a rigid surface. (Erdmann; 1998) discusses methods for determining pose and motion from a known geometry in the planar case. Lynch and Mason's work on planar manipulation shows that goal directed manipulation is possible in the presence of uncertainty in the contact pressure distribution. (Okamura; 1997) has shown work in exploration of object parameters using rolling motions, and motion planning for polyhedra on the plane.

Task

Stated formally, the task is to determine the geometry, mass distribution, and coefficient of friction of an unknown object as quickly as possible. We make no initial assumptions regarding the mass distribution, coefficient of friction, or geometry of the object.

The system can detect the pose of a single point on the object's surface, and can apply force at any point on the objects surface. We assume that the object tracking sensor has zero mean normally distributed noise in the measurement of each axis. The manipulation is performed by an idealized single point of contact manipulator. We assume that the manipulator tip has a high frictional coefficient, allowing it to apply forces to the object so long as the applied force opposes the surface normal at the point of contact. Further, we ignore all issues relating to arm kinematics or measures of manipulability and assume that the manipulator position is known to within a normally distributed noise term $p_m = \mathbf{p}_m + \sigma_m$, where $\mathbf{p}_m$ is the position of the manipulator's end effector and σ_m is the noise term. Likewise, the maximum force that the manipulator can apply is assumed independent of $\mathbf{p}_m$ such that $|\mathbf{f}_m| < \mathbf{f}_{\max}$. We assume the arm is equipped with a force sensor which makes measurements of the force in each axis at the manipulator tip.

We assume that the object being manipulated is rigid and that its interaction with the work surface can be modeled using coulomb friction with a uniform frictional coefficient. Furthermore, we assume that the work surface is a flat, infinite plane.

Approach

We learn about the world through experimentation. Given some uncertainty about the state of the world we design an experiment, such that observing the outcome of that experiment will reduce that uncertainty. This suggests a natural decomposition of the problem. First, a description of uncertainty about the world is needed. Once a representation of this uncertainty exists, a method for deciding which experiments should be carried out must be developed. Finally, once experimental results and a representation of uncertainty exist, a method for altering that uncertainty in light of the results needs to be developed.

An important first step is to recognize that experimentation does not necessarily necessitate intervention. Ideally, the system would learn as much as possible about the world regardless of the forces applied to the object. The choice of forces could then be made a completely separate problem concerned only with maximizing the amount of available information. In other words, a well designed learning subsystem will learn as much as possible given its observations regardless of whether the manipulator is actively carrying out a learning oriented manipulation, moving the object towards a goal pose, or simply watching the object without applying any forces. For this reason, both in the implementation and in the development of our algorithms, there is a distinct separation between learning from experiments and choosing what experiments to carry out.

Initially, a statistical representation for uncertainty was chosen, and the learning problem was approached using Bayesian inference. However, as the physics of the problem were explored in greater detail it was discover that the uncertainty in this problem cannot be modeled by a single valued probability distribution function. A new model of uncertainty based on finding bounds on a class of possible pdfs was developed that accommodates these difficulties in the physical model. This new model of uncertainty changes the approach to choosing experiments as well. A naïve, brute force planner augmented with behavioral heuristics was planned, but the change in uncertainty model propagated changes in this planning model as well.

Physics

In order to carry out experiment on the object and evaluate their results, a description of the physical models describing the behavior of the object is required. The motion of the rigid body is described by the familiar Newton-Euler equations

$$f = ma$$
$$\tau = I\alpha$$
Where a is linear acceleration, m is the mass of the object, f is the force applied to object, α is the angular acceleration, I is the inertial tensor, and τ is the applied torque. From this point on, force and torque will be combined into a 6x1 wrench vector as
$$w = \begin{bmatrix} f \\ \tau \end{bmatrix}$$
as such, we can rewrite the Newton-Euler equations into the more compact form
$$w = Ma$$
Where a is now a generalized acceleration in pose space. If we know the wrench applied to the object and its acceleration, then the value of the mass matrix, which has six free dimensions, is observable. In the simplest case of a particle floating in space the only source of applied wrench would be the manipulator. If we observe the acceleration and know the force on such a particle, then the mass is trivially computed as $\frac{f}{a}$. This simple case will be useful later in motivating the discussion of learning.

In the actual system there are three possible sources for a wrench applied to the object. These sources are the manipulator, gravity, and the work surface. As such we can expand the description of the physics to
$$w_{manip} + w_{surface} + w_{gravity} = Ma$$
Since the manipulation and gravity wrenches are easily found, our concern now is to investigate the nature of the $w_{surface}$ term in greater detail.

The wrench applied by the surface is due to both normal forces and friction forces. The normal force at any point on the object in contact with the work surface is $p(x)\hat{\mathbf{z}}$ where $p(x)$ is the pressure applied to the surface at the point x. This leaves the frictional load to be determined. From the assumption of Coulomb friction and the maximum power law (Mason, 2001) we know that if the object is stationary
$$|f_{load}| \leq \mu p(x)$$
where f_{load} is the force applied to the surface by the object at x and μ is the frictional coefficient. Furthermore, if the object is in motion, then
$$f_{load} \parallel v(x)$$
$$|f_{load}| = \mu p(x)$$
Where $v(x)$ is the velocity of the point x. This second set of equations derives from the maximum power law, which indicates that the projection of the frictional load applied by a point onto the velocity of that point is the maximum projection over all possible loads described by Coulomb's law. Essentially, if we push a point in a particular direction, its frictional load is in that direction as well.

The problem we now face is that p(x) is indeterminate (Mason, 2001, pg 130). We know that the total contact pressure must be equal in magnitude to the normal force, and that the pressure at each point must be greater than or equal to zero. Consider the wrench on the object due to contact at a particular point on the surface

$$w(x) = \begin{bmatrix} -f_{load}(x) + f_{normal} \\ -(CM - x) \times f_{load}(x) \end{bmatrix} = p(x) \begin{bmatrix} -\mu\hat{v}(x) + \hat{\mathbf{z}} \\ -(CM - x) \times (\mu\hat{v}(x) + \hat{\mathbf{z}}) \end{bmatrix}$$

Where *CM* is the location of the object's center of mass. The total wrench resulting from any possible pressure distribution over the contact surface is the convex hull of the wrenches resulting from concentrating all pressure at a single point on the contact surface.

This convex hull describes the set of possible contact wrenches which can be applied to the object and is the extent of the information we can have about the contact wrench under our assumed friction model.

We find that the structure of this convex hull corresponds to the type of motion the object is undergoing. The easiest case to consider is the case where the object is sliding across the surface without rotation. In this case $\hat{v}(x)$ is constant for all x in the contact region. Thus the force component of the wrench is known. Since the torque resulting from pressure at a given point is the cross product of a constant term with a term which varies linearly in the location of that point in the plane, the set of possible torques will lie in a two dimensional linear subspace of wrench space. This follows from the fact that

$$p(q)\begin{bmatrix} f \\ q \times f \end{bmatrix} + p(r)\begin{bmatrix} f \\ r \times f \end{bmatrix} = \begin{bmatrix} f(p(q) + p(r)) \\ [p(q)q + p(r)r] \times f \end{bmatrix} = [p(q) + p(r)]\begin{bmatrix} f \\ (uq + (1-u)r) \times f \end{bmatrix}$$

where u is a term used to linearly interpolate between q and r.

$$-(CM - x) \times (f_{contact}) = f_{contact} \times CM - f_{contact} \times x = \underset{3x1}{C} - \begin{bmatrix} 0 & -f_z & f_y \\ f_z & 0 & -f_x \\ -f_y & f_x & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} x = \underset{3x1}{C} - \underset{3x2}{S}\, x$$

The convex hull of the torques must therefore also lie in this same subspace. Furthermore, we can compute this convex hull directly by computing the torque resulting for concentrating pressure at each corner of the contact hull and taking the convex hull of those torques. We know that all torques resulting from concentrating pressure at all points lie in this convex hull because of the linearity of the transform between contact space and torque space. This same argument shows that any torque that can be produced by a continuous pressure distribution must also lie in this convex hull.

It is worth noting at this point that in the special cases where the contact hull is a line or point this argument still applies, but that the dimensionality of the torque space hull is reduced as well. That is, if the contact region's convex hull is a line segment then the possible torques lie in a one dimensional subspace, and if the contact with the work surface is a single point, then there is only a single possible torque. More generally, if the dimension of the contact area is reduced by n dimensions, then the dimension of the wrench hull is reduced by n dimensions as well, regardless of the type of motion the object is undergoing.

In the case where the object is sliding and rotating at the same time the convex hull becomes more complex. If we substitute $v(x)$ for $\hat{v}(x)$ then the force becomes linear in x and the torque becomes quadratic in x, since x appears as a term on both sides of the cross product. The forces then occupy a two dimensional subspace and the wrenches due to extremal pressure must lie on a two dimensional manifold embedded in wrench space but not necessarily in any particular lower dimensional subspace. The wrench produced by maximum pressure at a point in the contact hull is

$$w(\alpha) = f_{normal} \begin{bmatrix} -\alpha_y \\ \alpha_x \\ 1 \\ CM_y - \alpha_y - CM_z \alpha_x \mu \\ -CM_x + \alpha_x - CM_z \alpha_y \mu \\ (CM_x - \alpha_x^2 + CM_y \alpha_y - \alpha_y^2)\mu \end{bmatrix}$$

Where, for clarity, we have chosen the origin of the work surface to be the instantaneous center of rotation. The convex hull of these wrenches over the contact hull has this manifold as an outer surface, since the second partials have constant negative sign. The remaining surfaces of the convex hull could be analytically computed as well.

Since we have neglected to scale $v(x)$ by the inverse of its magnitude we must do so before drawing conclusions about the structure of the convex wrench hull for this motion case. To simplify this scaling we express the wrench equation with α given in radial coordinates.

$$w(\alpha) = f_{normal} \begin{bmatrix} -\mu \sin(\theta) \\ \mu \cos(\theta) \\ 1 \\ CM_y - CM_z \mu \cos(\theta) - r \sin(\theta) \\ -CM_x + r\cos(\theta) - CM_z \mu \sin(\theta) \\ \mu(-r + CM_x \cos(\theta) + CM_y \sin(\theta)) \end{bmatrix}$$

Note that this equation is linear in r and trigonometric terms of θ. Despite this, we were unable to find the surfaces of the convex hull of these wrenches analytically. However, we can approximate the convex wrench hull by taking linear combinations of the wrenches due full pressure at regularly spaced points on the boundary of the contact region and at the center of rotation. The linear combination of these vectors, such that the total weighting equals 1 expresses an approximation of the convex hull of the possible contact wrenches.

The error introduced by this convex hull can be expressed in terms of the sample density. We sample in radial coordinates, such that no sample point is more than $\frac{2\gamma}{r}$ radians away from its neighbors, where *r* is the maximum distance between the center of rotation and the convex contact hull. We sample points along the boundary of the contact region and at the center of rotation, if the center is inside the convex hull. Using Taylor's theorem we know that the approximation error at any point on the manifold the same radial distance from the origin as a sample point is less than

$$\frac{d^2w(p)}{d\theta^2}\left(p-p_{sample}\right)=\left\|\begin{bmatrix}\mu\sin(\theta)\\-\mu\cos(\theta)\\0\\CM_z\mu\cos(\theta)+r\sin(\theta)\\CM_z\mu\sin(\theta)-r\cos(\theta)\\\mu(-CM_x\cos(\theta)-CM_y\sin(\theta))\end{bmatrix}\left(\frac{\gamma^2}{r^2}\right)\right\|\le\left\|\begin{bmatrix}\frac{\mu\gamma^2}{r^2}\\\frac{\mu\gamma^2}{r^2}\\0\\\gamma^2\left(\frac{CM_z\mu}{r^2}+1\right)\\\gamma^2\left(\frac{CM_z\mu}{r^2}+1\right)\\\gamma^2\mu\left(\frac{|CM_x|+|CM_y|}{r^2}\right)\end{bmatrix}\right\|$$

Any point not at the same radius as a sample point has error linearly proportional to points at the same angle that are on a sample radius, since the second partial with respect to radius is zero.

$$|w(r,\theta)-approx(r,\theta)|\le\left|\begin{matrix}w(r_{sample},\theta)+(r-r_{sample})\frac{\partial w(r_{sample,}\theta)}{\partial r}-\\approx(r_{sample},\theta)+(r-r_{sample})\frac{\partial approx(r_{sample},\theta_{sample})}{\partial r}\end{matrix}\right|$$

which can be re-arranged to form

$$\left|\left(w(r_{sample},\theta)-approx(r_{sample},\theta)\right)+(r-r_{sample})\left(\frac{\partial w(r_{sample,}\theta)}{\partial r}-\frac{\partial approx(r_{sample},\theta_{sample})}{\partial r}\right)\right|$$

We have already established an upper bound on the magnitude of the first term in this sum. We can establish a bound on the second term by taking the partial with respect to θ

$$\frac{\partial w(r_,\theta)}{\partial r\partial\theta}=\begin{bmatrix}0\\0\\0\\-\cos(\theta)\\-\sin(\theta)\\0\end{bmatrix}$$

Therefore, the second term grows at a rate no greater than $r\theta$ and, since the theta distance between samples is proportional to $\frac{1}{r}$ the second term has a constant upper bound in terms of k and γ.

We have not, however, established a constant upper bound on the approximation error, since the upper bound on the first term goes to infinity as the radius goes to zero. To establish the upper bound we must introduce an additional stipulation to the sampling scheme. Specifically, we sample points at $r=0$ such that the difference in θ between any two points is less than some constant value. Using the second partial of wrench with respect to θ as computed above we can see that the error in the approximation grows at worst linearly in the θ distance between sample points. Since we have already established that the error in the second term of the approximation goes as $r\theta$ we need only establish an upper bound on the minimum distance to the first set of samples with non-zero radius to establish a constant upper bound on the error for the disk within this radius. If we take this minimum radius to be *k*, then the global upper bound exists and can be expressed in terms of *k* and γ. This assures us that a sampling based scheme for reconstructing the convex hull of possible wrenches will work well.

The final, and most complex, case we consider is the case where the object is not moving. In this case the maximum power law only establishes an inequality on the magnitude of the frictional load, but tells us nothing about its direction. Here again constructing the convex hull is difficult analytically, and so we resort to an approximation scheme. The sampling method is identical to the scheme used in the general motion case above, with an additional degree of freedom to accommodate the indeterminacy in load direction. For each point at which we could choose to concentrate pressure we sample several possible values for the direction of the force at that point, maintaining some maximum distance between a point and its nearest neighbor in terms of θ.

Further investigation of this convex hull will be motivated once we begin to investigate the statistical aspects of the problem.

Geometry

The geometry of the object is related to our description of the object's motion, as the geometry describes the contact area from which we draw possible pressure distributions. However, geometry is also related to the position of the object, in addition to its motion. Specifically, when the manipulator makes contact with the objects surface, we know that the point of contact is a point on the surface of the object. Additionally, the object cannot penetrate the work surface, so we know that it always lies in the half space above the surface. Finally we should note that the convex hull of the object's geometry can occupy the same set of poses with respect to the ground plane as the object itself.

Statistical Inference

We learn the properties of the object using Bayesian inference. If we let *b* represent a distribution over the state space in which the object is defined, then we can describe the effect of an observation *o* on this distribution as $p(b \mid o)$. Bayes rule gives

$$p(b \mid o) = \frac{p(o \mid b)p(b)}{p(o)}$$

We note that $p(o)$ is constant over *b*, and might assume that given our current belief state and a method for computing the probability of any observation under that belief state we can correctly incorporate new observations into our belief about the parameters of the object. However, as will become clear during the formulation of the particle filter, the indeterminacy in the physical model has important ramifications on the learning model as well.

In the single point of contact manipulation problem the observed values are the position of the object and the forces applied to it. We treat these as independent observations and consider the computation of the Bayesian posterior after each type of observation.

Representation

We represent the physical parameters of the object. The parameters which we store are the location of the center of mass, the coefficient of friction with the work surface, the mass, and the inertial tensor. Together these comprise 11 degrees of freedom.

We also represent the position, velocity, and acceleration of the object. These comprise 18 degrees of freedom.

The geometry is represented by a set of unit vectors that form at least a basis for $\mathbf{R}^3$. Typically, approximately 200 such vectors are used. For each of these vectors we define two planes, one with the vector as its normal and a second with the vector scaled by -1 as it's normal. The planes are uniquely defined by an offset associated with each which represents the minimum distance between the origin and the plane. We take the intersection of the interior half spaces of this set of planes to be the convex hull of the object. As noted earlier, knowing only the convex hull of the object is enough to determine the convex hull of the area of the object in contact with the work surface, and to describe the set of possible poses for the object.

Kalman Filter

The Kalman filter is an attractive first choice for the implementation of Bayesian inference. The Kalman filter takes advantage of the fact that the posterior distribution resulting from an observation on a normally distributed belief distribution is another normally distributed belief distribution in the case that the observation is linear in state space and has error which can be modeled as Gaussian white noise. This is, in essence, the multivariate case of the observation that the convolution of two Gaussians is a Gaussian.

Initially, when only the translation motion case had been considered, a method for using a Kalman filter based on a tensor valued normal distribution was considered. It was hoped that this Kalman filter would be able to represent a distribution on the two dimensional wrench subspace and convolute that distribution with an observation. Unfortunately, this approach does not scale to the more general motion case, and more importantly, it can not represent the bounds on the uncertainty accurately, as it constrains only to the entire subspace.

Consider any recursive filter with a state space of fixed dimension. Convoluting a continuous distribution on this space with the observed distributions on the convex wrench hull introduces two distinct regions in the state space. Over time, these regions can overlap so as to produce an unboundedly large number of distinct regions. The pdf in state space then cannot be represented with a fixed number of parameters, since information from every observation may still be extracted. We must conclude that no recursive filter can perfectly represent the posterior distribution under our assumptions.

Particle Filter

The particle filter is a statistical filtering algorithm that attempts to approximate the Bayesian posterior using a finite set of discrete points. We call each of these points a particle, and in a conventional particle filter associates some vector in state space and a probability mass with each particle. Each time an observation occurs, the probability of each particle given that observation is computed, and the individual weights are renormalized. Periodically, the particles undergo a resampling procedure which removes particles with very low probability and divides particles with higher probability into multiple particles. Finally, in systems where both actions and observations occur, the particles are moved after each action according to a model of the uncertainty produced by that action.

The value of a particle filter is that it can approximate arbitrary probability distributions. However, because the filter approximates a continuous distribution with a finite set of points, the particles need to be sufficiently dense in order to capture the distribution. Methods for adapting the number of samples over time exist (Fox; 2001), but are not addressed in this thesis.

As mentioned earlier, computing the upper bound after an observation given only bounds on the possible forces introduces new, non-trivial elements to the particle filter's update. Consider the equation describing Bayesian inference

$$p(b \mid o) = \frac{p(o \mid b)\,p(b)}{p(o)}$$

In a conventional particle filter the denominator on the right, $p(o)$, is constant across every particle in the filter. It's value is easily computed as

$$\frac{1}{\sum_{b \in particles} p(o \mid b)\,p(b)}$$

since the sum of the resulting probability mass is known to be one. However, when we consider the case of bounded probabilities, this normalization term is not constant across all particles. Furthermore, because the expected wrench is known only to within certain bounds, the resulting probability distribution cannot be known exactly either. Instead we must maintain an upper and lower bound on the probability of each particle such that any distribution which satisfies these bounds is a possible distribution.

Bounds on $p(o|b)$ can be found by determining the wrench inside the wrench hull that maximizes or minimizes the value of the multivariate normal distribution associated with the observed wrench. In the maximization case this becomes a quadratic programming problem where the covariance matrix of the observation's distribution is used as the objective function. We can use quadratic programming to optimize the non-linear likelihood function because under the mapping of the covariance matrix there is a isotone, bijective mapping between distance and likelihood. The minimization problem is simpler, since the distance is maximized for one of the vertices of the convex hull. We need only test each wrench on a facet of the hull and select the one with maximum distance to the expected wrench.

In practice the convex hull optimization is made difficult by the fact that the wrenches often occupy subspaces of indeterminate dimension. The implementation must first determine this subspace, then find a convex hull within this subspace, optimize within the subspace, then map the results back to full wrench space. Furthermore, for reasons that are explained later in this section, the normal force is assumed to be one, so the optimization is done on the hull of *w*' wrenches.

This ability to compute $p(o|b)$, while sufficient in a conventional particle filter is not enough to compute new bounds on the probability of a given particle. We wish to determine the upper bound on the probability of the particle under all possible distributions which satisfy the bounding condition. Without loss of generality, we consider only the upper bound computation.

$$\max(p(b \mid o) = \max\left(\frac{p(o \mid b)p(b)}{p(o \mid b)p(b) + \sum_{k \in particles \setminus b} p(k)p(o \mid k)}\right)$$

where both instances of $p(o|b)$ must have the same value. Furthermore, the value for the true normal force must be constant across the computation of the convex wrench hull for every particle. This coupling of the normal force used in calculating the numerator and denominator follows from the fact that p(o) reflects the relative value of the observation over all possible true states. The terms which are contained in the particles, like mass, are discretized and computed directly by the sum. However, the normal force is a combination of particle and observation terms, and therefore has an associated distribution. This means that a continuum of possible ground truths with differing likelihoods exists. Coupling the normal force allows us to express the maximum likeyhood bound over all these possible states.

$$\max(p(b \mid o)) = p(b)\max_{f_{normal}}\left(\frac{p_{\max}(o \mid b, f_{normal})}{p_{\max}(o \mid b, f_{normal}) + \sum_{k \in particles} p(k)p_{\min}(o \mid k, f_{normal})}\right)$$

A more intuitive interpretation of this equation would focus on the idea that the likelihood of a particle over all possible distributions is maximized at the point in state space where that particle is most likely in relation to every other particle. That point is distinct from the point in state space where the probability observation is maximized given that particle.

Having established a mathematical definition of the new lower and upper bounds, the question of how to efficiently compute these bounds now remains. As suggested in the previous equation, the convex hull optimization decouples from the rest of the bound search problem. For any particular normal force the upper bound is maximized by taking the maximum likelihood of *b* and the minimum likelihood of every other particle. Because the normal force scales the wrench hull linearly we can perform the convex hull optimization on the fully bounded *w*' hull, and then find the optimal value of the normal force in a second, decoupled, step. Performing this convex hull optimization on each particle for its particular *w*' hull results in a set of wrench vectors with normal forces of one.

Changing the normal force linearly scales these vectors. The likelihood of a particular particle's scaled wrench given an observation is expressed using the six dimensional multivariate normal distribution associated with that observation. The marginal distribution of any line through this multivariate distribution is a scaled one dimensional normal distribution. This observation allows us to reduce the optimization problem to optimizing a single variable in a relatively well structured space composed of normal distributions.

Specifically, we are trying to optimize the function

$$p(f) = \frac{n(\alpha_b f, \sigma_b)}{n(\alpha_b f, \sigma_b) + \sum_{k \in particles} n(\alpha_k f, \sigma_k)}$$

over all possible values of f. Taking the derivative of this function we find that is has zeros when:

$$\frac{dp(f)}{df} = 0$$

$$d\left(\frac{n(\alpha_b f, \sigma_b)}{n(\alpha_b f, \sigma_b) + \sum_{k \in particles} n(\alpha_k f, \sigma_k)}\right) = \frac{n(\alpha_b f, \sigma_b)}{n(\alpha_b f, \sigma_b) + \sum_{k \in particles} n(\alpha_k f, \sigma_k)}$$

$$\frac{n(\alpha_b f, \sigma_b)}{n(\alpha_b f, \sigma_b) + \sigma_b \sum_{k \in particles} \frac{n(\alpha_k f, \sigma_k)}{\sigma_k}} = \frac{n(\alpha_b f, \sigma_b)}{n(\alpha_b f, \sigma_b) + \sum_{k \in particles} n(\alpha_k f, \sigma_k)}$$

$$\sigma_b \sum_{k \in particles} \frac{n(\alpha_k f, \sigma_k)}{\sigma_k} = \sum_{k \in particles} n(\alpha_k f, \sigma_k)$$

$$\sum_{k \in particles} \left(\frac{\sigma_b}{\sigma_k} - 1\right) n(\alpha_k f, \sigma_k) = 0$$

We also see that the number of zeros is bounded to twice the number of particles, and that only one zero can lie between adjacent means. Thus, simple root finding applied to each interval will find a global maximum value in O(n) time, where n is the number of particles.

Planning

Given a method for representing the uncertainty about the state of the object, and a method for computing a new belief based on a set of observations, the next goal was to develop a way to choose which actions to perform in order to learn about the object as quickly as possible. If we had a conventional probability distribution over state space, the method we initially developed would be applicable.

This method is based on a set of pre-defined behaviors, which the planner selects between. Each behavior has a small number of parameters, so the entire search space is composed on less than fifty possible choices for an action. For each particle in the particle filter we could simulate the result of each behavior, collecting aggregate statistics for each behavior over the set of all particles. Then, a planning decision could be made based on these statistics. We considered using the volume of the ellipsoid corresponding to one standard deviation in the covariance matrix of the particle filter as a potential metric for learning.

Introducing the bounded particle filter makes computing these aggregate statistics more difficult. Instead, only bounds on the weight of any simulated results can be found. Finding the possible distribution out of the set of possible distribution which maximizes or minimizes the expectation of some metric of learning is not difficult in this case. We use an algorithm that assigns as much probability mass as possible to the particle with the lowest or highest value for the metric in question, then proceed to the next best particle and so on, until all the probability mass has been assigned.

This calculation gives bounds on the possible learning that could occur given a particular behavior. It is unclear how the program should choose between possible behaviors in cases where the expected value of one behavior is not definitely better than the other. It is temping to assume that the unknown information represented by the bounds is roughly uniform and select the action with the higher average or weighted average of its bounds. However, because we have absolutely no information about the distribution within those bounds, this is not a justifiable approach and can lead to cases where the robot consistently chooses a behavior which learns very slowly because it has a particularly high upper bound.

This issue also arises in the design of behaviors, where we need to use information from the particle filter. Currently only sliding and rolling behaviors are implemented, and these do not depend explicitly on knowledge of the object. Instead they use silhouette images generated by a camera mounted on the simulated manipulator to choose rays along which to push the object. However, it is unclear how more complex motion planning could be done. One approach might be to compute bounds on the expectation of the true state of the object using linear programming, then use those bounds to generate a plan which produces the desired result for all objects falling within those bounds.

Results

We have successfully implemented a dynamic simulation environment, the algorithm for computing optimal wrenches on *w*' hulls, behavior based manipulation, and the ability to simulate particles under a given behavior. We hope to complete work on the non-linear optimization step in the particle filter bounds computation shortly and show convergence of the particle filter in general cases. Currently the software can accurately determine the geometry of the convex hull of an unknown object and in very limited cases the particle filter may begin to converge to approximate dynamics parameters. However, because we currently do not couple the normal force in the bounds computation the computed bounds are much too large. Since our current formulation will represent only those distributions which are possible under the observations and none of the distributions which are not possible, the results with that filter will indicate whether the approach we have taken is feasible.

Conclusions and Future Work

The majority of our work is based on the indeterminacy of the contact pressure distribution. While this had yielded interesting mathematical results and uncertainty representations, it may not be the correct assumption to build a working system on. Most of our work is been a process of generalizing and refining the bounds that can be placed on different representations of the system. We have believed, and continue to hope, that the failure to converge is a result of bounds which are not tight enough, rather than the information simply not being available. Once the non-linear optimization in the particle filter is working we believe the only remaining reasons that the particle filter might not converge are insufficient sample density or insufficient assumptions in the physical model to obtain convergence. With respect to this latter possibility, we observe that when humans manipulate objects they appear to assume that pressure distributions are generally uniform, or at least have some heuristic description of the pressure distribution. These assumptions are not without justification, since human hands, and to some degree most objects and surfaces, are compliant, pressure distributions should tend to be somewhat uniform.

Perhaps by adding a compliant surface assumption, such as working on a rubber mat instead of a rigid floor, better bounds could be obtained. One easy way to test this idea would be to change the convex hull optimization constraints such that the weighting of any individual force is limited to some constant and the total weight is the inverse of that constant. Of course, the constant would need to be scaled depending on the sampling density, but if behavior and convergence improved with this alteration it would suggest a promising avenue for further research. Another approach to pressure indeterminacy is Lynch's work (Lynch; 1993) on estimating friction is also a potential approach, in that it could be applied to each surface, and a model for the pressure distribution on each side of the object developed. Integrating this approach with a changing geometric representation would not be a trivial problem.

That work does, however, lead to the idea of applying a similar approach to the end result of the learning. Perhaps a second layer of learning applied to the un-modeled response of the system within the expectation bounds we compute could help to determine behaviors. It would almost certainly avoid the failure cases where the system continually picks a bad behavior because of an attractive upper bound.

With more analysis of the sampling error it will be possible to apply Rao-Blackwellization to some of particle filter state space. Those dimensions which have simply defined (like the normal force) or minimal (like total mass) effects on the w' hull would be good candidates for this process. This will reduce the number of particles needed to get a useful sampling density.

In the planning stage we need to find a way to select representative samples from the distribution rather than sampling every particle. How to do this in a bounds-based particle filter is unclear, but could speed up execution by several orders of magnitude. Furthermore, the metric we use to measure learning may be a poor choice when the distribution is multimodal. A more sophisticated metric, perhaps entropy, should be used to handle these cases.

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