Tractability and Intractability in Model Checking

Rupak Majumdar

Max-Planck Institute for Software Systems

A Puzzle

UNITED STATES OF AMERICA
$1

Move #1

Move #2

UNITED STATES OF AMERICA
$1

Perform Moves #1 and #2 any number of times.
Keep all the coins at the end.

How much money can you make?

$100?

$1000?

$ 1 MILLION?

MORE MONEY THAN BILL GATES?

INFINITE?

Not Infinite…

Each move decreases the value in lexicographic ordering
So no matter what you do, the process will terminate

Not Infinite…

Each move decreases the value in lexicographic ordering
So no matter what you do, the process will terminate

Three Cups

UNITED STATES OF AMERICA
$1
UNITED STATES OF AMERICA
$1
UNITED STATES OF AMERICA
$1

Three Cups

(a, b, 0) → (a, 0, 2b) → (a-1, 2b, 0) →* (0, 2^a.b, 0)

(a, 0, 0) →* (0, 2^a, 0)

Four Cups

(a, b, 0, 0) →* (a, 0, 2^b, 0) → (a-1, 2^b, 0, 0)

(a, 0, 0, 0) →* (0, 2^^a, 0, 0)

Knuth's Up Arrow

$$a \text{ ^^ } 0 = 1$$

$$a \text{ ^^ } (b + 1) = a \text{ ^ } (a \text{ ^^ } b)$$

$$2 \text{ ^^ } 5 = 2 \text{ ^ } 65536 \sim 10 \text{ ^ } 20033$$

$$2 \text{ ^^ } 6 = 2 \text{ ^ } .$$

Bill Gates has < $10^10

Knuth's Up Arrow

a ^^(0) b = a . b

a ^^(n) 0 = 1 (if n>=1)

a ^^(n) b = a ^^(n-1) (a ^^(n) (b – 1))

(Ackermann, non-primitive recursive)

N Cups

(a, 0, …, 0) →* (0, 2^^(N-2) a, 0, …, 0)

Where is Model Checking?

Finite state spaces defined by simple transitions can be very large!

Decidable != Practical

Undecidable != Impractical

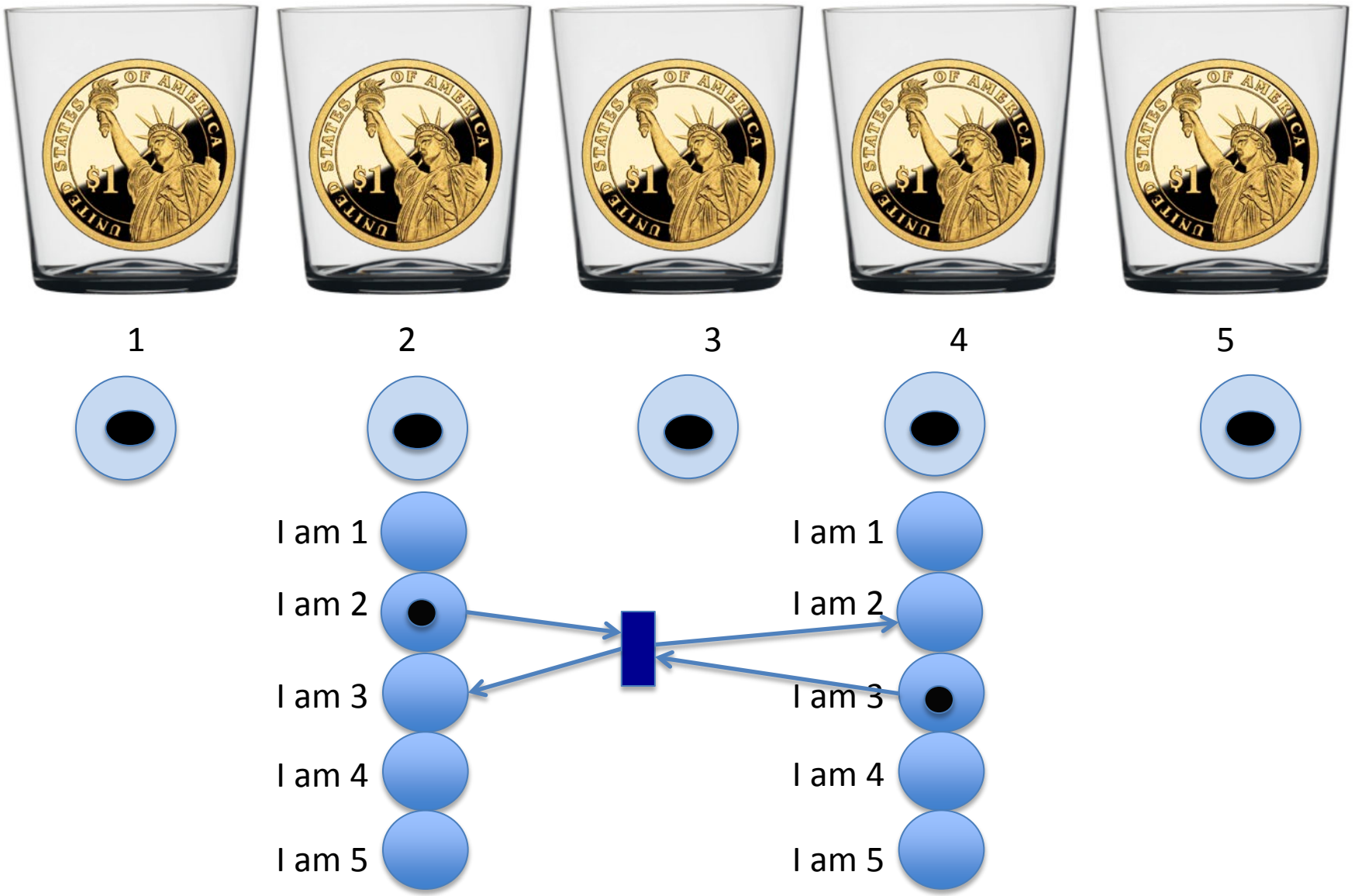
Oh and One More Thing…

Theorem: There is a family of Petri nets with finite but non-primitive recursive reachable state space

[Mayr & Meyer 1981]

Petri Net?

Move #2?



Thank You