Distributed Coordination In Multi-Agent Control Systems Through Model Predictive Control

THESIS PROPOSAL

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

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Abstract

This work concerns multi-agent control systems where the agents coordinate their actions without the help of a central coordinator. Each agent uses a model predictive control (MPC) strategy, viewing influences from neighbor subsystems as bounded disturbances in its local model. Agents exchange predictions of the bounds on their future state trajectories and incorporate this information into their local MPC problems. Minimax optimization is used to minimize the worst-case performance, and some additional constraints are incorporated to guarantee the feasibility of control actions. Parameterized feedback control policies are introduced in the MPC optimization to reduce conservativeness. The principal contributions of the proposed research are:

- A framework for multi-agent coordinated control systems with MPC strategy;
- A one-step delayed information exchange for coordination;
- Viewing interactions among subsystems as bounded disturbances with updated information;
- A new MPC optimization problem to incorporate bounded disturbances and model uncertainties;
- Introducing parameterized feedback policy in the MPC optimization to reducing conservativeness;
- The application of a set approximation algorithm for the proposed distributed control scheme;
- Computing the corresponding θ-control invariant sets;
- Incorporating set-membership estimation into the scheme to accommodate the uncertainties in state estimation;
- Information compensation for the time mismatch in asynchronous control;
- Techniques to compute the reachable sets for hybrid systems;
- Demonstration of the proposed framework for two application examples: generation control in power systems and plant-wide control in chemical plants.

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1 Introduction

This proposal concerns the decentralized control of large-scale dynamic systems comprised of several interacting subsystems as shown in Figure 1. Controllers, or *agents*, are designed to control the subsystems based on local measurements to achieve some local objectives. We consider applications where the distributed agents need to coordinate with each other to achieve certain global objectives, such as the stability of the whole system.

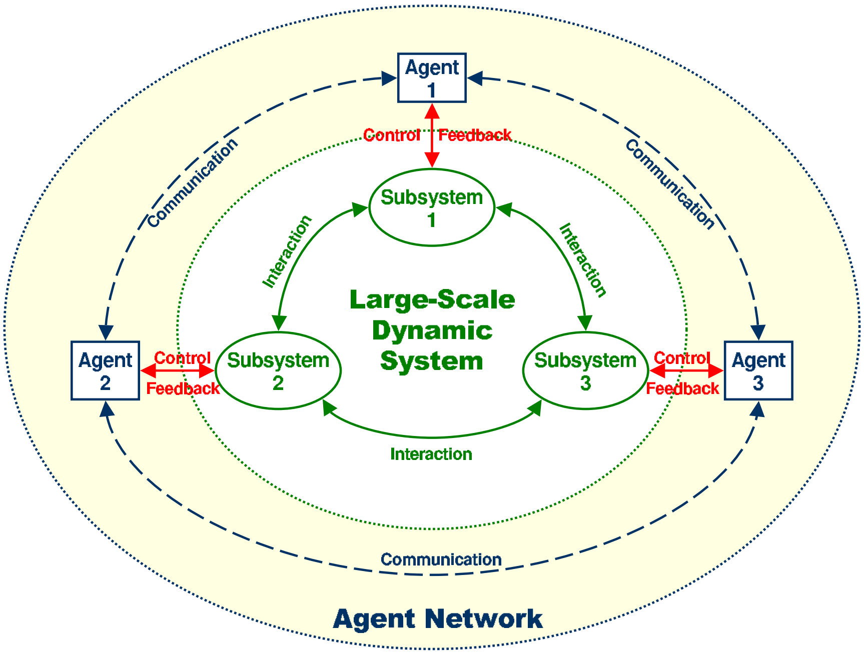

Figure 1: Agent-Controlled Large-Scale System

Many large-scale systems fit the scenario of Figure 1. Power systems offer one set of examples. Figure 2 shows a power system with a three areas, six generators and fourteen buses. Three agents, A, B and C, take charge of the load-frequency control in each area. Their objectives are to optimize generation allocation and maintain the power balance between the power generation and the power consumption. Because these agents are owned by different organizations, they are not willing to divulge all local information. But the stability of the whole system is of concern to all the agents.

Chemical plants offer another set of examples, for instance the Tennessee Eastman (TE) process shown as Figure 3. The operation of the system requires the coordination among four operation units: the reactor, the compressor, the separator and the stripper. The control objective is to maintain the production rate and composition at set-points and keep other variables within specified

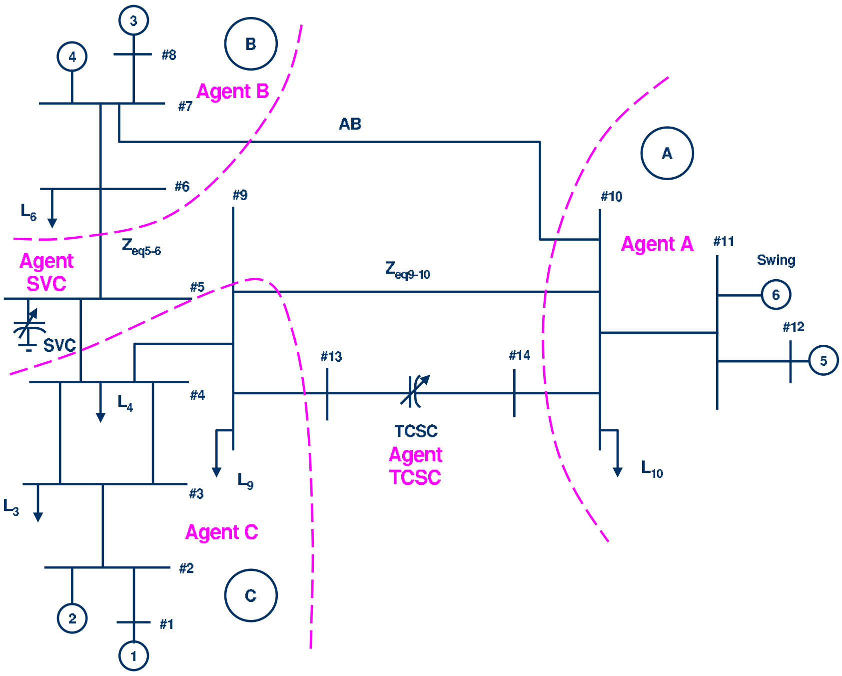

Figure 2: ABC System

limits. Typically, a controller is designed for each of these four units. As higher performance requirements are proposed, explicit coordination among the controllers is called for. To avoid the high cost in building a centralized control unit, a attractive solution is to introduce information sharing to achieve coordination.

In most large-scale system applications, like the above two examples, the agents need to take into account the influences from other parts of the whole system when they make decisions. Several approaches have been proposed to design decentralized control systems [40, 41]. Robust decentralized control is a widely used method [18, 31, 39], where the influences from other parts are considered as disturbances. Since there is no coordination during control, the control actions are often too conservative using this approach. Hierarchical decentralized control is an important method to achieve coordinated control [17]. In this case, a centralized controller is designed to optimize the parameters in decentralized controllers based on a simplified global model. The problem with this method is that a model of the global system is required, which is hard to develop, or even unavailable, in some applications.

In this work, we consider systems where distributed controllers only have local models, objectives and constraints. However, global objectives of concern to all controllers, such as system stability, require coordination among the distributed agents. In this research, we propose a new coordination

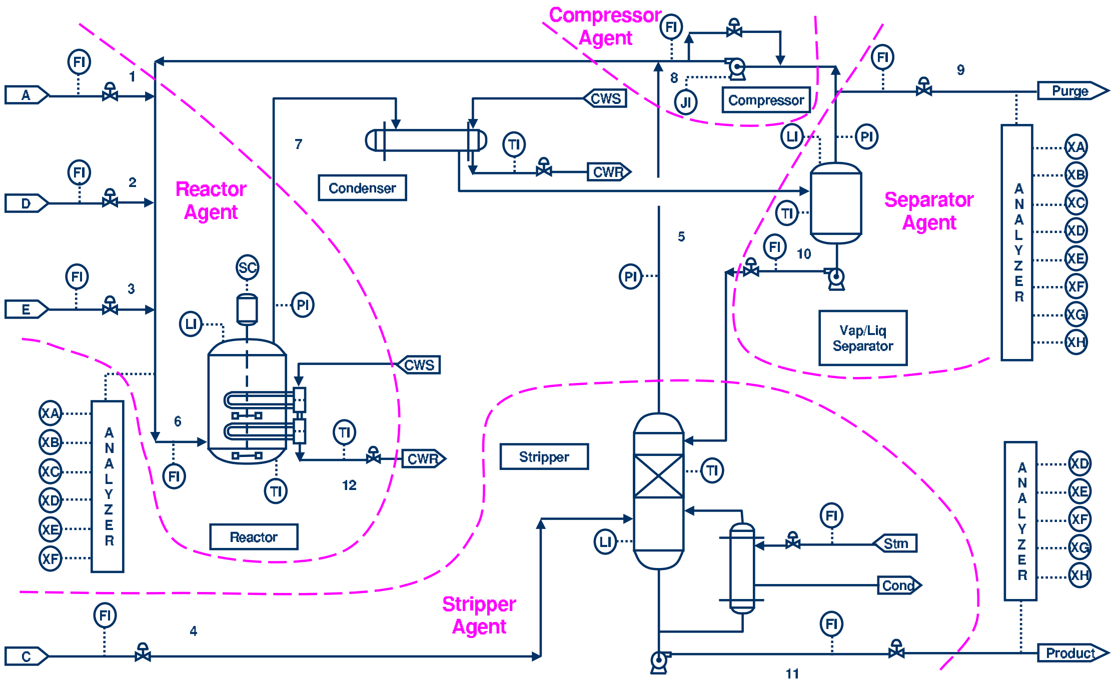

Figure 3: Tennessee Eastman Process

scheme for decentralized control in which the agents exchange dynamic information about their future local state trajectories. At each stage, the agents' decisions are based on the updated information, including the local measurements and the information from other agents.

To handle such complex coordinated control problems, we propose to use *model predictive control* (MPC), also called *receding horizon control* (RHC), in each agent because it is an optimal control strategy that incorporates operating constraints explicitly [8, 14, 32]. At each stage, all agents solve local MPC control problems to find the control actions. Figure 4 gives the iteration process for each agent. Between two consecutive control decisions, they exchange their predictions about the local future behaviors. Such information is incorporated in local MPC problems to estimate the influences from other parts of the system.

Since the agents do not have global information, the influences from other subsystems appear as disturbances in the local MPC problems. Consequently, the robust MPC approach can be applied. We will explore methods for exchanging information to make the controls less conservative than the typical robust decentralized control approaches so as to accommodate stronger interactions among the subsystems.

In the context of the proposed scheme for decentralized control, we address the following problems:

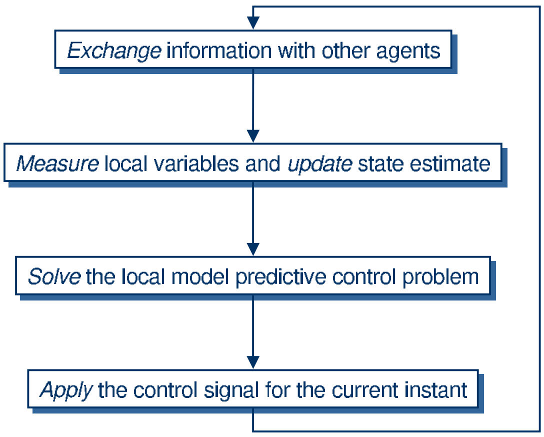

Figure 4: Local Iteration in Agents

- The communication scheme for coordination;
- Method to handle interaction;
- The formulation of MPC problem to achieve coordination;
- The method to reduce the conservativeness in control actions;
- Stability of the system by the proposed scheme;
- A set approximation method in computation;
- Computation of the controlled invariant set;
- Uncertainties in state estimation;
- Reachable set computation in hybrid systems;
- Information compensation in asynchronous control;
- Demonstrations of the proposed framework using simulation examples.

The proposal is organized as follows. Section 2 proposes an optimization problem to achieve distributed coordination for decentralized control. Section 3 reviews previous works in the related areas: minimax and feedback MPC, set approximation, and MPC for hybrid systems. Section 4 presents techniques for solving the distributed coordination problem and gives the results obtained thus far. Section 5 illustrates the proposed scheme for a three-solenoid example. Section 6 lists the research issues for the future work in the Ph.D. program.

2 Problem Description

2.1 Distributed Control Problem

In this research, we will develop a distributed control scheme, where each agent solves its local control problem without a central arbitrator. The influences from other subsystems are represented as bounded disturbances in the local model and the agents try to find a performance-guaranteed control action based on their local measurements and the information from other agents. We assume that each agent knows only its local model, objective and constraints. For coordination, the agents broadcast bounds on the future values of the local states at each step after they apply local control actions. We propose to formulate a minimax MPC problem for each agent, similar to methods used in centralized MPC to handle bounded uncertainties [4, 28, 37].

To describe our approach, we first consider the single-agent control problem. Consider a system with the dynamics

$$z(k+1) = F(z(k), u(k), w(k)) \tag{1}$$

and the operating constraints

$$G(z(k), z(k+1), u(k), w(k)) \leq 0 \tag{2}$$

where $z \in \mathcal{Z}$ is the system state vector, $u \in \mathcal{U}$ is the agent's input (manipulated variables) to the system, and $w \in \mathcal{W} \subset \mathbb{R}^{n_w}$ is the vector of external disturbances.

The model of the system used by the agent is of the form

$$x(k+1) = f(x(k), u(k), v(k)) \tag{3}$$

with the constraints

$$g(x(k), x(k+1), u(k), v(k)) \leq 0 \tag{4}$$

where $x \in \mathcal{X}$ is the state vector of the model, and $v \in \mathcal{V}$ is the disturbance signal in the model, which represents the external disturbances and the model uncertainties.

In this proposal, we first consider perfect state estimation, which means that the agent knows its state from the measurements. We let $\mathcal{G} : \mathcal{Z} \to \mathcal{X}$ denote the mapping from the state variables in the real world to those in the agent's model. In the proposed future work, imperfect state estimation will be considered by using set-valued estimators, called *set-membership state estimators* [36], which give the set of all possible values of the state given the measurement.

The agent formulates an optimization problem using a *parameterized control policy* $u(j) = h(x(j), \theta(j))$, rather than an open-loop control sequence as used in traditional MPC. $\theta(j)$ is a vector of *control policy parameter* characterizing the feedback policy at time j. The form of $h(\cdot, \cdot)$ is fixed by the designer and the values of $\theta(j)$ are the variables of optimization. When h is independent of $x(k)$, $h(\theta(j))$ is an *open-loop control policy*; when all $\theta(j)$ have the same value for all j, it becomes a *time-invariant state feedback control policy.* Following the standard MPC philosophy, the agent solves the a min-max parameterized optimization problem given below with the constraints on states and the bounds of the uncertainties given by

$$\mathcal{C} = \{\mathcal{X}(1), \mathcal{X}(2), \cdots, \mathcal{X}(N-1), \mathcal{V}(0), \mathcal{V}(1) \cdots, \mathcal{V}(N-1)\}.$$

where $\mathcal{X}(j) \subseteq \mathcal{X}$, $j = 0, \cdots, N-1$, are the constraints on the state, which may also be updated from stage to stage and $\mathcal{V}(j) \subseteq \mathcal{V}$, $j = 0, \cdots, N-1$, are bounds on disturbances and can be updated by system identification techniques for model uncertainties or from some information source. For simplicity, the control prediction horizon and the state prediction are assumed to be the same, N. The problem solved by the agent at each stage is stated in Figure 5. The constraints with quantified variables make the problem time consuming. In [4], Bemporad proposes a method to eliminate quantified variables and thus simplifies the optimization problem.

The information required to formulate $\mathcal{P}1$ at stage k is

$$\mathcal{I}^k_{\mathcal{P}1} = \left\{J^k(\cdot,\cdot,\cdot,\cdot), f^k(\cdot,\cdot,\cdot), g^k(\cdot,\cdot,\cdot), x^k_0, \Xi^k(0), \cdots, \Xi^k(N-1), \mathcal{C}^k\right\} \tag{5}$$

All components in $\mathcal{I}^k_{\mathcal{P}1}$ are not necessarily the same at each instant. $x^k_0 = \mathcal{G}(z(k))$ is the current state obtained through estimation. And as mentioned above, the constraint set $\mathcal{C}$ can vary from time to time. The parameter constraints $\Xi^k(1), \cdots, \Xi^k(N-1)$ may be updated when other components in $\mathcal{I}^k_{\mathcal{P}1}$ are updated.

2.2 Model Requirement

The first issue to be addressed is the relationship between the agent's model and the real system. What guarantees that a feasible solution of $\mathcal{P}1$ is feasible for the real system? To ensure that the

$\mathcal{P}$1: ($\mathbf{M^2P}(\mathcal{C})$ Optimization)

$\min_{\Theta} \max_{V} J(\Theta, V)$

where

$\Theta = \{\theta(0), \theta(1), \cdots, \theta(N-1)\}$

$V = \{v(0), v(1), \cdots, v(N-1)\}$

subject to

$x(j+1) = f(x(j), u(j), v(j)),\ j = 0, 1, \cdots, N-1$

$u(j) = h(x(j), \theta(j)),\ j = 1, 2 \cdots, N-1$

$\theta(j) \in \Xi(j), j = 0, 1, \cdots, N-1$

$v(j) \in \mathcal{V}(j), j = 0, 1, \cdots, N-1$

$x(0) = x_0$

$\tilde{x}(j+1) = f(\tilde{x}(j), h(\tilde{x}(j), \theta(j)), \tilde{v}(j)) \in \mathcal{X}(j+1), \forall \tilde{v}(j) \in \mathcal{V}(j), j = 0,1,\cdots,N-1$

$g(\tilde{x}(j), f(\tilde{x}(j), h(\tilde{x}(j), \theta(j)), \tilde{v}(j)), h(\tilde{x}(j), \theta(j)), \tilde{v}(j)) \leq 0, \forall \tilde{v}(j) \in \mathcal{V}(j), j = 0,1,\cdots,N-1$

Figure 5: Min-Max Parameterized Optimization with Constraint Set $\mathcal{C}$

agent's decisions are feasible and useful, we require that the model the agent uses is an *abstraction* of the real dynamics, defined as follows. Here, we extend the concept of abstraction in [12].

Definition 2.1 (Abstraction) *The model (3)-(4) is an* **abstraction** *of the system dynamics (1)-(2), if $\mathcal{G}(\mathcal{Z}) \subseteq \mathcal{X}$ and for all $x \in \mathcal{X}$ and $u \in \mathcal{U}$ satisfying*

$$g(x, f(x, u, v), u, v) \leq 0, \forall v \in \mathcal{V}, \tag{6}$$

the following are true for all $z \in \mathcal{G}^{-1}(x)$,

$$G(z, F(z, u, w), u, w) \leq 0, \forall w \in \mathcal{W}, \tag{7}$$

$$\mathcal{G}(F(z, u, \mathcal{W})) \subseteq f(x, u, \mathcal{V}). \tag{8}$$

$\mathcal{G}$ is called the **abstraction function** *and the inverse mapping $\mathcal{G}^{-1} : \mathcal{X} \to 2^{\mathcal{Z}}$ is called the* **concretization function**.

When the model in the agent is an abstraction of the real dynamics, everything in the real world is anticipated. If the current state is known, the next state will be sure to fall in the predicted reachable set of the next stage no matter what the actual disturbance is. The abstraction model helps the agent to compute controls that are truly feasible for the real system.

Definition 2.2 (Feasible Control Law) *A control law* $u(k) = \mathcal{H}(x(k), k)$ *is a* **feasible control law** *for the model (3)-(4) if for any initial state* $x(0) \in \mathcal{X}$ *and any disturbance sequence* $v(k) \in \mathcal{V}$, $k = 0, 1, \cdots$,

$$u(k) \in \mathcal{U} \text{ and } g(x(k), x(k+1), \mathcal{H}(x(k), k), v(k)) \leq 0, k = 0, 1, \cdots, \tag{9}$$

where $x(k+1) = f(x(k), \mathcal{H}(x(k), k), v(k))$*. The control law* $u(k) = \mathcal{H}(\mathcal{G}(z(k)), k)$ *is* **feasible** *for the system (1)-(2) if for any initial state* $z(0) \in \mathcal{Z}$ *and any disturbance sequence* $w(k) \in \mathcal{W}$, $k = 0, 1, \cdots$,

$$u(k) \in \mathcal{U} \text{ and } G(z(k), z(k+1), \mathcal{H}(\mathcal{G}(z(k)), k), w(k)) \leq 0, k = 0, 1, \cdots, \tag{10}$$

where $z(k+1) = F(z(k), \mathcal{H}(\mathcal{G}(z(k)), k), w(k))$, $k = 0, 1, \cdots$.

In [20], we proved that if the model in the agent is an abstraction of the system, the control law generated by the $\mathrm{M}^2\mathrm{P}(\mathcal{C})$ is feasible for the real system.

For a system controlled by M agents, the control action applied to the system is the composite action of all the agents. The disturbance signal in the local model includes the model uncertainties, the external disturbances and the influence from other subsystems. In the following discussion, subscript i denotes the variables and functions related to the agent i. In this proposal, we consider the case where all agents are synchronized. In the future research, we will take into account asynchronous distributed agents. For the distributed control, we use the following distributed version of abstraction.

Definition 2.3 (Distributed Abstraction) *Models in* M *agents*

$$x_i(k+1) \quad = \quad f_i(x_i(k), u_i(k), v_i(k)), i = 1, 2, \cdots, M \tag{11}$$

with local constraints

$$g_i(x_i(k), x_i(k+1), u_i(k), v_i(k)) \leq 0 \tag{12}$$

compose a **distributed abstraction** *of the real dynamics (1)-(2) if* $\mathcal{G}_i(\mathcal{Z}) \subseteq \mathcal{X}_i$, $i = 1, 2, \cdots, M$, *and for all* $\left[x_1^T x_2^T \cdots x_M^T\right]^T \in \mathcal{X}_1 \times \mathcal{X}_2 \times \cdots \times \mathcal{X}_M$ *and* $\left[u_1^T u_2^T \cdots u_M^T\right]^T \in \mathcal{U}_1 \times \mathcal{U}_2 \times \cdots \times \mathcal{U}_M$ *satisfying*

$$g_i(x_i, f_i(x_i, u_i, v_i), u_i, v_i) \leq 0, \forall v_i \in \mathcal{V}_i, i = 1, 2 \cdots, M, \tag{13}$$

the following are true for all $z \in \mathcal{G}_1^{-1}(x_1) \bigcap \mathcal{G}_2^{-1}(x_2) \bigcap \cdots \bigcap \mathcal{G}_M^{-1}(x_M)$,

$$G(z, F(z, u, w), u, w) \leq 0, \forall w \in \mathcal{W} \tag{14}$$

$$\mathcal{G}_i(F(z, u, \mathcal{W})) \subseteq f_i(x_i, u_i, \mathcal{V}), i = 1, 2, \cdots, M \tag{15}$$

where

$$u = \left[u_1^T u_2^T \cdots u_M^T\right]^T \in \mathcal{U}. \tag{16}$$

$\mathcal{G}_i : \mathcal{Z} \to \mathcal{X}_i$ *is called the* **local abstraction function** *and the inverse mapping* $\mathcal{G}_i^{-1} : \mathcal{X}_i \to 2^{\mathcal{Z}}$ *is the* **local concretization function**.

In [20], we showed that if the models in all agents compose a distributed abstraction of the system, all local control actions generated by the distributed control laws feasible for the local models compose control actions to the system. In the following discussion, we assume that local models in distributed agents compose a distributed abstraction of the system. Then, this feasible control actions the agents get is feasible to the real world.

3 Previous Work

3.1 Minimax MPC

Model Predictive Control (MPC), also called *receding horizon control* (RHC), is a control scheme where the control input is obtained by solving an optimal control problem over a finite horizon. In the standard formulation, an open-loop optimal control problem is formulated and solved, based on the state of the system. Only the first control in resulting control sequence is applied to the physical system. The controls for future instants are obtained by repeating this process. MPC is very popular in process control because it is an optimal control heuristic that incorporates operating constraints explicitly [8, 14, 32]. Several papers have been published on decentralized or distributed MPC following the standard formulation [1, 2, 9, 10, 16, 21, 30]. In this work, some variations are introduced to make less conservative decisions and guarantee the feasibility of local decisions.

In MPC literature, min-max optimization is used to find control actions which optimize the worst-case performance under bounded uncertainties, including external disturbances and model mismatches. In [15] and [42], the min-max MPC problem is formulated using open-loop control policies. Since only a single open-loop control sequence is optimized over all disturbance realizations, this approach often meets feasibility difficulties. In [24] and [37], feedback control strategies are introduced into the MPC formulation. Kothare [24] incorporates a linear time invariant feedback policy into the formulation of an infinite horizon min-max MPC formulation. Scokaert and Mayne [37] add a logic constraint to the optimal control problem, which requires the same control action for the same value of state variables, to implicitly add the feedback control policy. In [4], Bemporad introduces affine feedback control policy into the MPC optimization to make control actions less conservative for the system with bounded disturbances. In this work, we follow the idea in [4] to handle bounded uncertainties by introducing parameterized MPC problem.

3.2 Set Calculus

In the proposed scheme, agents must predict their future reachable sets and provide the estimates to other agents. There are two popular methods to estimate the reachability set: the ellipsoidal method [25, 26, 27] and the polyhedral method [3, 11, 13, 34, 35]. In the ellipsoidal method, the closed set is approximated by a set of ellipsoids, which are described by quadratic functions. The greatest virtue of the ellipsoidal method is that it avoids the curse of dimension in approximating high-dimension sets. But the complexity in computation prevents it from being widely used. Although the polyhedral method has difficulties in higher dimensions (beyond 3 or 4), people tend to use it for its simplicity in computation because each hyperplane is described by a set of linear constraints. All previous research in this area focuses on evaluating the performance of the system and checking the specification for reachability, so accuracy is of the first importance. In our work, the set approximation is used for on-line coordination among controllers, thus the simplicity and the speed of the computation are the most important things. Although some researchers proposed to use some special polyhedra to avoid difficulties in higher dimension problems, such as parallelotope [23] and orthogonal polyhedra [7], there is no efficient way to find a single such polyhedron as an optimal approximation. We propose an easy way to compute a single ellipsoid as an optimal approximation.

3.3 Control in Hybrid System

In many complex systems, hybrid properties need to be considered. The common method to design control scheme for hybrid system is first to design controllers for dynamics of continuous variables in each mode of the system and then to design discrete controllers based on a high-level discrete event model. In [5, 38], an MPC strategy was introduced to design control scheme for hybrid systems though mixed-integer optimization. Both continuous variables and discrete variables are taken into account simultaneously. Following the same idea, we propose to extend our framework to handle hybrid properties of the system. In the work thus far, we have focused on continuous-state systems. In the future work, we will consider the problem in distributed control of hybrid system.

4 Min-Max Feedback DMPC

This section summarizes the results presented in [20] and [19]. We assume in the discussion that the uncertainties in the local model for an agent are the influences from other subsystem. The proposed method can be extended to handle other external disturbances and model uncertainties.

4.1 Constraint Updating

In our scheme, the agents model the influences from other parts of the system as disturbances. A straightforward method to achieve coordination is to have all agents predict the future reachable sets of the local state variables and broadcast these sets to neighbor agents for them to estimate the bounds of the interactions. This means that the agents update the uncertainty bounds in their local optimization problem in the following way:

$$\mathcal{V}_i^k\left(j\right)=\hat{\mathcal{X}}_1\left(k+j|k-1\right)\times\cdots\times\hat{\mathcal{X}}_{i-1}\left(k+j|k-1\right)\times\hat{\mathcal{X}}_{i+1}\left(k+j|k-1\right)\times\cdots\times\hat{\mathcal{X}}_M\left(k+j|k-1\right), \tag{17}$$

where $j=0,1,\cdots,N-1$ and $\hat{\mathcal{X}}_i\left(k+j|k-1\right)$ is the future reachable set predicted at the stage $k-1$. And the corresponding disturbance variable is

$$v_i\left(j\right)=\left[\tilde{x}_1^T\left(j\right)\cdots\tilde{x}_{i-1}^T\left(j\right)\tilde{x}_{i+1}^T\left(j\right)\cdots\tilde{x}_M^T\left(j\right)\right]^T, \tag{18}$$

To make the approach clear, we consider constraint updating for a single agent and omit the subscript. As the bounds on disturbance signals are updated at each stage, the decisions of the agent depend on the incoming information from other agents, or more generally from any possible source. To ensure that the decisions are correct, the following constraints on the updated information need to be satisfied:

$$\mathcal{V}^k\left(j\right)\subseteq\mathcal{V}^{k-1}\left(j+1\right), j=0,1,\cdots,N-2, \text{ and } \mathcal{V}^k\left(N-1\right)\subseteq\mathcal{V}, \tag{19}$$

with the initial bounds $\hat{\mathcal{V}}^0(j) = \mathcal{V}$, $j = 0, 1, \cdots, N-1$. If (19) is violated, then the previous information missed some possible values of disturbance signal. As a result, the previous decision can lead to infeasibility because some possible disturbances are not considered.

To guarantee that the set information passed to other agents satisfies the requirement (19) for the updated information, the agents need to put the following requirements on the prediction:

$$\hat{\mathcal{X}}(k+j|k) \subseteq \mathcal{X}(k+j|k-1), j = 0, 1, \cdots, N-2, \text{ and } \hat{\mathcal{X}}(k+N|k) \subseteq \mathcal{X}, \tag{20}$$

with the initial predicted reachable sets $\hat{\mathcal{X}}(j|-1) = \mathcal{X}$, $j = 0, 1, \cdots, N-1$. $\hat{\mathcal{X}}(k+j|k)$ denotes the future reachable set predicted recursively by the controller at the control step k using the set-valued state equations, given by

$$\hat{\mathcal{X}}(k+j+1|k) = \left\{ x | x = f\left(x', h\left(x', \theta^{k*}(j)\right), v\right), x' \in \hat{\mathcal{X}}(k+j|k), v \in \mathcal{V}^k(j) \right\}, j = 0, 1, \cdots, N-1 \tag{21}$$

with $\hat{\mathcal{X}}(k|k) = \{\hat{x}(k)\}$. Let $\Theta^{k*} = \left\{\theta^{k*}(0), \theta^{k*}(1), \cdots, \theta^{k*}(N-1)\right\}$ be the solution to the $\mathrm{M^2P}(\mathcal{C})$ optimization at the stage k. The simplest way to satisfy the requirement (20) is to update the state constraints in the following way:

$$\mathcal{X}^k(j) = \hat{\mathcal{X}}(k+j|k-1), j = 1, 2, \cdots, N-1, \tag{22}$$

Following the approach developed in [29] for centralized control, we introduce the control invariant set to the MPC problem so that a initial solution can be constructed based on the optimal solution at the previous stage. For the model (3) and (4), with a variation of the notion in [22], $\mathcal{T}^x \subseteq \mathcal{X}$ is said to be a θ-*control invariant set* if there exists a $\theta^{\mathcal{T}^x} \in \Xi$ such that

$$x \in \mathcal{T}^x \Longrightarrow f\left(x, h\left(x, \theta^{\mathcal{T}^x}\right), v\right) \in \mathcal{T}^(x), \forall v \in \mathcal{V}. \tag{23}$$

Throughout the remainder of the proposal, the set $\mathcal{T}^x$ will denote a θ-control invariant set. And it is not difficult to see that $\mathcal{T}^z = \mathcal{G}^{-1}(\mathcal{T}^x)$ is the corresponding θ-control invariant set of the system (1) and (2). At each control step, the controller solves the $\mathcal{P}1$ problem with the end state constraint

$$\mathcal{X}(N) = \mathcal{T}^x. \tag{24}$$

This end constraint not only helps the agent to construct initial feasible solutions, but also helps the agent to achieve the stability of the system.

By constraint updating (17), (22) and (24), each agent has a local *min-max feedback DMPC* ($\mathrm{M^2F}$-DMPC) problem.

4.2 Set Approximation

Since the precise set representation computations are impractical or even impossible, an approximation method needs to be used. Generally, the inclusion relation between two sets cannot be maintained under set approximation, which means $\mathcal{X} \subseteq \widetilde{\mathcal{X}}$, $\mathcal{Y} \subseteq \widetilde{\mathcal{Y}}$ and $\mathcal{X} \subseteq \mathcal{Y} \nRightarrow \widetilde{\mathcal{X}} \subseteq \widetilde{\mathcal{Y}}$, where $\widetilde{\mathcal{X}}$ and $\widetilde{\mathcal{Y}}$ are approximations of $\mathcal{X}$ and $\mathcal{Y}$, respectively. So, with the set approximation, the feasibility of M^{2}F-DMPC is threatened. To avoid this side effect, the following set approximation process at the control step k is proposed as follows.

Step 1: Let $j = 0$ and calculate the set approximation $\widetilde{\hat{\mathcal{X}}}(k|k)$ of the set $\hat{\mathcal{X}}(k|k)$.

Step 2: If $\widetilde{\hat{\mathcal{X}}}(k|k) \nsubseteq \widetilde{\hat{\mathcal{X}}}(k|k-1)$, let $\widetilde{\hat{\mathcal{X}}}(k|k) = \widetilde{\hat{\mathcal{X}}}(k|k-1)$.

Step 3: Let $j = j+1$ and calculate the set approximation $\widetilde{\hat{\mathcal{X}}}(k+j|k)$ of the set $\hat{\mathcal{X}}(k+j|k)$ definition by the equation (21).

Step 4: If $\widetilde{\hat{\mathcal{X}}}(k+j|k) \nsubseteq \widetilde{\hat{\mathcal{X}}}(k+j|k-1)$, let $\widetilde{\hat{\mathcal{X}}}(k+j|k) = \widetilde{\hat{\mathcal{X}}}(k+j|k-1)$ or $\widetilde{\hat{\mathcal{X}}}(k+N|k-1) = \widetilde{\mathcal{T}}^x$ if $j = N$.

Step 5: If $j = N$, stop; otherwise, go to Step 3.

This scheme is suitable for any set approximation method. For LTI systems, we propose an ellipsoid method to approximate the future reachable sets in [20].

4.3 Current Results

Definition 4.1 *Θ is said to be a* feasible solution *to the $M^2P(\mathcal{C})$ problem if all constraints are satisfied. The $M^2P(\mathcal{C})$ problem is said to be* feasible *if there exists a feasible solution Θ.*

Theorem 4.1 *If the $M^2P(\mathcal{C})$ problem with updated state constraints and bound on uncertainties is feasible at the first step $k = 0$, then it is feasible at all control steps $k \geq 0$ with updated state constraints and bounds on uncertainties satisfying equations (22) and (19). And furthermore, $x(k) \in \mathcal{T}^x$, i.e. $z(k) \in \mathcal{G}^{-1}(\mathcal{T}^x)$, for $k = N, N+1, \cdots$.*

Theorem 4.1 shows that that the stability of the system is ensured by updating the state constraints following the equations (22). This is not the stability in the sense of asymptotical stability.

For LTI system, if we update the θ-control invariant sets as described below, the asymptotical stability can be achieved. Theorem 4.1 is based on the precise set calculation. The feasibility of M^{2}F-DMPC and the stability of the system is maintained by the proposed set approximation scheme.

Theorem 4.2 (Feasibility with Set Approximation) *With the above set approximation scheme, if M^{2}F-DMPC is feasible at the first step $k = 0$, it is feasible at all control steps $k \geq 0$. Furthermore, the state of the system z goes into $\bigcap_{i=1}^{M} \mathcal{G}_i^{-1}(\mathcal{T}_i^x)$.*

For LTI systems, agents can update the end constraint, i.e. the θ-control invariant set at each step, in the following way

$$\mathcal{T}_i^{x,k} = \begin{cases} \alpha^{k/N}\mathcal{T}_i^x & k \text{ is a multiple of } N; \\ \mathcal{T}_i^{x,k-1} & \text{otherwise.} \end{cases}, \tag{25}$$

where $\mathcal{T}_i$ is the initial θ-control invariant set in agent i and $0 < \alpha < 1$ is selected such that $\mathcal{T}_i \subseteq \alpha\mathcal{X}_i$. Following this procedure, the asymptotic stability of the whole LTI system is achieved.

Theorem 4.3 *If the M^{2}F-DMPC problems for LTI system for all agents are feasible at the first control step $k = 0$, then, with the θ-control invariant set updating procedure (25), the model is asymptotically stable at the point $x = 0$ which means that the system goes to $\bigcap_{i=1}^{M} \mathcal{G}_i^{-1}(0)$ asymptotically.*

5 Solenoid Example

Consider the example shown in Figure 6, consisting of four springs with the spring constants k and three identical solenoids with the masses m, each of which is controlled by an MPC controller solving $\mathcal{P}1$, with the parameterized control policy.

The state variables $x_{i,1}$ and $x_{i,2}$, $i = 1, 2, 3$, are the position and velocity of the i^{th} solenoid, respectively. The zero-order hold, with sampling time $T_s = 0.2$s, is used to get the discrete-time model with $m = 1$kg and $k = 10$N/m. Figure 7 shows the future reachable set predicted by three agents at the step $k = 0$ when the open-loop control policy is used in the $\mathcal{P}1$ optimization. The solid ellipsoids are the predicted reachable sets and the dashed ellipsoids form the state constraints.

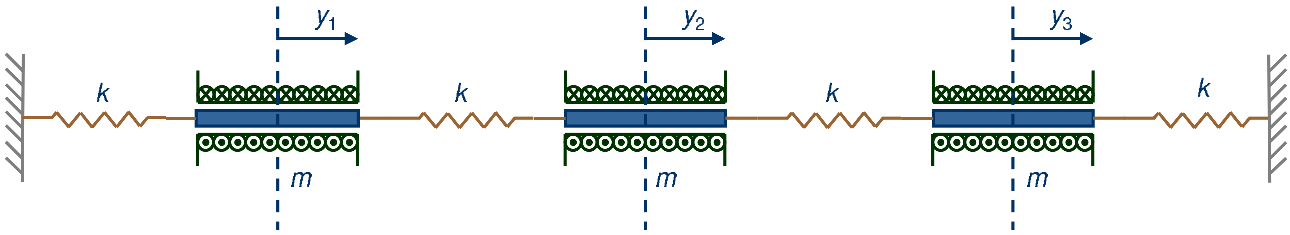

Figure 6: A solenoid example.

$\hat{\mathcal{X}}_i(1|0)$, $i = 1, 2, 3$, in the upper plots give the reachable sets at step 1 and $\hat{\mathcal{X}}_i(2|0)$, $i = 1, 2, 3$, in the lower plots give the reachable sets at step 2. All computations are through the ellipsoidal method in [20]. Since the open-loop control sequence can only manipulate the centers of the reachable sets, we move the centers of all future reachable sets to the original point because it is easier to tell whether the optimization problem can be feasible or not. We can see that no matter what the agent 2 selects as the open-loop control sequence, it is possible of the state $x_2(2)$ to violate the state constraint.

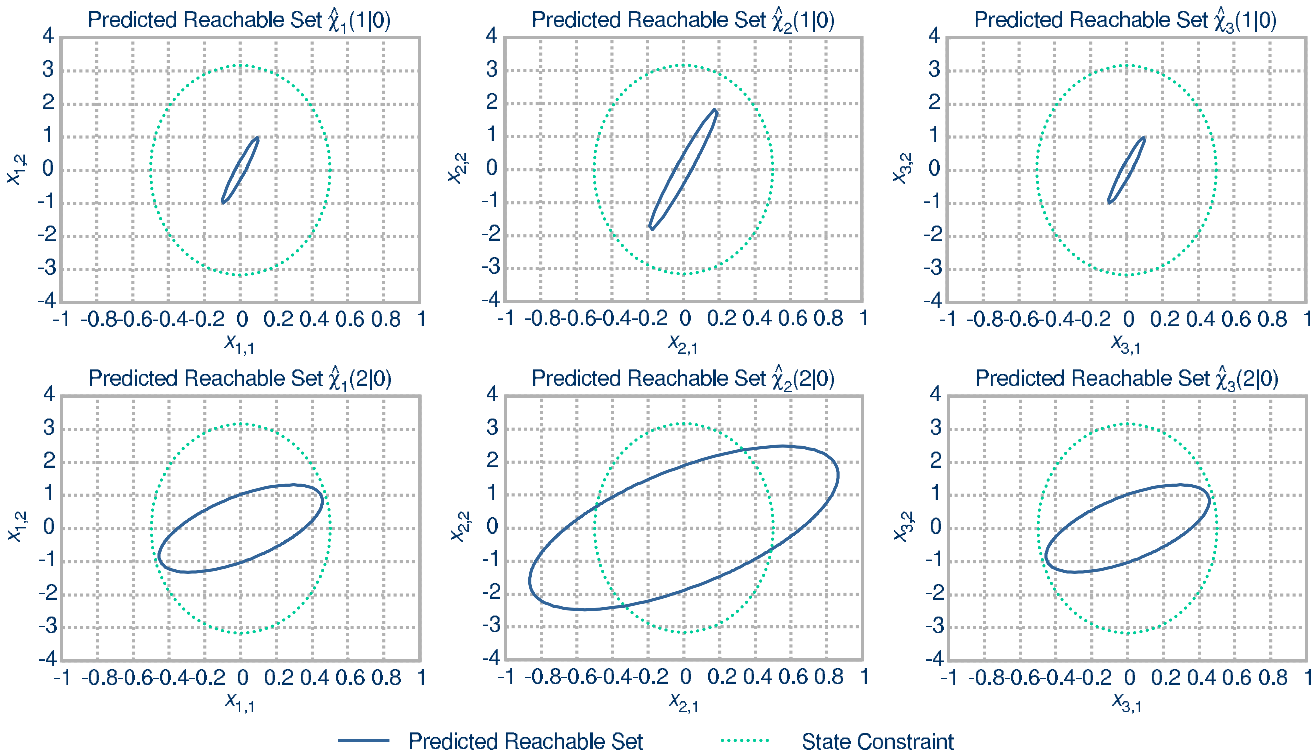

Figure 7: Feasibility Difficulty of Open-Loop Control Policy.

In the following we evaluate the performance of the proposed the decentralized control scheme by using parameterized feedback control policy as follows

$$u(j) = K(j)\,x(j) + u^0(j)\,. \tag{26}$$

In the simulation, the prediction horizon is set to be 2 and the initial conditions are

$$\begin{bmatrix} x_{1,1}(0) \\ x_{1,2}(0) \end{bmatrix} = \begin{bmatrix} x_{2,1}(0) \\ x_{2,2}(0) \end{bmatrix} = \begin{bmatrix} x_{3,1}(0) \\ x_{3,2}(0) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}.$$

We simulated 20 steps and the Figure 8 gives the simulation results. The upper three figures give the responses of the three sub-systems. The curves with crosses are positions and the curves with circles are velocities of those plates. The lower three plots demonstrate that the predictions of the corresponding agents which are used as the dynamic information by the neighbor agent(s). The dotted ellipsoids in the three lower plots are the physical state constraints. The solid curves and the dashed ones show the predicted reachable set $\hat{\mathcal{X}}_i(k+1|k)$ and $\hat{\mathcal{X}}_i(k+2|k)$ respectively, $i = 1, 2, 3$ and $k = 1, \cdots, 20$. We can see, by using the parameterized feedback policy, the feasibility problem is avoided. Each sub-system goes back to the equilibrium point and the uncertainties in the predictions of the controllers decrease during the control process.

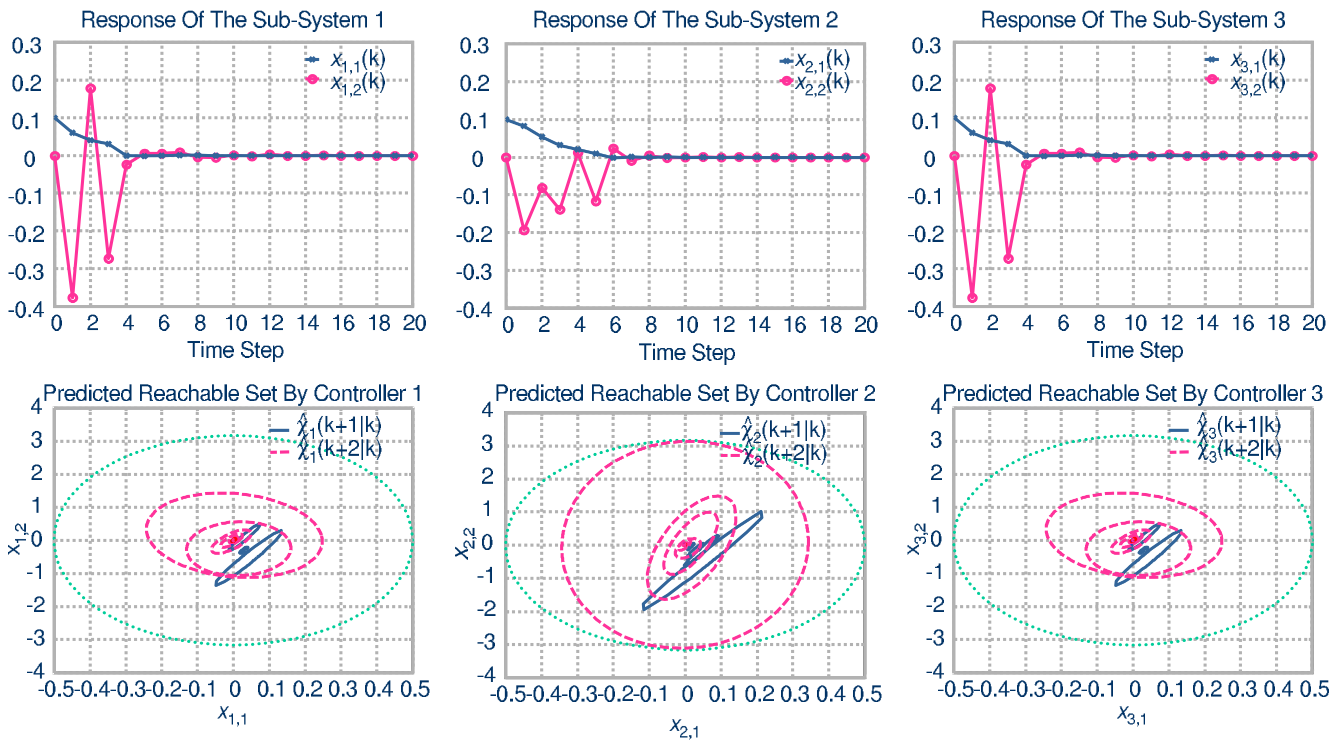

Figure 8: Simulation results.

6 Future Research and Plan

The remaining dissertation research will focus on four areas, namely, computational issues, extensions to hybrid systems, asynchronous agents, and demonstrations through simulations of applications.

6.1 Computational Issues

6.1.1 θ-Control Invariant Set

To implement the proposed scheme, the computation of the θ-control invariant set is unavoidable. In [22], Kerrigan and Maciejowski presented the basic principle for computing control invariant sets given feasible control sets. In our scheme, the maximal θ-control invariant set needs to be computed given the parameterized control policy and the feasible control set. We will develop a procedure find the proper form of the parameterized control policy and compute the corresponding θ-control invariant sets.

6.1.2 Set-Membership Estimation

In the work to date, we assume that the local state is available. In practice, there are uncertainties in the information about the state. In [6, 36], a recursive ellipsoid-based set-membership estimation was proposed to give all possible values of the state. This scheme will be extended for the proposed distributed MPC scheme.

6.2 Hybrid Systems

Although the work till now considered only the continuous system, all methods and results are applicable to hybrid systems. When hybrid properties are taken into account, the agent contains different continuous dynamic models and different controllers for different modes shown as Figure 9

We use discrete state variables to described different modes of the system and discrete control variables and discrete disturbance variables to model the event inputs and the event disturbances.

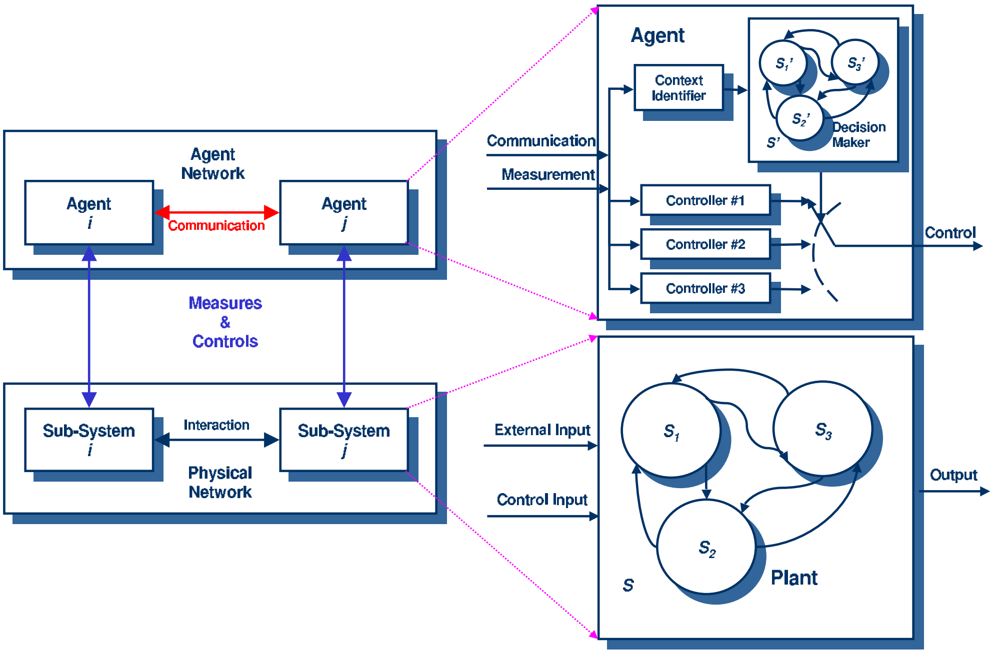

Figure 9: Structure of Distributed Control for Hybrid System

The state, control and disturbance variables become:

$$z = \left[z_c^T z_d^T\right]^T \in \mathcal{Z}_c \times \mathcal{Z}_d, u = \left[u_c^T u_d^T\right]^T \in \mathcal{U}_c \times \mathcal{U}_d \text{ and } w = \left[w_c^T w_d^T\right]^T \in \mathcal{W}_c \times \mathcal{W}_d. \tag{27}$$

Correspondingly, the variables in the agent's model become:

$$x = \left[x_c^T x_d^T\right]^T \in \mathcal{X}_c \times \mathcal{X}_d \text{ and } v = \left[v_c^T v_d^T\right]^T \in \mathcal{V}_c \times \mathcal{V}_d. \tag{28}$$

As discrete variables appear in the formulation of the proposed optimization problem $\mathcal{P}1$, the optimization problem becomes a minimax, parameterized mixed-integer optimization problem as in [5, 38].

Besides the mix-integer programming problem, a challenging problem faced by each agent in controlling hybrid systems is the prediction for the future reachable set. For continuous-state systems, the reachable sets at any time are connected sets. But in hybrid system, every time when a reachable set reaches a switching surface, it may split into two parts. Generally, the reachable set for each future time point is no longer a connected set. If the prediction horizon is long, the number of sets will be very large for the end time point. This will greatly increase the computation and communication loads. The future research will focus on the computation and representation of future reachable set.

6.3 Asynchronous Agents

In the previous discussion, we assume that all agents work synchronously at the same rate. But in very large-scale system, agents generally have local clocks. Thus, the agents need to include time information in their communications. Even so, the information from other agents cannot be applied directly to the optimization problem. For example, since the agent i works at the time sequence $t_0^i, t_1^i, t_2^i, \cdots$, it needs the information about the states of other subsystems at those time points. But, because the agent j works at the time sequence $t_0^j, t_1^j, t_2^j, \cdots$, it sends the state information of those time points to agent i. Therefore, some method is needed to compensate such time mismatch. In the future work, we will introduce additional disturbance signal to model the errors caused by such time mismatch.

6.4 Applications

To demonstrate and evaluate the proposed method for decentralized control, we will develop simulations of real applications. Power system and chemical plant will be two example systems.

6.4.1 Load-Frequency Control in ABC System:

First, we will apply our scheme to the distributed *automatic generation control* (AGC) and use the system shown in Figure 2 as the example system. The three agents, A, B and C, control the set-point of power generation in their local areas. The objective of AGC is to maintain the frequency and the power flow through each transmission line. The agents will coordinate their actions following our scheme to achieve better transient performance of the system.

6.4.2 Plant-Wide Process Control in TE Process:

Secondly, we will apply our scheme to plant-wide control in chemical industry and use the TE process shown as Figure 3 as the example. The four agents, the reactor, the compressor, the separator and the stripper, coordinate to maintain the production rate and composition at set-points and keep other variable within the specified limits. A simplified model in [33], including 26 state variables, 10 manipulated variables and 23 output variables, will be used for the demonstration.

6.5 Tentative Timetable

In the future research of my Ph.D. program, the contributions are expected in the following aspects:

1. The computational issues in the M^2F-DMPC scheme;
2. The reachable set computation for the hybrid system;
3. The information correction techniques for asynchronous agents;
4. Implementation of the scheme to some benchmark applications.

By the end of this Ph.D. program, a complete implementable scheme is expected to be constructed for distributed coordination in multi-agent control system. The scheme will be illustrated by some real applications rather than the toy example in this proposal. Figure 10 is the timetable of the future work.

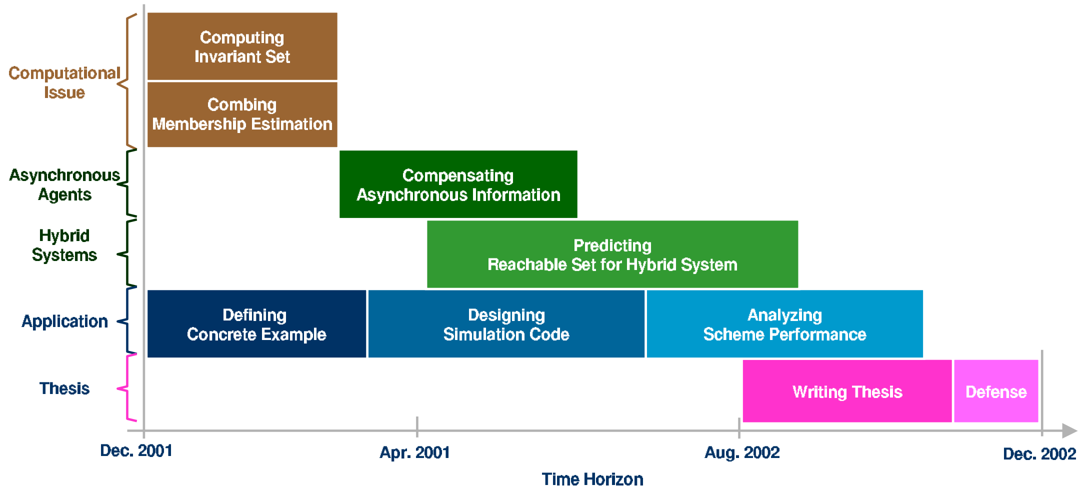

Figure 10: Time Schedule for Future Work

References

[1] M. Aicardi, G. Casalino, R. Minciardi, and R. Zoppoli. On the existence of stationary optimal receding-horizon strategies for dynamic teams with common past information structure. *IEEE Transactions on Automatic Control*, 37:1767–1771, 1992.

[2] M. Baglietto, T. Parisini, and R. Zoppoli. Neural approximators and team theory for dynamic routing: a receding-horizon approach. In *Proceedings of the* 38^{th} *Conference on Decision and Control*, pages 3283–3288, 1999.

[3] B.R. Barmish, Y. Lin, and J. Sankaran. Reachability of targets subject to disturbances bounded by convex sets. *International Journal of Control*, 28(5):771–783, 1978.

[4] A. Bemporad. Reducing conservativeness in predictive control of constrained systems with disturbances. In *Proceedings of the 37th IEEE Conference on Decision and Control*, 1998.

[5] A. Bemporad and M. Morari. Control of systems integrating logic, dynamics and constraints. *Automatica*, (3):407–427, 1999.

[6] D.P. Bertsekas and I.B. Rhodes. On the minimax reachability of target sets and target tubes. *Automatica*, 7:233–247, 1971.

[7] O. Bournez, O. Maler, and A. Pnueli. Orthogonal polyhedra: representation and computation. In *Hybrid Systems: Computation and Control*, pages 46–60. 1999.

[8] E.F. Camacho and C. Bordons. Model predictive control in the process industry. In *Advances in Industrial Control*. 1995.

[9] E. Camponogara. *Controlling networks with collaborative agents*. PhD thesis, Carnegie Mellon University, 2000.

[10] E. Camponogara, D. Jia, B.H. Krogh, and S. Talukdar. Distributed model predictive control. *Accepted for publication in IEEE Control System Magazine*, 2001.

[11] A. Chutinan and B.H. Krogh. Verification of polyhedral invariant hybrid automata using polygonal flow pipe approximations. In *Hybrid Systems: Computation and Control*, pages 76–90. 1999.

[12] T. deAlfaro, T.A. Henzinger, and F.Y.C. Mang. Abstract interpretation of game properties. In *Proceedings of the* 7^{th} *International Static Analysis Symposium*, pages 220–239, 2000.

[13] L. Farina and L. Benvenuti. Polyhedral reachable set with positive controls. *Mathematics of Control, Signals and Systems*, 10(4):364–380, 1997.

[14] C.E. Garcia, D.M. Prett, and M. Morari. Model predictive control: Theory and practice - a survey. *Automatica*, 25(3):335–348, 1989.

[15] H. Genceli and M. Nikolaou. Robust stability analysis of constrained l_1-norm model predictive control. *Journal of AIChE*, 39(12):1954–1965, 1993.

[16] G. Georges. Decentralized adaptive control for a water distribution system. In *Proceedings of the* 3^{rd} *IEEE Conference on Control Applications*, pages 1411–1416, 1994.

[17] A.N. Hagras and Z. Binder. Three level cooperative decentralized control. for interconnected dybamic systems. In *Proceedings of the IFAC Symposium on Large Scale Systems Theory and Applications*, pages 149–156, 1981.

[18] M.C. Han and Y.H. Chen. Decentralized control of nonlinear uncertain systems with bounded uncertainties. In *Proceedings of the* 30^{th} *Conference on Decision and Control*, pages 321–326, 1991.

[19] D. Jia and B.H. Krogh. Distributed model predictive control with ellipsoid approximation. submitted to ACC 2002.

[20] D. Jia and B.H. Krogh. Distributed control through minimax feedback mpc with one step delayed communication. Technical report, Department of Electrical and Computer Engineering, Carnegie Mellon University, 2001.

[21] D. Jia and B.H. Krogh. Distributed model predictive control. In *Proceedings of the American Control Conference*, pages 2767–2772, 2001.

[22] E.C. Kerrigan and J.M. Maciejowski. Invariant sets for constrained nonlinear discrete-time systems with application to feasibility in model predictive control. In *Proceedings of the 39th IEEE conference on decision and control*, 2000.

[23] E.K. Kostoukova. State estimation for dynamic systems via parallelotopes: optimization and parallel computations. *Optimization Methods and Software*, (9):269–306, 1999.

[24] M.V. Kothare, V. Balakrishnan, and M. Morari. Robust constrained model predictive control using matrix inequalities. *Automatica*, (10):1361–1379, 1996.

[25] A. Kurzhanski and I. Vályi. Ellipsoidal techniques for dynamic systems: the problem of control synthesis. *Dynamics and Control*, 1(4):357–378, 1991.

[26] A. Kurzhanski and I. Vályi. Ellipsoidal techniques for dynamic systems: control synthesis for uncertain systems. *Dynamics and Control*, 2(2):87–111, 1992.

[27] A. Kurzhanski and I. Vályi. *Ellipsoidal Calculus for Estimation and Control*. Birkhauser, 1996.

[28] J.H. Lee and Z. Yu. Worse-case formulations of model predictive control for systems with bounded parameters. *Automatica*, 33(5):763–781, 1997.

[29] H. Michalska and D.Q. Mayne. robust receding horizon control of constrained nonlinear systems. *IEEE Transactions on Automatic Control*, 38(11):1623–1633, 1993.

[30] S. Oschs, S. Engell, and A. Draeger. Decentralized vs. model predictive control of an industrial glass tube manufacturing process. In *Proceedings of the 1998 IEEE International Conference on Control Applications*, pages 16–20, 1998.

[31] P.R. Pagilla. Robust decentralized control of large-scale interconnected systems: general interconnections. In *Proceedings of the 1999 American Control*, pages 4527–4531, 1999.

[32] S.J. Qin and T.A. Badgwell. An overview of industrial model predictive control technology. In *Chemical Process Control - V*. 1996.

[33] N.L. Ricker and J.H. Lee. Nonlinear model predictive control the tennessee eastman challenge process. *Computer and Chemical Engineering*, 19(9):961–981, 1995.

[34] V.G. Rumchev. Constructing the reachability sets for positive linear discrete-time systems: the case of polyhedra. *Systems Science*, 15(3):11–20, 1989.

[35] V.G. Rumchev and A. Grzech. Asymptotic reachable sets for discrete-time positive linear systems. In *Proceedings of the* 13^{th} *International Conference on Systems Science*, pages 15–18, 1998.

[36] F.C. Schweppe. Recursive state estimation : unknown but bounded errors and system inputs. *IEEE Transactions on Automatic Control*, AC-13:22–28, 1968.

[37] P.O.M. Scokaert and D.Q. Mayne. Min-max feedback model predictive control for constrained linear systems. *IEEE Transactions on Automatic Control*, 43(8):1136–1142, 1998.

[38] O. Stursberg and S. Engell. Optimized start-up procedures of processing systems. In *Proceedings of* 6^{th} *IFAC Symposium on Dynamics and Control of Processing Systems*, 2001.

[39] V. Vesely, A. Kozakova, and D. Rosinova. Robust decentralized control of nonlinear systems with disturbance attenuation. In *International Conference on Control*, pages 1542–1547, 1994.

[40] D.D. Šiljak. *Decentralized Control of Complex Systems.* Academic Press, 1990.

[41] D.D. Šiljak. Decentralized control and computations: status and prospects. *Annual Reviews of Control*, 20:131–141, 1996.

[42] Z.Q. Zhang and M. Morari. Robust stability of constrained model predictive control. In *Proceedings of American Control Conference*, pages 379–383, 1993.