Applications of Spectral Algorithms

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Abstract

Spectral algorithms exploit information on the graph spectrum to gain computational speedups. Advances in spectral algorithms, such as spectral sparsifiers and fast symmetric diagonally dominant (SDD) system solvers, have created very powerful tools for the algorithmist. In this thesis, we would study the applications of spectral algorithms on a selection of problems where different classical algorithms are incorporated into a common spectral framework. Particularly, problems which can be represented as undirected graph are studied. The key advantages of such an approach is that it allows a common data structure, the graph laplacian, and a common subroutine, the SDD solver, to be shared across the various algorithms. In the application portion of the thesis, we using the grpah optimization framework in a spectral setting to perform various image processing tasks such as image restoration and segmentation. By apprioperiate choice of parameters, the graph optimization framework is capable of expressing classical signal processing and control theory algorithms such as guassian low pass filters. Also we show that it can be a viable upstream preprocess which can significantly boost the performance of downstreams processes such as segmentation.

1 Introduction

1.1 Previous Work

Currently, the state of the art solver approximate flow algorithms involve solving electrical flows [CKM$^+$11] [KMP10]. This involves solving optimization problems by computing multiple feasible electric flows to a graph and using the multiplicative weight schemes to solve for desired norms on the solution. In this thesis, we show that many graph algorithms have a natural formulation in spectral graph framework.

2 Formulation

In the spectral graph approach, the problems are presented as optimization problems on a weighted undirected graph that has an associated set of reweighting function on subsets of edges which we call clusters.

2.1 Graph Optimization

We solve problems in the following form

$$\min_X \sum_{c \in C} \sqrt{w_c \sum_{(i,j) \in E} w_{ij}(x_i - x_j)^2}$$

$$\text{subject } X \lceil S = s$$

Informally speaking, S is the set of vertices which takes in the input. Hence the $X \lceil S = s$ since the values s are fixed. X is the set of vertices which are our output. The edge set E is determined by our belief on the vertices. Also the reweighting function w_c, w_{ij} is used to expressed the task which we are going to solve. Through difference choices of the weights, we can express different norms for different applications. Currently, we can express $L1, L2, L_\infty$ norms in this framework.

2.2 Spectral Cuts

$$\text{find x, } Lx = \lambda Dx$$

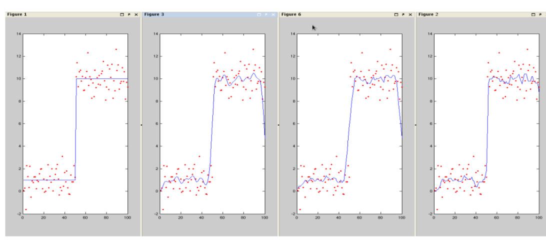
Solving the above system of equation gives the k-spectral cuts. This is the same problem as spectral clustering. However, instead of doing an expensive spectral decomposition, we can iteratively compute the k eigenvalues and eigenvectors using the SDD solver. Also we can refine the spectral cuts using spectral rounding approaches which are stuided in [KMT09]

3 Applications

Currently, we are applying this approach to image processing problems, even though it can be easily adapted to any undirected graph based problem. We are currently working on conversion of direct graphs problem to undirected graphs problem.

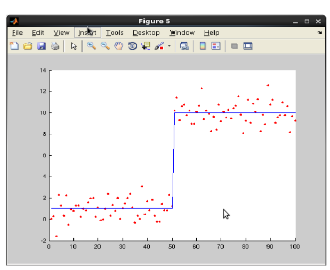
3.1 Signal Processing

In classical signal filtering approaches, prior belief of the noise structure is encoded in the choice of the filter. However, classical filters are unable to express mixtures of priors due to the lack of a framework. For example, given a step signal corrupted by gaussian noise,



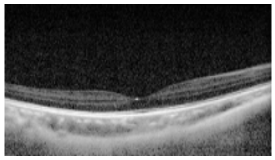
Noisy Signal Mean Filter Gaussian Low Pass Median Filter

We can encode a Gaussian belief on the the noise structure by using a $L2$ norm on the clusters running across the $X - S$ and encode a laplacian belief on the edges within X. We are able to achieve the following result.

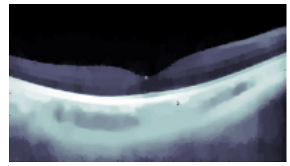


Note that the solution recovered simulatenously statisfies both beliefs on the noise and signal structure. This has many applications in both signal processing and control theory.

3.2 Image Denoising



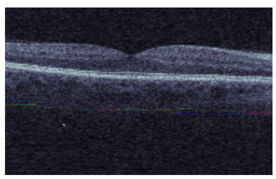
Noisy optical coherence tomography image of retina



denoise image that preserve key features.

In this application, using $L1/L2$ total variation minimization algorithms we are able to denoise images obtain from optical coherence imaging of the retina. Using a combination of the $L1/L2$ reweighting schemes, remove noise while preserving the sharpness between the nerve fiber layers. The nerve fiber layers are of clinical importance as anomalities often are indication of diseases.

3.3 Spectral Segmentation

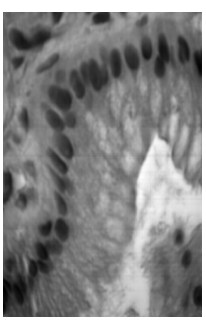


OCT image of retina

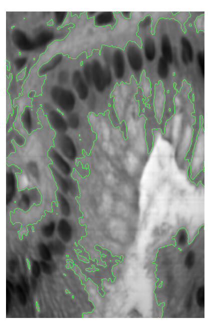
top 4 cuts of the OCT image

Using the spectral cut algorithm, we are able to obtain segmentations of very thin nerve fiber layers. This approach is superior over edge based segmentation techniques as we can incorperate different priors on the segmentation. In this example, we incorperate the prior that nerve fibers are thin horizontal strips.

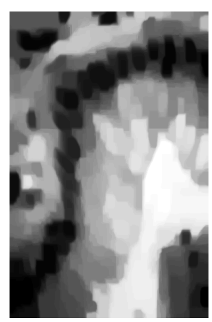
3.4 Segmentation Guide



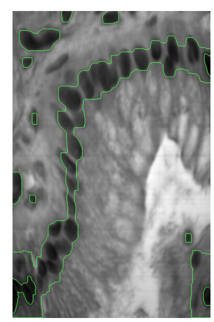
Original lung tissues



Naive segmentation using MATLAB



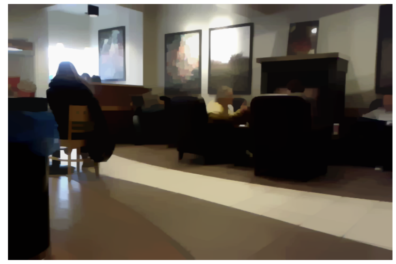
Aggressive denoising



Same segmentation algorithm

If specialized segmentation algorithm are preferred, we can use the denoising result as a guide for the downstream segmentation algorithm. Since denoised result remove local noise while maintaining global features such as long edges, it has a tendency of boosting the accuracy of the downstream algorithms. In this example, MATLAB's builtin segmentation algorithm is used to segment the nucleus (the black dots) of lung cell in a lung scan. Since the algorithm uses only local morphological operators, it is naive and and not robust. However, by doing denoising and using the denoise image as a guide, we can get closer segmentations.

3.5 Color Correction

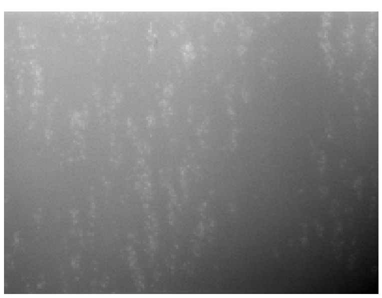
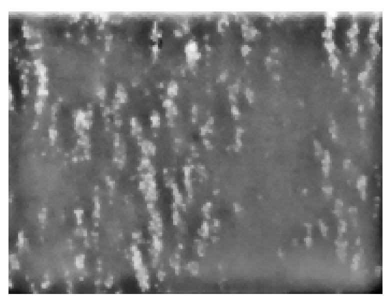
Color image Globally stable features

By setting the apprioriate ratio between the reweighting schemes of the input layer and the output layer, we are able compute globally stable features of images such a global illumination. These are features that likely to remain unchange under local transformation and they appear different scale spaces. Previous approaches that compute such features uses gaussian filtering which is an $L2$ optimization essentially. Our approach allows the preservation of sharp features such as edges. Such features are of interest to applications such as SIFT and object tracking.


newpaper scan with visual artifect corrected image

In addition to grayscale images, we can treat color images as graphs with vertices in 3 dimensional space, where each pixel is a combination of the RGB channel. We can reformulate the denoising problem as color correction if there are visual artifect that are present in a single color channel. This would have applications in optical character recognition tasks as the characters are preserve while large artifects, such as the yellowing of newspaper, are corrected.

3.6 Signal Boosting

blood platelet with visual artifects and occlusions boosted image

Combing all the approaches, we can boost weak signals by iteratively removing global features while accentuating local features. However, the algorithm is able to distinguish noise from true local features, such as blood platelets in the above example.

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