Distributed Coordination In Multi-Agent Control Systems Through Model Predictive Control

Ph.D. Thesis Proposal

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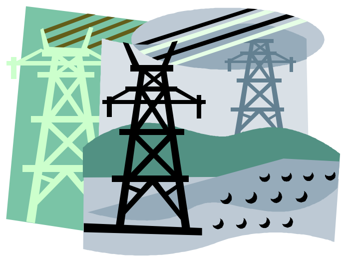
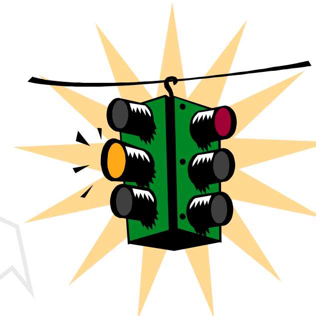
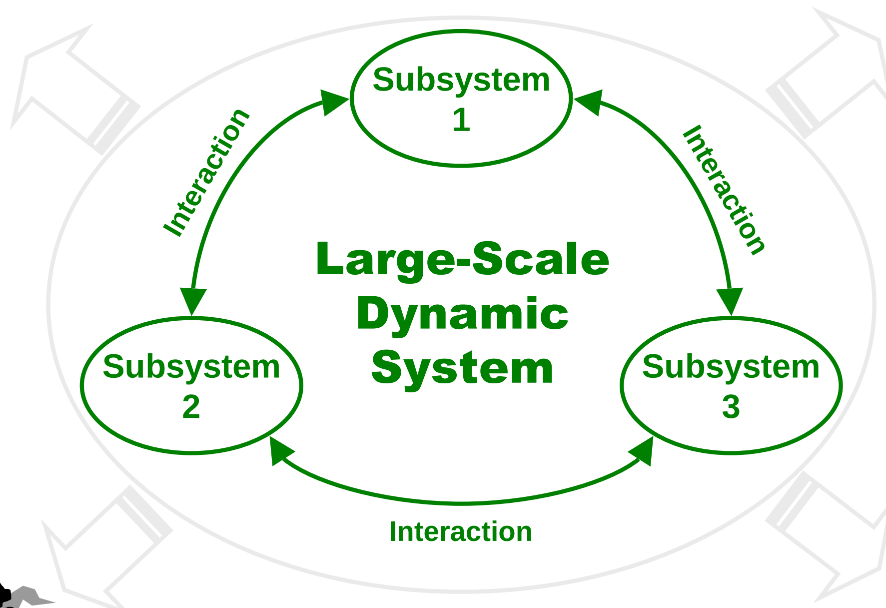
Motivating Applications

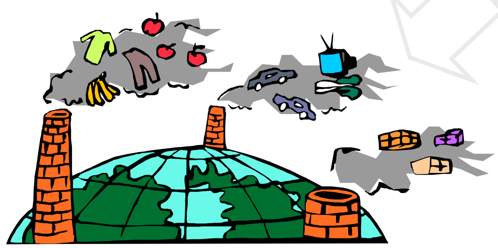


Electric Power Systems

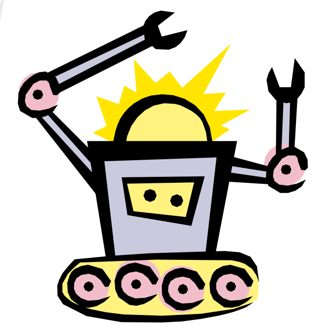
Traffic Systems







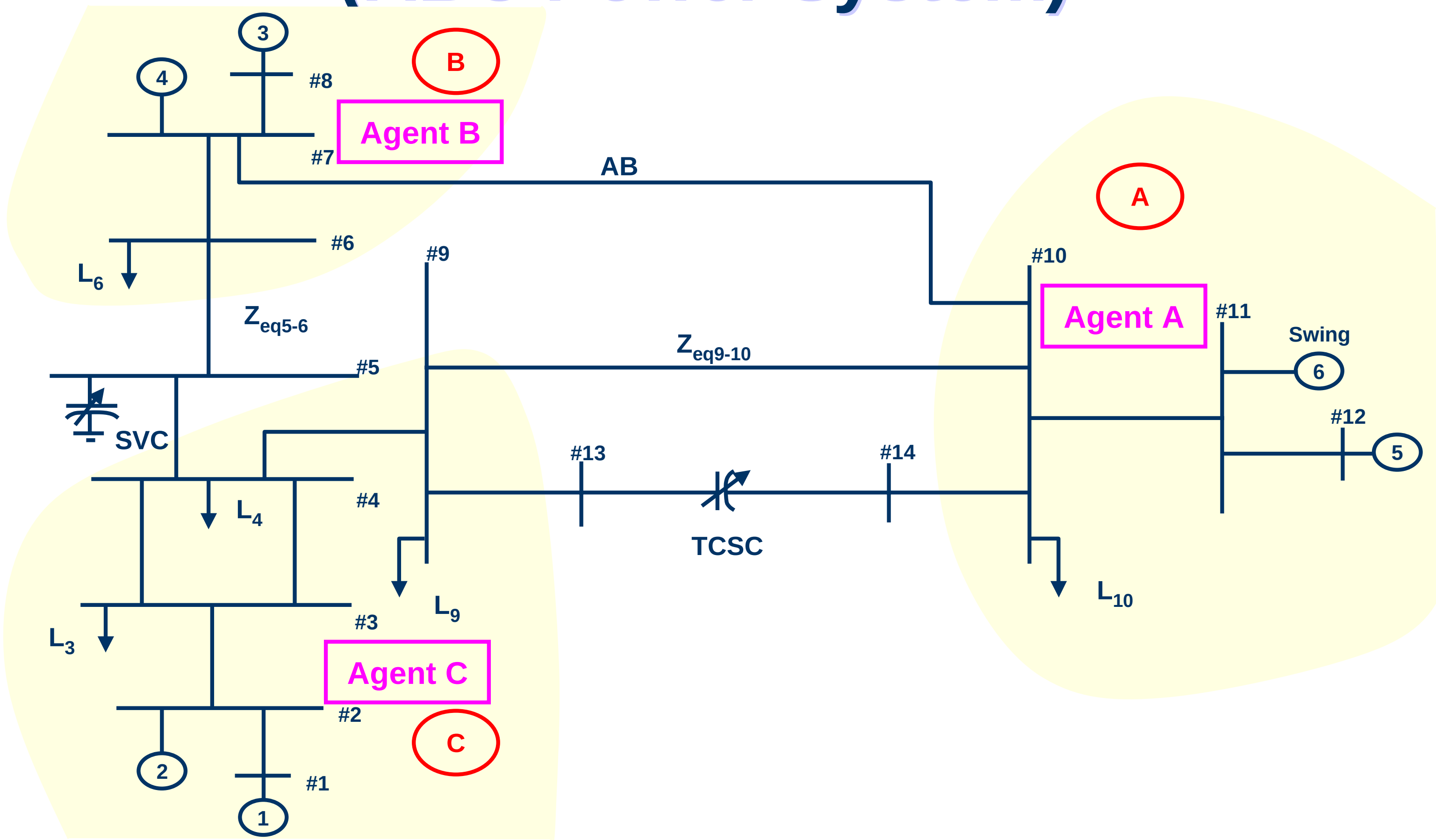
Decentralized Control



Manufacturing Plants

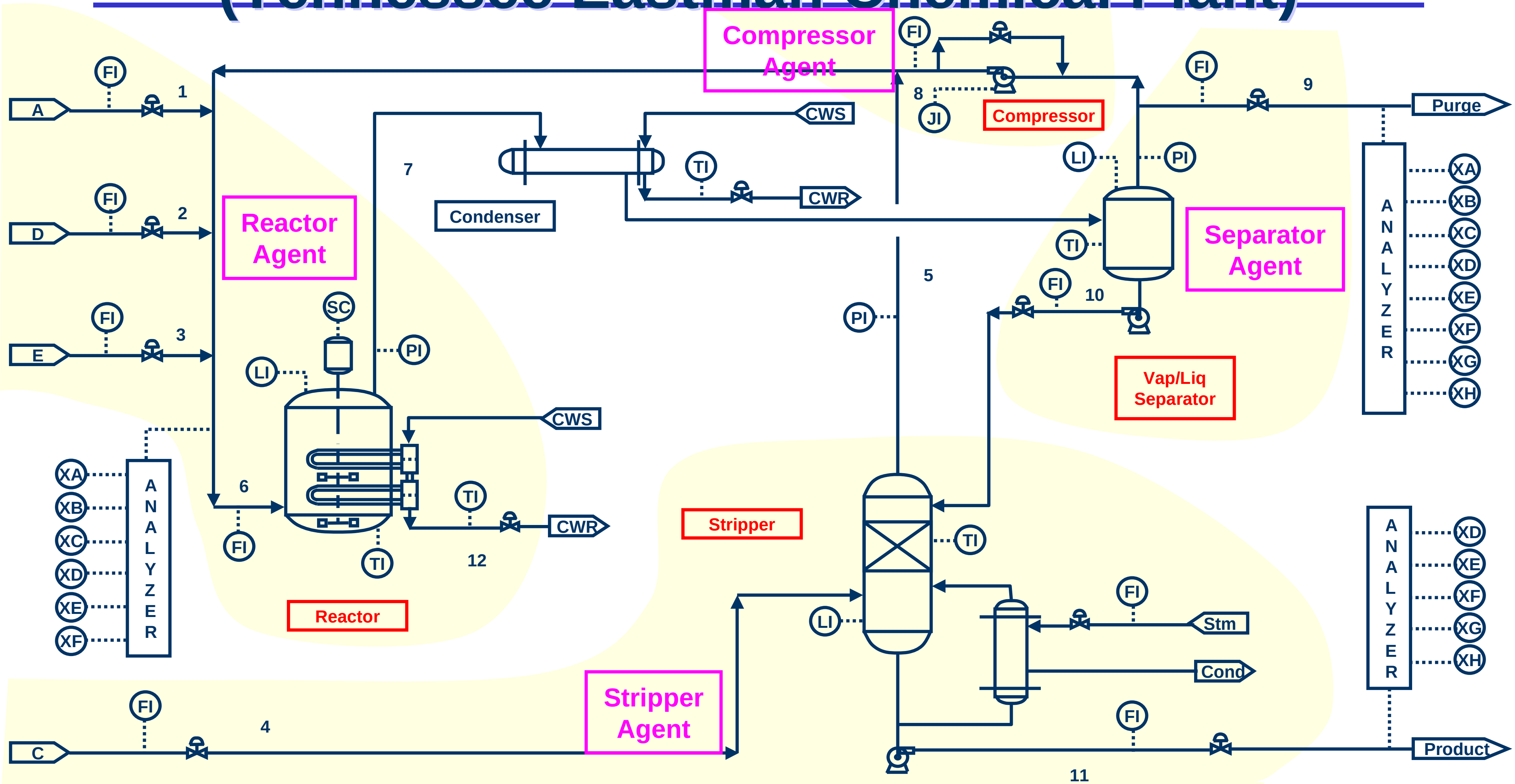
Multi-Robot Systems

Automatic Generation Control (ABC Power System)



- To optimize the generation allocation
- To maintain the balance between the power generation and the power consumption
- To maintain the power flow between areas at the scheduled value

Plant-Wide Process Control (Tennessee Eastman Chemical Plant)



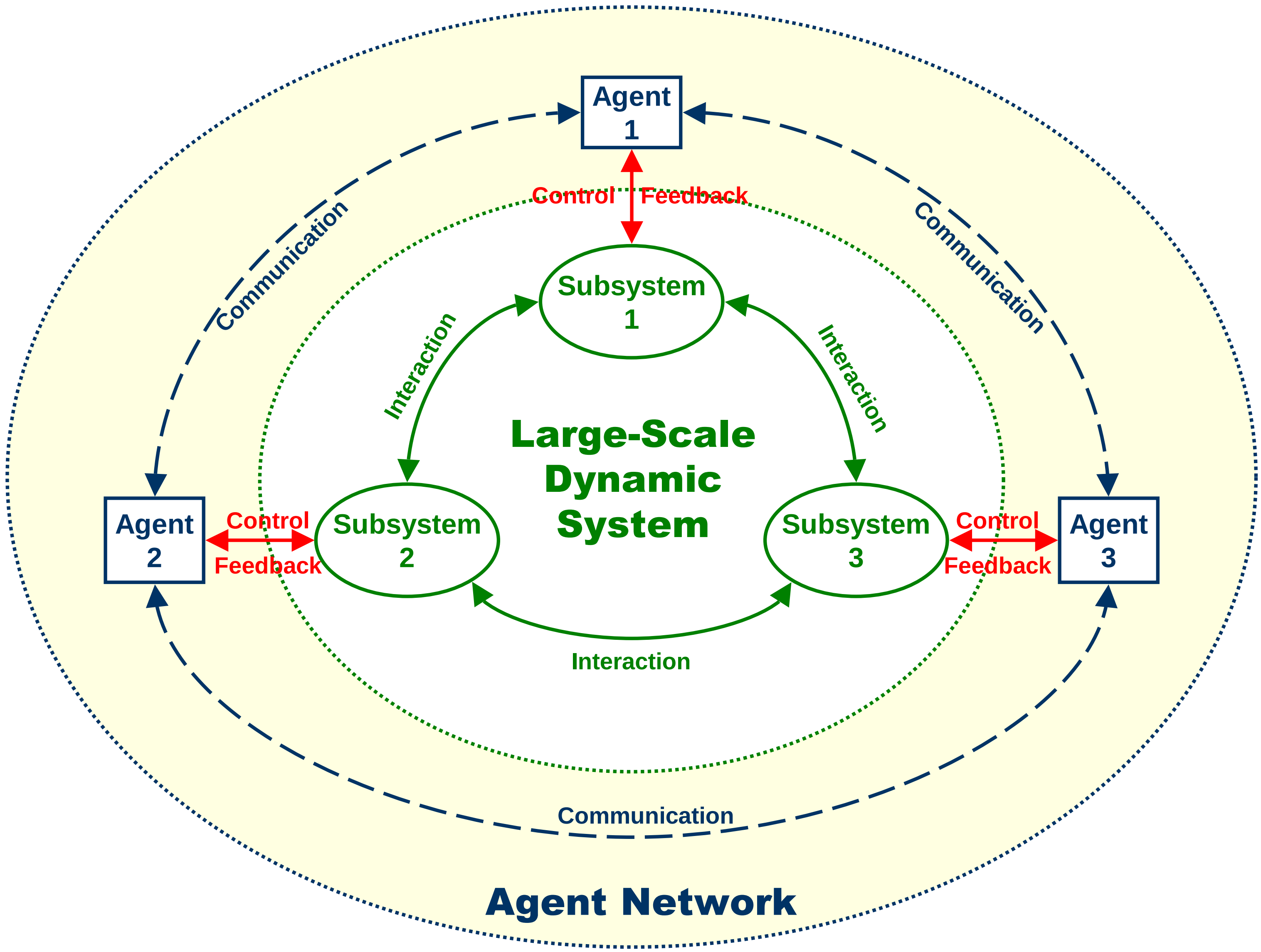
- To maintain the production rate and composition at the set-points
- To keep other variables within specified limits

Motivating Considerations

- Decentralized control actions without coordination are typically conservative;
- For existing decentralized control systems, introducing coordination among decentralized controllers is cheaper than rebuilding a centralized control system;
- Systems controlled by decentralized controllers are more survivable than those controlled by centralized controllers;
- Hierarchical control requires models of the global systems, which is hard to develop, or even unavailable.

Dynamic Information Sharing Among Decentralized Controllers

Structure of Multi-Agent Control System



Research Objective

To develop a fully distributed coordination mechanism for decentralized control

- No agent manipulates the whole system;
- Each agent has a model of only its local subsystem;
- Online information exchange helps to achieve coordination.

Problems Considered in this Research

- Framework
  - Communication Scheme for Coordination
  - Method to Handle Interaction
  - Formulation of MPC Optimization to Achieve Coordination
- Approaches
  - Method for Reducing the Conservativeness in Control Actions
  - Feasibility of the Control Decision
  - Stability of the System
  - Set Approximation Method
  - Computation of Control Invariant Set
  - Uncertainties in State Estimation
  - Information Compensation in Asynchronous Control
  - Reachable Set Computation in Hybrid Systems
- Demonstrations

Outline

- Problem Description
- Previous Work
- Approaches
- An Example
- Future Work

Outline

- **Problem Description**
  - ***Iteration Process***
  - ***Minimax Parameterized Optimization***
- Previous Work
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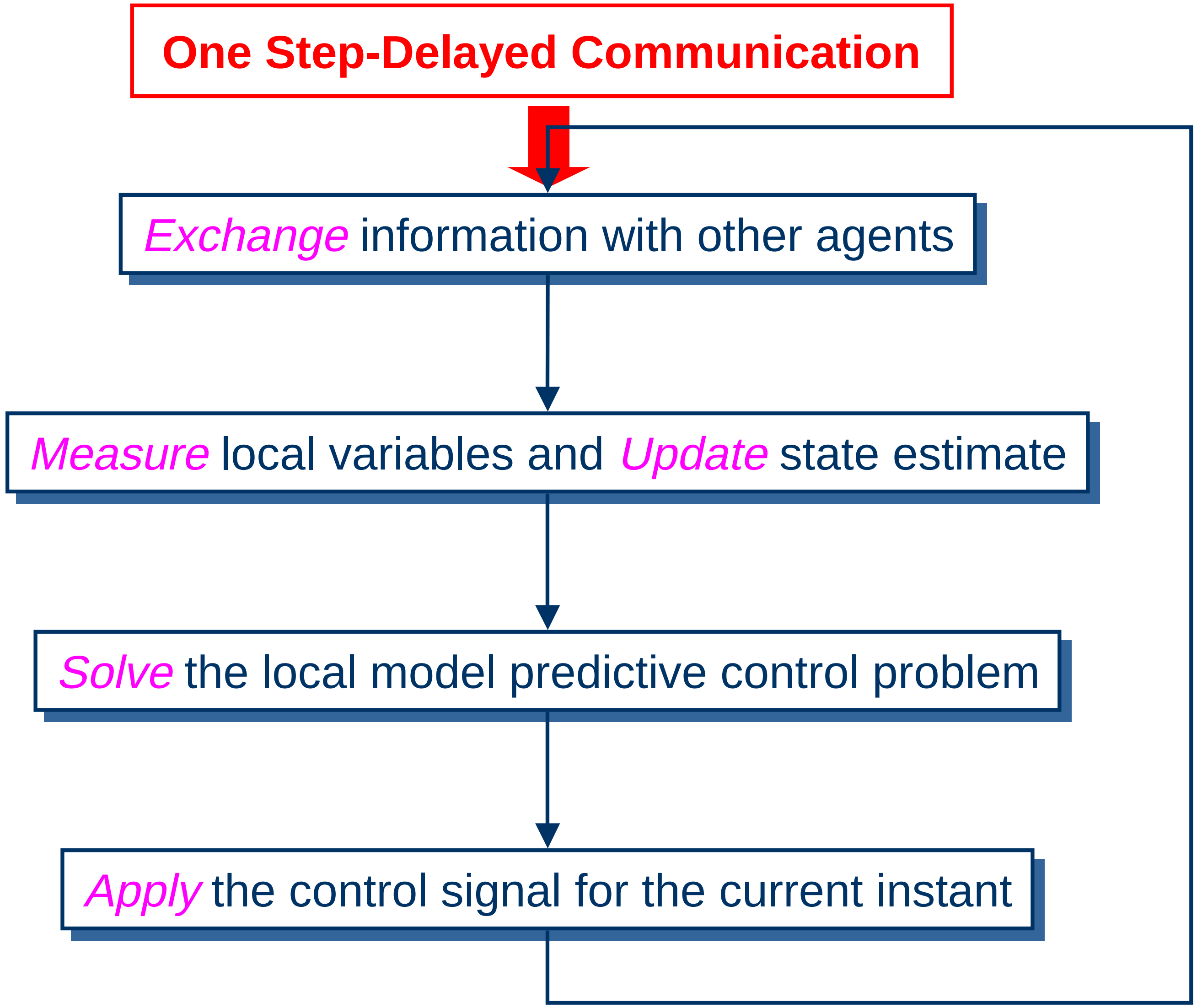
Iteration Process



Min-Max Parameterized Optimization with Constraint Sets *C* ($M^2P(C)$)

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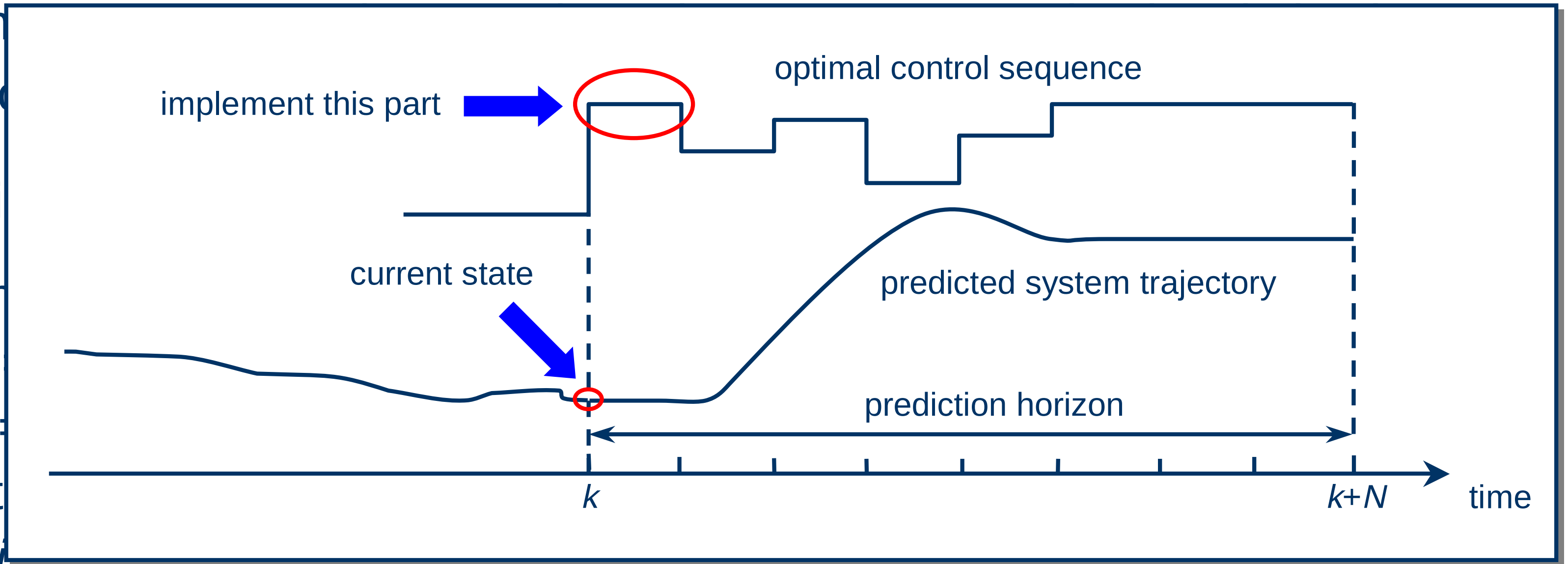


$x(j$ ... N-1

$u(j) = h(x(j), \theta(j)),\ j = 0, 1, \ldots, N\text{-}1$ — *Parameterized Control Policy*

$\Theta \in \Xi$

$v(j) \in \mathcal{V}(j),\ j = 0, 1, \ldots, N\text{-}1$

$x(0) = x_0$

for all $\tilde{v}(j) \in \mathcal{V}(j),\ j = 0,1,\ldots,N\text{-}1$

$\tilde{x}(j+1) = f(\tilde{x}(j), h(x(j), \theta(j)), \tilde{v}(j)) \in \mathcal{X}(j)$

$g(\tilde{x}(j), h(\tilde{x}(j), h(\tilde{x}(\tilde{j}), \theta(j)), \tilde{v}(j)), h(\tilde{x}(j), \theta(j)), \tilde{v}(j)) \leq 0$ — *Quantified Constraints*

Model Requirement

The model in the agent MUST be an ***abstraction*** of the real dynamics.

Model in Agent

$x(k+1)=f(x(k),u(k),v(k))$

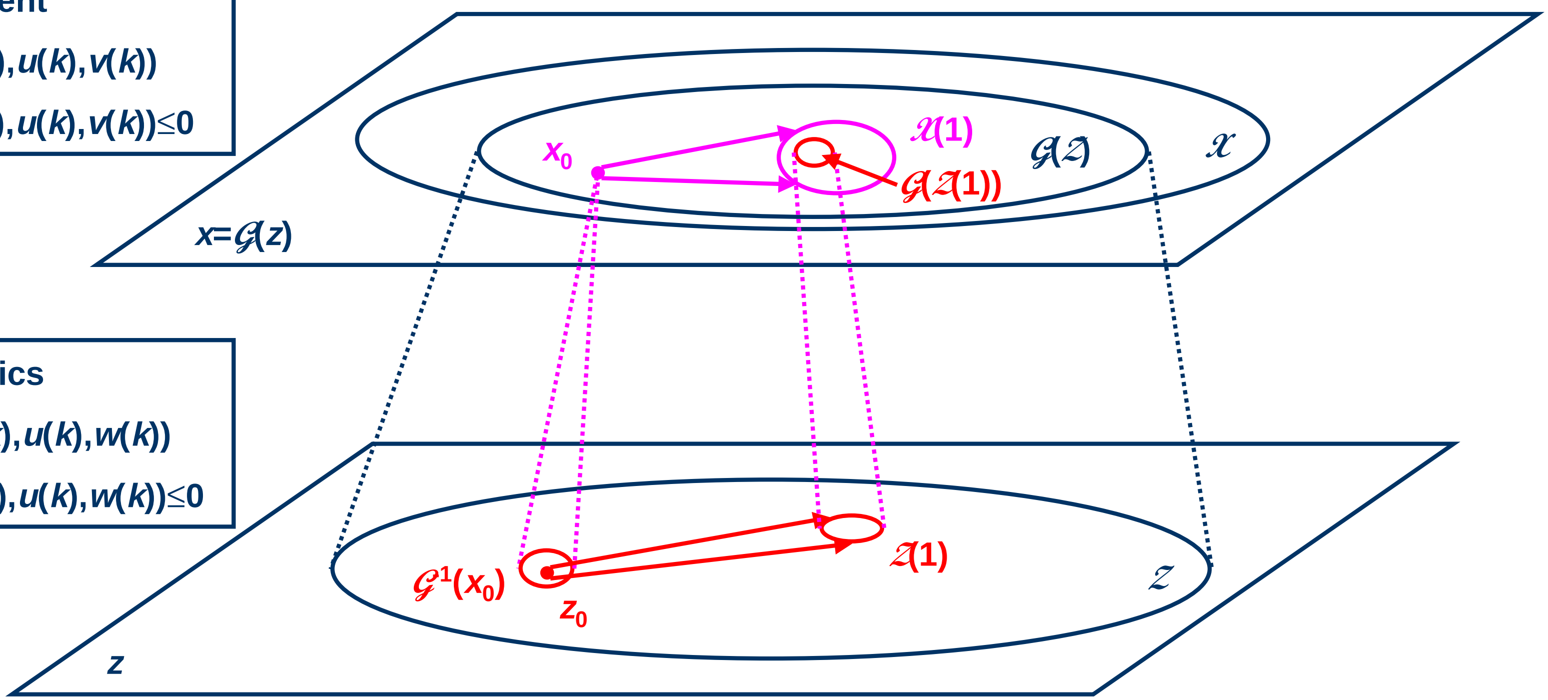
$g(x(k),x(k+1),u(k),v(k))\leq 0$

Real Dynamics

$z(k+1)=F(z(k),u(k),w(k))$

$G(z(k),z(k+1),u(k),w(k))\leq 0$

State Space of The Agent Model



State Space of The Real Dynamics

Outline

- ✓ Problem Description
- **Previous Work**
- Approaches
- An Example
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Previous Works

- Model Predictive Control (MPC)
  - *Min-Max MPC*

    [Genceli and Nikolaou, 1993, Zhang and Morari,1993]
  - *Feedback MPC*

    [Bemporad, 1993, Scokaert and Mayne, 1998]
- Set Approximation
  - *Polyhedral Method*

    [Barmish, Lin and Sankaran, 1978, Farina and Benvenuti,1997, Chutinan and Krogh, 1999]
  - *Ellipsoidal Method*

    [Kurzhanski and Valyi, 1991, 1992, 1996]
- Control of Hybrid Systems
  - *MPC for Hybrid Systems*

    [Bemporad and Morari, 1999, Stursberg and Engell, 2001]

Outline

- ✓ Problem Description
- ✓ Previous Work
- **Approaches**
  - ***Methods for Feasibility***
  - ***Set Approximation***
  - ***Current Results***
- An Example
- Conclusion
- Future Work

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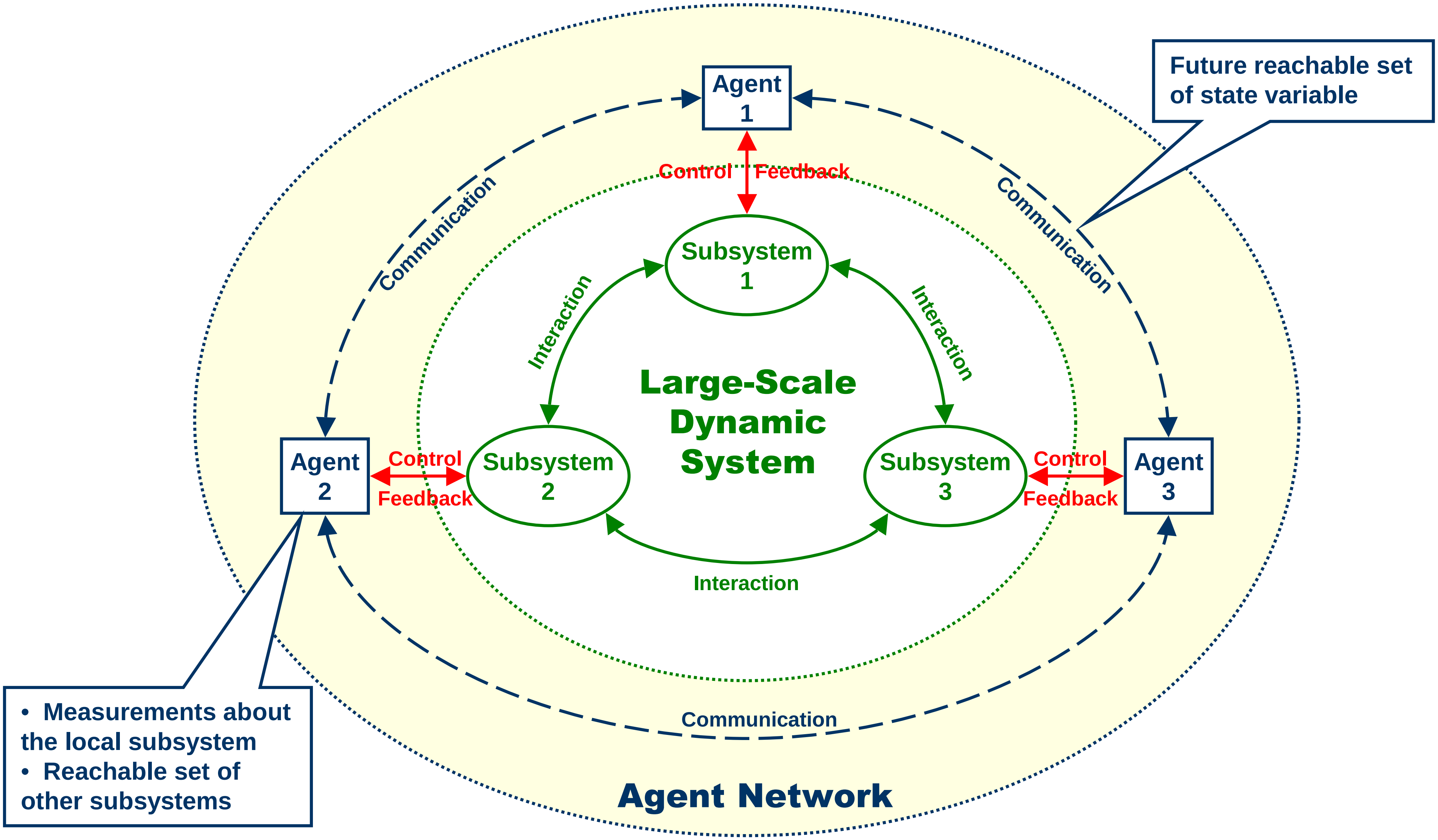
Information Flow in Multi-Agent Control System



Feasibility Problem

At stage k-1, the quantified constraints are satisfied with $\mathcal{V}^{k-1}(j)$:

for all $\tilde{v}(j) \in \mathcal{V}^{k-1}(j)$, $j = 0,1,\ldots,N-1$

$\tilde{x}(j+1)=f(\tilde{x}(j),h(\tilde{x}(\mathrm{j}),\theta^{k-1*}(j)),\tilde{v}(j)) \in \mathcal{X}(j)$

$g(\tilde{x}(j),h(\tilde{x}(j),h(\tilde{x}(j),\theta^{k-1*}(j)),\tilde{v}(j)),h(\tilde{x}(\mathrm{j}),\theta^{k-1*}(j)),\tilde{v}(j)) \leq 0$

At stage k, we check these conditions with $\mathcal{V}^{k}(j)$:

for all $\tilde{v}(j) \in \mathcal{V}^{k}(j)$, $j = 0,1,\ldots,N-1$

$\tilde{x}(j+1)=f(\tilde{x}(j),h(\tilde{x}(\mathrm{j}),\theta^{k-1*}(j)),\tilde{v}(j)) \in \mathcal{X}(j)$

$g(\tilde{x}(j),h(\tilde{x}(j),h(\tilde{x}(j),\theta^{k-1*}(j)),\tilde{v}(j)),h(\tilde{x}(\mathrm{j}),\theta^{k-1*}(j)),\tilde{v}(j)) \leq 0$

Requirements On Updating Uncertainty Bounds

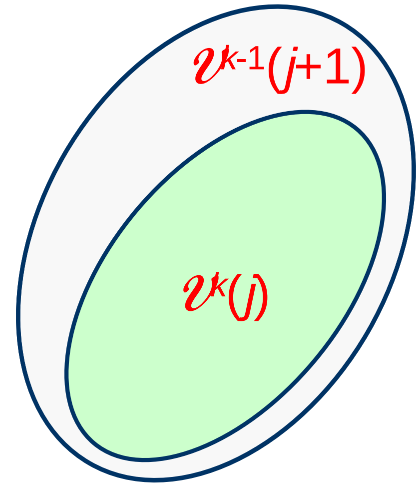
Updating Uncertainty Bounds:

$\mathcal{V}^k(j) \subseteq \mathcal{V}^{k-1}(j+1),\ j = 0,1,\ldots,N-2$

$\mathcal{V}^k(N-1) \subseteq \mathcal{V}$

Initial Setting of Uncertainty Bounds:

$\mathcal{V}^0(j) = \mathcal{V},\ j = 0,1,\ldots,N-1$



Updating Uncertainty Bounds:

$$\mathcal{V}_i^k(j) = \prod_{\substack{l=1 \\ l\neq i}}^{M} \hat{\mathcal{X}}_l(k+j\,|k-1),\ j = 0,1,\ldots,N-1$$

Initial Setting :

$$\mathcal{V}_i^0(j) = \prod_{\substack{l=1 \\ l\neq i}}^{M} \mathcal{X}_l,\ j = 0,1,\ldots,\ N-1$$

Full-State Communication

Requirements On Predicting Reachable Sets

***Updating State Constraints*:**

$\hat{\mathcal{X}}(k+j|k) \subseteq \hat{\mathcal{X}}(k+j\,|k\text{-}1),\ j = 1, 2, \ldots, N\text{-}1$

$\hat{\mathcal{X}}(k+N|k) \subseteq \mathcal{X}$

***Predicting Future Reachable Sets*:**

$\hat{\mathcal{X}}(k+j\,|k) = \{x\,|\,x = f(x',h(x',\theta^{k*}(j)),v), x' \in \hat{\mathcal{X}}(k+j\text{-}1|k), v \in \mathcal{V}^k(j\text{-}1)\},\ j = 1,2,\ldots,N$

$\hat{\mathcal{X}}(k|k) = \{x(k)\}$

***Updating State Constraints*:**

$\mathcal{X}^k(j) = \hat{\mathcal{X}}(k+j\,|k\text{-}1),\ j = 1, 2, \ldots, N\text{-}1$

$\mathcal{X}^k(N) = \mathcal{X}$

***Initial Setting*:**

$\mathcal{X}^0(j) = \mathcal{X},\ j = 1, 2, \ldots, N$

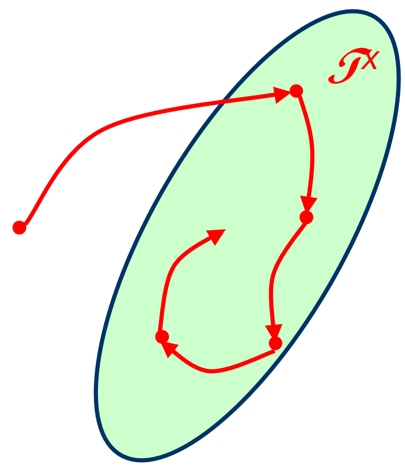
θ-Control Invariant Sets

***θ-Control Invariant Set*:**

$x \in \mathcal{T}^x \Rightarrow f(x, h(x, \theta^{\mathcal{T}x}), v) \in \mathcal{T}^x$ for all $v \in \mathcal{V}$

***End State Constraints*:**

$\mathcal{X}(N) = \mathcal{T}^x$



The initial solution can be constructed from the previous solution.

$\theta^k(j) = \theta^{k-1*}(j+1)$, j = 0,1,…,$N$-2 and $\theta^k(N-1) = \theta^{k-1*}(N-1)$

Min-Max Feedback DMPC (M^2F-DMPC)

$$\min_{\Theta_i} \{ \max_{V_i} J_i(\Theta_i, V_i) \}$$

where

$$\Theta_i = \{\theta_i(0), \theta_i(1), \ldots, \theta_i(N-1)\}$$

$$V_i = \{v_i(0), v_i(1), \ldots, v_i(N-1)\}$$

$$v_i(j) = [\hat{x}_1^T(j) \ldots \hat{x}_{i-1}^T(j)\ \hat{x}_{i+1}^T(j) \ldots \hat{x}_M^T(j)]^T$$

s.t.

$$x_i(j+1) = f_i(x_i(j), u_i(j), v_i(j)),\ j = 0,1,\ldots,N-1$$

$$u_i(j) = h_i(x_i(j), \theta_i(j)),\ j = 0,1,\ldots,N-1$$

$$\Theta_i \in \Xi_i$$

$$v_i(j) \in \mathcal{V}_i(j),\ j = 0,1,\ldots,N-1$$

$$\mathcal{V}_i(j) = \prod_{\substack{l=1 \\ l \neq i}}^{M} \hat{\mathcal{X}}_l(k+j \mid k-1)$$

$$x_i(0) = x_{i0}$$

For all $\tilde{v}_i(j) \in \mathcal{V}_i(j),\ j = 0,1,\ldots,N-2$

$$\tilde{x}_i(j+1) = f_i(\tilde{x}_i(j), h_i(\tilde{x}_i(j), \theta_i(j)), \tilde{v}_i(j)) \in \hat{\mathcal{X}}_i(k+j|k-1)$$

$$\tilde{x}_i(N) = f_i(\tilde{x}_i(N-1), h_i(\tilde{x}_i(N-1), \theta_i(N-1)), \tilde{v}_i(N-1)) \in \mathcal{T}_i^X$$

$$g_i(\tilde{x}_i(j), h_i(\tilde{x}_i(j), \theta_i(j)), \tilde{v}_i(j)) \leq 0$$

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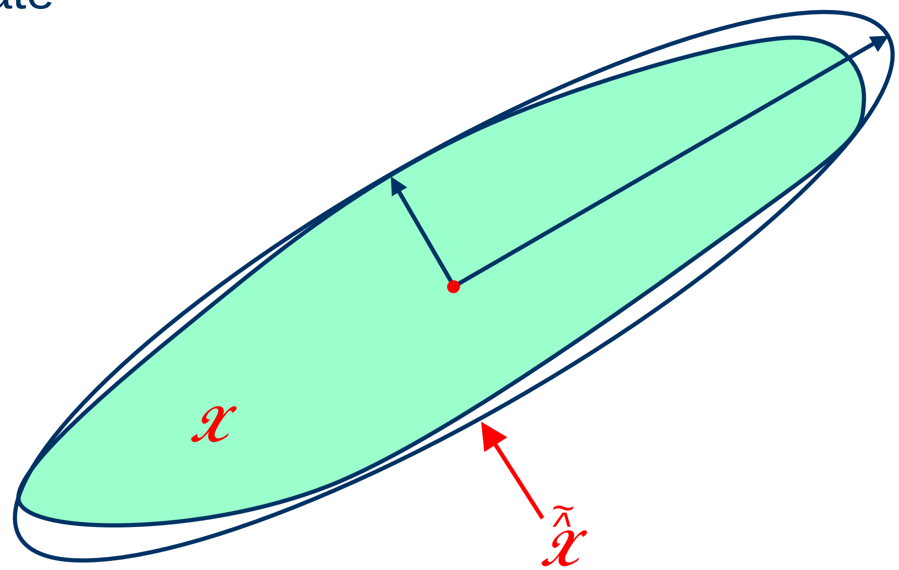
Ellipsoidal Approximation

Ellipsoidal method is used to approximate

reachable sets:



$$\tilde{\hat{\mathcal{X}}}(k+j|k) = \varepsilon(q(k+j|k), Q(k+j|k))$$

q : the ***center*** of the ellipsoid;

Q : the ***configuration matrix*** of the ellipsoid.

Use a single ellipsoid

Optimize the sum of the square of semi-axes

Set Approximation

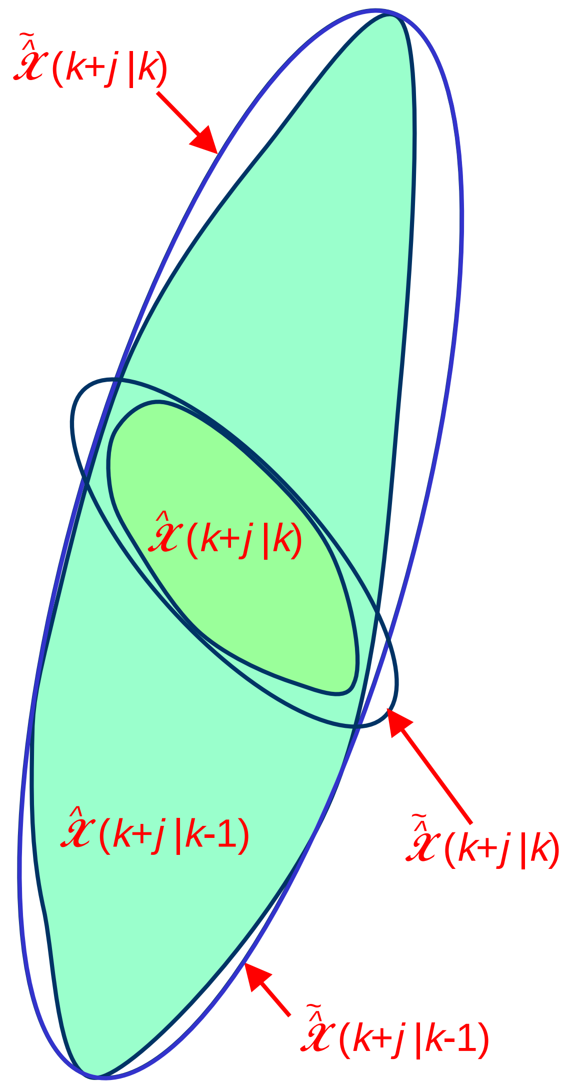
Since the set inclusion is not invariant under set approximation, the following steps are used to maintain set inclusion relationship.

Step 1: Let $j = 0$ and calculate the set approximation $\tilde{\hat{\mathcal{X}}}(k|k)$ of the set $\hat{\mathcal{X}}(k|k)$ such that $\tilde{\hat{\mathcal{X}}}(k|k) \subseteq \tilde{\hat{\mathcal{X}}}(k|k\text{-}1)$;

Step 2: Let $j = j+1$ and calculate the set approximation $\tilde{\hat{\mathcal{X}}}(k+j\,|k)$ of the set $\hat{\mathcal{X}}(k+j\,|k)$

Step 3: If $\tilde{\hat{\mathcal{X}}}(k+j\,|k) \subseteq \tilde{\hat{\mathcal{X}}}(k+j\,|k\text{-}1)$, let $\tilde{\hat{\mathcal{X}}}(k+j\,|k) = \tilde{\hat{\mathcal{X}}}(k+j\,|k\text{-}1)$ or $\tilde{\hat{\mathcal{X}}}(k+N\,|k\text{-}1) = \mathcal{T}$;

Step 4: If $j = N$, stop; otherwise, go to Step 2.



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Current Results – Bounded Stability

$M^2P(C)$

If the $M^2P(C)$ optimization is feasible at the first step $k = 0$, then it is feasible at all control steps $k \geq 0$ with updated state constraints and uncertainty bounds. And furthermore, $x(k) \in \mathscr{T}$, i.e. $z(k) \in \mathscr{G}^{-1}(\mathscr{T}^{x})$, $k = N, N+1, \ldots$

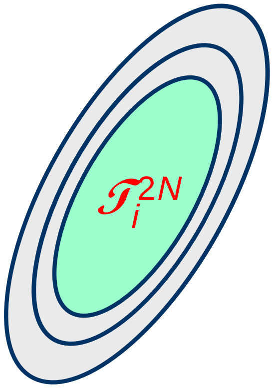
M^2F-DMPC with Set Approximation

With the set approximation method, if M^2F-DMPC is feasible at the first step $k = 0$ in all agents, it is feasible at all control steps $k \geq 0$. Furthermore, the state of the whole system z goes into $\cap \mathscr{G}_i^{-1}(\mathscr{T}_i^{x})$.

Current Results – Asymptotical Stability

In LTI systems, we use the following method to update the end constraint

$$\mathcal{T}_i^k = \begin{cases} \alpha^s \mathcal{T}_i^0, \ k = sN, \ s \geq 0 \text{ is an integer;} \\ \mathcal{T}_i^{k-1}, \text{ otherwise.} \end{cases}$$



M²F-DMPC with Updated End Constraints

If M²F-DMPC in LTI is feasible at the first step $k = 0$ in all agents, it is feasible at all control steps $k \geq 0$ with the updated end constraints. Furthermore, the system goes to $\cap \mathcal{G}_i^{-1}(0)$ asymptotically.

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An Example

Control Object

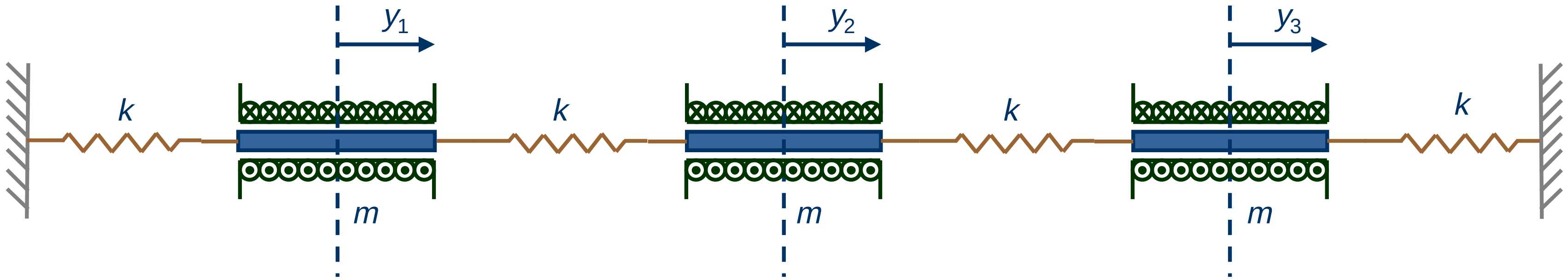
- To keep the solenoids at the equilibrium points

State Variables

- x_i^1: the position of the i th solenoid;
- x_i^2: the velocity of the i th solenoid.

Control Variables

- u_i: the magnetic force applied to the i th solenoid



Study Cases

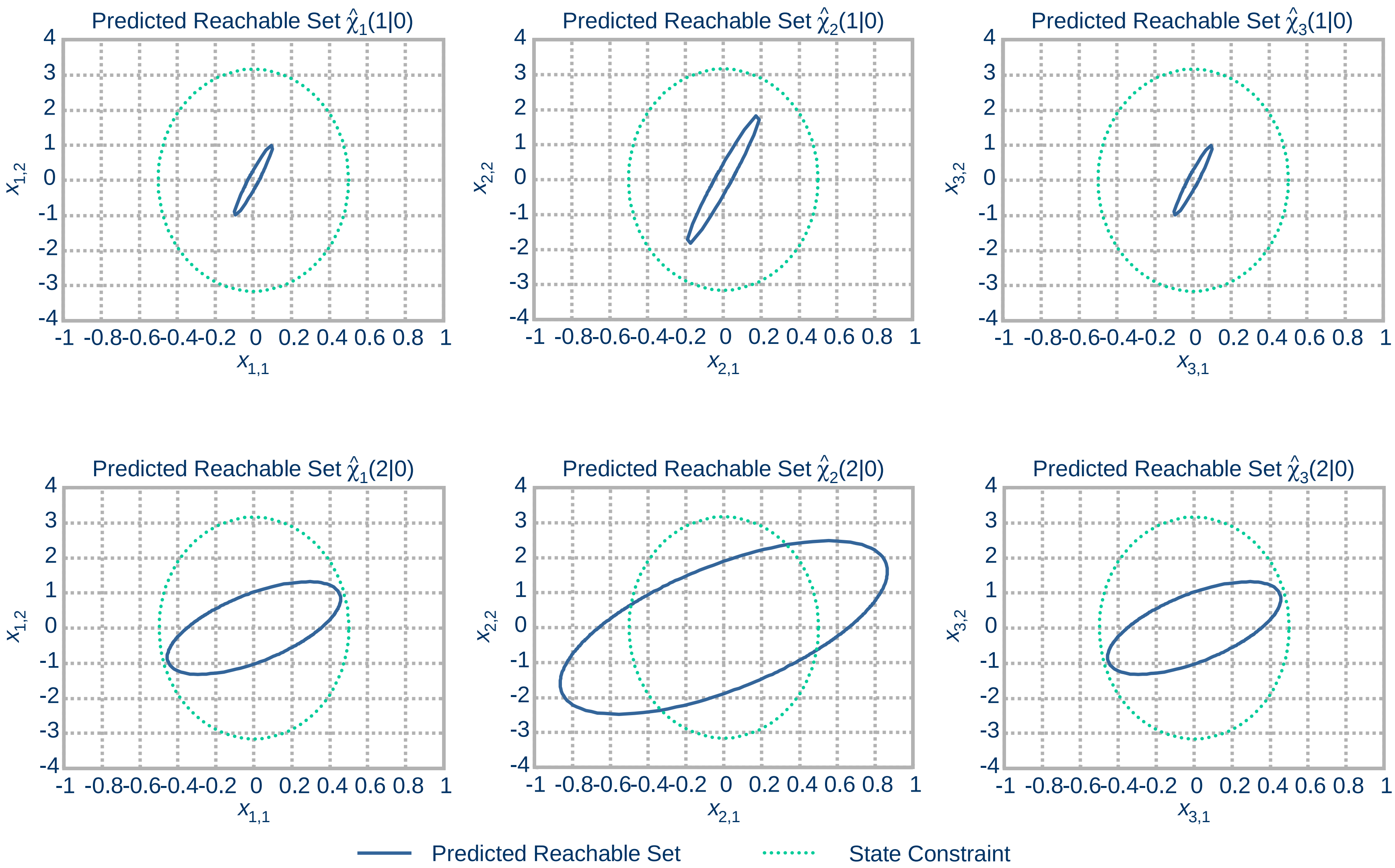
- Parameterized Open-Loop Control Policy in Optimization:

$$u(j) = u^0(j)$$

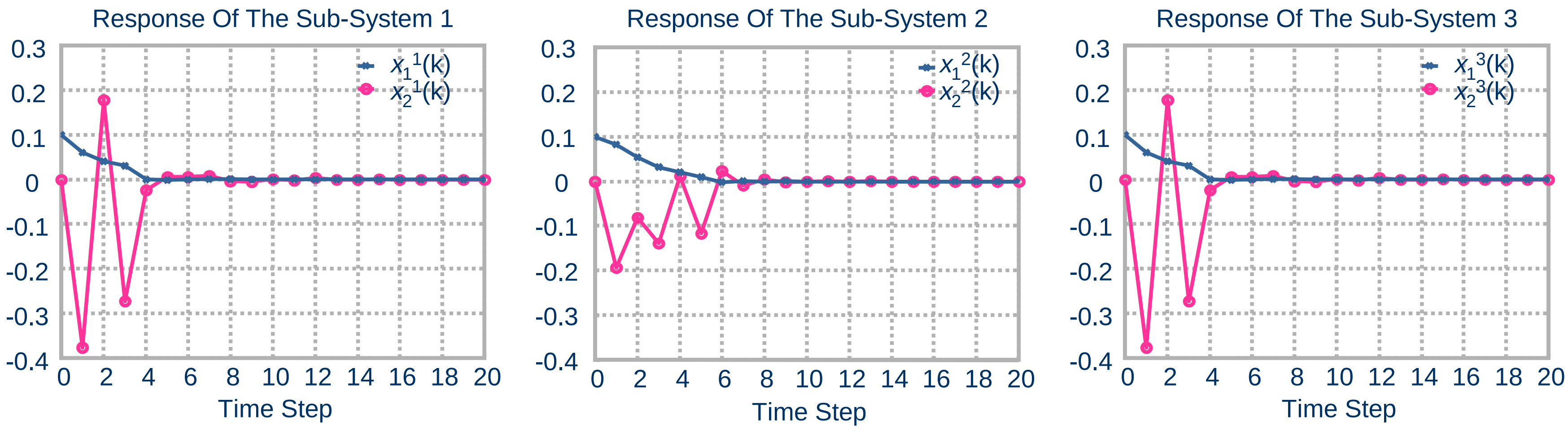
- Parameterized Feedback Control Policy in Optimization:

$$u(j) = K(j)x(j)+u^0(j)$$

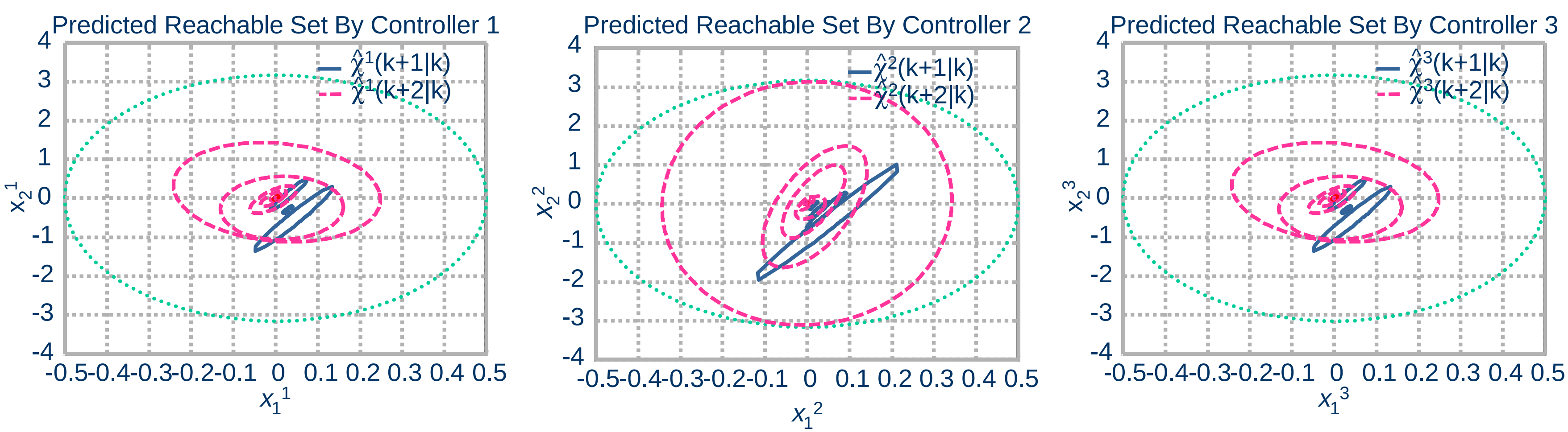
Simulation Results (Parameterized Open-Loop Policy)



Simulation Results (Parameterized Feedback Policy)



state trajectories of three sub-system



state predictions of three sub-system

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Contributions to Date

Framework

- Communication Scheme for Coordination
- Method to Handle Interaction
- Formulation of MPC Optimization to Achieve Coordination

Approaches

- Method for Reducing the Conservativeness in Control Actions
- Feasibility of the Control Decision
- Stability of the System
- Set Approximation Method
- Computation of Control Invariant Set
- Uncertainties in State Estimation
- Information Compensation in Asynchronous Control
- Reachable Set Computation in Hybrid Systems

Demonstrations

Problems Considered In Research

- Computational Issues
  - θ-Control Invariant Set
  - Set-Membership Estimation
- Asynchronous Agents
  - Information Compensation
- Hybrid systems
  - Reachable Set Prediction
- Applications
  - Automatic Generation Control (ABC Power System)
  - Plant-Wide Process Control (The Tennessee Eastman Plant)

Time Schedule