Quantum Information and Game Theory

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Abstract

This thesis investigates areas in game theory where concepts from quantum information may be applied and vice versa. The nature of correlations that exist in quantum entanglement serve as an additional resource in the games we consider. We present a reformulation of some of the well known violations that quantum theory predicts in the framework of game theory. This approach allows us to show a general structure present in such games. We show how given a game and an entangled quantum state, one can go about enumerating all the cases for which a quantum advantage exists. We further explore connections these games have to multi-prover interactive proof systems.

I) Introduction

The fact that quantum information is useful in game theory was first pointed out by Meyer in [1]. Today there exist various sources in the literature that investigate the advantages of using quantum information in specific games, e.g. [2] & [3]. In this paper we begin in Section II by presenting a reformulation of some well known quantum paradoxes and violations under the framework of game theory. The insight gained by constructing these games allows us to consider the general structure of such games for any given experimental setup. Although the examples discussed in Section II present conclusive evidence of the advantages of using quantum information in such scenarios, they may not necessarily be considered to be of much practical interest. Section IV deals with the computational aspects of determining the cases in which quantum entanglement is actually useful in a given game. This is based on Pitowsky & Svozil's work on enumerating all the Bell Inequalities of a given experimental setup [4]. In Section V we present better motivated examples by investigating connections between these games and multi-prover interactive proofs.

II) Bell Inequalities, Quantum Paradoxes and Games

Bell Inequalities are logical tests, where classical logic and quantum theory give different predictions. These differences occur essentially due to the correlation functions that one can extract from entanglement which are not possible to construct classically. Our interest lies in the possibilities of using these correlations in games. We present here three examples that have been constructed by abstracting away the actual physical experiment and using the correlations that are obtained as a result of these experiments. They are based on Mermin's presentation in [5], [6] & [7]. In all of the games below, the players are allowed to agree on a strategy before the game begins and make any preparation before they are taken to "remote and isolated booths", to play the game.

II.1) Bell Inequalities

This game is directly based on Mermin's example of the Bell inequalities in [5]. It takes advantage of the violation of one of Bell's inequalities by a maximally entangled qubit pair. There is a slightly erroneous discussion of a similar setup in [8]. Two players are allowed to communicate and agree on a strategy before the game begins. Once they have been isolated in their remote booths, each of them receives an input chosen iid uniformly from {A, B, C}. They privately observe their input and then simultaneously say either "Yes" -- '1' or "No" -- '0'. The payoffs are determined by the following rules:

- If the players receive same input and they agree in their output (both say yes or both say no), then both players lose and incur a loss of –900 each (The large negative number is arbitrary)

- If players receive different input and they agree in their output then both players win and receive a payoff of 9 each

- If the players disagree in their output, both of them receive 0 as a payoff

The Table *II.a* represents the payoff matrix for the game. Due to the huge loss incurred when players receive same input, there are no Nash equilibriums that contain the event in a mixed strategy. Complementary pure strategies, (which the players can decide upon initially), which do not lead to loss are:

	Same Input	**Different Input**
Same Output	**-900**	**9**
Diff Output	0	0

Table II.a

Table II.b

Player1	**A B C**	**Player2**	**A B C**
I	0 0 0		1 1 1
II	0 0 1		1 1 0
III	0 1 0		1 0 1

IV	0 1 1	1 0 0
V	1 0 0	0 1 1
VI	1 0 1	0 1 0
VII	1 1 0	0 0 1
VIII	1 1 1	0 0 0

Strategies (i) & (viii) lead to an expected payoff of 0, & (ii) – (vii) lead to an expected payoff of 4, which form the optimal Nash equilibrium solutions for the game and also the correlated equilibriums. Note that if the player inputs are limited to two, then the pure strategies maximize the expected payoff. ((½ * ½) + (½ * ½))*9 = 4.5. However, for the game under consideration, we obtain a payoff of (1/3)*(1/3 + 1/3 + 2/3)*9 = 4, whereas the maximum possible payoff is (1/3)*(2/3 + 2/3 + 2/3)*9=6

For their quantum strategy, the players share the maximally entangled state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \qquad (1)$$

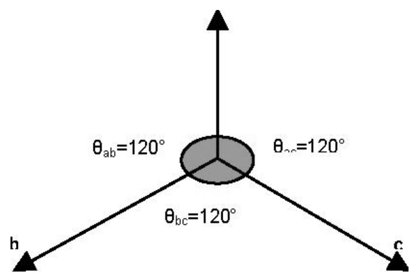


Fig. II.a

Table II.c

	Same Input	**Different Input**
Same Output	**0**	**0.75**
Diff Output	1	0.25

They decide before hand to measure in the basis {a, b, c} based on the input they receive, as indicated in figure *II.a*. So, they might choose to measure along a if input is A, along b if input is B and along c if input is C. When the players measure in the same basis (i.e. when they receive the same input) measurement results would always be different and hence the players would avoid the loss of (–900). Calculating the probabilities, we note that exactly one fourth of the measurements will yield different results for measurement along different axis and the remaining three fourths would have the same output result. So, the expected payoff is: (2/3)*(3/4)*9=4.5 > 4 (the best classical payoff). The probabilities for each case are given in Table *II.c.* We may even be able to obtain better co-ordination if the players share an ensemble of EPR pairs or qutrits.

II.2) GHZ Paradox

This game consists of a team of three players. Each player receives an input chosen from {A, B}. Each player must give an output, which is limited to one of only two possibilities: "0" or "1." One of the rules of the game is that either all three players will receive input A or only one player will receive A as input and the other two will receive B. The team wins if the number of 0 outputs is odd (one or three) in the case when everyone receives 'A' as input, and is even (zero or two) in the case of one A and two B inputs.

Assuming that the four possible combinations of inputs (i.e. A_1, A_2, A_3; A_1, B_2, B_3; B_1, A_2, B_3; and B_1, B_2, A_3) are received with the same frequency, no classical protocol allows the players to win the game in more than 75% of the runs. For instance, a simple strategy that allows them to win in 75% of the runs is that each player always outputs 1 when input A is received and 0 if input B is received. However, quantum mechanics provides a method to always win the game.

The method for always winning is the following. The players share a large number of three-qubit systems in the GHZ state:

$$\left|GHZ\right\rangle = \frac{1}{\sqrt{2}}(\left|y_0, y_0, y_0\right\rangle + \left|y_1, y_1, y_1\right\rangle) \qquad (2)$$

where:

$$\left|y_o\right\rangle = \frac{1}{\sqrt{2}}(\left|z_o\right\rangle + i\left|z_1\right\rangle) \qquad \left|z_o\right\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad \left|z_1\right\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\left|y_1\right\rangle = \frac{1}{\sqrt{2}}(\left|z_o\right\rangle - i\left|z_1\right\rangle)$$

In case a player received input A, he performs a measurement on his qubit in the Z basis and outputs 0 if the outcome corresponds to $\left|z_o\right\rangle$ and outputs 1, if the outcome corresponds to $\left|z_1\right\rangle$. In case a player received input B he performs a measurement in the X basis where $\left|x_o\right\rangle = \frac{1}{\sqrt{2}}(\left|z_o\right\rangle + \left|z_1\right\rangle)$ and $\left|x_1\right\rangle = \frac{1}{\sqrt{2}}(\left|z_o\right\rangle - \left|z_1\right\rangle)$ and outputs 0, if the measurement results corresponds to $\left|x_0\right\rangle$ and outputs 1, if result is $\left|x_1\right\rangle$. This strategy allows the team to always win the game. Writing out the GHZ state in the four equivalent forms in (3), (4), (5) & (6) makes this explicit.

It can be inferred from equation (3) that, if all players measure in the Z basis, then either all of them will obtain z_0 or one will obtain z_0 and the other two will obtain z_1. Analogously, it can be inferred from Eqs. (4) – (6) that, if one player measures Z and the other two measure X, then either all of them will obtain 1, or one will obtain 1 and the other two will obtain 0. The probabilities for these events are given in Table *II.d*

$$
\begin{aligned}
|GHZ\rangle &= \frac{1}{2}(|z_0, z_0, z_0\rangle - |z_0, z_1, z_1\rangle \\
&- |z_1, z_0, z_1\rangle - |z_1, z_1, z_0\rangle) \qquad (3) \\
&= \frac{1}{2}(|z_0, x_0, x_1\rangle - |z_0, x_1, x_0\rangle \\
&- |z_1, x_0, x_0\rangle - |z_1, x_1, x_1\rangle) \qquad (4) \\
&= \frac{1}{2}(|x_0, z_0, x_1\rangle - |x_0, z_1, x_0\rangle \\
&- |x_1, z_0, x_0\rangle - |x_1, z_1, x_1\rangle) \qquad (5) \\
&= \frac{1}{2}(-|x_0, x_0, z_1\rangle - |x_0, x_1, z_0\rangle \\
&- |x_1, x_0, z_0\rangle - |x_1, x_1, z_1\rangle) \qquad (6)
\end{aligned}
$$

Table II.d

Output	Input A_1, A_2, A_3	Input A_1, B_2, B_3	Input B_1, A_2, B_3	Input B_1, B_2, A_3
000	**0.25**	0	0	0
001	0	**0.25**	**0.25**	**0.25**
010	0	**0.25**	**0.25**	**0.25**
011	**0.25**	0	0	0
100	0	**0.25**	**0.25**	**0.25**
101	**0.25**	0	0	0
110	**0.25**	0	0	0
111	0	**0.25**	**0.25**	**0.25**

II.3) Hardy's Paradox

The game using Bell inequalities is based on 2 players with three possible inputs and the one based on GHZ has three players with two inputs. The game based on Hardy's paradox is simpler since it has two players with just two inputs. So, it is the simplest case where one could expect entanglement to be useful.

The game is setup similar to the last two games. Two players receive on of two possible inputs {A, B} and they have to respond by signaling either "0" or "1". The players incur a huge loss if:

- The players receive different inputs and both of them output 1

- Both the players receive input A and both of them output 0

The players win if:

- Both the players receive input B and both of them output 1

Table II.e

Output	Input AA	Input AB	Input BA	Input BB
00	**0**	0.15	0.15	0.64
01	0.375	0.225	0.625	0.135
10	0.375	0.625	0.225	0.135
11	0.25	**0**	**0**	**0.09**

Now, classically there is no pure strategy that would allow the players to avoid the losses and at the same time win when possible. However, using the experimental setup from Hardy's paradox allows the players to assign a probability > 0 (but < 0.1) to the winning event. The probabilities for the Hardy state used in [7] are given in Table *II.e.* Note that all the losing events have a zero probably whereas the winning event has a probability of 0.09.

III) General Game Structure

The results of the above examples are summarized in Table *III.a.* Note that we have restricted out analysis to one particle per player for one run of each game. There might be advantages of using an ensemble of entangled particles in a given game.

Table III.a

Scheme	#Particles	#Players	#Inputs	#Output
Bell	2	2	3	2
GHZ	3	3	2	2
Hardy	2	2	2	2

The games as discussed in the above examples are co-operative ones, i.e. the players can communication (before the game begins) to come up with a joint strategy to maximize their payoff. This seems to be a common aspect of the games constructed to utilize a specific shared entangled system. Now, given an entangled quantum state that exhibits violation of classical correlations for certain events, one can always construct a game that utilizes the violation by setting such events to be the winning events. So, if we are able to enumerate all such violations (Bell Inequalities) that exist for a given experimental setup, we have essentially identified all the cases in which that state may provide advantage in a classical game.

To generalize our formulation consider figure *III.a* introduced by Prof. Griffiths [18]

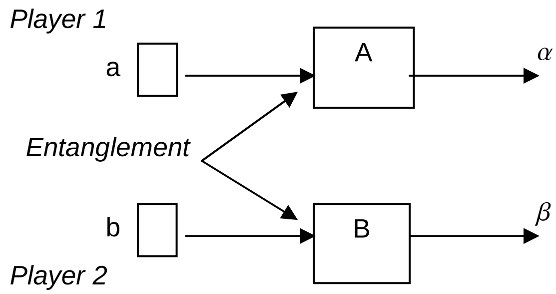


Figure III.a

The labels *(a)* and *(b)* refer to the degrees of freedom the players enjoy in choosing their measurement orientation. In another sense they correspond to the number of possible player types. So for example in Bell's case they correspond to the three possible player types. A and B then are the local operations / measurements that they choose to perform on their particle. The outputs α and β then take the possible output values that players can signal. Note that there is no communication allowed between the players once they have been taken to their "remote booths". It is obvious how the above setup generalizes to n players with various possible values of input and output types.

In the classical case (i.e. if no entangled particle is shared), we can establish the following relationship:

$$\sum_{\lambda} P(\alpha,\beta \mid a,b,\lambda) = \sum_{\lambda} P(\alpha \mid a,\lambda)P(\beta \mid b,\lambda)P(\lambda) \qquad (7)$$

One may think of the hidden variable λ as an "instruction set" that the particles carry with them when they are emitted from their source. These instructions determine the state the particle would be found in, given the measurement that is carried out. For quantum states we enforce a weaker constraint, i.e.

$$P(\alpha \mid a,b) = P(\alpha \mid a) \qquad (8)$$

$$P(\beta \mid a,b) = P(\beta \mid b) \qquad (9)$$

These equations imply that output α is independent of input *(b)* and output β is independent of input *(a).* There exist quantum states that satisfy these conditions but are not separable as required by (7). However, note that there are cases which satisfy conditions (8) and

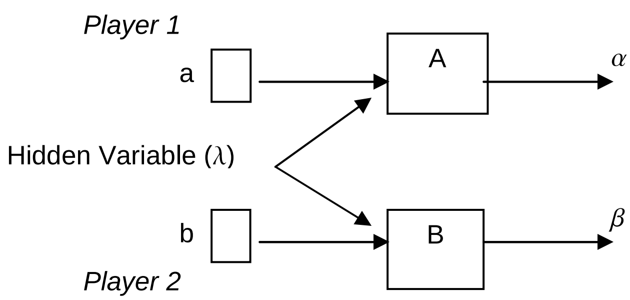
(9) but are not allowed by Quantum Physics.

IV) Computing all the Inequalities

Pitowsky and Svozil present a general method for the derivation of all Bell inequalities for each given experimental setup in [4]. The general setup that they consider is similar to the one discussed earlier in Section III with the important distinction that we replace our entangled particle with the notion of a Hidden Variable. The schematic for this is shown in Figure *V.a.* It can be shown that this setup is similar to the one considered by Pitowsky (although it is not trivial to do so).

In order to understand Pitowsky's formulation, consider the setup in Fig. *V.a.* Let the variables a, b, α and β take on two possible values from {0, 1}. Then there are four elementary events, i.e. a = 0, a = 1 corresponding to Player 1's choice of measurements and b = 0, b = 1 corresponding to Player 2's measurement choices.

Figure V.a



In order to derive all the inequalities for this case, list all the 2^4 = 16 instructions that the particles can carry in this scenario. In addition we also consider the following conjunctions of these propositions (a=0 AND b=0), (a=0 AND b=1), (a=1 AND b=0) and (a=1 AND b=1). The reason we do not consider (a=0 AND a=1) is essentially because these are two are mutually exclusive events in our experimental setup and hence do not contribute to the analysis. Now we obtain Table *V.a*:

a=0	a=1	b=0	b=1	a=0&b=0	a=0&b=1	a=1&b=0	a=1&b=1
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1

1	**0**	**0**	**0**	**0**	**0**	**0**	**0**
1	**0**	**0**	**1**	**0**	**1**	**0**	**0**
1	**0**	**1**	**0**	**1**	**0**	**0**	**0**
1	**0**	**1**	**1**	**1**	**1**	**0**	**0**
1	**1**	**0**	**0**	**0**	**0**	**0**	**0**
1	**1**	**0**	**1**	**0**	**1**	**0**	**1**
1	**1**	**1**	**0**	**1**	**0**	**1**	**0**
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**

Table V.a

The first four columns are essentially the instructions carried by the particles in the hidden variable model. Each row in this table forms a vector in an eight dimensional real space. Denote by C the convex hull of the sixteen vertices taken as vertices in this space. The dual representation of this hull is given by the intersection of a finite number of half-spaces that may be represented as linear inequalities. The above procedure can be applied to any general experimental setup that takes the form discussed in this section. In [13] and [14] Pitowsky shows that deciding whether a vector p is in this polytope is NP-Complete. However, in [4] and [16] use available linear programming packages [15] and Mathematica to solve for small cases of this problem such as GHZ.

Note that this approach allows us to test for all possible violations that occur in an experimental setup given a specific quantum state. We can simply perform a search on these inequalities using random input states until we encounter violations, hence allowing us to enumerate all such cases. The gives a somewhat systematic approach to identifying cases in which quantum information proves useful.

V) Multi-Prover Interactive Proof Systems

The aim in this section is to present better motivated examples on using quantum information by developing connections to multi-prover interactive proof systems. Two-prover interactive proof systems were introduced by Ben-Or, Goldwasser, Kilian and Wigderson in [9]. Formally, a multi-prover interactive proof system for a language L is a game between a verifier and $k \geq 2$ provers that interact on a common input in a way that satisfies the following properties:

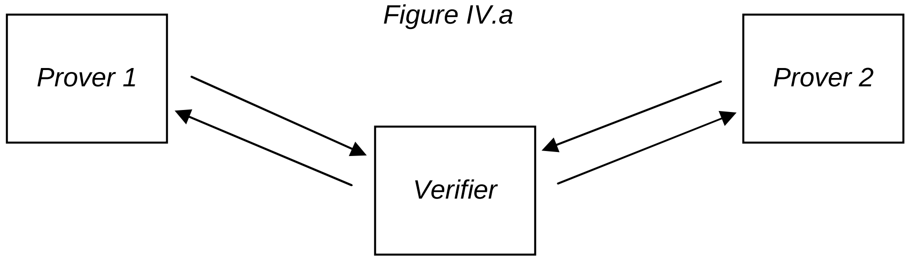
- The verifier's strategy is a probabilistic polynomial time procedure
- The only interaction allowed is between the verifier and each of the provers. (No communication can take place between the provers once the game has begun).
- A prover does not know the message exchanged between the verifier and any other prover.
- **Completeness:** For every $x \in L$, there exists a prover strategy P such that when

interacting on the common input x, the prover P convinces the verifier with probability at least 2/3 that $x \in L$

- **Soundness:** For every $x \notin L$, when interacting on the common input x, any prover strategy P convinces the verifier with probability at most 1/3.

We take the provers to be computationally unbounded. Now, the setup for a 2-prover system would be similar to the situation in Fig. *IV.a.*

Figure IV.a



Here we investigate what happens in these proof systems when the provers can share entanglement. Note that one may now think of the two provers as the two players in a game similar to what was discussed in preceding sections. The setup obtained is similar to the one shown in Fig. *IV.b.* We now give an example of a two-prover one round proof system that is classically sound but become unsound when provers are allowed quantum strategies.

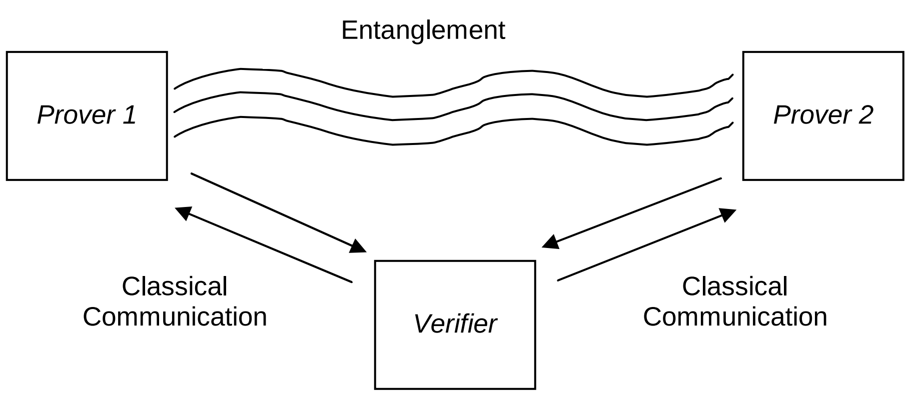


Figure IV.b

IV.1) 2-Coloring a Cycle

Here, the two provers try to convince a verifier of 2-coloring a cycle of length N in a graph such that all the adjacent nodes in the cycle are of different colors, except nodes which 1 and N which should have the same color. This is trivial when N is odd (just assign the coloring) and impossible when N is even. However in the quantum case, even when N is even the provers can convince the verifier that they know such a coloring with a high probability based on the scheme in [10].

The verifier asks prover 1 for the color of a random node and prover 2 the color of one of the adjacent nodes. If the provers succeed in giving the correct answers to the verifier in repeated experiments (with a new random edge chosen) then the verifier would be convinced that they know such a coloring. For the classical provers, the probability to fail is at least 1/N. So, the probability to convince the verifier 5N times is given by:

$$p_{classical} = (1 - \frac{1}{N})^{5N} \approx e^{-5} \approx 0.01 \qquad (10)$$

The quantum provers can do much better by sharing an EPR pair. When a player is asked the color of a bead *i,* he measures the spin component in the direction θ_i n the x-z plane which makes an angle $\theta_i = \frac{\pi}{N}$ with the z axes. The probability to succeed 5N times using this scheme is:

$$p_{quantum} = (1 - \sin^2 \frac{\pi}{2N})^{5N} \approx (1 - \frac{\pi^2}{4N^2})^{5N} \approx e^{\frac{-5\pi^2}{4N}} \qquad (11)$$

So, for N = 100, the quantum strategy allows almost a 90% chance to succeed, compared to a 1% for the classical provers. Cleve discusses a slightly different version of the same scheme for Odd Cycle games in [11].

IV.2) 3-SAT Proof System

This example is given in [11], which is based on a slight variation of the “Magic Square” discussed in [12]. Here the authors construct a specific instance of 3-SAT, where the resulting formula is not satisfiable, but for which there exists a perfect quantum strategy using a two-prover proof system. The “Magic Square” game essentially relies on the fact that there does not exist a 3 x 3 binary matrix such that each row has even parity and each column has odd parity. Now, player 1 is asked to fill in the values of a row or a column, and player 2 is asked to fill in a single entry corresponding to one of the three entries given to player 1 (chosen randomly). There is no classical strategy that would allow the players to obtain this correlation in every run of the game. However, using a quantum strategy player 1 would always be able to meet the parity conditions and player 2’s answer would always be consistent to player 1’s solution.

The generally used two-prover proof system for 3-SAT is similar to the setup of the game discussed above. The verifier sends Prover 1 a clause in the formula and Prover 2 a variable from that clause. Prover 1 returns an assignment to the variables that satisfy the given clause and Prover 2 must return a value for the received variable which is consistent with Prover 1's assignment. Now, let the variables x_{00}, x_{01}, x_{02}, x_{10}, x_{11}, x_{12}, x_{20}, x_{21}, x_{22} correspond to the 3 x 3 binary matrix.

$$
\begin{aligned}
&(\bar{x}_{00} \vee \bar{x}_{01} \vee \bar{x}_{02}) \wedge (\bar{x}_{00} \vee x_{01} \vee x_{02}) \wedge (x_{00} \vee \bar{x}_{01} \vee x_{02}) \wedge (x_{00} \vee x_{01} \vee \bar{x}_{02}) \wedge \\
&(\bar{x}_{10} \vee \bar{x}_{11} \vee \bar{x}_{12}) \wedge (\bar{x}_{10} \vee x_{11} \vee x_{12}) \wedge (x_{10} \vee \bar{x}_{11} \vee x_{12}) \wedge (x_{10} \vee x_{11} \vee \bar{x}_{12}) \wedge \\
&(\bar{x}_{20} \vee \bar{x}_{21} \vee \bar{x}_{22}) \wedge (\bar{x}_{20} \vee x_{21} \vee x_{22}) \wedge (x_{20} \vee \bar{x}_{21} \vee x_{22}) \wedge (x_{20} \vee x_{21} \vee \bar{x}_{22}) \wedge \\
&(x_{00} \vee x_{10} \vee x_{20}) \wedge (\bar{x}_{00} \vee \bar{x}_{10} \vee x_{20}) \wedge (x_{00} \vee \bar{x}_{10} \vee \bar{x}_{20}) \wedge (\bar{x}_{00} \vee x_{10} \vee \bar{x}_{20}) \wedge \\
&(x_{01} \vee x_{11} \vee x_{21}) \wedge (\bar{x}_{01} \vee \bar{x}_{11} \vee x_{21}) \wedge (x_{01} \vee \bar{x}_{11} \vee \bar{x}_{21}) \wedge (\bar{x}_{01} \vee x_{11} \vee \bar{x}_{21}) \wedge \\
&(x_{02} \vee x_{12} \vee x_{22}) \wedge (\bar{x}_{02} \vee \bar{x}_{12} \vee x_{22}) \wedge (x_{02} \vee \bar{x}_{12} \vee \bar{x}_{22}) \wedge (\bar{x}_{02} \vee x_{12} \vee \bar{x}_{22})
\end{aligned}
$$

The parity conditions for the first row are satisfied if and only if $x_{00} \oplus x_{01} \oplus x_{02} = 0$. There are similar restrictions on the remaining rows are columns. The above 24 clauses are enough to express the six parity conditions. Even though this formula is unsatisfiable, the quantum strategy from "Magic Square" game may be used to convince the verifier with certainty that this instance is satisfiable.

VI) Summary

This paper presented a general framework in terms of co-operative games that may be useful in understanding quantum entanglement. We note that all the cases where Bell inequalities are violated can be considered as games between players where by sharing an entangled state they are able to improve their expected payoff. The winning events are determined by whether the players are able to obtain certain correlations in their output based on their private input signals. Enumerating the Bell inequalities for a given experimental setup allows us to search for these quantum advantages.

One immediate possibility for future work is to determine a more refined approach to search for violations given the list of inequalities produced by the method in Section IV. This also allows for numerical studies of these games, hence possibly simplifying the search for generating useful multi-player quantum games [17].

The convex polytope discussed in Section IV is contained within a larger region that is allowed by quantum physics. How could we go about determining the surface of this region?

Appendix

A bimatrix game consists of two players (I and II) each of whom has a finite number of actions (pure strategies) from which to choose. Let x be a mixed strategy (probabilistic distribution over the rows and columns) of player I (the row player) and y a mixed strategy of player II (the column player).

Strategy x is a m-vector, where $x_i \geq 0$ and $\sum_{i=1}^{m} x_i = 1$ $\quad x = \begin{pmatrix} x_1 \\ x_2 \\ .. \\ x_m \end{pmatrix}$

Strategy y is a n-vector with similar constraints. The payoffs for the strategies of I and II are given by two m x n matrices A and B respectively. Assume WLOG that matrix elements for A and B are positive. This is unrestrictive because all the entries in A and B can be made positive by adding sufficiently large scalars. This doesn't affect the equilibrium solutions.

The entry a_{ij} is the payoff to player I, when he plays his i-th pure strategy and the opponent (player II) plays the pure strategy j. According to the mixed strategies x and y, the entry a_{ij} contributes to the expected payoff of player I with weight $x_i y_j$. So, the expected payoff for player I is (adding up all the entries of A weighted by the corresponding entries of x and y)

$$\sum_{ij} x_i y_j a_{ij} = \sum_i x_i \sum_j a_{ij} y_j = x^T A y$$

Similarly, the expected payoff of for player II is x^T By. A Nash equilibrium is a pair of strategies (x, y) such that neither I nor II have an incentive to change strategy, i.e.

$$x^T A y \geq x'^T \ A y \quad \& \quad x^T B y \geq x^T B y' \qquad \text{for all vectors x' and y'}$$

Such an equilibrium solution always exists. So, given the matrices A and B, the problem is to find a mixed strategy equilibrium. It is not known whether there is a polynomial algorithm for this problem.

A correlated equilibrium is nothing more than a Nash Equilibrium where each player may receive a private signal before the game is played. The players may base their choices on the signals received. A correlated equilibrium results when each player realizes that the best he can do is to follow the recommendation, provided that all the other players do likewise.

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