When Model Checking Met Deduction

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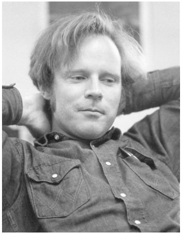
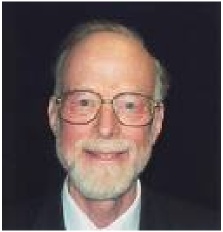
Alan Turing

It is of course important that some efforts be made to verify the correctness of assertions that are made about a routine. There are essentially two types of method available, the theoretical and the experimental. In the extreme form of the theoretical method a watertight mathematical proof is provided for the assertion. In the extreme form of the experimental method, the routine is tried out on the machine with a variety of initial conditions and is pronounced fit if the assertions hold in each case.

Alan Turing (quoted by D. MacKenzie in *Risk and Reason*)

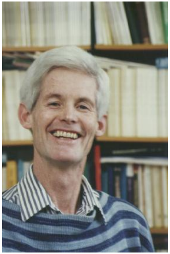
Floyd, Hoare, and Dijkstra

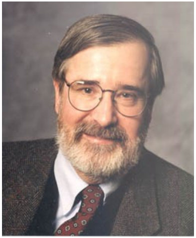




Skip	$\{P\}skip\{P\}$
Assignment	$\{P[\overline{e}/\overline{y}]\}\overline{y} := \overline{e}\{P\}$
Conditional	$\dfrac{\{C \wedge P\}S_1\{Q\} \quad \{\neg C \wedge P\}S_2\{Q\}}{\{P\}C \ ?\ S_1 \ :\ S_2\{Q\}}$
Loop	$\dfrac{\{P \wedge C\}S\{P\}}{\{P\}while\ C\ do\ S\{P \wedge \neg C\}}$
Composition	$\dfrac{\{P\}S_1\{R\} \quad \{R\}S_2\{Q\}}{\{P\}S_1; S_2\{Q\}}$
Consequence	$\dfrac{P \Rightarrow P' \quad \{P'\}S\{Q'\} \quad Q' \Rightarrow Q}{\{P\}S\{Q\}}$

Cook Completeness and Clarke Incompleteness [Apt'81]





7.1 Clarke's Incompleteness Result

A satisfactory treatment of procedures having procedures as parameters is impossible in full generality within the framework of Hoare's logic. This rather astonishing result was proved by Clarke [9] and is the contents of the following theorem.

THEOREM 4. *There exists no Hoare's proof system which is sound and complete in the sense of Cook for a programming language which allows*

1. *procedures as parameters in procedure calls,*
2. *recursion,*
3. *static scope,*
4. *global variables in procedure bodies, and*
5. *local procedure declarations.*

The Role of Fixpoints

- Dijkstra's predicate transformer semantics is key to the completeness argument.
- Verifying $\{P\}S\{Q\}$ reduces to showing
  1. $\{wlp(S)(Q)\}S\{Q\}$, which is always valid, and
  2. $P \implies wlp(S)(Q)$.
- $wlp(while\ C\ do\ S)(Q) = \nu X.(\neg C \wedge Q) \vee (C \wedge wlp(S)(X))$.
- Incompleteness is related to the undecidability of the Halting problem over finite interpretations for the given class of programs.

Fixpoints to Model Checking

- Temporal logics have a fixpoint characterization.

$$\mathbf{EF}p = \mu Y.p \vee EXY$$

$$\mathbf{EG}p = \nu Y.p \wedge EXY$$

- For finite-state systems, the states can be classified in a bounded number of steps.
- With symbolic representations (Binary Decision Diagrams, Difference-Bounded Matrices), the image computations and equivalence checks can be done using logic operations.
- Predicate abstraction allowed logic to sneak in even further through finite-state over-approximations of infinite-state behavior.
- Bounded model checking, k-induction, and interpolation lean more heavily on deduction than model checking.

Bradley's Algorithm

- Bradley's algorithm works by Conflict Directed Reachability (CDR) captured by the abstract system below.[1]
- Given a transition system $M = \langle I, N \rangle$, let $M[X] = I \sqcup N[X]$, where $N[X]$ is the image and $N^{-1}[X]$ is the preimage.
- The state of the algorithm, initially $n = 0$, $Q_0 = I$, $C_0 = \emptyset$, consists of
  1. Inductive candidates (sets of clauses) $Q_0, \ldots, Q_n$:
     1. $Q_0 = I$
     2. $Q_i \sqsubseteq Q_j \sqcap P$ for $i < j \leq n$
     3. $N[Q_i] \sqsubseteq Q_{i+1}$
  2. Counterexample candidates (sets of cubes) $C_0, \ldots, C_n$, where each C_i is a set of symbolic counterexamples: for each (cube) $R \in C_i$
     1. $R = \neg P$ and $i = n$, or there is an S in $C_{i'}$, $R \sqsubseteq N^{-1}[S]$, where $i' = n$ if $i = n$, and $i' = i + 1$, otherwise.
     2. $Q_j \sqsubseteq \neg R$ for all $j < i$.
     3. $R \sqcap Q_i$ is nonempty, i.e., $Q_i \not\sqsubseteq \neg R$.

[1] Thanks to Aaron Bradley, Dejan Jovanović, and Bruno Dutertre for feedback.

Abstract Conflict Directed Reachability

- **Fail:** If C_0 is nonempty, $\neg P$ is reachable.
- **Succeed:** If $Q_i = Q_{i+1}$ for some $i < n$, we have an inductive weakening of P.
- **Extend:** If C_i is empty for each $i \leq n$, add Q_{n+1} such that $M[Q_n] \sqsubseteq Q_{n+1}$ and $C_{n+1} = \emptyset$, if $Q_{n+1} \sqsubseteq P$, and $C_{n+1} = \{\neg P\}$, otherwise.
- **Refine:** Check $N[Q_i] \sqsubseteq \neg R$ for some R in C_{i+1}, where C_i is empty, for $j \leq i$:
    1. **Strengthen:** If the query succeeds, find an $\widehat{R}$ weakening R that is relatively inductive: $M[Q_i \sqcap \neg\widehat{R}] \sqsubseteq \neg\widehat{R}$: conjoin $\neg\widehat{R}$ to each Q_j for $1 \leq j \leq i+1$, move any $S \in C_{i+1}$ such that $Q_{i+1} \sqsubseteq \neg S$ (including R) to C_{i+2} if $i+1 < n$.
    2. **Reverse:** If the query fails with counterexample s, weaken s to S such that $S \sqsubseteq N^{-1}[R]$ and add S to C_i.
- **Propagate:** Whenever Q_i is strengthened, strengthen Q_{i+1} with Q where $M[Q_i] \sqsubseteq Q$, move any $R \in C_{i+1}$ such that $Q_{i+1} \sqsubseteq \neg R$ to C_{i+2} if $i+1 < n$.

Model Checking Becomes Deduction

- Automated verification through model checking of temporal formulas was very successful for essentially finite-state systems.
- Many systems could be reduced to tractable finite-state systems through abstraction, composition, and a little deduction.
- In the Handbook article, *deductive* is interpreted (narrowly) as syntax-directed, and *model checking* (broadly) as based on semantic unfolding.
- "Bounded model checking", k-induction, and interpolation, as practised, are really *deductive* approaches.
- Techniques like Bradley's CDR are squarely deductive, but owe a lot to model checking.
- Ed's contributions have radically advanced verification as a whole, and not merely model checking.

"Happy Retirement, Ed"

We obviously have a long way to go, so it's good that Ed will soon have few other distractions.