Decentralized Model Predictive Control (DMPC)

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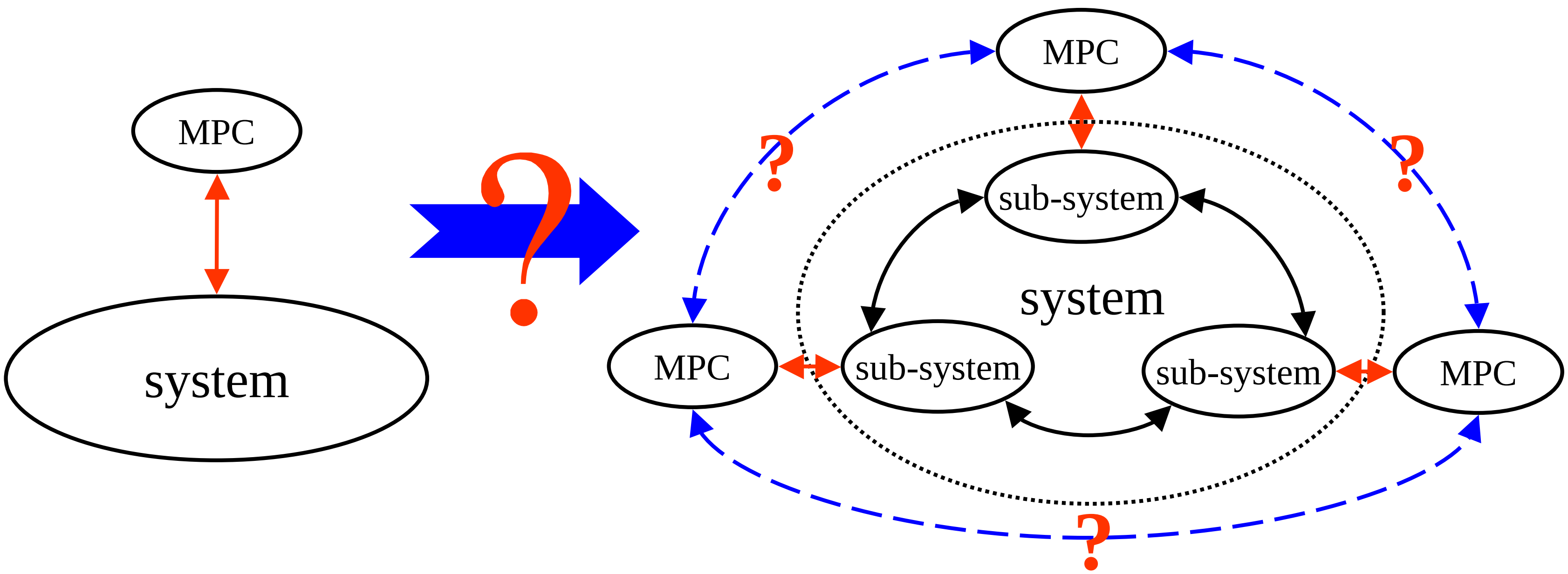
Objective

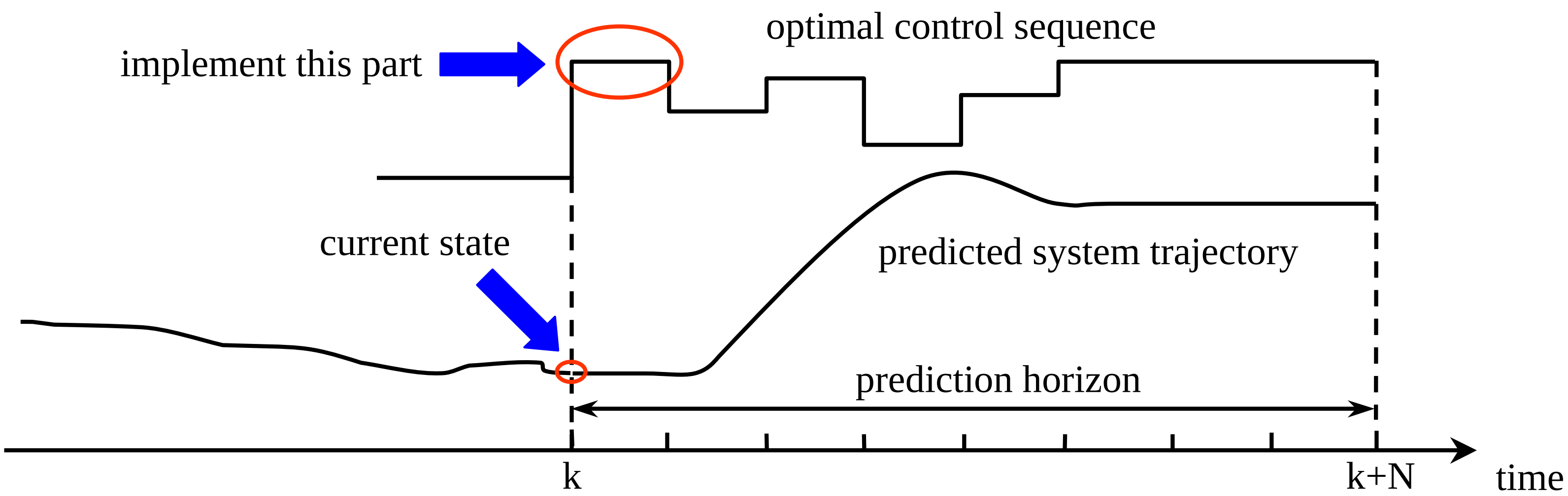


- To extend Model Predictive Control (MPC) to decentralized control of large scale systems
- To develop some methods for controllers to coordinate their control actions by themselves

Outline

- Model Predictive Control (MPC)
- The Decentralized MPC (DMPC) Scheme
- Example: Load Frequency Control Problem
- Summary & Open Problems

Model Predictive Control (MPC)



At each control instant:

- Optimal open-loop control inputs are calculated for some prediction horizon;
- Future states of the system are predicted by using a model of the system;
- The control signal for the current instant is applied to the system.

MPC Formulation

optimal control problem at control instant k

$$\min_{U(k)} J\left(X(k), U(k)\right)$$

where,

$$X(k) = \left\{\hat{x}(k+1 \mid k), \cdots, \hat{x}(k+N \mid k)\right\} \qquad U(k) = \left\{\hat{u}(k \mid k), \cdots, \hat{u}(k+N-1 \mid k)\right\}$$

s.t.

$$\hat{x}(k+j+1 \mid k) = F\left(\hat{x}(k+j \mid k), \hat{u}(k+j \mid k), \hat{v}(k+j \mid k)\right) \qquad (j = 0, \cdots, N-1)$$

$$G\left(\hat{x}(k+j \mid k), \hat{u}(k+j \mid k), \hat{v}(k+j \mid k)\right) \leq 0 \qquad (j = 0, \cdots, N-1)$$

$$G_N\left(\hat{x}(k+N \mid k), \hat{v}(k+N \mid k)\right) \leq 0$$

$$\hat{x}(k \mid k) = x(k)$$

x: state variable of the system u: control input to the system

v: external input to the system N: prediction horizon

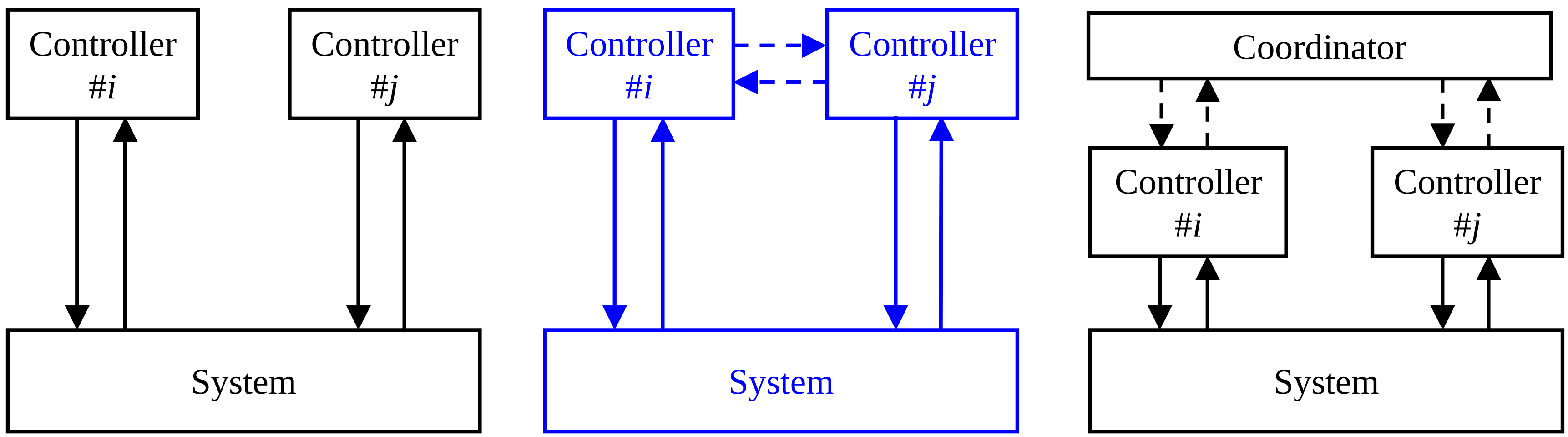
$\hat{x}$, $\hat{u}$ and $\hat{v}$ are predicted values

Why Use MPC ?

☺ "Feedback" open-loop optimal control strategy

☺ Easy to incorporate operating constraints

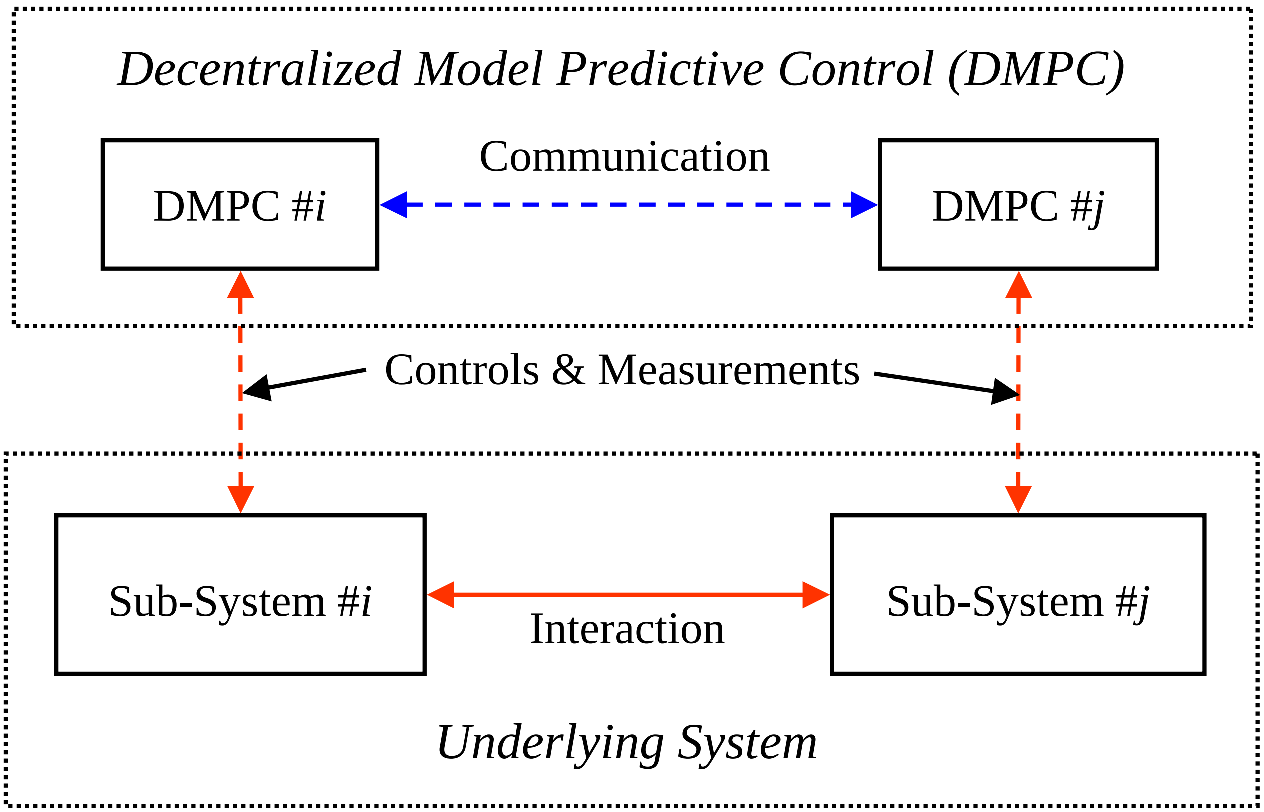
Decentralized Control Schemes



Completely Decentralized Control | Partially Decentralized Control | Hierarchically Decentralized Control

- Sandell, N. R., Varaiya, P., Athans, M. and Safonov, M. G., "Survey of Decentralized Control Methods for Large Scale Systems", *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 2, pp. 108-128, 1978
- Siljak, D. D., "Decentralized Control and Computations: Status and Prospects", *Annual Review of Control*, Vol. 20, pp. 131-141, 1996

Decentralized Model Predictive Control (DMPC)



DMPC Scheme

Exchange information with other controllers

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Measure the local sub-system

↓

Solve the local model predictive control problem

↓

Apply the control signal for the current instant

Assumptions

- ☑ Linear Time-Invariant (LTI) system
- ☑ Local effects of control input
- ☑ Full knowledge about local part and interaction
- ☑ Local operating constraints
- ☑ Working simultaneously

Stability Method for MPC

Approaches:

Add stability constraints to the optimal control problem.

- Stability constraints on the final state
  - Keerthi, S. S. and Gilbert, E. G., Optimal Infinite-Horizon Feedback Control Laws for a General Class of Constrained Discrete-Time Systems: Stability and Moving Horizon Approximation, *Journal of Optimization Theory and Application*, Vol. 57, pp. 265-293, 1988
  - Michalska, H. and Mayne, D. Q., Robust Receding Horizon Control of Constrained Nonlinear Systems, *IEEE Transactions on Automatic Control*, Vol. 38, pp. 1623-1632, 1993
  - Kwon, W. H.and Byun, D. G., Receding Horizon Tracking Control as a Predictive Control and its Stability Properties, *International Journal of Control*, Vol. 5, pp.1807-1824, 1989

- Stability constraints on the state at next instant
  - Cheng, X. and Krogh, B. H., Stability-Constrained Model Predictive Control for Nonlinear Systems, *Proceedings of the 36th Conference on Decision and Control*, 1997

Stability Of DMPC

For LTI system, if the model can be expressed by the following canonical form

$$\begin{bmatrix} x_i^1(k+1) \\ x_i^2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & I_{ii} \\ A_{ii}^1 & A_{ii}^2 \end{bmatrix} \begin{bmatrix} x_i^1(k) \\ x_i^2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B_i^2 \end{bmatrix} u_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{n} \begin{bmatrix} 0 & 0 \\ A_{ij}^1 & A_{ij}^2 \end{bmatrix} \begin{bmatrix} x_j^1(k) \\ x_j^2(k) \end{bmatrix} \quad i = 1,2,\cdots,n$$

the system can be stable under the decentralized model predictive control with following stability constraints

$$\left\|\hat{x}_i(k+1|k)\right\|^2 \leq \left\|x_i(k)\right\|^2 - \beta_i \left\|x_i^1(k)\right\|^2 \qquad 0 < \beta_i < 1 \qquad i = 1,2,\cdots,n$$

Stability Of DMPC (con'd)

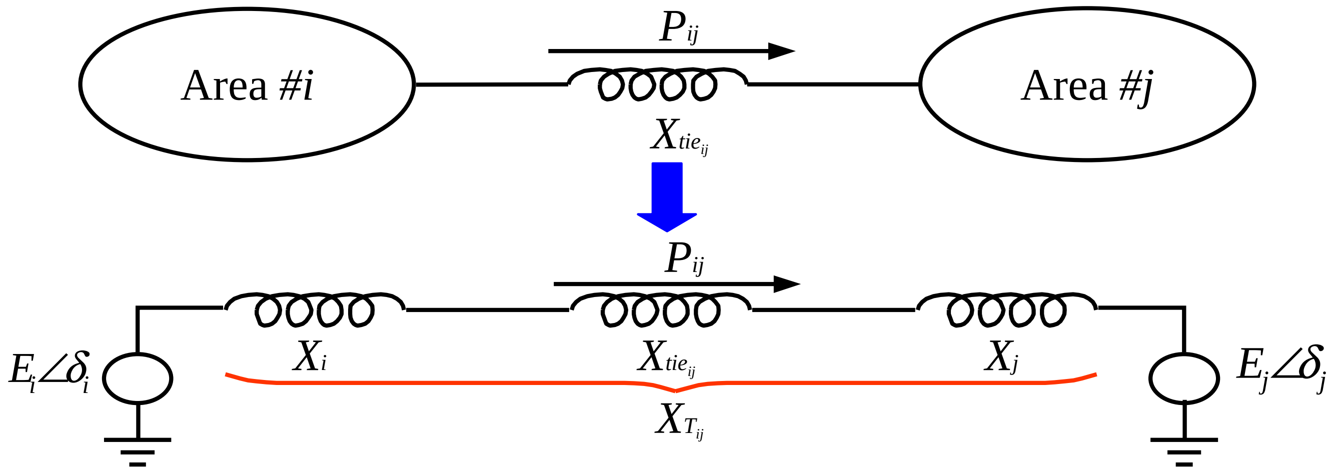
If each sub-system satisfies the matching condition

$$span(A_{ij}) \subset span(B_i) \quad i = 1,2,\cdots,n \quad j = 1,2,\cdots,n \quad j \neq i$$

The overall system can be transformed to the canonical form by a block-diagonal matrix

$$P = diag(P_1, P_2, \cdots, P_n)$$

Example: Load Frequency Control



- To keep the frequency of the system constant
- To maintain the power interchange as scheduled

local states: $\Delta\delta_i \quad \Delta f_i$	local control input: ΔP_{gi}
performance index:	$J=\sum_{i=1}^{n}\left\{\sum_{k=0}^{K}\left[\sum_{\substack{j=1\\ j\neq i}}^{n} s_{ij}\left(\Delta\delta_i(k)-\Delta\delta_j(k)\right)^2+q_i\Delta f_i^{\,2}(k)+r_i\Delta P_{gi}^2(k)\right]\right\}$

Simulation Study

1. System performance in two cases

   - Case 1: Euler Approximate Model

   $$X(k+1) = (I + T_s A) X(k) + T_s B U(k)$$

   - Case 2: Discrete Equivalent Model

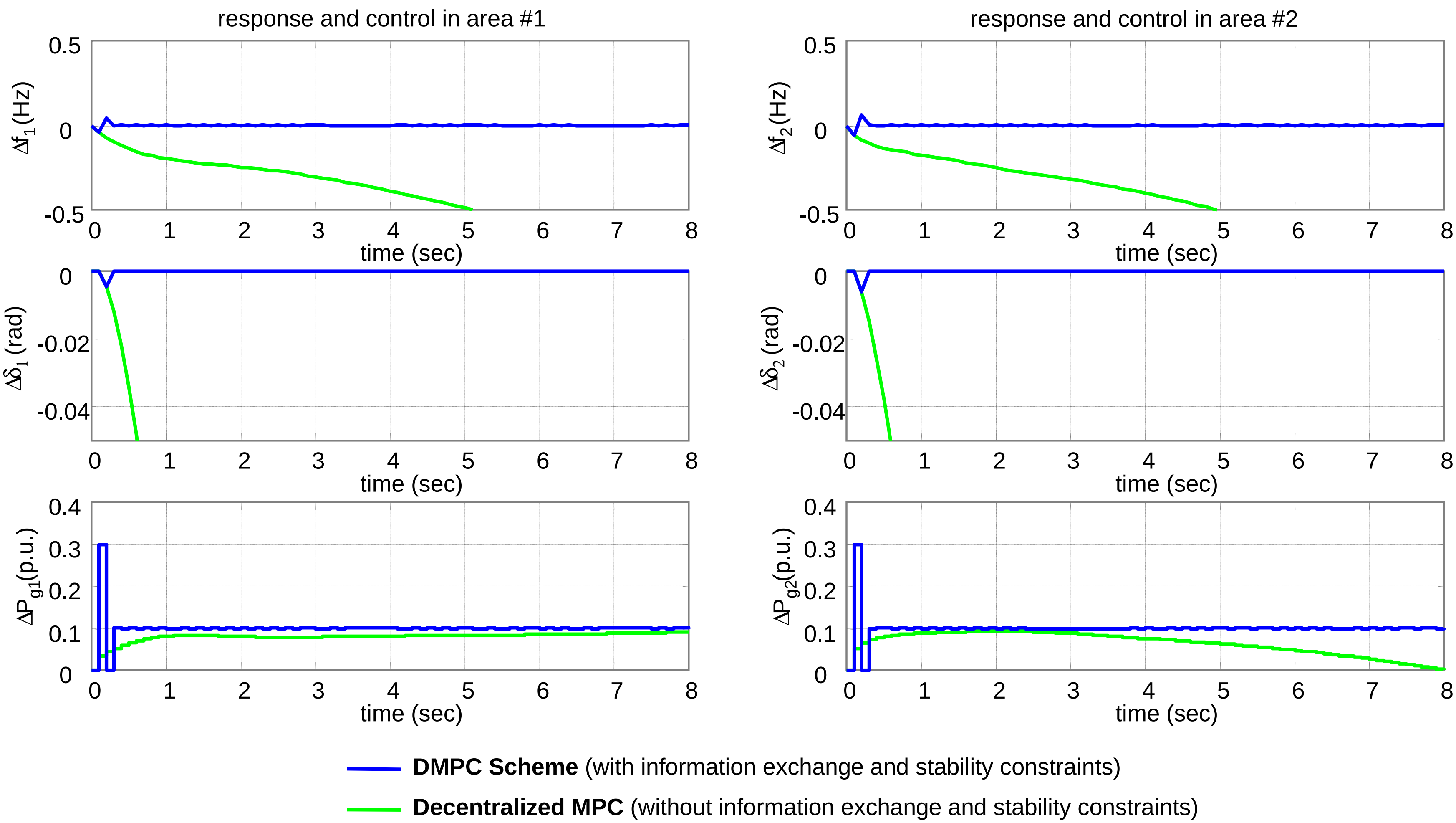
   $$X(k+1) = e^{T_s A} X(k) + \left(\int_0^{T_s} e^{A\tau} d\tau B \right) u(k)$$

2. Results for different stability constraints

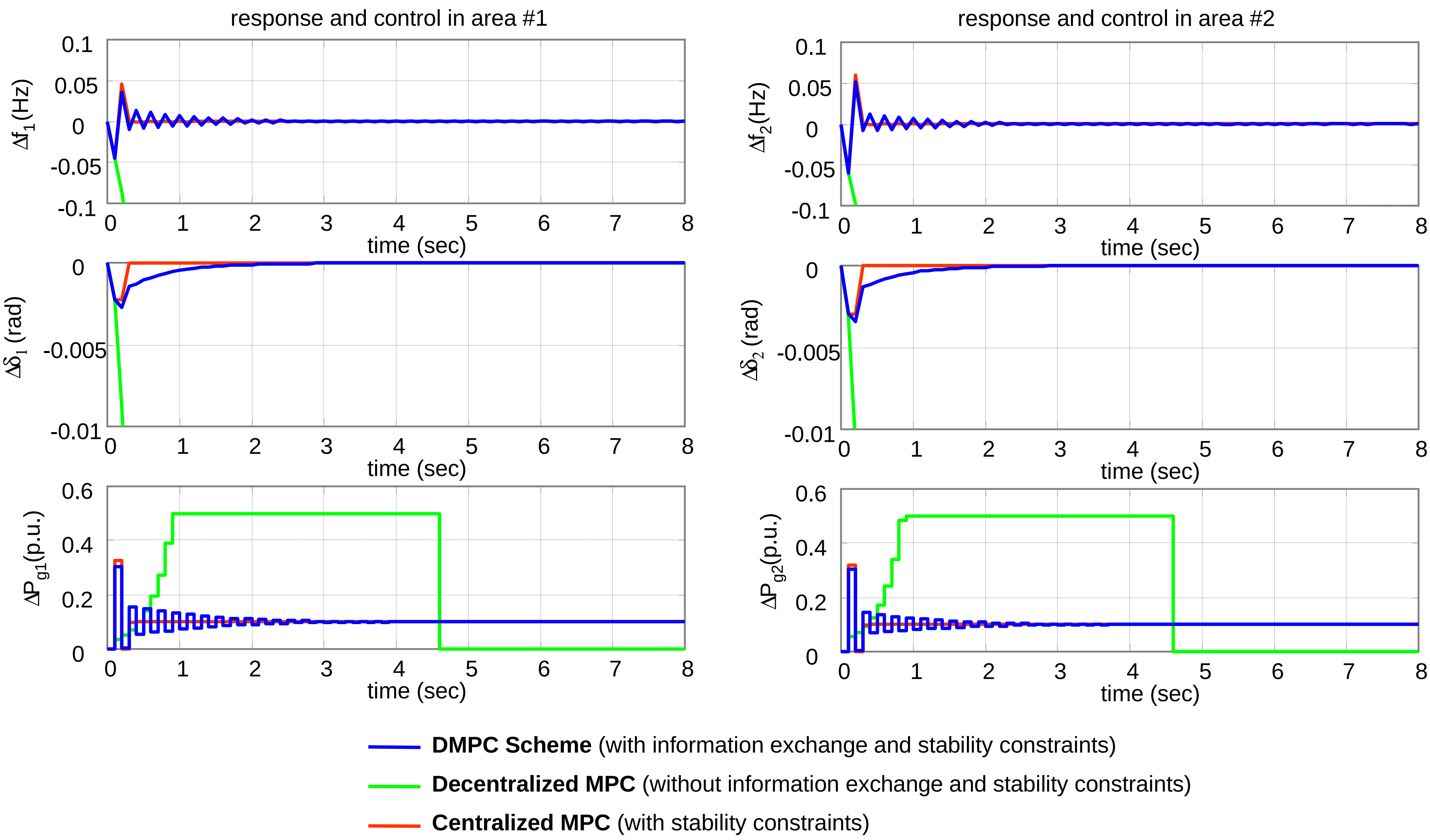
   $$\left\| \hat{x}_i(k+1 \mid k) \right\|^2 \le \left\| x_i(k) \right\|^2 - \beta_i \left\| x_i^1(k) \right\|^2$$

3. Effects of different prediction horizons

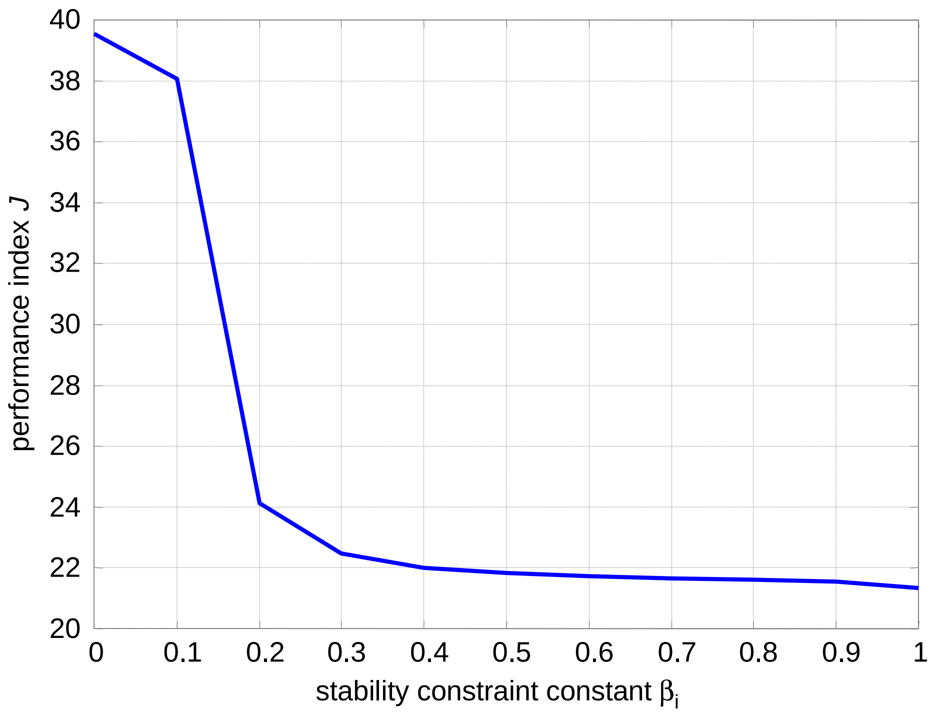
Case 1: Euler Approximate Model



Case 2: Discrete Equivalent Model

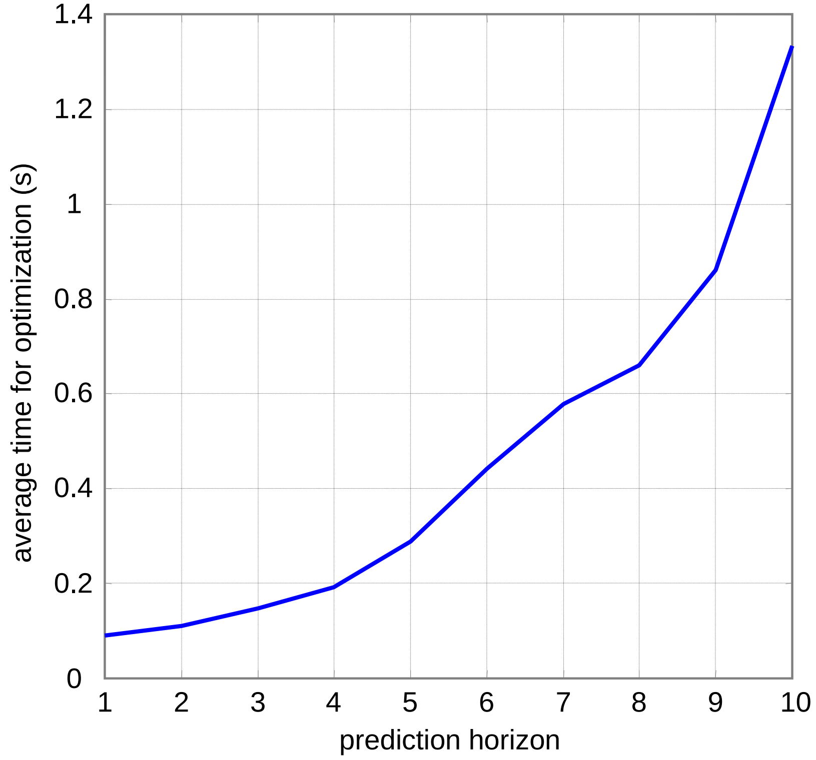


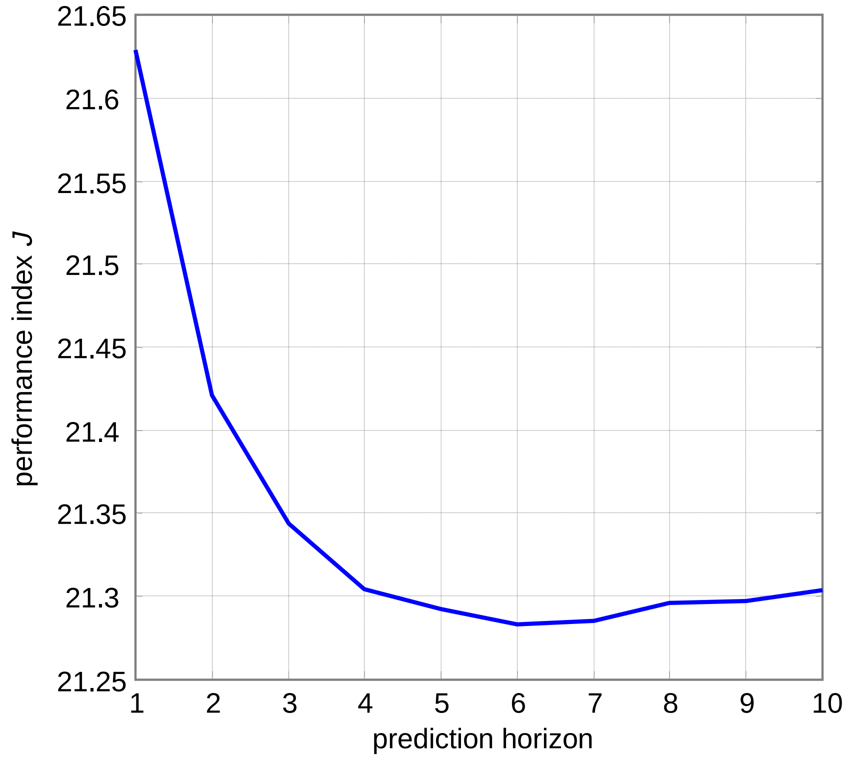
Results For Stability Constraints



stability constraints: $\left\|\hat{x}_i(k+1\,|\,k)\right\|^2 \le \left\|x_i(k)\right\|^2 - \beta_i\left\|x_i^1(k)\right\|^2$

Effects Of Prediction Horizons





Summary

Implement Model Predictive Control (MPC) in decentralized control system

- ✓ Model predictive controllers coordinate by themselves, instead of by a centralized coordinator;
- ✓ Stability of this decentralized model predictive control scheme is guaranteed for systems with certain structure.

Open Problems

😐 Imperfect Model and Unknown Disturbances

😐 Coupling among Constraints

😐 Inaccessibility of Some State Variables

😐 Nonlinear Properties of Systems