Bayes Network

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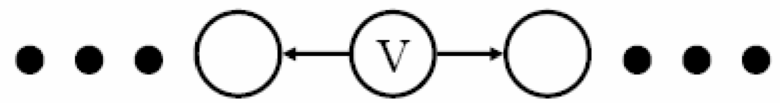
Recap

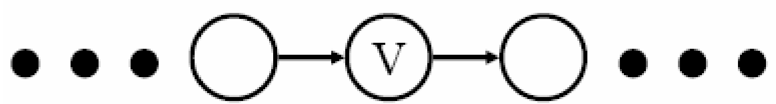
- A path $X_1 - X_2 - \cdots - X_k$ is an active trail when variables $\mathbf{Z} \subseteq \{X_1, \ldots, X_n\}$ are observed if for ***EACH*** consecutive triplet in the trail:
  - $X_{i-1} \rightarrow X_i \rightarrow X_{i+1}$ and X_i is NOT observed
  - $X_{i-1} \leftarrow X_i \leftarrow X_{i+1}$ and X_i is NOT observed
  - $X_{i-1} \leftarrow X_i \rightarrow X_{i+1}$ and X_i is NOT observed
  - $X_{i-1} \rightarrow X_i \leftarrow X_{i+1}$ and X_i is observed, or one of its descendents is observed

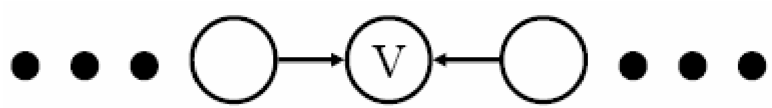
Recap

- Theorem: variables $\mathbf{x}_i$ and $\mathbf{x}_j$ are independent given $\mathbf{z} \subseteq \{X_1,\ldots,X_n\}$ if there is ***NO active trail*** between $\mathbf{x}_i$ and $\mathbf{x}_j$ when variables $\mathbf{z} \subseteq \{X_1,\ldots,X_n\}$ are observed.
- In other words, every trail between $\mathbf{x}_i$ and $\mathbf{x}_j$ is ***NON-active*** when $\mathbf{z} \subseteq \{X_1,\ldots,X_n\}$ are observed

NON-Active Trail

- There exists a node *V*, such that
  - *V* is observed, and
  - 

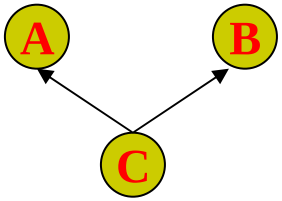
- OR there exists a node *V*, such that
  - *V* is observed, and
  - 

- OR there exists a node *V*, such that
  - **Neither** *V* **nor** any of its descendant is observed, and
  - 

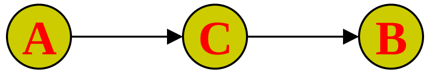
(V-structure)

Examples

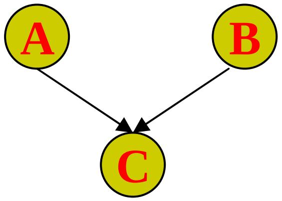
- Trail from A to B



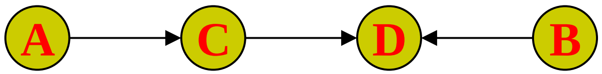
Z=[] Active
Z=[C] Non-Active



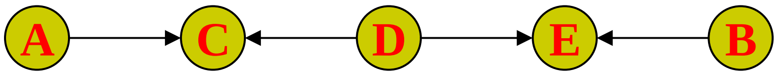
Z=[] Active
Z=[C] Non-Active



Z=[] Non-Active
Z=[C] Active

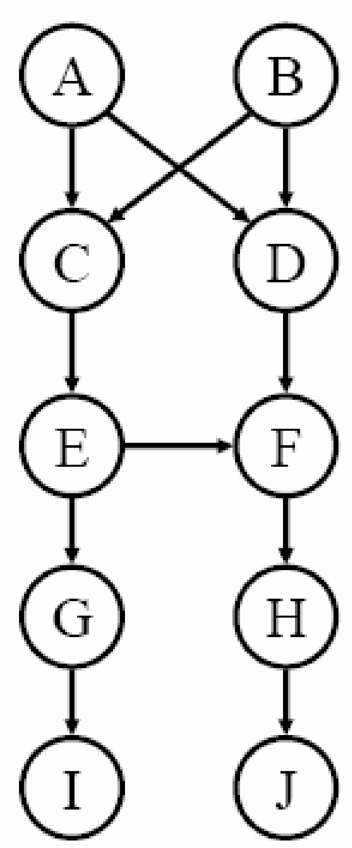


Z=[] Non-Active
Z=[C] Non-Active
Z=[D] Active
Z=[C, D] Non-Active



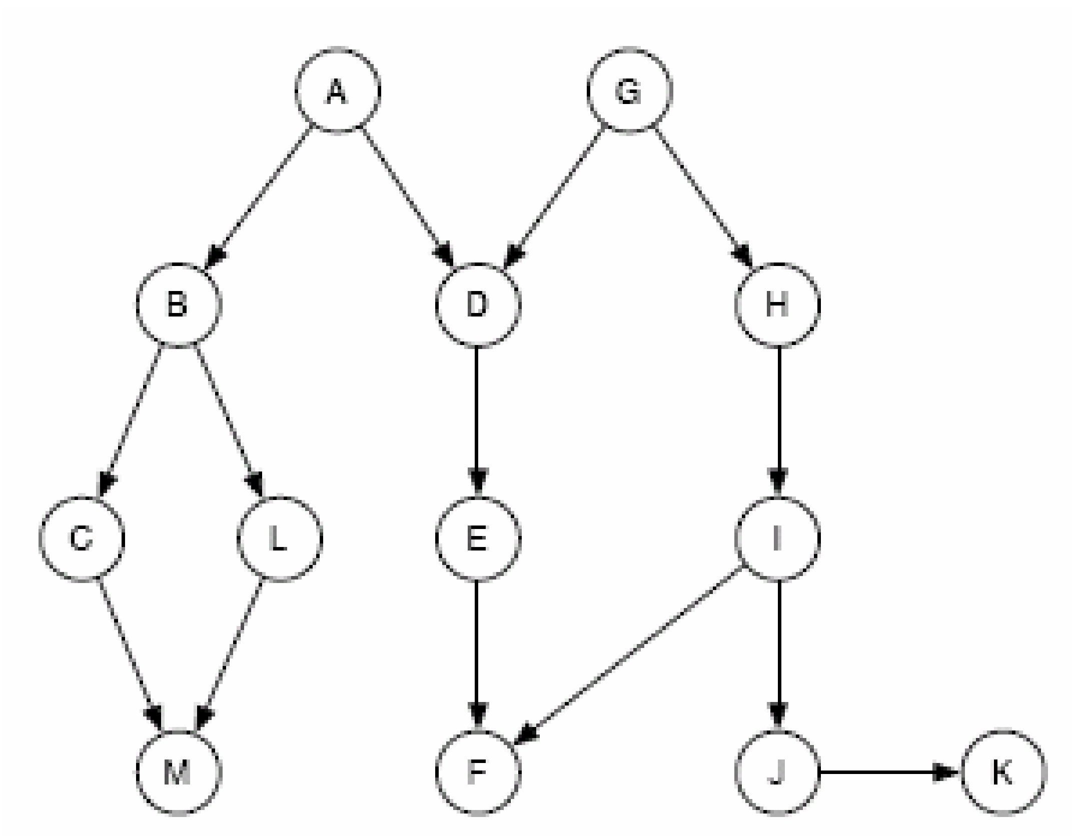
Z=[] Non-Active
Z=[C] Non-Active
Z=[D] Non-Active
Z=[C, E] Active
Z=[C, D, E] Non-Active

Test Your Understanding



- I<C, {}, D>? ✗
- I<C, {A}, D>? ✗
- I<C, {A, B}, D>? ✓
- I<C, {A, B, J}, D>? ✗
- I<C, {A, B, E, J}, D>? ✓

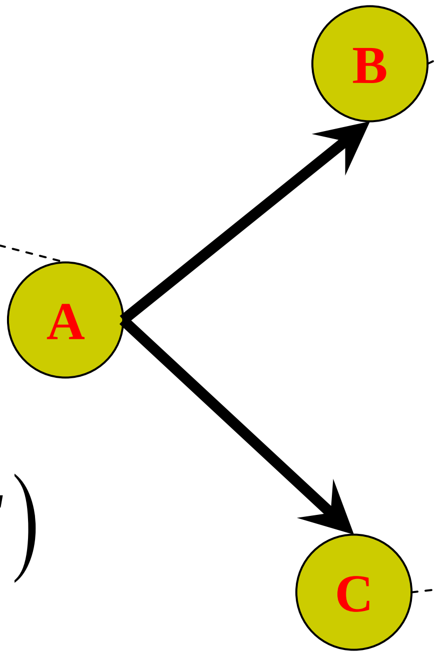
Test Your Understanding



- $I\langle B,\{E\},F\rangle$?
- $I\langle G,\{F, I\},K\rangle$?
- $I\langle D,\{G\},I\rangle$?
- $I\langle B,\{A, F\},H\rangle$?

Inference

T	F
0.2	0.8



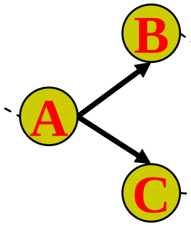
A	T	F
T	0.3	0.7
F	0.25	0.75

A	T	F
T	0.37	0.63
F	0.21	0.79

$$P\left(B=T \middle| C=T\right)$$

Inference

T	F
0.2	0.8



A	T	F
T	0.3	0.7
F	0.25	0.75

A	T	F
T	0.37	0.63
F	0.21	0.79

- $$P(B=T \mid C=T) = \frac{P(B=T, C=T)}{P(C=T)}$$
- $$P(B=T, C=T) = \sum_A P(A, B=T, C=T)$$

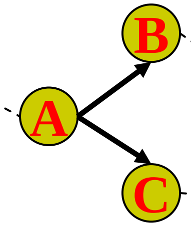
$$= \sum_A P(A) P(B=T \mid A) P(C=T \mid A)$$

$$= 0.2 \times 0.3 \times 0.37 + 0.8 \times 0.25 \times 0.21$$

$$= 0.0642$$

Inference

T	F
0.2	0.8



A	T	F
T	0.3	0.7
F	0.25	0.75

A	T	F
T	0.37	0.63
F	0.21	0.79

- $P(C=T)=P(B=T,C=T)+P(B=F,C=T)$
- $P(B=F,C=T)=\sum_A P(A,B=F,C=T)$

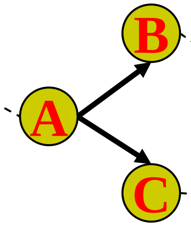
$$=\sum_A P(A)P(B=F|A)P(C=T|A)$$

$$=0.2\times 0.7\times 0.37+0.8\times 0.75\times 0.21$$

$$=0.1778$$

Inference

T	F
0.2	0.8



A	T	F
T	0.3	0.7
F	0.25	0.75

A	T	F
T	0.37	0.63
F	0.21	0.79

- $$P(B=T \mid C=T) = \frac{P(B=T, C=T)}{P(C=T)}$$

$$= \frac{P(B=T, C=T)}{P(B=T, C=T) + P(B=F, C=T)}$$

$$= \frac{0.0642}{0.0642 + 0.1778}$$

$$= 0.2653$$