# Graphical Numerical Inference

a.k.a Brain Surgery for Excel

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***Abstract*—Excel's drag and auto-fill feature works for most simple numerical cases like addition. However, it fails when someone gives it a checkerboard pattern with 1s and 0s and tries to extend the pattern. Excel is unable to expand this obvious pattern because its entire inference is based on a static snapshot of the final data. Graphical Extrapolating Numerical Inferencer for Excel (GENIE), on the other hand, takes a dynamic approach by monitoring how the sequence is being filled. It will then try to figure out how the user is filling up the entries. After that, it picks up where the user has left off and fills in the rest of the entries.**

## I. Introduction

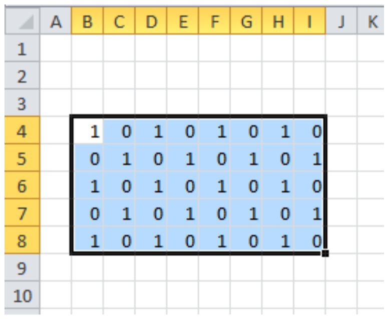
The drag-fill feature in Excel is trying to look for a trend in what the user is doing and fill the empty grids according to this trend. In essence, if the user has a predictable pattern, the program is trying to guess it. However, this feature in Excel has several major limitations making it useless except for doing the simplest of tasks. This paper tackles most of these limitations by looking at the data given from a dynamic point of view rather than a static one. The program that we have created monitors the way the user inputs the data in order to more accurately guess what the user's intention is. Excel's drag-fill feature only allows expansion in one direction (right or down). It does not work if the user wants to expand it to the top or to the left. Furthermore, Excel cannot infer data that is not given in a rectangular grid. This is because Excel assumes that the data is entered in a column-wise manner or a row-wise manner. However, this assumption may not hold for some data. Our program, the Graphical Extrapolating Numerical Inferencer for Excel (GENIE) does not make such assumptions and is hence able to have more accurate predictions. We hope that this program can offer a new perspective of how a spreadsheet program can be. Specifically, it is one that takes on a more proactive role when it comes to data extrapolation.

Numerical inference is not a solved problem. To properly describe the problem that we are trying to solve, we need to first define what it means to for a pattern to be inferable. We classify a sequence as inferable if and only if there exists a state machine with finite states such that if I run the state machine (possibly for infinitely many steps), then the sequence that we produce is the sequence that the user has in mind. There are many pre-existing methods for inferring patterns. However, there is no set of methods that can infer all possible inferable sequences. Therefore, websites like the Online Encyclopedia of Integer Sequences (OEIS) exist. It functions likes a huge database of integer sequences.

## II. Examples

### A. Checkerboard Pattern

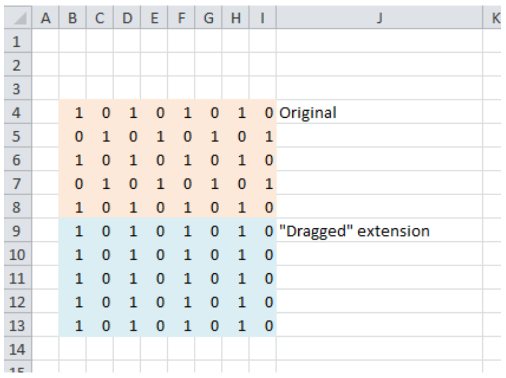
Let's start off with a simple example: The user wants to enter a checkerboard pattern into excel that consists of 1s and 0s, in the following manner:



| | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 5 | | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | | |
| 6 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 7 | | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | | |
| 8 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |

Figure 1

If you fill up a checkerboard pattern in Excel as above (Figure 1), and you try to drag the borders of this selection in attempts to expand the checkerboard, this is what you will get:



| | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | Original | |
| 5 | | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | | |
| 6 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 7 | | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | | |
| 8 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 9 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | "Dragged" extension | |
| 10 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 11 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 12 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 13 | | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | | |
| 14 | | | | | | | | | | | |

Figure 2

Instead of continuing the checkerboard pattern that the user wants, Excel merely duplicates the last line 5 times. Furthermore, the data was entered in a row-wise fashion.

### B. *Diagonal Inputs*

Besides predicting the pattern a user has in mind, GENIE can also help users infer patterns like the following:

| | **n=1** | **n=2** | **n=3** | **n=4** | **n=5** |
|---|---|---|---|---|---|
| **m=1** | **1** | **2** | **4** | **7** | **11** |
| **m=2** | **3** | **5** | **8** | **12** | |
| **m=3** | **6** | **9** | **13** | | |
| **m=4** | **10** | **14** | | | |
| **m=5** | **15** | | | | |

Figure 3

This table will gradually be filled in a diagonal order, starting at with 1 on the top left hand corner, followed by 2 and 3 on the next diagonal, 4, 5 and 6 on the next diagonal and so on. The program will observe and try to continue the trend by filling up the rest of the table. When the user is done entering the data in, he can click on infer and GENIE will produce the output desired.

## III. Previous Work

This thesis continues on Sam Tetruashvili's past work on inductive inference of integer sequences. In his paper, he mentions a novel spreadsheet program as possible future work. This thesis explores this possibility theoretically and provides a two-dimensional integer-based implementation of a spreadsheet like program. My research partner, Aaron Snook, provides an implementation of an integer inference algorithm that is sometimes used by my program to infer data in the cells. While his program can infer a large number of sequences, it requires a large input in order for the desired output to be inferred. However users of spreadsheet programs are seldom patient enough to input 100 data points before their data can be accurately inferred. This thesis attempts to fix this problem by exploring integer inference based on *minimal* points.

## IV. Goals

There are two main goals of GENIE

1. Inference - A program that can mimic a spreadsheet program for integers and infer continuations for the data.
2. Performance - It must be able to do the above with *minimal* inputs. By minimal, we mean that given a sequence, if it takes an average human *X* data points to successfully infer the continuation, it should take the program at most 1.5*X* data points to infer the same.

## V. Methodology

There are two main stages in GENIE: Deciphering the input and generating the output.

### A. *Deciphering the Input*

There are two kinds of input: Positional and value. By positional input, we refer to the order that the cells are being chosen when the user is entering in his data. For example, if he is keying in his data row by row, then the positional input is just row by row. In Figure 3, the user is keying in his data in diagonals so the positional input becomes *<(x, y), (x+1, y), (x, y+1), (x+2, y), (x+1, y+1), (x, y+2), …>*. By value input, we refer to the actual values that the user is entering in. For example, in Figure 3, the user enters *<1, 2, 3, 4, 5, 6, 7, …>* in this order and hence the value input sequence is the same.

#### *a) Positional Input*

Positional Input can be grouped into two categories depending on whether the input makes sense if we split up the rows and columns to consider them individually. For example, in the example of the n by 8 checkerboard, let us represent each 1 with a click and each 0 without a click. And we will fill in the grid in a row by row order. So the first row will be filled left to right with each alternating grid clicked, and then the second, and then the third, and so on. If we consider the row-coordinates of the grids that are clicked, we will see: <1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4,…> Now if we consider the column-coordinates, we will see: 1, 3, 5, 7, 2, 4, 6, 8, 1, 3, 5, 7, 2, 4, 6, 8, … Notice how the two sequences have predictable patterns independent of the other. This case falls into the first category – Analyze row and column independent of each other.

However, consider a spiral that looks like the following:

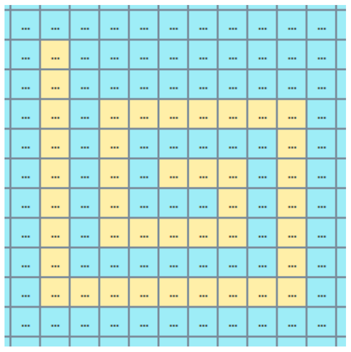


Figure 4

If we look at the row coordinates, we get this: 9, 9, 9, 10, 11, 11, 11, 11, 11, 10, 9, 8, 7, 7, 7, 7, 7, 7, 7, 8, 9, 10, 11, 12, 13, 13, 13, 13, 13, 13, 13, 13, 13, 12, 11, 10, 9, 8, 7, 6, 5. This pattern, compared to the previous, is not as clear. However, if we look at how the coordinates are moving as a 2-tuple, the motion is a lot more obvious. R, R, D, D, L, L, L, L, U, U, U, U, R, R, R, R, R, R, D, D, D, D, D, D, … This falls into the second category – Analyze displacements as 2-tuples.

### b) *Value Input*

Since this GENIE is modeled after a spreadsheet program like Excel, users can also choose to enter data into the cells as they are selecting them. The data that they are entering will also be interpreted as an integer sequence. This sequence can be considered on its own, independent of where it is being entered, or as a function of the position of the cell. For example, consider figure 6. Data is entered in the following fashion into the grid <1, 2, 2, 3, 4, 3, 4, 6, 6, 4, …>.

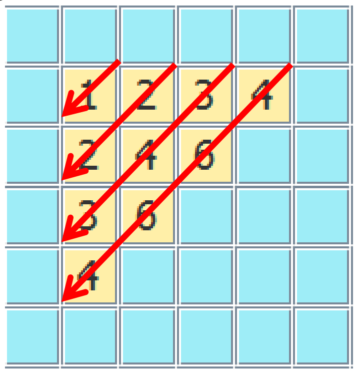


Figure 6

If we consider the sequence entered independent of the position of the data, we will get <1, 2, 2, 3, 4, 3, 4, 6, 6, 4, …>, which is a very hard sequence to infer (especially with so little points. However, if we consider the value entered as a function of *x* and *y*, we can see that the value of the grid is just *x* * *y*. GENIE currently does not have this feature.

## B. *Generating the Output*

### *a) Analyze row and column independent of each other*

For the first category of sequences, we can only look at the row coordinates and column coordinates separately. We will then pass the row (or column) coordinates through a database of patterns which will then determine if the input is an instance of any of the classes of patterns in the database. Since each class of pattern in the database is essentially a state machine waiting for the input to fill in the constants. If the pattern does not match the state machine, then that particular state machine will return false for the given input. Each of the state machines in the database will individually assess whether the pattern matches its template. For example, the following is an example of a state machine that accepts input which accepts sequences $i - p = d$, where $i$ is the input that is currently being red, $p$ is the previous input, and $c$ is some natural number. The starting state, $a_0$, must be a natural number too.

The example on the next page is a template for the state machine that is described above. There is one variable stored in every state except the fail state, which is denoted by "F". The program is initiallized with the variables set to null. When the first input is read, a0 will be updated and d will be updated once. From then on, only p will be updated. If we have to ever update d, we will end up in the "F" state and the machine will terminate. If we read until the end of the input and never get to the "F" state, then the input given matches the given machine. This machine is then run forever to produce the continuation for the given input.

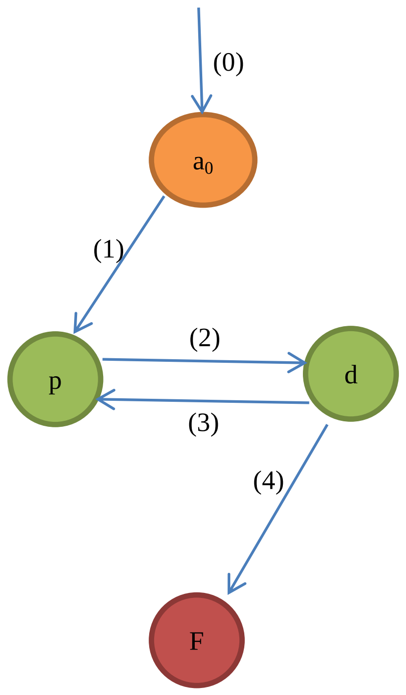


Figure 5

(0) Upon reading the first input, we update $a_0$ with the first number. For the example given, $a_0$ will be set to 3. Set *i*, the variable that keeps track of the last input, to $a_0$.

(1) We will move to the "*p*" state which stands for print. We will set the value at *p* to $a_0$.

(p) At state p, we will set *p:= i*. Then we will print out the value of *p*.

(2) We will then read the next input, *i*, and move to *d*.

(d) If the variable at *d* is unset, it will be set to *i – a0*. Else, we will check whether *d == i – p*. If *i* is null (we have reached the end of the input), set *i := p+d,* keeping the value of *d* intact.

(3) If *d == i-p*, then we will move back to state *p*.

(4) If *d != i-p*, then we will move to the "*F*" state.

(F) Do nothing.

To generate the continuation, all we have to do is continue running the state machine with null input.

There are 11 state machines in total in GENIE. Further details are in the appendix.

*b) Analyze displacement from previous entry*

1. We will first transform the sequence of inputs from coordinates to displacements from the previous point. In the example of the spiral that is shown above, we will get : R, R, D, D, L, L, L, L, U, U, U, U, R, R, R, R, R, R, D, D, D, D, D, D, … where R = (1, 0), L = (-1, 0), U = (0, 1) and D = (0, -1).
2. We will then map each of these displacements to a natural number. Hence, the sequence will now become 1, 1, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, …
3. This sequence will then be passed through state machines 1-11 to find a continuation.
4. The continuation will be in the form of natural numbers of we will then maps these back to the displacement form.
5. Lastly, we will find out the last input point that we have and calculate the next points based on the current point and the displacement from the current point.

## VI. Results

We have conducted an experiment to test how well GENIE satisfies Goals 1 and 2 as listed in the goals section. We asked 100 volunteers to give one pattern each. If their pattern is similar to what someone else gave previously, they are asked to think of another one. While they are inputting the test pattern, we asked another volunteer also observe the sequence. When the human observer is confident that he knows exactly how to continue the sequence, he will say "stop" and we will proceed to take note of the number of entries that he needs to infer the data. We will then re-enter the data into the computer and test for the minimum number of entries GENIE needs to infer the data.

To test for Goal 1, we just have to enter as many points as many points as necessary for GENIE to infer the sequence. If it is able to do that, then we will move on to test for the next goal.

To test for Goal 2, we will compare the minimum number of entries GENIE needs to infer the data with the number of entries the human observer needs to infer the data.

Here are some results:

Percentage of sequences that successfully inferred:

GENIE: 78%
Excel: 2%
Human: 98%

Percentage of data points GENIE needs to successfully infer a sequence (Compared to a human)

Average: 127%
Max: 150%
Min: 50%

## VII. Conclusion

From our results above, GENIE performs satisfactorily when it comes to meeting our goals. For most sequences, GENIE can infer them in under a second. This means that GENIE can definitely be a base for a useful extension to existing spreadsheet programs like Excel.

## VIII. Future Work

1. Non-integer inputs

A constraint for GENIE right now is that it can only take in numerical inputs in the cells. Extending this to take in all forms of inputs, cell numbers and reals for example, will be able to more realistically demonstrate the possibility of GENIE being an alternative to the conventional spreadsheet programs.

2. Solver for mathematical problems

GENIE can be further extended into a problem solver for mathematical problems. Many math problems can be reduced into a 1-D or 2-D array of numbers. GENIE can be used to extrapolate these data to produce more data points, and even guess the closed-forms for the solutions using programs like Wolfram|Alpha.

## IX. References

1. N. J. A. Sloane. The On-Line Encyclopedia of Integer Sequences.
http://www.research.att.com/njas/sequences/

2. Wolfram | Alpha
http://www.wolframalpha.com

3. Sam Tetruashvili and Manuel Blum. Inductive Inference of Integer Sequences
http://www.cs.cmu.edu/afs/cs/user/mjs/ftp/thesis-program/2010/abstracts/tetruashvili-EA.pdf

## X. Appendix

### *a) State machines*

State Machine 1:

- Recognizes sequences where consecutive terms differ by a fixed amount.
- Examples
  - 1, 2, 3, 4, 5, 6, ...
  - 3, 6, 9, 12, 15, 18, …
  - 19, 17, 15, 13, 11, …

State Machine 2:

- Recognizes sequences of repeated intervals.
- Examples
  - 2, 6, 3, 1, 4, 2, 6, 3, 1, 4, …

State Machine 3:

- Recognizes blocks of numbers of fixed lengths.
- Examples
  - 1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, …
  - 4, 4, 4, 4, 7, 7, 7, 7, 10, 10, 10, 10, 13, 13, 13, 13, …

State Machine 4:

- Recognizes patterns of increasing / decreasing intervals
- Examples
  - 3, 3, 5, 3, 5, 7, 3, 5, 7, 9, …
  - 3, 5, 3, 7, 5, 3, 9, 7, 5, 3, …
  - 12, 10, 8, 6, 4, 2, 10, 8, 6, 4, 2, …
  - 2, 4, 6, 8, 10, 12, 2, 4, 6, 8, 10, 2, 4, 6, 8, …

State Machine 5:

- Recognizes blocks of same numbers repeated for increasing / decreasing intervals
- Examples
  - 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, …
  - 8, 8, 8, 8, 8, 8, 7, 7, 7, 7, 7, 6, 6, 6, 6, 5, …

State Machine 6:

- Recognizes increasing / decreasing snake-like patterns
- Examples
  - 1, 2, 2, 1, 1, 2, 3, 3, 2, 1, 1, 2, 3, 4, 4, 3, 2, 1, …
  - 1, 2, 3, 4, 5, 6, 6, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1, …

State Machine 7:

- Checks for interleaved sequences where if we extract the individual sequences, these individual sequences will fit into one of the state machines 1 to 6.
- Examples
  - 1, 1, 3, 1, 5, 1, 7, 1, 9, 1, … [odds interleaved with ones

State Machine 8

- Recognizes pattern that appear in "squares"
- Examples
  - 1, 2, 3, 2, 3, 4, 5, 6, 4, 5, 6, 4, 5, 6, 7, 8, 9, 10, 7, 8, 9, 10, 7, 8, 9, 10, 7, 8, 9, 10, …

State Machine 9

- Recognizes the pattern where the first number is being repeated once, the next two numbers repeated twice, the next three numbers repeated thrice, etc.
- Examples
  - 1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10, 10, 10, …

State Machine 10

- This is technically not a state machine. It searches for this sequence in the online encyclopedia of integer sequences for a hit and a continuation.
- Examples
  - 1, 1, 2, 3, 5, 8, 13, 21, …
  - 2, 3, 5, 7, 11, 13, 17, 19, …

State Machine 11

- This state machine caters to noise at the start of a sequence and then runs state machines 1-10 on the de-noised version.
- Examples
  - 120, 284, 1, 2, 3, 4, 5, 6, …